Measuring String Tension

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Tennis players like to test the tension of their strings by hitting them and listening to the pitch of the resulting "ping". Guitar players pluck their guitar strings to listen to the pitch. A high note is heard at high string tension and a low note is heard at low string tension. Pitch refers to the vibration frequency, which is the number of times per second the string vibrates back and forth. The vibration frequency of tennis strings is typically between 500 Hz and 700 Hz. It is not easy to measure the frequency without equipment such as a microphone and an oscilloscope, but players can at least hear by ear whether one pitch is higher or lower than another. Various frequency "pings" can be heard on the string test page so you can make this sort of comparison.

The vibration frequency of a stretched string depends on its length as well as the string tension, and it also depends on its mass. Thin strings are lighter and vibrate faster than thick strings of the same material, and short strings vibrate faster than long strings. For a guitar string, the vibration frequency of the fundamental mode (the lowest frequency of the string, and the one that determines its pitch) is given by

$$f = \sqrt{\frac{T}{4mL}} \tag{1}$$

where T is the string tension in Newton, m is its mass in kg and L is its length in metres. The same formula can be used to calculate the vibration frequency of the strings in a racquet. The strings in a racquet vary in length, but because they are woven together they all vibrate together at the same frequency. If they weren't woven together and if they were all at the same tension then the short strings would vibrate faster than the long strings. When they are woven together the vibration frequency is essentially the average frequency of all the separate strings. The average frequency in this case is given by Eq. (1) where m is the mass of a string of length $L = \sqrt{A}$ where A is the area of the string plane. If the racquet head was a square with sides of length L then the string area would be L^2 and the length of every string would be L. Eq. (1) works with an error of only about 1% even if the racquet head is circular or if it is oval shape, provided we take the average string length to be $L = \sqrt{A}$. The theory behind this result is too complicated for a web page, but can be found in the 2001 Sports Engineering paper by Cross and Bower listed on the publications page. This paper also contains some experimental results, showing that a racquet strung at any given tension can drop in tension by as much as 30% within the first hour and even while it is being strung.

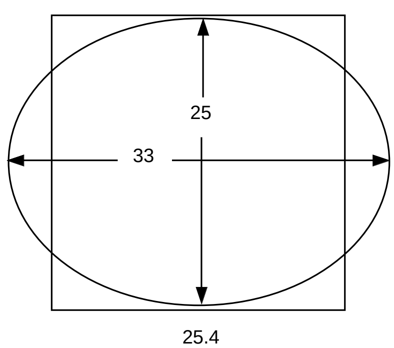
To measure a change in string tension

It would be useful for players to know whether there has been a sufficient drop in the string tension in their racquet to warrant a restring. Professional players restring their racquets every day during a tournament. The average player should get a restring at least once a year, depending on how often the racquet is used. A rule of thumb is to get a restring when the tension has dropped by about 10%. That is, when the tension has dropped from say an initial 60 lb to a value of around 60 - 6 = 54 lb.

The change in tension is easier to measure than the actual tension. According to Eq. (1), the vibration frequency of the strings is proportional to the square root of the string tension. If you do a simple calculation on this, you will find that a 10% drop in tension leads to a 5% drop in frequency. If the strings in a freshly strung racquet vibrate at say 600 Hz, and the tension later drops by 10%, then the frequency will drop by 30 Hz to 570 Hz. This is easy to pick by ear, provided you can remember what 600 Hz sounds like when you later check your strings. To assist in this task, I have included 20 different "ping" sounds on the strings page, varying from 500 Hz to 700 Hz in 20 Hz steps.

To measure actual string tension

Racquet heads are not square and they are not circular. They are approximately elliptical or oval shape. If the longest main string has length S and the longest cross string has length C then the area of the whole string plane is given by $A = \pi SC/4$. If the head was circular with radius R then $S = C = 2R$ and the area would be πR^2. For a typical racquet, S is about 33 cm, C is about 25 cm so $A = 3.14 \times 33 \times 25/4 = 648$ cm^2. The average string length in this case is $L = \sqrt{648} = 25.4$ cm. In other words, the area of the oval shape string plane is the same as that of a square racquet head with sides of length 25.4 cm, as indicated in the drawing. Furthermore, the string plane will vibrate at the same frequency as a single string of length 25.4 cm, provided it is strung at the same tension.

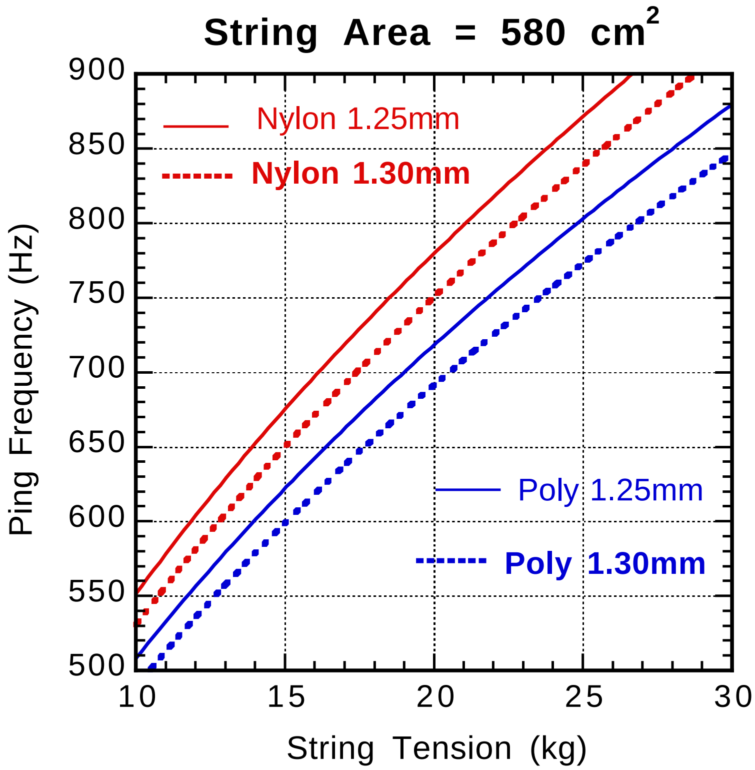


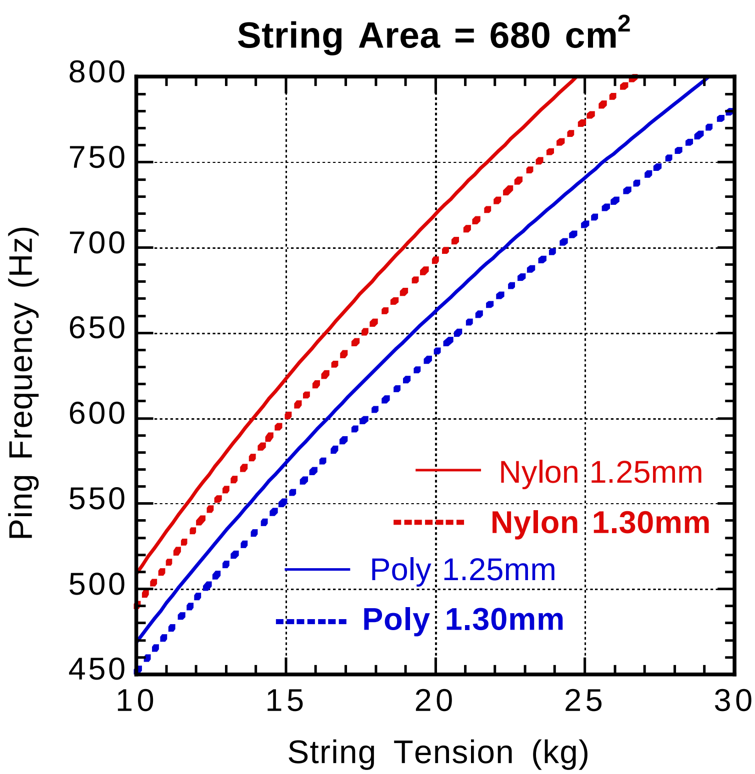
The mass of a given length of string can be worked out from its density ρ and its volume V. Nylon and kevlar have a density of about 1.14 gm/cm^3, while natural gut and polyester have a density of about 1.34 gm/cm^3. A string of length L and diameter D has a volume $V = \pi LD^2/4$. The mass of the string is given by $m = \rho V$. For example, a 1.30 mm = 0.13 cm diameter nylon string of length 30 cm has a volume V = 0.398 cm^3 and its mass is $m = 1.14 \times 0.398 = 0.454$ gm. The total mass of all the strings in a racquet is typically about 15 gm = 0.015 kg.

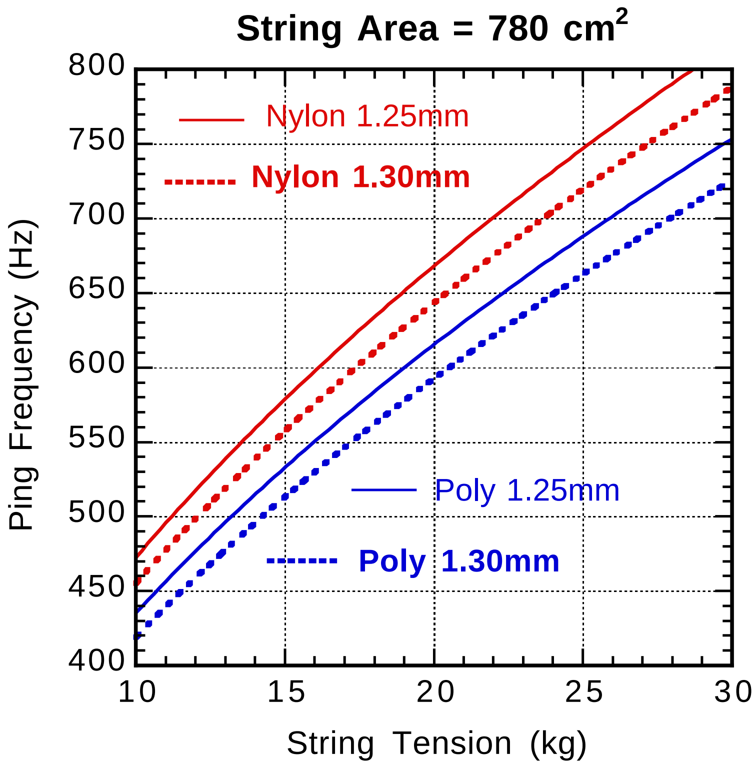
We can use Eq. (1) to work out the string tension in a racquet, after it has been strung, by measuring the pitch or the frequency of the string plane. Players and racquet stringers measure string tension in kg or in lb, but we need to change that to Newton in Eq. (1). The conversion is 1 kg = 2.205 lb = 9.8 Newton, or 1 lb = 4.444 Newton.

Frequency vs String Tension

Calculations of the ping frequency vs tension are shown in the following three graphs for three different head sizes. Each graph shows four curves, two for nylon (1.25 and 1.30 mm diameter) and two for polyester (1.25 and 1.30 mm diameter).


String Area = 580 cm2
Nylon 1.25mm
Nylon 1.30mm
Poly 1.25mm
Poly 1.30mm
Ping Frequency (Hz)
String Tension (kg)
900
850
800
750
700
650
600
550
500
10
15
20
25
30


String Area = 680 cm2
Nylon 1.25mm
Nylon 1.30mm
Poly 1.25mm
Poly 1.30mm
Ping Frequency (Hz)
String Tension (kg)
800
750
700
650
600
550
500
450
10
15
20
25
30


String Area = 780 cm2
Nylon 1.25mm
Nylon 1.30mm
Poly 1.25mm
Poly 1.30mm
Ping Frequency (Hz)
String Tension (kg)
800
750
700
650
600
550
500
450
400
10
15
20
25
30