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Ball Trajectories

Factors Influencing the Flight of the Ball

By Rod Cross

Introduction

A good tennis player knows instinctively how hard to hit a ball and at what angle to get the ball over the net and into the court so that the ball lands at just the right spot. This comes from years of practice. Players might be surprised to know just what they are doing in terms of the actual ball trajectories. For example suppose you hit a forehand at the baseline so that the ball lands on your opponent's baseline 78 feet away. If you hit the same ball but 1% faster it will land 18 inches beyond the baseline. If you hit the ball one degree higher it will land about 6 feet beyond the baseline, depending on the initial speed and angle. If you hit the ball one degree further to the left it will land 16 inches further to the left.

When serving a ball at moderate to high speed, the ball must be served a few degrees down from the horizontal. Too far down and the ball will hit the net. Not enough and the ball will be long. The range of possible angles is only about two degrees, and it gets smaller if the ball is served faster or by a shorter player (Brody, 1987). The range of angles increases if the ball is served with topspin. Typical ball trajectories are shown below, together with an explanation of the trajectories.

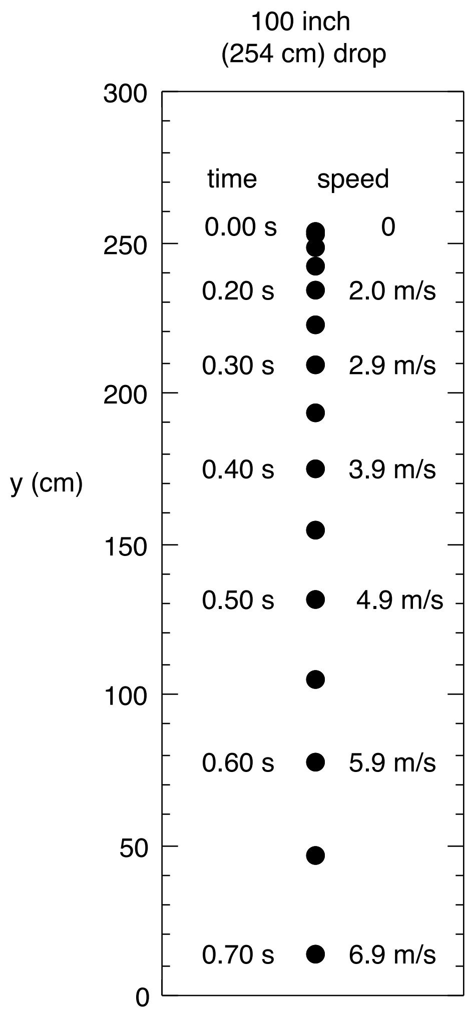

Figure 42.1

Examples of Ball Trajectories

(a) Dropping a Ball (Free Fall)

Suppose that a ball is dropped from a height of 1.0 m (about waist height) and is allowed to fall to the ground. Provided that it is not hit or thrown then it starts out with zero speed. It will accelerate downwards due to the force of gravity. If we ignore air resistance, then the vertical distance (y) travelled in time t is given by $y = gt^2/2$ where $g = 9.8$ m/s^2 is the acceleration due to gravity. After = 0.1 s, y = 0.049 m and after 0.2 s, y = 0.196 m. The ball will hit the ground at t = 0.452 s. On the way down, the speed increases and is given by v = gt. At t = 0, v = 0. At t = 0.1, v = 0.98 m/s. Just before it hits the ground, at t = 0.452 s, v = 4.43 m/s. The corresponding results for a ball dropped from a height of 100 inches (2.54 m) is shown in Figure 42.1. Air resistance is very small in this case since the ball speed is small. If allowance is made for air resistance, a ball dropped from a height of 100 in would hit the ground at 0.725 s instead of 0.720 s.

(b) Horizontal hit with free fall

Now suppose that the ball is hit at 30 m/s (67 mph) from a height of 1.0 m so that it starts moving in a horizontal direction, parallel to the ground, as shown in Figure 42.2. It is easy to calculate where the ball will land provided we ignore the force on the ball due to air resistance. Then the only force acting on the ball is the gravitational force pulling the ball vertically towards the ground. The ball will accelerate downwards and change its vertical speed, but there is no change in the horizontal speed since there is no force in the horizontal direction. In other words, the ball will keep moving horizontally at 30 m/s until it hits the ground. It starts out with zero speed in the vertical direction, and moves downwards a vertical distance $y = gt^2/2$, as in our first example. It therefore takes 0.452 s to hit the ground. During that time, it travels a horizontal distance x = 30 x 0.452 = 13.56 m.

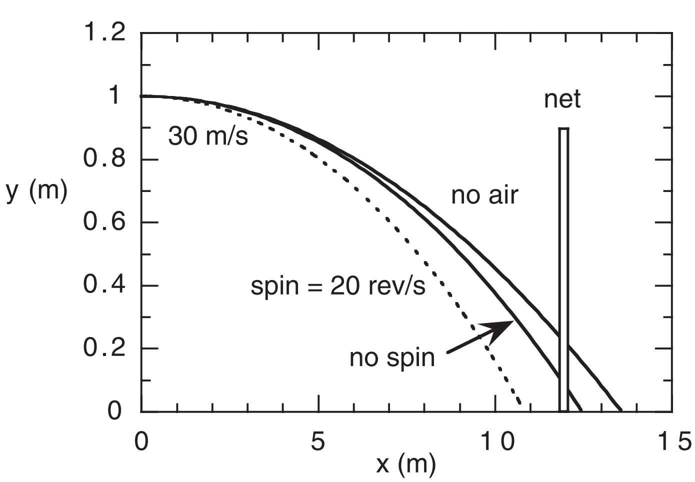

Figure 42.2

In this case air resistance does make a difference since the ball is moving fairly fast and since the air resistance increases as the ball speed increases (Table 42.1).

If the ball has no spin then it will actually hit the court after 0.470 s after traveling a horizontal distance of 12.46 m. The average horizontal speed is 12.46/0.47 = 26.5 m/s. The horizontal speed drops slightly as the ball moves through the air, and so does the final vertical speed since air resistance exerts a force backwards and upwards along the path of the ball. If the ball has topspin, there is an additional force that acts at right angles to the path of the ball, in a downwards and backwards direction in this case. This force is called the Magnus force. For example, if the ball is spinning at 20 revolutions/sec, the ball will hit the court after 0.410 s after traveling a horizontal distance of 10.85 m.

Table 42.1. Trajectories With Same Launch Speed and Angle Under Various Conditions

Conditions	Velocity	Time to Bounce	Travel Distance
No air no spin	30 m/s	0.452 secs	13.56 m
Air no spin	30 m/s	0.470 secs	12.46 m
Air w/ topspin (20rev/s)	30 m/s	0.410	10.85

The distance from the baseline to the net is 39 feet (11.887 m) and the height of the net in the middle is exactly 3 feet (0.9144 m). A ball hit horizontally at 30 m/s will not clear the net even if it starts at a point 1.0 m off the ground. To get the ball over the net it has to be hit upwards at a certain angle to the horizontal. It can't be hit too steeply or it will land past the baseline at the other end of the court. There is a certain range of angles that will get the ball over the net and inside the baseline. The range of angles is relatively small.

(c) Hitting at Various Launch Angles

Figure 42.3 shows the trajectories of a ball hit from a height of 1.0 m, at 30 m/s and at various angles to the horizontal, including the effects of air resistance. In order to just clear the net, a ball hit without spin must be hit upwards at 4.0^{o} to the horizontal. If the ball has topspin at 20 revolutions/sec, it must be hit upwards at 5.5^{o} to the horizontal. If a ball is hit without spin at 8.1o to the horizontal, it lands on the far baseline. If the ball has topspin at 20 revolutions/sec, it must be hit upwards at 11.9^{o} to land on the baseline. The range of available angles is 4.1^{o} without spin or 6.4^{o} with topspin.

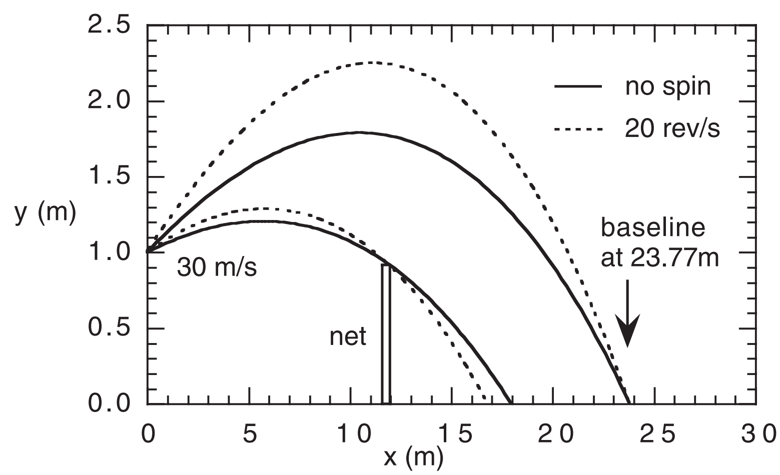

Figure 42.3

Drag Force

(a) Drag force pushes ball backwards and slows it down.

A ball traveling through the air experiences a backwards force due to the fact that the air pressure on the front of the ball is larger than the force at the back of the ball. This force is called the drag force. The force is large because air is heavy. Not as heavy as water, but a room full of air contains about 90 kg or 200 lb of air. When a tennis ball is at rest, air pressure exerts a force of 150 lb on the front of the ball and 150 lb on the back of the ball. The ball doesn't collapse since the air inside the ball exerts a similar force outwards. When a ball is traveling through the air, the force on the rear side of the ball is typically about 0.2 lb lower than the force on the front side, depending on the ball speed.

(b) Formula for drag force.

The flow of air around a tennis ball is turbulent at all ball speeds of interest in tennis, due to the relatively rough surface of the ball. This actually simplifies the analysis of ball motion since the drag coefficient remains constant for all speeds of interest. Other balls used in sport are generally smoother, but this complicates the analysis since the drag coefficient usually decreases suddenly at a sufficiently high ball speed (an effect known as the drag crisis). The drag force is proportional to the ball speed squared and is given by the formula

$$F = C_d\, A\, d\, v^2/2 \tag{42.1}$$

where $d = 1.21\ kg/m^3$ is the density of air, $A = \pi R^2$ is the cross-sectional area of the ball, R is the radius of the ball, v is the ball speed and C_d is the drag coefficient. The only factor here that can be influenced by the player is the ball velocity. If the ball was a flat, circular disk, the drag coefficient would be 1.0. But a ball has a rounded nose which streamlines the air flow and reduces the drag. For a relatively new tennis ball, C_d is about 0.55. A used ball experiences a slightly lower drag force since it has a smoother surface. A standard new ball of radius 3.3 cm therefore experiences a backwards force

$$F = 0.55 \times 0.00342 \times 1.21 \times v^2/2 = 0.00114 \times v^2 \quad \text{Newton (N).}$$

For example, if v = 10 m/s then F = 0.114 N. If v = 20 m/s then F = 0.456 N. If v = 30 m/s then F = 1.026 N. We can compare this with the force of gravity F = mg acting downwards on the ball. Since $g = 9.8\ m/s^2$ and m = 0.057 kg for a standard tennis ball, mg = 0.057 x 9.8 = 0.559 N. The drag force is as large as the gravitational force even at moderate ball speeds. At a very fast serve speed of 60 m/s, the drag force is 7.3 times bigger than the gravitational force. As a result, a ball served at high speed slows down rapidly through the air and lands on the court at about 75% of the serve speed. In fact, a ball served at any speed lands on the court at about 75% of the serve speed. A ball served at a low speed experiences a small drag force, but it spends a relatively long time slowing down.

Magnus Force

(a) Force due to spin

An additional force arises if the ball is spinning. A spinning ball sets the air around it in motion in a thin layer near the surface of the ball. The flow of air around the ball is altered in such a way that the air pressure on top of a spinning ball is decreased if the ball has topspin and is increased if the ball has backspin. As a result, a ball that travels horizontally experiences a force downwards if it has topspin or upwards if it has backspin. If the ball spins about a vertical axis then the force causes the ball to swerve sideways. The force due to spin is called called the Magnus force and it always acts at right angles to the drag force and to the spin axis. (see Figure 42.4). Consequently, if a ball with topspin is rising upwards at an angle to the court, the Magnus force tends to push the ball down onto the court and it pushes it forwards in a direction parallel to the surface. If a ball with topspin is falling towards the court surface, the Magnus force pushes the ball downwards and backwards. A ball with topspin therefore falls onto the court at a steeper angle than a ball without topspin. Conversely, a ball with backspin tends to float through the air and falls to the court at a shallow angle.

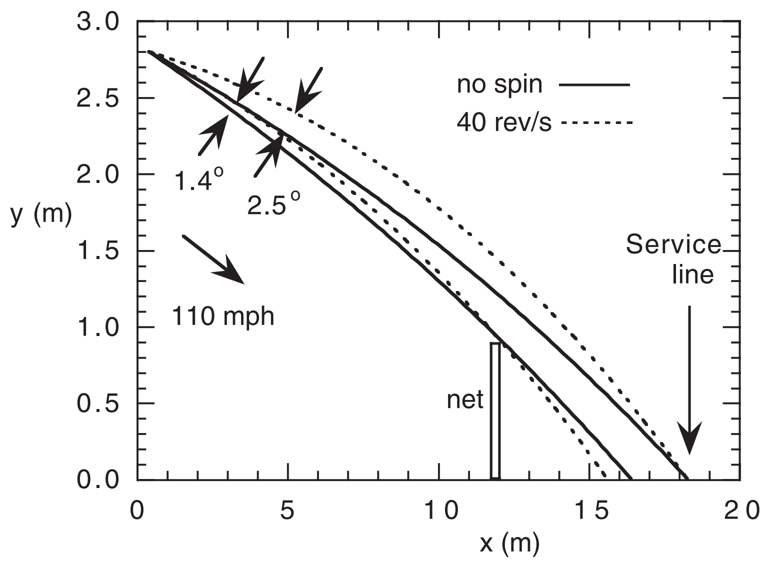

Figure 42.4

(b) Formula for the Magnus force.

The magnitude of the Magnus force on a tennis ball is given by

$$F = C_L A d v^2/2 \qquad (42.2)$$

where C_L is called the lift coefficient since the ball experiences a vertical lift force if it has backspin and is given by

$$C_L = 1/[2 + (v/v_{spin})] \qquad (42.3)$$

where $v_{spin} = R\omega$ is the peripheral speed of the ball, R is its radius and ω is the angular speed about a horizontal axis perpendicular to the path of the ball. The angular speed varies typically from about 100 to about 500 radians/sec which translates to a spin between 16 and 80 revolutions per second (1 rev/sec = 6.28 radians/sec). Suppose that v = 30 m/s, ω = 300 rad/s and R = 0.033 m. Then v_{spin} = 9.9 m/s, v/v_{spin}= 3.03 and C_L = 0.20. The lift coefficient in this case is smaller than the drag coefficient, but not a lot smaller. The lift coefficient never gets to be as large as the drag coefficient. At most, C_L = 0.5, which is the limit when v_{spin} is much larger than v.

Ball spin also affects the drag coefficient (Stepanek, 1985). A good fit to the experimental data is given by

$$C_d = 0.55 + 1/[22.5 + 4.2(v/v_{spin})^{2.5}]^{0.4}$$

which indicates that C_d increases from 0.55 to 0.84 as v_{spin} increases from zero to a value much larger than v.

Serve Trajectories

Armed with the relevant values of the lift and drag coefficients, one can calculate the trajectory of a tennis ball through the air for any initial set of conditions. The equations describing the trajectory are given in the Appendix to this chapter. Some typical serve trajectories are shown in Figure 42.5 for a ball served at 110 mph (177 kph) from a height of 2.8 m (9 feet 2 inches). A person of height h serves the ball from a height typically about 1.5h. For example, a person of height 6 feet usually serves the ball from a point about 9 feet above the court, even if he or she tosses it a lot higher. A high ball toss

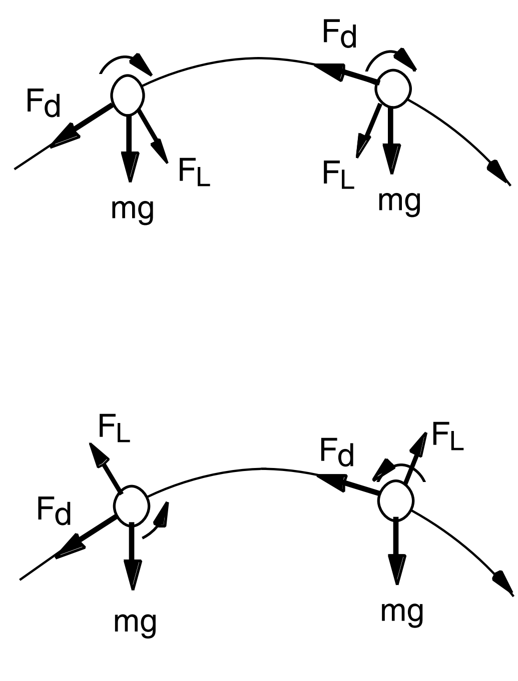

Figure 42.5

allows the server enough time to sight the ball and judge its position accurately. A high ball toss also allows the server to maintain good balance while the tossing arm is lifted upwards and the serving arm is moved backwards. If the toss is too high, the ball might be thrown too far forwards or backwards.

The trajectories in Figure 42.5 were calculated for a serve down the center line so that the ball just cleared the net or so that it landed on the service line. Results are given for a perfectly flat serve with no spin and for a serve with topspin at 40 rev/sec. One of the advantages of serving with topspin is that the available range of serve angles is increased (from 1.4^o to 2.5^o in this case). The ball also approaches the court at a steeper angle and kicks up at a higher angle. The results in Figure 42.5 are typical of a first serve. If the serve speed is reduced to 85 mph and the spin increased to say 50 rev/s for a second serve, the range of available angles increases to 3.8^o.

A surprising result is that the range of available angles for a serve is almost the same for a serve in the far corner of the service court. Even though the net is 1.7 inches (43 mm) higher along that path, the distance from the server to the far corner is 18 inches (457 mm) longer. As a result, the range of available angles is reduced by only about 0.1^o. It is therefore just as easy to serve into the far corner as to serve down the center line. The hard part is to serve even wider than the far corner. There is, however, one advantage of serving towards the far corner. If you aim for the center line and miss in the horizontal direction, then you have only a 50-50 chance that the ball will be in. If you aim towards the far corner and miss in the horizontal direction then the ball can still land in, provided you serve the ball short.

To compensate for possible horizontal direction errors, you should never aim exactly down the center line, but a bit towards the receiver. Just how far is something that you need to determine for yourself. Try serving down the line a few times and see how far you miss (on average). That's how far you should aim away from the line.

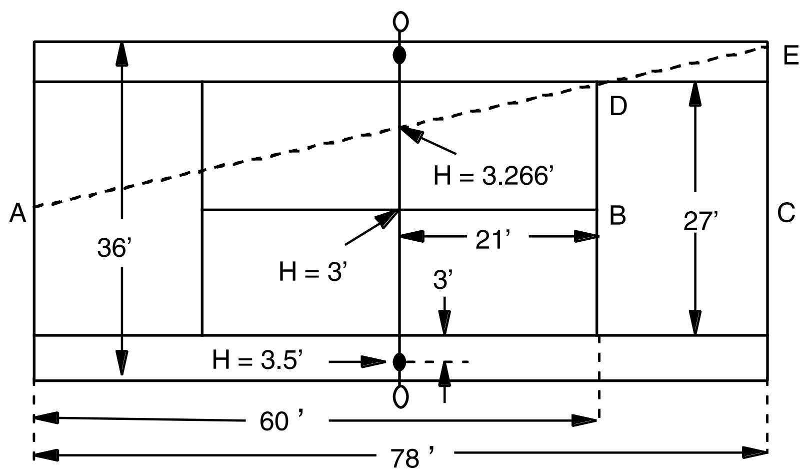

Figure 42.6

Appendix: Trajectory Equations

Suppose that a tennis ball of mass m and radius R is traveling at speed v and at an angle theta upwards from the horizontal. The velocity in the horizontal direction is then $v_x = v \cos(\theta)$ and the velocity in the vertical direction is $v_y = v \sin(\theta)$. The forces acting on the ball are mg downwards, $F_d = C_d Adv^2/2$ backwards and $F_L = C_L Adv^2/2$ upwards and at right angles to the path of the ball (assuming the ball has backspin). The equations of motion are then

$$md_{vx}/dt = -F_d\cos(\theta) - F_L \sin(\theta)$$

$$md_{vy}/dt = F_L\cos(\theta) - F_d \sin(\theta) - mg$$

which can written as

$$d_{vx}/dt = -kv\,(C_d\, v_x + C_L\, v_y) \qquad (43.a1)$$

$$d_{vy}/dt = kv\,(C_L\, v_x - C_d\, v_y) - g \qquad (43.a2)$$

where $k = d.\pi.R^2/(2m)$. If the ball has topspin, the sign in front of C_L must be changed in each of these equations. These equations can be solved numerically, but care is needed to avoid numerical errors. A good check is to solve for a purely vertical drop without spin. In that case the equation of motion is

$$dv/dt = -kC_d\, v^2 - g \qquad (42.a3)$$

which can be solved analytically to check the numerical result. For example, if m = 57 gm, g = 9.8, d = 1.21 kg/m^3, diameter = 2R = 65 mm, C_d = 0.50, initial speed = 15 m/s downwards starting at y = 30 m, then the ball hits the ground at y = 0 at speed v = 21.006 m/s after 1.6199 sec.

The essential features regarding the horizontal motion of a tennis ball can be described analytically if one ignores the small vertical component of the ball speed. Consider a case where a ball is traveling horizontally at speed v without spin, and where the equation of motion in the horizontal direction has the form

$$dv/dt = -k\, C_d\, v^2 \qquad (42.a4)$$

The ball will subsequently develop a velocity component in the vertical direction due to the gravitational force, but this component is typically much smaller than the horizontal component and can be neglected as a first approximation. Since the drag force increases with the velocity squared, one might expect that the drag force on a ball would have a significantly greater effect at higher ball speeds. However, the kinetic energy of the ball also increases with the velocity squared. As a result, the percentage change in ball speed as a result of the drag force, over a given distance, does not depend on the ball speed. Equation (42.a4) can be integrated directly to show that

$$v/v_o = 1/(1 + k\, Cd\, s) \qquad (42.a5)$$

where v_o is the initial speed and $s = v_o t$ is the distance the ball would travel at speed v_o in a time t. For a standard 66 mm diameter, 57 gm tennis ball, k C_d = 0.020 m-1 when C_d = 0.55. Over the distance s = 17.888 m from a point 40 cm in front of the baseline to the opposite service court line, the ball speed drops to v = 0.737v_o, regardless of the initial speed. For example, if v_o = 160 kph, then v = 118 kph, which agrees well with the numerical solution (113.6 kph) . The numerical solution takes into account both the horizontal and vertical motion of the ball.

Equation (42.a5) can also be integrated directly to show that the time taken to travel a horizontal distance x is given by

$$t = [\exp(k\ C_d\ x) - 1]/(k\ C_d\ v_o) \qquad (42.a6)$$

For a 66 mm diameter tennis ball, the time taken to travel from a point 40 cm in front of the baseline to the opposite service line is therefore $t = 21.5/v_o$. If v_o = 44.44 ms (160 kph), then t = 0.484 s, which also agrees closely with the numerical solution (t = 0.487 s).

References

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