Mechanics of swinging a bat

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Measurements on the swing of a baseball bat are analyzed to extract the basic mechanics of the swing. The force acting on the bat is determined from the velocity of the center of mass, and the angular velocity of the bat provides additional information on the couple exerted by the two hands. The motion of the bat was calculated for other force-couple combinations to determine their effects on the swing of the bat. It was found that a couple is needed to start the swing, and a large opposing couple is required near the end of the swing to prevent the bat rotating through an excessive angle before it impacts with the ball.
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I. INTRODUCTION

Swinging a bat, like walking, is a task that is much easier to perform in practice than to describe in terms of the relevant mechanics. Part of the problem is that many body segments are involved, with the result that a complete analysis of the swing is a complex problem involving both the biomechanics of the batter and the mechanics of the bat.[1–3] The swing of a bat can be analyzed approximately by treating the system as a double pendulum, in which case all body segments collapse into a single forearm pivoting about an axis at one end. The other end of the forearm is attached by a hinge to the implement being swung.[4–9]

In this paper the biomechanics of the problem is largely ignored by focusing on the physics of swinging a bat, the main question being, "What forces and torques, applied at the handle end, are required to swing a bat and in what directions do they act?" Despite the fundamental nature of this question and the fact that baseball is the national pastime in the United States, this question has not been examined quantitatively. However, it has been considered in relation to the swing of a golf club.[10,11]

Visual observation of the action of a right-handed batter indicates that the bat is swung in a roughly circular arc using the left hand to pull along the handle while the right hand pushes approximately at right angles to the bat. This observation suggests that the left hand provides the necessary centripetal force, while the right hand acts to increase the bat rotation and translation speed. In fact, the situation is more complicated than this simple model suggests, involving several stages. Initially, the elbows are bent and the bat is swung close to the body at relatively low speed in a small radius arc. During this stage, the direction of the force on the handle is in the opposite direction to the direction of motion of the handle. As the bat speed increases, both arms straighten, the arc radius increases, and the bat swings to be approximately in line with each arm at the instant the bat collides with the ball. During the latter stage of the swing, both arms pull on the handle in a direction almost at right angles to the motion of the handle. A similar sequence of events is observed when swinging almost any object, including a golf club, cricket bat, tennis racquet, axe, and hammer, but we focus in this paper on the swing of a bat.

To provide a realistic model of the swing of a bat, the motion of a baseball bat was first determined experimentally by filming a particular swing in order to calculate the magnitude and the direction of the force on the handle. The torque required to generate the observed angular acceleration of the bat was also determined, providing additional information on the couple arising from equal and opposite forces applied to the handle. The inverse problem was then considered. That is, a different combination of forces and torques was assumed to calculate the effect on the motion of the bat. A surprising result is that a large negative couple is required late in the swing to prevent the bat from rotating through an excessive angle before it impacts with the ball.

II. EXPERIMENTAL DATA

Data on the swing of a baseball bat were obtained by filming a right-handed player from the Sydney University baseball team using a video camera mounted about 4 m above his head. The camera was operated at 25 frames/s (f/s) with an exposure time of 2 ms to minimize blurring of each image. Each frame consists of two interlaced images. The two images were separated using SWINGER PRO software[12] to capture the swing action at an effective frame rate of 50 f/s.[13] The video was taken in a laboratory, the batter swung at an imaginary ball, and the batter kept both feet firmly planted on the floor at marked positions under the camera. The batter was instructed to swing the bat as fast as possible and to swing in a horizontal plane at waist height after releasing the bat from his shoulder. Because the camera was mounted above his head, and only a single camera was used, the results described in the following refer to the horizontal components of the bat velocity and the forces and torques acting on the handle.

The batter swung several different bats at maximum speed to determine how his swing speed varied with bat mass. One particular swing, using an 871 g, 840 mm long Louisville Slugger R161 wood bat, was selected for further analysis. The center of mass of the bat was located 560 mm from the knob end, the barrel diameter was 66.7 mm (2 5/8 in.), and its moment of inertia about an axis through the center of mass, I_{cm}, was measured to be 0.039 ± 0.001 kg m^2.

A. Bat positions and rotation axes

The positions of the bat at 20 ms intervals are shown in Fig. 1, starting at time $t=0$ when the bat begins its forward rotation. The batter stood with the line joining his two feet aligned parallel to the laboratory x axis, and swung the bat at an imaginary ball incident in a direction parallel to the x axis. The ball would therefore have been struck at a time near t

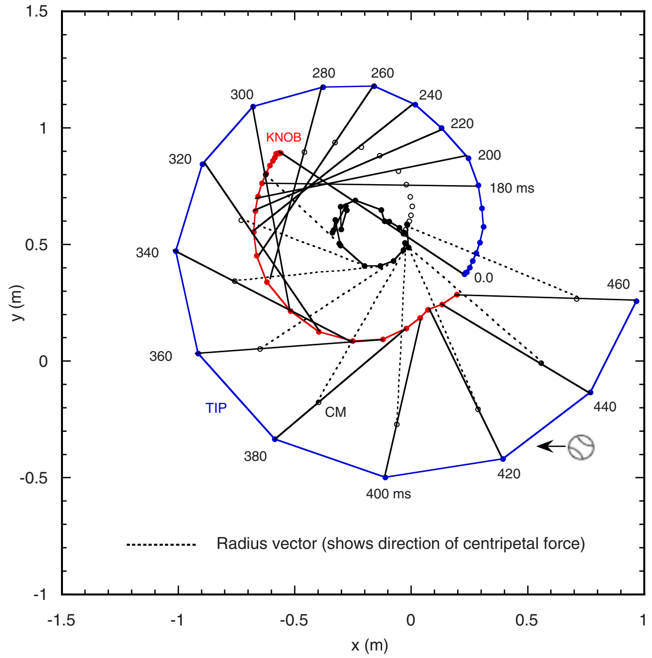

Fig. 1. Observed positions of the bat at 20 ms intervals. The tip of the bat moves along the outer spiral path, while the knob end follows the inner, semicircular path. The small innermost circle of solid black dots denotes the instantaneous center of curvature of the path followed by the bat center of mass (open circles). The direction of the centripetal force on the bat is perpendicular to the path followed by the center of mass, as indicated by the dashed lines.

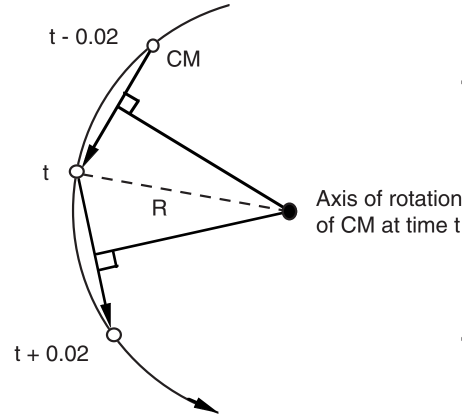

Fig. 2. Geometrical construction used to determine the radius of curvature R at time t of the path followed by the bat center of mass. Velocity vectors are shown with arrowheads.

=400 ms, the bat having rotated through an angle of about 300°. The bat initially rotated about a fixed axis in the handle, near the batter's right shoulder. During this time interval the batter did not move his arms with respect to his trunk, but rotated his legs, hips, and shoulders to start the swing. The batter's head remained fixed to within ≈30 mm during the entire swing. The initial axis of rotation of the bat can be identified in Fig. 1 as the intersection point of the three images of the bat at times $t=0$, $t=180$, and $t=200$ ms. The bat rotated about this axis during the interval $0<t<200$ ms. After this time, the rotation axis of the bat moved to a point well outside the bat and then moved to a point near the knob end of the bat just before impact with the ball.

A more relevant axis in terms of the force on the bat is the axis of rotation of the bat center of mass. This axis is the center of curvature of the path followed by the center of mass. The axis is shown in Fig. 1 by the sequence of solid dots and was located using the geometrical construction shown in Fig. 2. The axis was taken as the intersection point of the radius vectors drawn in directions perpendicular to the velocity vectors of the bat center of mass connecting the position of the center of mass of the bat from one frame to the next. Each radius vector was constructed to pass through the midpoint of the corresponding velocity vector, as shown in Fig. 2. This procedure fits a circle passing through three sequential locations of the bat center of mass, the center of the circle being taken as the instantaneous axis of rotation of the center of mass at time t. The dashed lines in Fig. 1 join the bat center of mass to its corresponding axis point and therefore represent the direction of the centripetal force on the bat, as well as the magnitude, R, and direction of the radius vector $\mathbf{R}$.

Each part of the bat rotates at angular velocity ω about an axis through the bat center of mass while the center of mass follows the spiral path indicated in Fig. 1. In a reference frame attached to the bat center of mass, each part of the bat rotates in a circular orbit about a fixed axis through the center of mass. In the laboratory frame of reference the bat rotates about an axis that is not attached to the bat center of mass, although the angular velocity ω remains the same in both reference frames. For example, suppose that the knob end of the bat rotates backward at 10 m/s relative to the center of mass, while the center of mass moves forward at 10 m/s in the laboratory frame. The knob is at rest in the laboratory frame, so the axis of rotation of the bat is at the knob end. We can therefore define ω as $d\beta/dt$, where β is the angle between the long axis of the bat and the laboratory x axis, in which case the torque, τ, on the bat about an axis through the bat center of mass is given by $\tau=I_{\rm cm}d\omega/dt$. The task of the batter is to rotate the bat center of mass along the spiral path indicated in Fig. 1, while simultaneously rotating the bat at angular velocity ω so that the barrel lines up at a suitable angle to impact the ball. In Fig. 1 the barrel extends from the tip of the bat to the center of mass. We will refer to ω as the angular velocity of the bat. The angular velocity of the bat center of mass about the axis of rotation of the bat center of mass is defined in Sec. III.

B. Forces acting on the bat

The velocity V of the bat center of mass was obtained by plotting the x and y coordinates of the center of mass (measured in the laboratory frame) as functions of time and by fitting fifth order polynomials during the time intervals $0<t<300$ ms and $280<t<500$ ms. This procedure improved the overall fit (compared with a single fit over the entire time interval) because the velocity changed relatively slowly during the first time interval and more rapidly during the second interval. The V_x and V_y velocity components were obtained by differentiating the polynomial functions to calculate both the velocity $V=(V_x^2+V_y^2)^{1/2}$ and the acceleration components dV/dt and V^2/R.

The two time intervals also coincided with a change in the batting action that affected the overall accuracy of the measured bat speed. As indicated in Fig. 1, the apparent length of the bat varied during the swing, as a result of two effects.

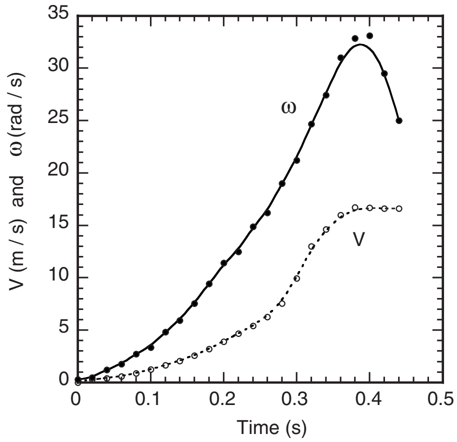

Fig. 3. Measured values of the angular velocity of the bat ω, and the linear velocity of the center of mass V. The smooth curves are best fit curves.

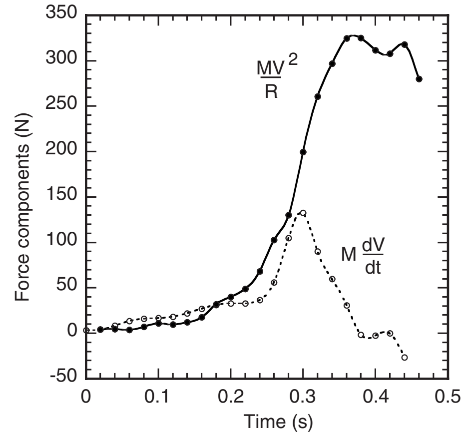

Fig. 4. The perpendicular force components MdV/dt and MV^2/R determined from the results shown in Figs. 1 and 3. The smooth curves are best fit curves.

During the first time interval, the bat was swung at about shoulder height and appeared on film to be longer (because it was closer to the camera) than that during the second time interval while the bat was being swung at about waist height. This effect was offset to some extent by the fact that the bat was initially inclined at an angle of about 30° to the horizontal plane and was then rotated to swing in a horizontal plane during the latter stages of the swing. The velocity measurements were scaled to the known bat length during both time intervals, resulting in an estimated measurement error in the magnitude of the bat velocity of about 10% during the first time interval and an error of about 5% during the second time interval. These errors could have been reduced by using additional cameras to measure the incline angle of the bat, but such a measurement would have added considerably to the complexity of the experiment. If a batter were to swing in a two-dimensional, inclined plane, then an alternative improvement would be to mount a single camera above and forward of the batter's head so that the camera axis is aligned perpendicular to the swing plane.

The angular velocity of the bat was calculated directly from the raw data as $\omega = \Delta\beta/\Delta t$, where $\Delta\beta$ is the change in the rotation angle of the bat from one frame to the next during the time interval $\Delta t = 0.02$ s. The rotation angle of the bat was determined to within $\pm 1°$ from the video. The angular velocity is shown in Fig. 3, together with the result for V.

In order to swing a bat the batter exerts a net force and a net torque on the handle end using both hands. He does what he needs to do, by instinct. From a physics point of view it helps to resolve the net force into two familiar, perpendicular components. One component is the centripetal force MV^2/R, where M is the mass of the bat. The other component, acting parallel to the velocity vector, has magnitude MdV/dt. The two components are shown in Fig. 4. The component MdV/dt acts in a direction perpendicular to the dashed lines in Fig. 1 and acts to increase the bat center of mass speed. In Sec. III we consider the inverse problem, assuming that the only information available is the magnitude and direction of force acting on the handle. To calculate the motion of the bat it is convenient to specify the resultant force as a function of time. Figure 5 shows the resultant force, F, on the bat from the experimental data, and the angle α between F and the longitudinal axis of the bat, derived from the results in Figs. 1 and 4.

C. Torque acting on the bat

The resultant force on a bat does not act along a line through the bat center of mass. Rather, the force is applied at the handle end of the bat. The force is distributed by the two hands over a length of about 200 mm along the handle, but can be regarded as being applied at a point about 100 mm from the knob, or at a distance $d = 0.46$ m from the bat center of mass. The resultant force generates a torque that can usefully be regarded as the sum of the two separate torques generated by the two perpendicular force components. If γ is the angle between the longitudinal axis of the bat and the radius vector, $\mathbf{R}$, then the resulting torque about an axis through the bat center of mass is given by $\tau_F = \tau_A + \tau_B$, where $\tau_A = (MV^2/R)d \sin\gamma$ is the torque arising from the centripetal force and $\tau_B = -(MdV/dt)d \cos\gamma$ is the torque arising from the MdV/dt force component. The torque component τ_B is negative throughout the swing, meaning that on its own it would cause the bat to rotate in the wrong direction. In ad-

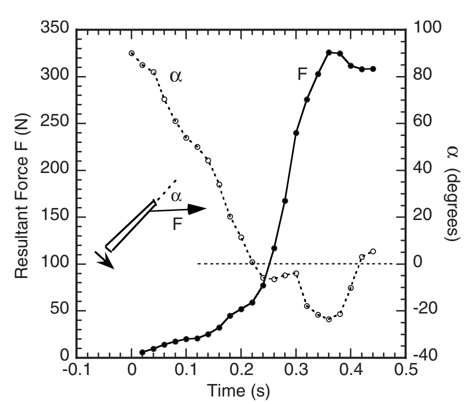

Fig. 5. The resultant force F and the angle α defined in the inset, determined from the results shown in Figs. 1 and 4. The experimental data points are connected by straight line segments. At the start of the swing, while $\alpha > 0$, the torque due to F acts in the "wrong" direction and the batter must apply a positive couple to rotate the bat in the correct direction.

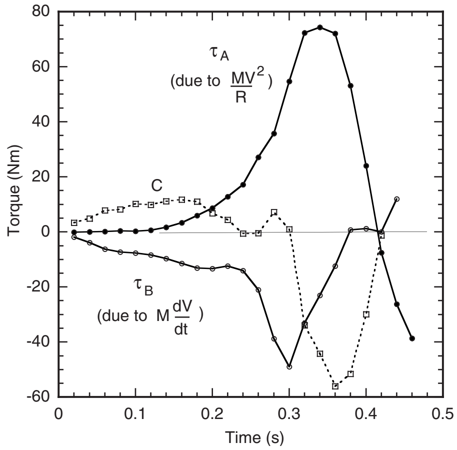

Fig. 6. The torque components τ_A, τ_B, and C acting about an axis through the bat center of mass. The experimental data points are connected by straight line segments. The net torque on the bat $(C+\tau_A+\tau_B=I_{\mathrm{cm}}d\omega/dt)$ is less than 6 Nm throughout the swing.

dition, the two hands apply a couple C, with the result that the net torque acting about an axis through the bat center of mass is given by

$$\tau_F + C = \tau_A + \tau_B + C = I_{\mathrm{cm}} d\omega/dt. \qquad (1)$$

Because $d\omega/dt$ is about 100 rad/s^2 during most of the swing, the net torque on the bat is about 3.9 Nm. However, the three components of that torque are all much larger than 3.9 Nm in magnitude.

Equation (1) can be used to determine the couple acting on the bat, given that the other terms can be determined experimentally. The resulting value of C is shown in Fig. 6, together with the contributions τ_A and τ_B. At the beginning of the swing τ_A is small because the centripetal force is small. Near impact at $t \approx 400$ ms, τ_A drops to zero when the line of action of the centripetal force passes through the handle. At other times during the swing, τ_A is the main component of the total torque on the bat, reaching a peak value of 75 Nm near the end of the swing.

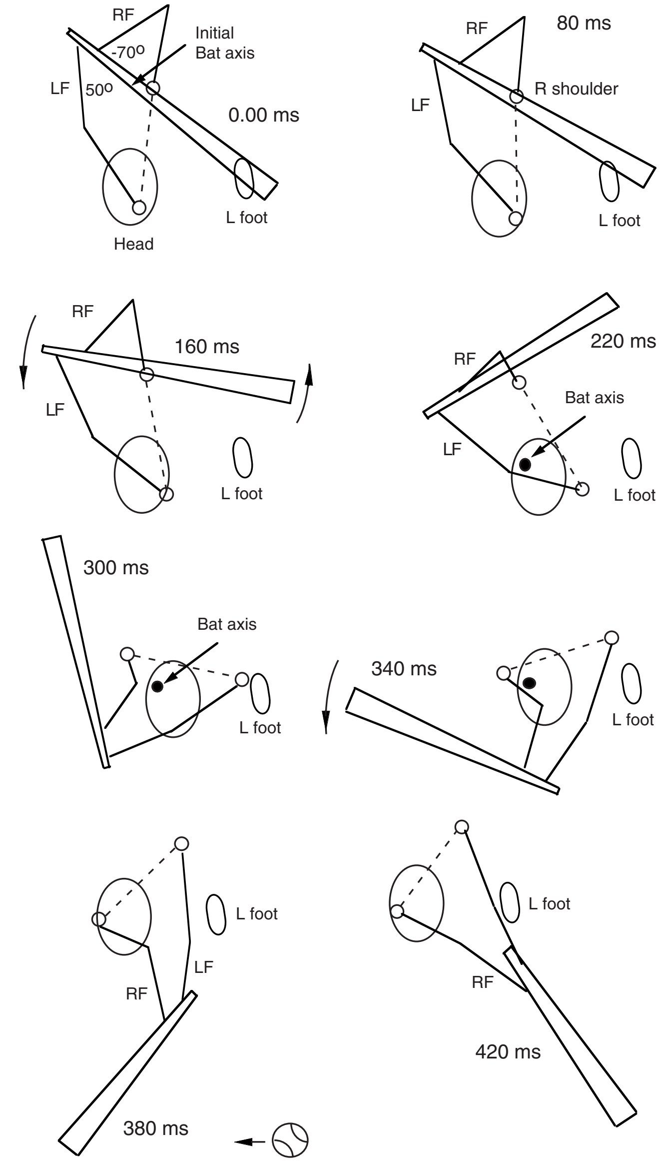

Fig. 7. Overhead view of the swing at eight times showing the positions of the bat, the four arm segments (LF=left forearm, RF=right forearm), the batter's head and shoulders, and his stationary left foot.

D. Angular displacement of bat and body segments

Figure 7 shows the positions of the batter's arms, his head, left foot, and markers on each shoulder at eight times during the swing. Each arm is bent at the elbow in a different manner. The left arm remains relatively straight during the whole swing, while the right arm remains bent at the elbow during most of the swing cycle, straightening out just as the batter impacts the ball. This sequence of events is common to all batters, as can be seen by observing batters in action or by analyzing slow motion film of batters at web sites such as YouTube. The reason for the different behavior of the left and right arms is related to the geometry of the swing. As shown in Fig. 7, the horizontal distance between the left shoulder and the bat handle is larger than that between the right shoulder and the bat handle, so the right elbow must necessarily bend at a greater angle.

The angular velocity of the left forearm increased to a maximum of 16 rad/s at $t=350$ ms and then decreased to almost zero when the forearm was in line with the bat near $t=420$ ms. The angular velocity of the right forearm also increased to a maximum of 16 rad/s, reaching maximum angular velocity near $t=420$ ms. Both forearms were approximately at right angles to the bat near $t=340$ ms, indicating that the batter deliberately delayed relaxing his wrists until just before impact with the (imaginary) ball, by which times both forearms were approximately in line with the bat. By locking the wrists in this manner, the batter was able to apply a positive couple to the bat early in the swing, and a negative couple later in the swing to delay the rapid rotation of the bat until just before impact.

E. Force exerted by each arm

From the initial positions of the arms in Fig. 7 it might appear that the left arm is pulling and the right arm is pushing on the handle, because the handle is rotating counterclockwise. However, the bat center of mass moves in the opposite direction to the handle end during the initial part of the swing, so the net transverse force on the handle must act in the opposite direction to the direction of motion of the

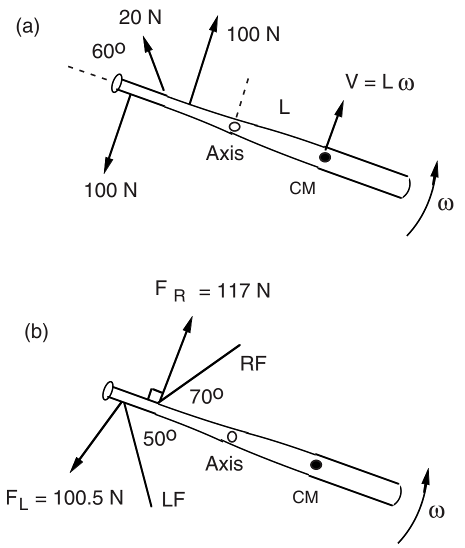

Fig. 8. (a) At $t=100$ ms there is a net force $F=20$ N on the bat plus a couple $C=10$ Nm. F and C act on the handle near the left end of the bat. The knob at the far left end helps to prevent the bat from slipping out of the batter's hands. (b) The force exerted by each arm, assuming that the left arm exerts the small force component in a direction parallel to the long axis of the bat.

handle. Both arms therefore pull in opposite directions on the handle, the right arm exerting a greater pull force than the left arm.

The magnitude and direction of the transverse force exerted by each arm can be estimated by combining information on the net force, the applied couple, and the axis of rotation of the bat. Consider the situation in Fig. 8(a), which shows the forces exerted by the two arms at $t=100$ ms, early in the swing. The net force on the bat is 20 N and it acts at an angle $\alpha=60°$ to the long axis of the bat. In addition, the two arms apply a couple $C=10$ Nm, which can be represented by equal and opposite 100 N forces acting at right angles to the bat and spaced 0.1 m apart. As a first approximation, each arm therefore exerts an equal and opposite transverse force of about 100 N on the bat. This estimate can be improved by considering the location of the rotation axis of the bat.

The rotation axis of the bat was located at a distance $L=0.3$ m from the bat center of mass for the first 200 ms of the swing. The net force component acting in a direction parallel to the long axis of the bat does not contribute to the torque acting on the bat. We can assume that the net transverse force and the torque on the bat arise from two oppositely directed force components F_1 and F_2, acting perpendicular to the bat and spaced a distance 0.1 m apart. In that case the net force $F_2-F_1=MdV/dt$, where $dV/dt=Ld\omega/dt$. Because the net torque is equal to $I_{\mathrm{cm}}d\omega/dt$, it is easy to show that $F_2/F_1=1.18$ when $L=0.3$ m and $I_{\mathrm{cm}}=0.039$ kg m^2. The force exerted by each arm must therefore be approximately as shown in Fig. 8(b). The only uncertainty is whether it is the left or the right arm that supplies the small force component acting parallel to the long axis of the bat, or whether each arm contributes about equally.

A similar analysis can be applied later in the swing. For example, at $t=340$ ms the net force on the bat is 300 N acting at $\alpha=-20°$ to the long axis of the bat, the main component being due to the centripetal force. In this case the torque due to F acts in the correct direction to increase the angular velocity of the bat, but it is opposed by a negative couple,

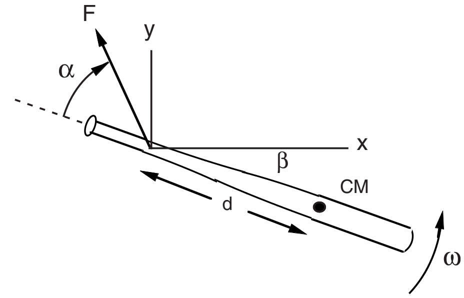

Fig. 9. Geometry used to calculate the swing of a bat, showing the position of the bat and the direction of the force on the handle near the start of the swing. In this position α is taken to be positive and β is taken to be negative. As the bat rotates in a counterclockwise sense, α decreases and β increases (to a positive value when the barrel is above the x axis) at a rate $\omega=d\beta/dt$.

$C=-45$ N, with the result that the total torque on the bat is much reduced and the bat approaches its maximum angular velocity at this time. The couple can be represented by two equal and opposite 450 N forces spaced 0.1 m apart, acting in directions perpendicular to the bat. Because the transverse force on the bat is $300 \sin 20°=102$ N, the left arm exerts a transverse push force of 399 N and the right arm exerts a transverse pulling force of 501 N. In addition, both arms together exert a pulling force along the handle of $300 \cos 20°=282$ N.

The large negative couple applied late in the swing is not only counterintuitive, but appears to be inconsistent with the action of the arms shown in Fig. 7. From the direction of motion of the arms during the interval 300–380 ms in Fig. 7, it appears that the left forearm is pulling on the handle and the right forearm is pushing, as if the batter is deliberately attempting to increase the angular velocity of the bat. However, this interpretation is not consistent with the experimental data. The largest torque component on the bat is due to the centripetal force. By itself, that torque component would cause the bat to rotate much more rapidly than it actually does, with the result that the bat would rotate through an excessive angle by the time it arrives at the impact point. If the bat were allowed to rotate at such a high speed, the handle would push firmly on the batter's left hand and tend to pull out of his right hand. The reaction force exerted by the batter is such that the left hand pushes on the handle and the right hand pulls on the handle, thereby generating the large negative couple that restricts the total torque on the bat to a value less than 6 Nm throughout the swing.

III. BAT SWING MODEL

The model used to calculate the swing of a bat in a two-dimensional (x,y) plane is shown in Fig. 9. Bats are not normally swung in a horizontal plane, but tend to be swung in a plane inclined to the horizontal to project the ball upward or downward. The angle α is the angle between the applied force F and the longitudinal axis of the bat, and the angle β is the angle between the longitudinal axis of the bat and the x axis. The positive direction of α is defined as shown in Fig. 9. When α is positive, the torque due to F acts in the "wrong" direction, meaning that the batter needs to exert a positive couple to rotate the bat in the "right" (counterclockwise) direction. The equations of motion are

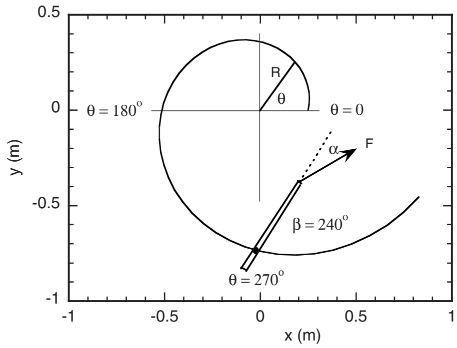

Fig. 10. The spiral path $R=0.25e^{0.23\theta}$ followed by the bat center of mass. The bat is shown arriving at $\theta=270°$, having rotated to an angular position where $\beta=240°$. If the bat struck the ball in this position, the ball would head toward first base rather than straight back toward the pitcher. The force F is shown acting at a positive angle α to the long axis of the bat.

$$\frac{d^2x}{dt^2} = -(F/M)\cos(\alpha-\beta), \tag{2}$$

$$\frac{d^2y}{dt^2} = (F/M)\sin(\alpha-\beta), \tag{3}$$

$$\frac{d^2\beta}{dt^2} = (C - Fd\sin\alpha)/I_{\rm cm}, \tag{4}$$

where d is the distance between the bat center of mass and the point of application of F on the handle, taken to be 0.46 m in the following. The simplicity of the three equations of motion stands in stark contrast to the complex set of relations normally used to describe the double pendulum model,[6] and allows the basic mechanics to be extracted in a more transparent manner.

Equations (2)–(4) could be used to examine a variety of circular motion problems, including the discus and hammer throw in athletics and the acceleration from rest of an object approaching uniform circular motion. Uniform circular motion results when $C=0$ and $\alpha=0$. To study the swing of a bat, Eqs. (2)–(4) were solved numerically for conditions similar to the measured swing. Two approaches were used to find the solutions. The first approach was to specify $F(t)$, $\alpha(t)$, and $C(t)$ and then calculate the bat trajectory. It was found that the calculated trajectory was very sensitive to small changes in all three functions. A change of only 1% or 2% in the magnitude of any one function caused the bat center of mass to follow a path that missed the incoming ball by about 0.5 m, either by spiraling inward toward the batter or by spiraling outward toward first base. An alternative method of solving Eqs. (2)–(4) was adopted by specifying the trajectory in order to determine the required forces and torques, in a manner similar to that used to analyze the experimental data.

A good fit to the experimentally observed bat center of mass trajectory is the logarithmic spiral shown in Fig. 10, defined by

$$R = 0.25e^{0.23\theta}, \tag{5}$$

where (R, θ) are the polar coordinates of the bat center of mass with respect to a fixed laboratory axis. In Fig. 1 the best fit axis is located at $x=-0.25$ m and $y=0.55$ m, near the center of the small circle of solid dots. The x,y coordinates of the bat center of mass are then $x=R\cos\theta$ and $y=R\sin\theta$. For example, at $t=0$ the bat center of mass is located at $R=0.25$ m and $\theta=0$. In Fig. 1 the bat center of mass arrives at $\theta=\pi/2$ rad at $t=0.25$ s, where $x=0$ and $y=R=0.359$ m (with respect to the polar coordinate axis). If we use $dR/dt=0.23R\,d\theta/dt$, we find

$$V_x = \frac{dx}{dt} = (0.23\cos\theta - \sin\theta)R\frac{d\theta}{dt}, \tag{6}$$

$$V_y = \frac{dy}{dt} = (0.23\sin\theta + \cos\theta)R\frac{d\theta}{dt}, \tag{7}$$

and the linear velocity of the bat center of mass is

$$V = (V_x^2 + V_y^2)^{1/2} = 1.026R\frac{d\theta}{dt}. \tag{8}$$

Differentiation of Eqs. (6) and (7) gives the acceleration components $a_x=dV_x/dt$ and $a_y=dV_y/dt$, from which we can calculate the resultant force $F=M(a_x^2+a_y^2)^{1/2}$ and the angle $\alpha-\beta=\sin^{-1}(Ma_y/F)$, for a given $\theta(t)$ function. The $\alpha(t)$ or $\beta(t)$ functions can be chosen arbitrarily. If one of these functions is specified, the other is determined by the value of $\alpha-\beta$, while the required couple is determined by Eq. (4).

The centripetal force was determined using the geometrical construction in Fig. 2 to locate the center of curvature of the spiral path and to determine the magnitude and direction of the corresponding radius vector. The center of curvature does not coincide with the $(x=0,\ y=0)$ origin in Fig. 10. At $\theta=180°$ for example, a straight line drawn perpendicular to the spiral path would pass below the origin, as can be seen by inspection. The slope of that line can be determined analytically from the slope dy/dx of the line tangential to the spiral path at $\theta=180°$. The spiral path can be expressed in the form $y^2=R^2-x^2$, where R is given by Eq. (5). The intersection point of the radius vectors drawn perpendicular to the spiral path shows that the center of curvature traces out a path that is essentially the same as that traced out by the small circle of dots in Fig. 1. Consequently, the direction of the centripetal force for the spiral trajectory in Fig. 10, at any point along the trajectory, is essentially the same as that shown in Fig. 1.

IV. MODEL CALCULATIONS

Two examples are now considered where the bat rotation angle increases from $\beta=-33°$ at $t=0$ to about $\beta=270°$ at $t=0.40$ s, while the bat center of mass rotates from the initial $\theta=0$ position to $\theta=270°$ at $t=0.4$ s. In this manner the bat rotates so that the bat is aligned approximately along the y axis when it collides at $x=0$ and $t=0.4$ s with a ball incident parallel to the x axis. These parameters were chosen to fit the experimental data in Fig. 1. Figure 11 shows calculated results for a bat swung with constant angular velocity, and Fig. 12 shows an example where the bat was swung with constant angular acceleration. Details of the calculations are given in the Appendix.

A bat is not normally swung at constant angular velocity, but it is instructive to consider this special case to examine some of the effects of the spiral trajectory. In Fig. 11 the bat center of mass rotates along a spiral path at a constant angular velocity $d\theta/dt=11.78$ rad/s. The velocity V increases in an approximately linear fashion from 3.0 m/s at $t=0$ to 8.9 m/s at $t=0.4$ s, and the force F increases from 32.2 N (at

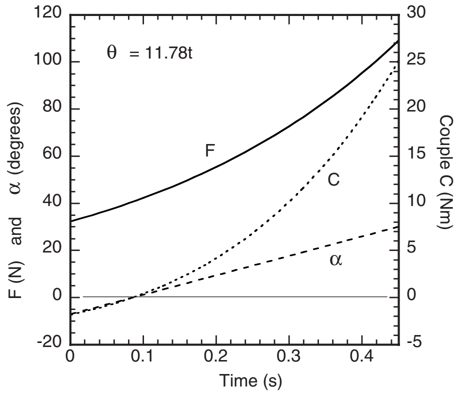

Fig. 11. F, α, and C vs time for uniform spiral motion under conditions where $\theta=\beta=270°$ at $t=0.4$ s, leading to good alignment of the bat with respect to the incoming ball at $t=0.4$ s.

$t=0$) to 95.3 N (at $t=0.4$ s). For uniform circular motion V and F would remain constant in time, but in the present case there is a factor of 2.96 increase in both V and F due to the factor of 2.96 increase in R from $t=0$ to $t=0.4$ s. A significant positive couple is required to swing the bat toward the end of the swing, because the centripetal force on the bat tends to rotate the bat in the "wrong" direction. At the beginning of the swing a small negative couple is required to swing the bat because the net force on the bat tends to rotate the bat too fast to maintain a constant angular velocity.

If a bat is swung at constant angular velocity along a spiral path, the bat speed at the impact point is relatively low. A larger impact speed results if the angular speed, $d\theta/dt$, increases during the swing, assuming that the bat is swung over the same time interval. In Fig. 12 the linear speed of the bat center of mass increased to 17.87 m/s at the impact position (starting from rest) or at about twice the linear speed of the constant angular velocity case, and the force F increased to 406 N at $t=0.4$ s.

The results in Fig. 12 provide a good fit to the experimental data in Figs. 5 and 6, although a better fit can be obtained with a higher order $\theta(t)$ function. A batter is not constrained to accelerate a bat at a uniform rate. Nevertheless, the essence of the swing is captured in Fig. 12, and the uniform acceleration model provides a good basis for analyzing other swings. Similarly, batters do not always swing a bat at the

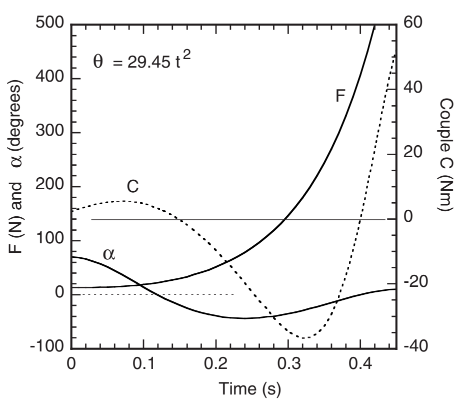

Fig. 12. F, α, and C vs time for uniformly accelerated spiral motion, for a case where $C=0$ at $t=0.4$ s. These results compare well with the experimental data shown in Figs. 5 and 6.

maximum speed possible, nor does the bat swing in a strictly two-dimensional plane. Sometimes, batters hold the bat farther up the handle to obtain improved control of the swing. An extension of the present model to consider these additional features is beyond the scope of the present paper, but could form the basis of some interesting student projects.

V. DISCUSSION

The small positive couple required to start the swing of a bat or a club is well known,[4–9] and is commonly described as a consequence of the player cocking the wrists to maintain a fixed angle of about 90° between the implement and the forearms. The couple required to maintain this fixed angle decreases as the bat accelerates, due to the increasing torque resulting from the increasing centripetal force on the bat. When the required couple drops to zero, a batter or golfer could then continue to swing the bat or club by relaxing the wrists slightly (while still maintaining a firm grip on the handle), in which case the wrists would behave as a relatively free hinge. A similar result is obtained with a mechanical double pendulum. A mechanical double pendulum can be provided with a stop to ensure that the angle between the two arms cannot exceed 90°. If the two arms of the pendulum are connected by a free hinge, then the stop exerts a couple on the lower arm at the start of the swing, but the couple drops to zero during the swing, and then remains zero while the two arms swing freely.[4] However, the two arms of a mechanical double pendulum do not necessarily line up at the bottom of the swing, as is required for the swing of a bat.

A bat requires a large negative couple near the end of the swing so that the bat lines up correctly on impact with the ball. The latter feature has been noted by Vaughan[11] in relation to the measured swing of a golf club, but it has not received the attention it deserves. Instead, most authors concentrate on the effect of varying the wrist torque, thereby giving a misleading picture of the torque required to swing a bat. For example, Jorgensen[6] found that a slightly increased club head speed results if the wrists apply a negative torque of 2.8 Nm for 0.1 s to hinder the uncocking process. Spriggings and Neal[8] found that the club head speed could be increased by 9% by applying a positive wrist torque of 20 Nm for 0.1 s just prior to impact with the ball. Both of these pictures are misleading because the authors invoke a wrist torque to apply the initial positive couple to start the swing, and then ignore the large negative couple required to complete the swing. The large negative couple could presumably be increased or decreased by using the wrists in an active rather than passive manner, but the contribution of the wrist torque to the total couple has not been thoroughly investigated. The couple exerted by the two arms, pulling and pushing in opposite directions on the handle, is likely to be much larger than the couple exerted by the wrists. Using one wrist alone, most batters would have difficulty supporting a 2 kg mass at the end of a horizontal bat, indicating that the total wrist torque is unlikely to exceed about 30 Nm, using maximum effort. In practice, the wrists tend to be relaxed near the end of the swing, in which case the wrist torque might be only about 10 Nm. In contrast, the negative couple required to swing the bat in Fig. 1 was almost 60 Nm.

VI. CONCLUSIONS

Experimental and theoretical results concerning the swing of a baseball bat indicate that a batter must apply a small positive couple to start the swing and a large negative couple to complete the swing. The mechanics of this process can be modeled purely in terms of the forces and torques acting on the bat, without being concerned with the biomechanics of the problem. In this case it is convenient to assume that the center of mass of the bat rotates in a logarithmical spiral with constant angular acceleration. It would be interesting to use such a model to explore a number of further questions regarding the swing of a bat not considered in the present paper. One question is how a batter might maximize the velocity at the impact point on the bat. The question is whether the batter should attempt to maximize the angular velocity of the bat, or whether it is more important to maximize the linear velocity of the center of mass. Another question is whether a particular batter might benefit by using a lighter or heavier bat or a bat with a larger or smaller moment of inertia. The model developed in this paper will assist in providing partial answers, but is unlikely to provide complete answers because the forces and torques applied to a bat depend on the forces and torques exerted by all the various body segments used by the batter. In that respect an important issue that still has to be resolved is the extent to which the wrists play an active role in providing the initial and final couples required to swing a bat.

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APPENDIX: MODEL PARAMETERS

The results in Fig. 11 were obtained by assuming that $d\theta/dt = 11.78$ rad/s. In this case the angle $\alpha - \beta = 0.452 - 11.78t$. If we assume that β increases at a uniform rate (from $-33°$ at $t=0$) to $270°$ at $t=0.4$ s, then $\beta = 13.221t - 0.576$ rad and $\alpha = 1.440t - 0.124$ rad. The initial force therefore acts at an angle $\alpha = -0.124$ rad$=-7.1°$ to the long axis of the bat and rotates to an angle $\alpha = 25.9°$ at $t=0.4$ s. Because $d^2\beta/dt^2 = 0$, the couple on the bat is given from Eq. (4) by $C = Fd \sin \alpha$, regardless of the moment of inertia of the bat. The resulting values of F, α, and C are shown in Fig. 11.

An alternative solution is possible where the centripetal force exerts no torque on the bat at $t=0.4$ s, in which case a smaller couple can be used to rotate the bat at constant angular velocity $\omega = d\beta/dt$. However, the bat then rotates at a smaller rate (with smaller ω) and arrives at the impact point at $t=0.4$ s aligned at an angle $\beta = 244°$.

The results in Fig. 12 were obtained with $d^2\theta/dt^2 = 58.9$ rad/s so that the bat center of mass would arrive at $\theta = 4.712$ rad$=270°$ at $t=0.4$ s. In this case $\alpha - \beta$ decreases in an approximately linear fashion from 1.794 rad at $t=0$ to -4.285 rad at $t=0.4$ s. The required couple depends on the assumed $\alpha(t)$ or $\beta(t)$ function. For the solution shown in Fig. 12 it was assumed that the bat accelerated from rest to a maximum angular velocity $\omega = 30$ rad/s at $t=0.4$ s, in a manner similar to that shown in Fig. 3. Such a result is obtained for $\beta = -0.576 - 42.7t^2 + 329.83t^3 - 367.81t^4$. This function was chosen so that $\alpha = 0$ at $t=0.4$ s, in which case $C=0$ and $\beta = 4.285$ rad at $t=0.4$ s. The other coefficients of $\beta(t)$ were chosen so that $d\beta/dt = 0$ at $t=0$, while $d\beta/dt = 30$ rad/s and $d^2\beta/dt^2 = 0$ at $t=0.4$.

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[12]Swinger software can be purchased at ⟨www.webbsoft.biz⟩.
[13]A new digital camera (Casio EX-F1) became available in 2008. It would be ideal for experiments of this type because it can film at rates from 30 to 1200 f/s and is similar in price to a normal video camera.