Lives of the Stars Lecture 2: Atoms and quantum mechanics

– A (gentle) introduction

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Spring 2016

THE UNIVERSITY OF SYDNEY

Quantum mechanics is one of the major triumphs of physics in the last century. It underpins much technology we take for granted: computers, transistors, lasers.

Perhaps more surprising is that the explanation for how stars work is intimately connected with how atoms work.

“The frontiers of science have moved far from the experience of ordinary persons. Unfortunately, we have never developed a way to bring people along as informed tourists of the vast terrain we have conquered, without training them to become professional explorers.”

from “The Big Crunch” by David Goodstein
http://www.its.caltech.edu/~dg/crunch_art.html

In tonight's lecture

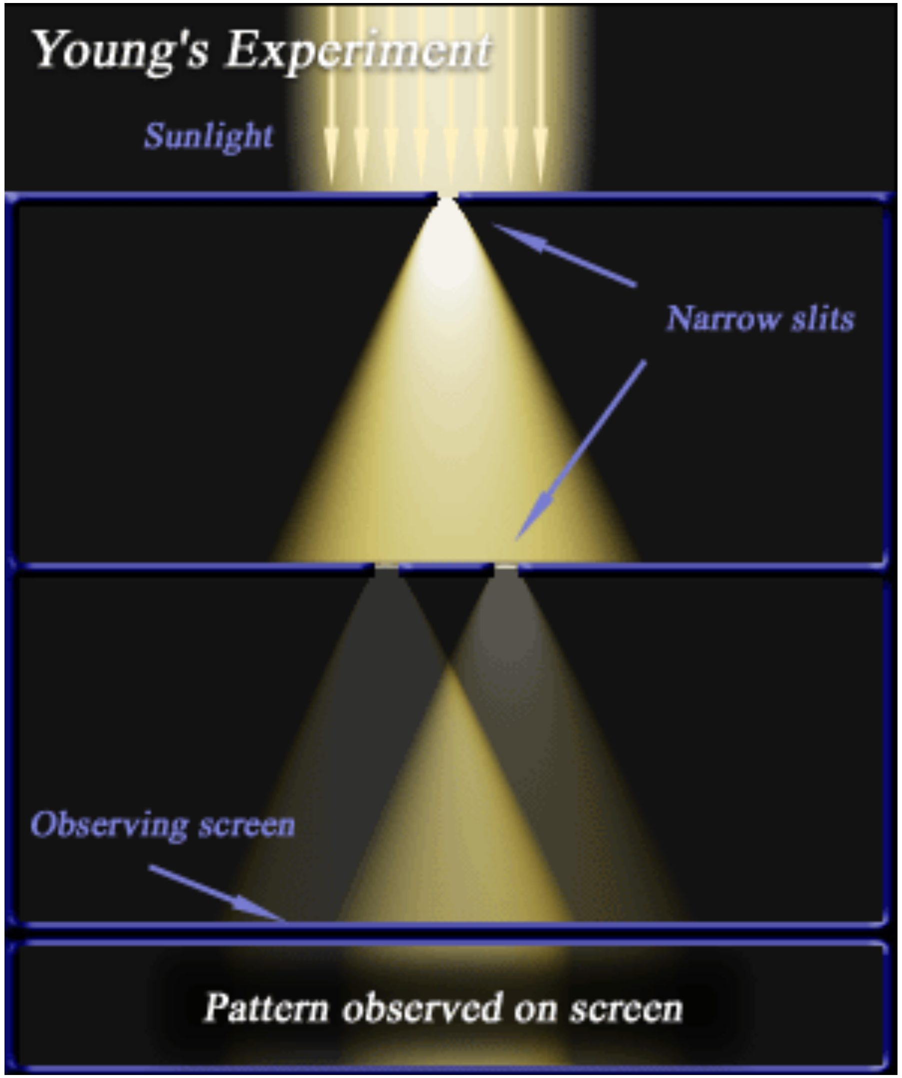
- Quantum weirdness
  – the nature of light and particles
- Atoms
  – what's going on in the nucleus, and how atoms stay together (or not)
- The electrons
  – how orbiting electrons determine the nature of the elements

Richard Feynman once said that the one question we cannot ask when we study quantum mechanics is

"But how can it be like that?"

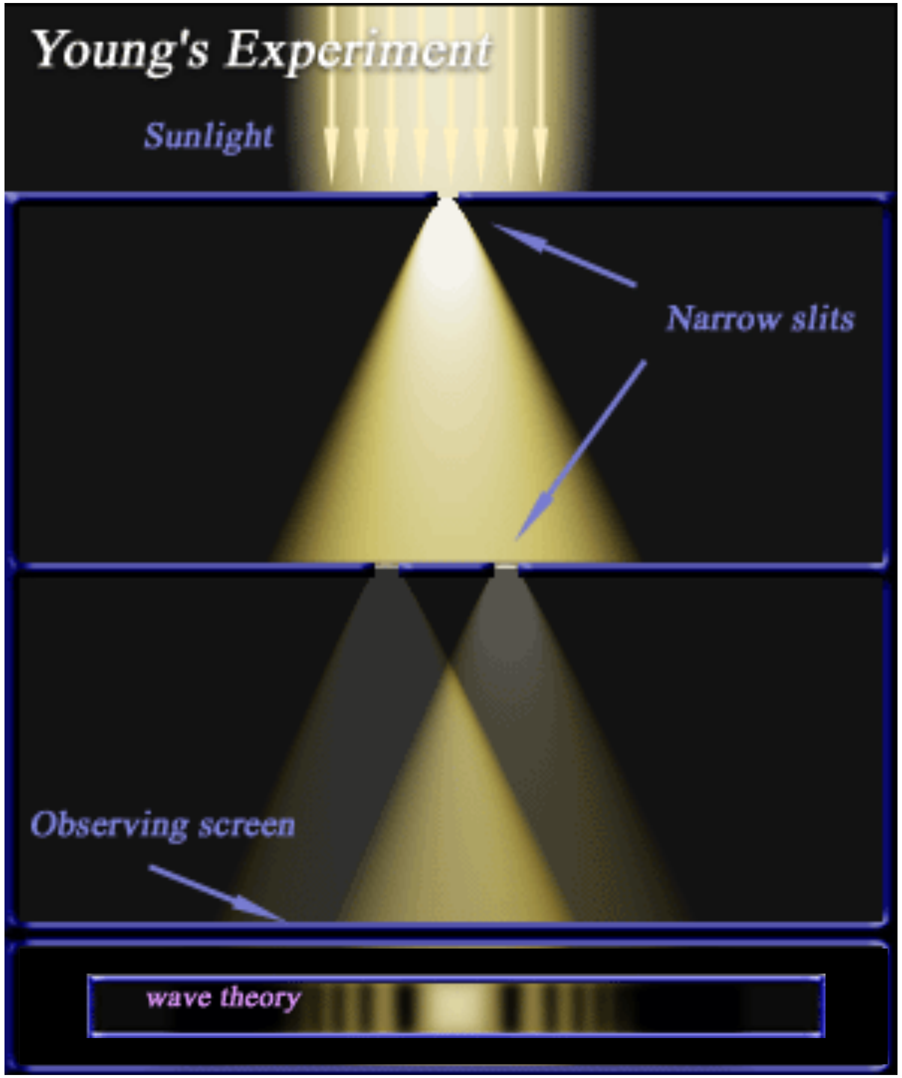
As will quickly become apparent, objects at the scale of atoms *do not behave* like anything we have any experience with. There are no analogies we can draw which will illuminate why things behave the way they do. In essence, no-one understands QM!

Wave-particle duality



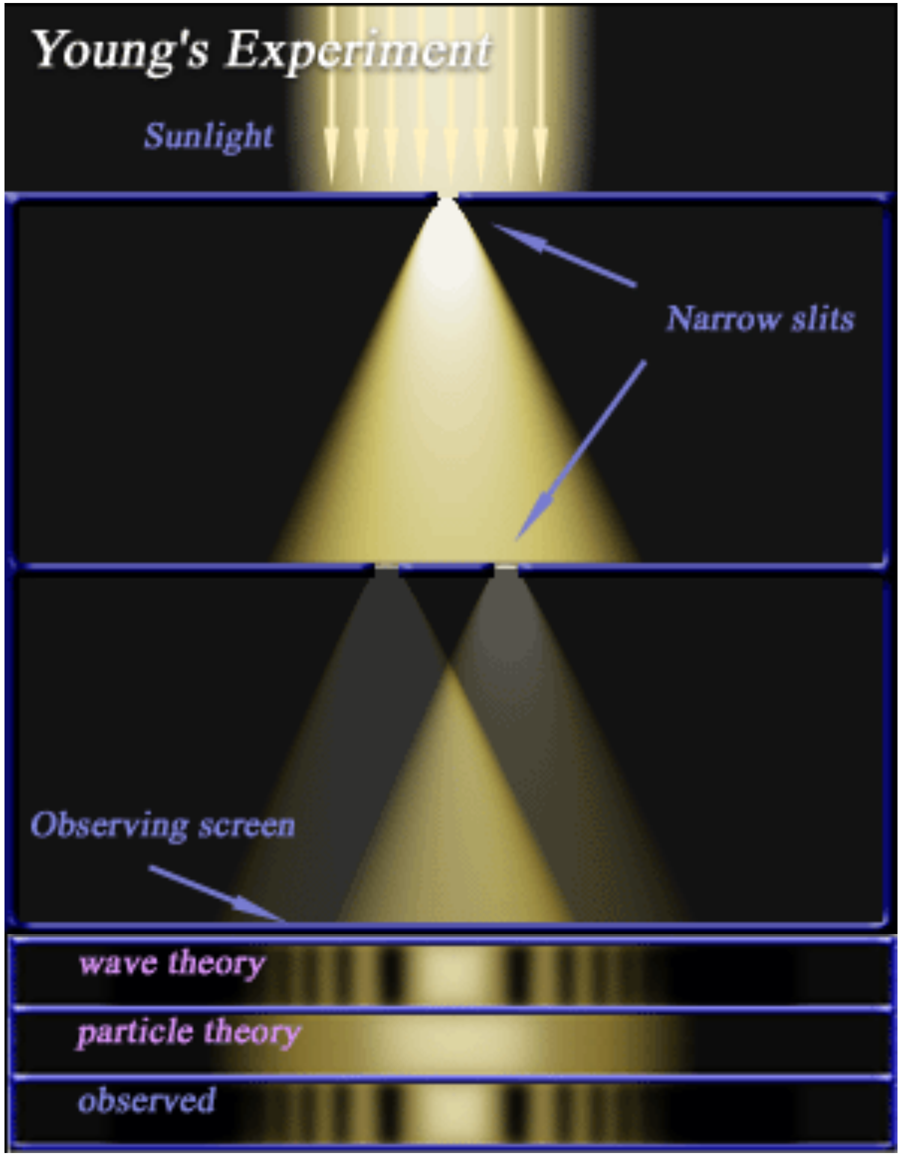
In around 1805, Thomas Young performed an experiment in order to resolve whether light was a wave or a particle.

He shone light through two narrow slits and observed the resulting pattern.



If light were made of *particles*, we would expect the pattern to just be the overlap of the individual patterns, brightest in the centre.

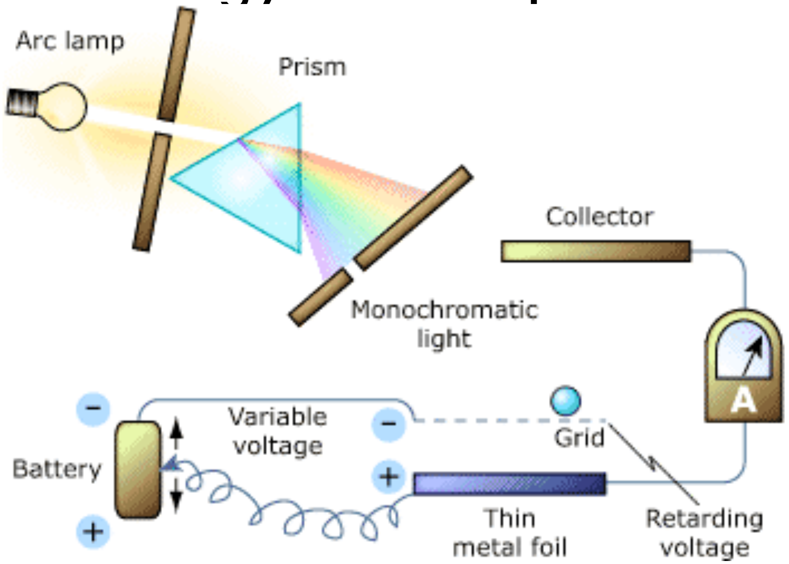
If light were made of *waves*, we would expect fringes, with alternating dark and light patches where crests and troughs reinforced or cancelled each other.



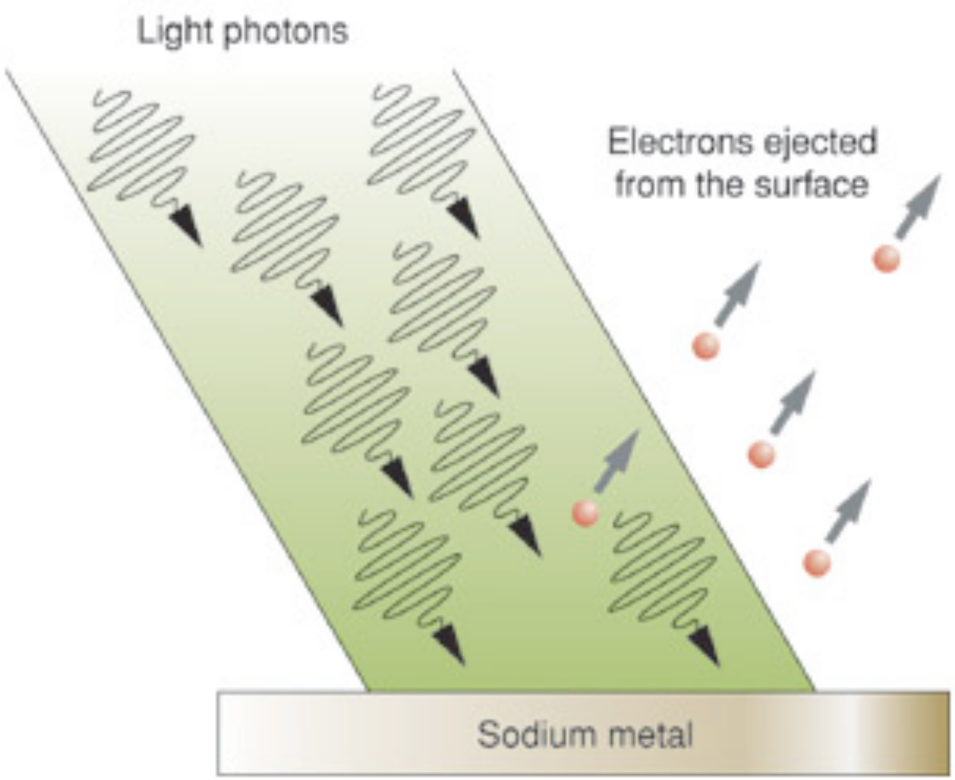
Young found that an interference pattern was produced. Hence light must be a wave.

But, Einstein showed that light must *also* be a particle! He was working to explain the *photo-electric effect*. If you shine ultraviolet light onto a negatively charged metal, it loses its charge as electrons are ejected from the surface. So far, so good: the light is providing the energy to eject electrons from the metal.

However, increasing the intensity of the light had no effect on the energy of the ejected electrons: more electrons were ejected, but at the same speed. Moreover, light with a low frequency (red light) ejected *no* photons at all, no matter what the intensity of the light.



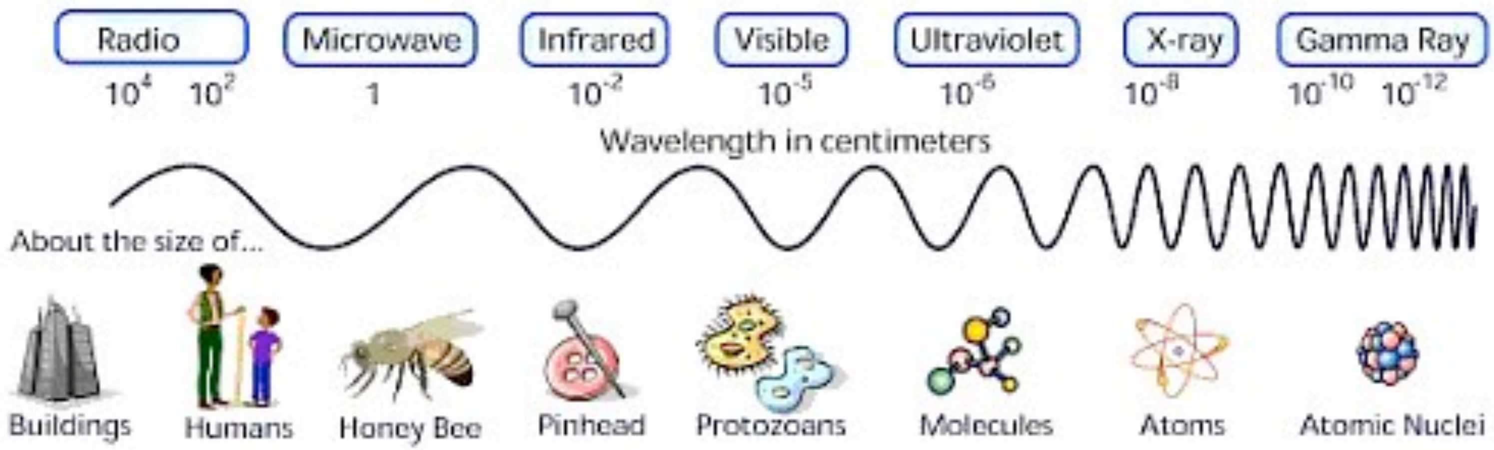
In one of his three famous papers in 1905, Einstein provided the solution to this problem: If light was *quantised*, so that it comes in little bundles, then in order to eject an electron, a bundle had to have enough energy to overcome the energy binding the electron to the surface. Photons in red light did not have enough energy, so ejected no electrons; as the frequency of the light increased, each photon had more energy so the electrons were ejected faster.



He had showed that light can *also* act as a particle.

Thus light is a wave, so we can define its *wavelength* and *frequency*, which are related by

$$\lambda f = c$$



But **at the same time**, light is also a particle, called a *photon*, which has an energy.

The energy of the photon is related to the frequency of the wave by

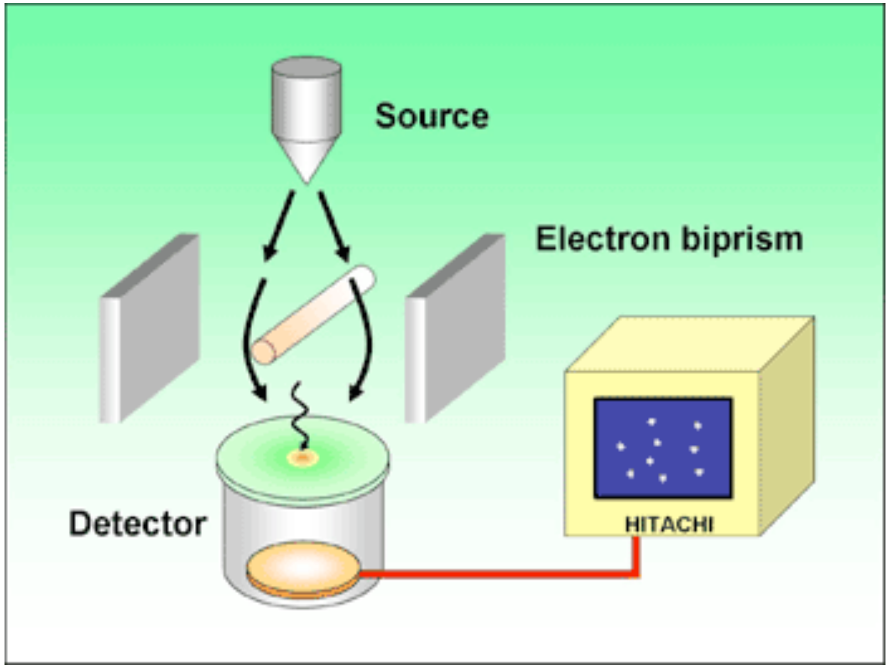
$$E = hf$$

It turns out that *everything* behaves like both a wave and a particle at the quantum level: this is known as *wave-particle duality*.

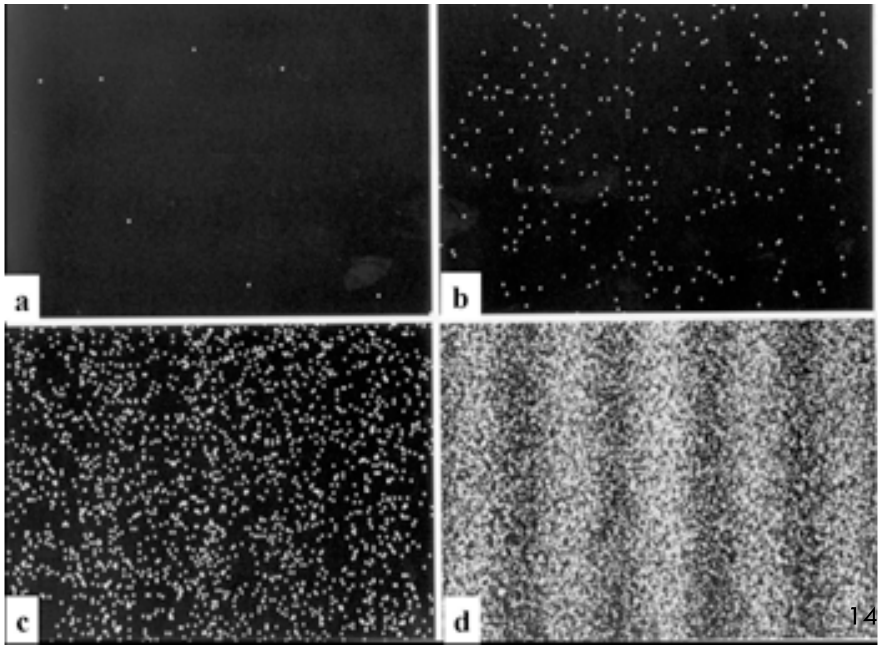
JJ Thomson showed that electrons are particles: the negative charge in the atom comes in discrete bits.

His son, GP Thomson, showed that if you shine a beam of electrons through a double slit, you get an interference pattern → electrons are also waves!

This is despite the fact that you detect the electrons as particles when they hit the phosphorescent screen.



What's more, this modern version of the experiment shows that electrons give an interference pattern *even when there's only one electron in the apparatus at once*.



Electrons are counted one by one; as the total number grows, interference fringes are clearly seen.

Clearly the idea that a single electron can pass through both slits at once is a problem for the classical view!

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Heisenberg went further, and said it is impossible for us to determine which slit the electron went through. There is a fundamental limitation to what we can measure, even in principle:

uncertainty in position x uncertainty in momentum ~ h

This is known as *Heisenberg's Uncertainty Principle*.

h is Planck's constant, and is a very small number*, so for every day situations the uncertainty is negligible.

But for tiny masses like electrons, it means we can *never* know both exactly how fast an electron is moving and where it is, no matter how clever our experiment.

All we can do is say where an electron *probably* is.

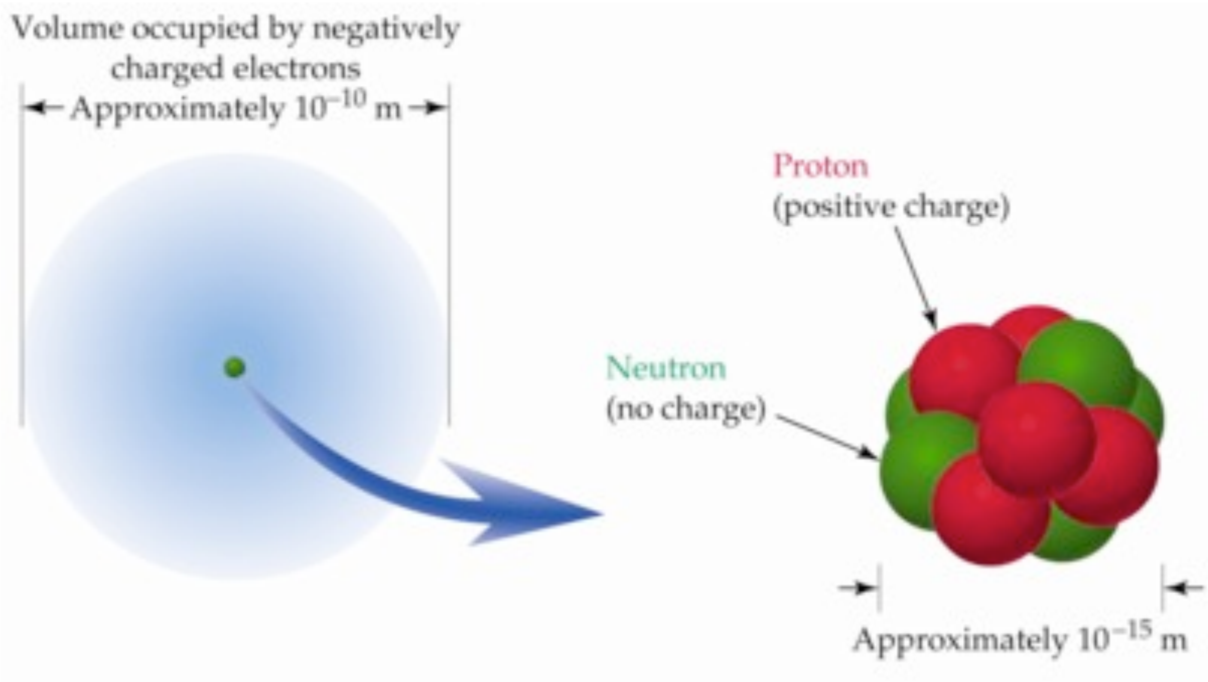
* $h = 6.626 \times 10^{-34}$ m^2 kg/s, in case you care.

Anyone who is not shocked by quantum theory has not understood it.

– Niels Bohr

Nuclei

Atoms have nearly all the mass and all of the positive charge concentrated into a tiny central region called the nucleus. Most of the atom is empty space!



The nucleus contains *protons* and *neutrons*, both of which are about 2000 times heavier than an electron. The proton has a charge equal and opposite to that of the electron; the neutron has no charge.

particle	mass	charge
proton	1836.15 m_e = 1.00727647 u	+1
neutron	1838.68 m_e = 1.0086647 u	0
electron	1 m_e = 0.000549 u	–1

In addition, each particle has an *anti-particle*: a twin particle, identical but with the opposite electric charge.

electron ⇔ positron

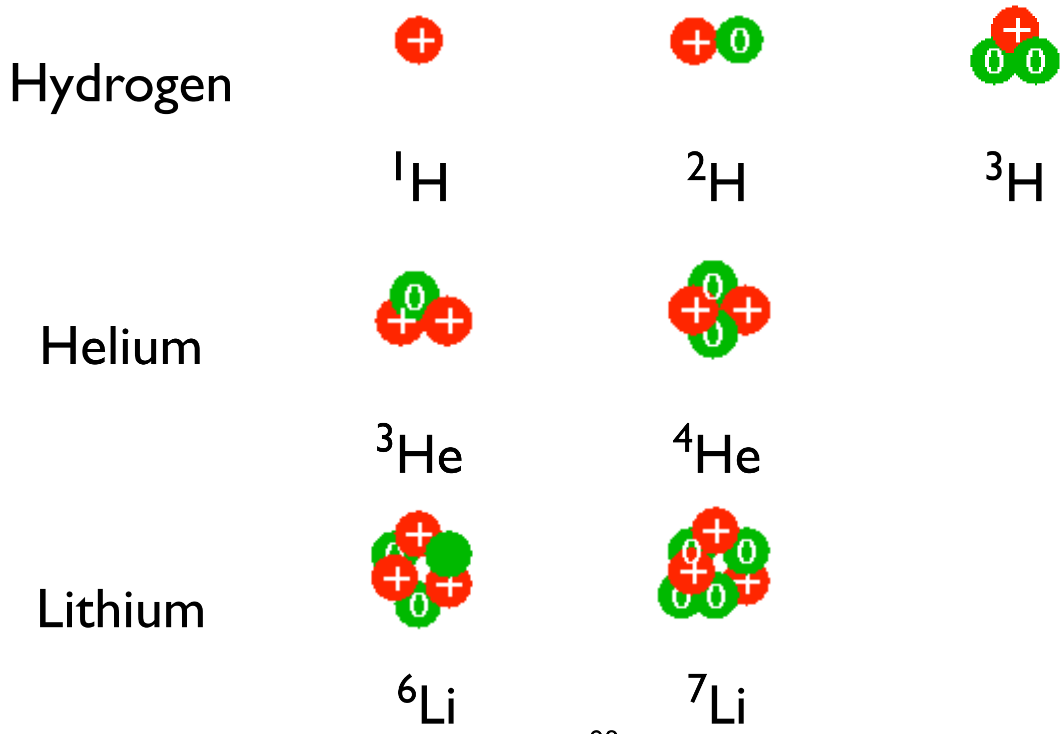
proton ⇔ antiproton

neutron ⇔ antineutron

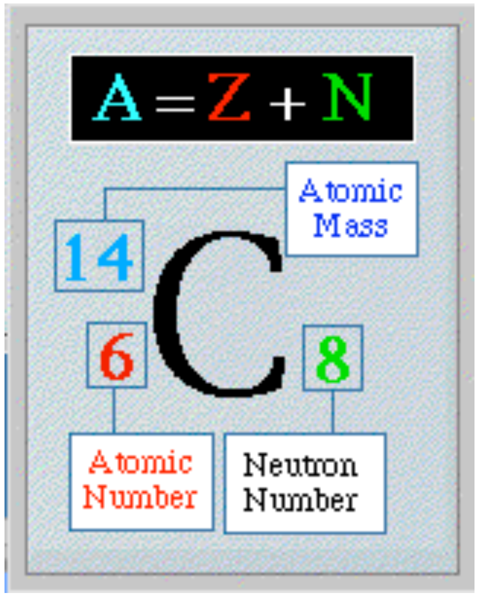
When matter and antimatter meet, they annihilate, completely. When particles are produced, they are produced in matter-antimatter pairs, like $e^- - e^+$.

The protons in the nucleus repel each other because of the electromagnetic force, so how does the nucleus stay together? The proton and neutron are held together in the nucleus by the *strong nuclear force*, which is much stronger than the electromagnetic force. It is, however, extremely short-range, being felt only over distances the size of the nucleus (10^{-15} m).

The number of protons (or, equivalently, the number of electrons) determines the chemical nature of the atom: which element it is. The same element can have different numbers of neutrons but the same number of protons: these are chemically equivalent, and are called *isotopes*.

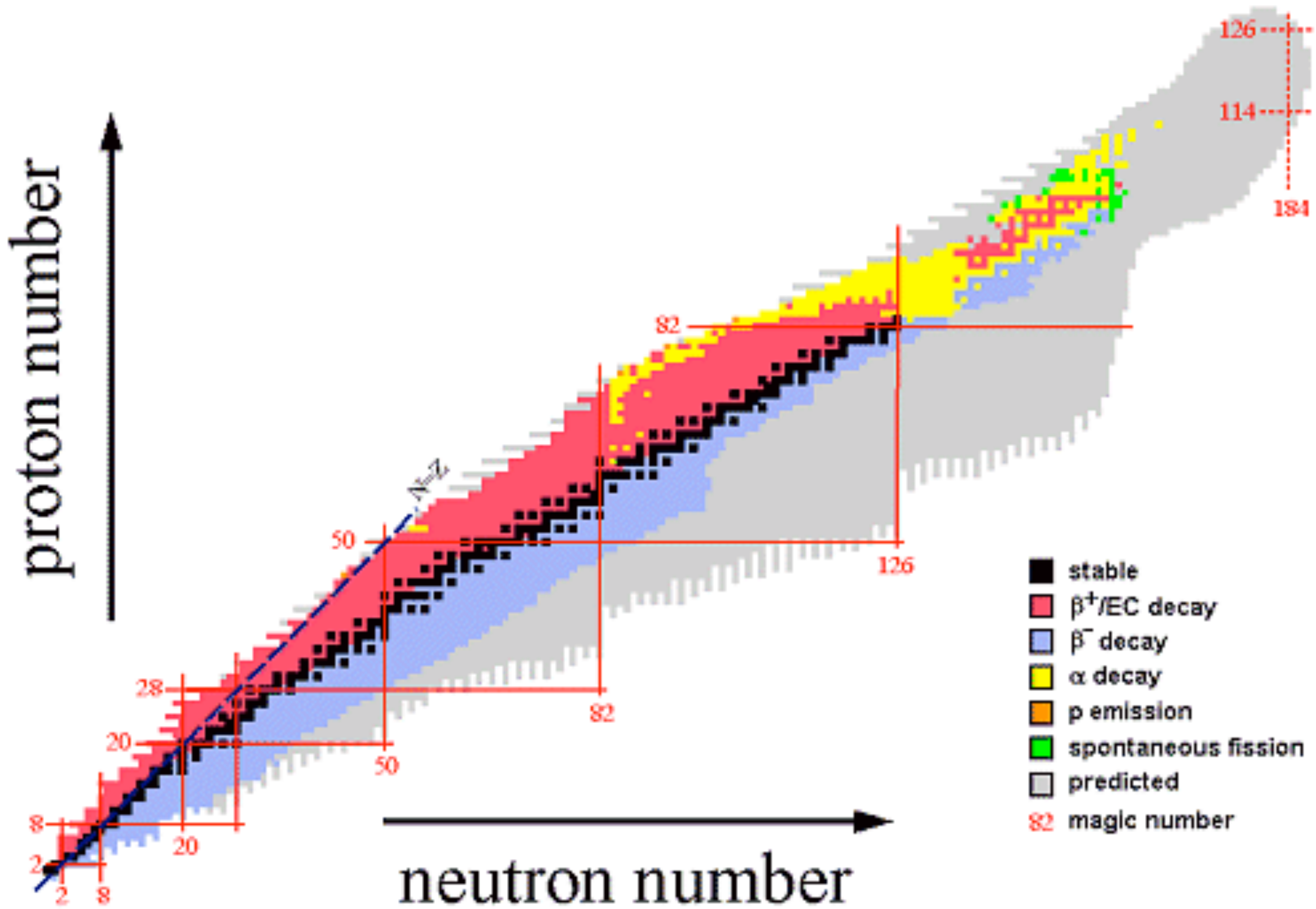


The number of protons in the nucleus, Z, is called the *atomic number*. The number of neutrons in the nucleus is denoted by N. Since the proton and the neutron have almost the same weight, the atomic mass of the nucleus, A, is equal to $Z + N$.

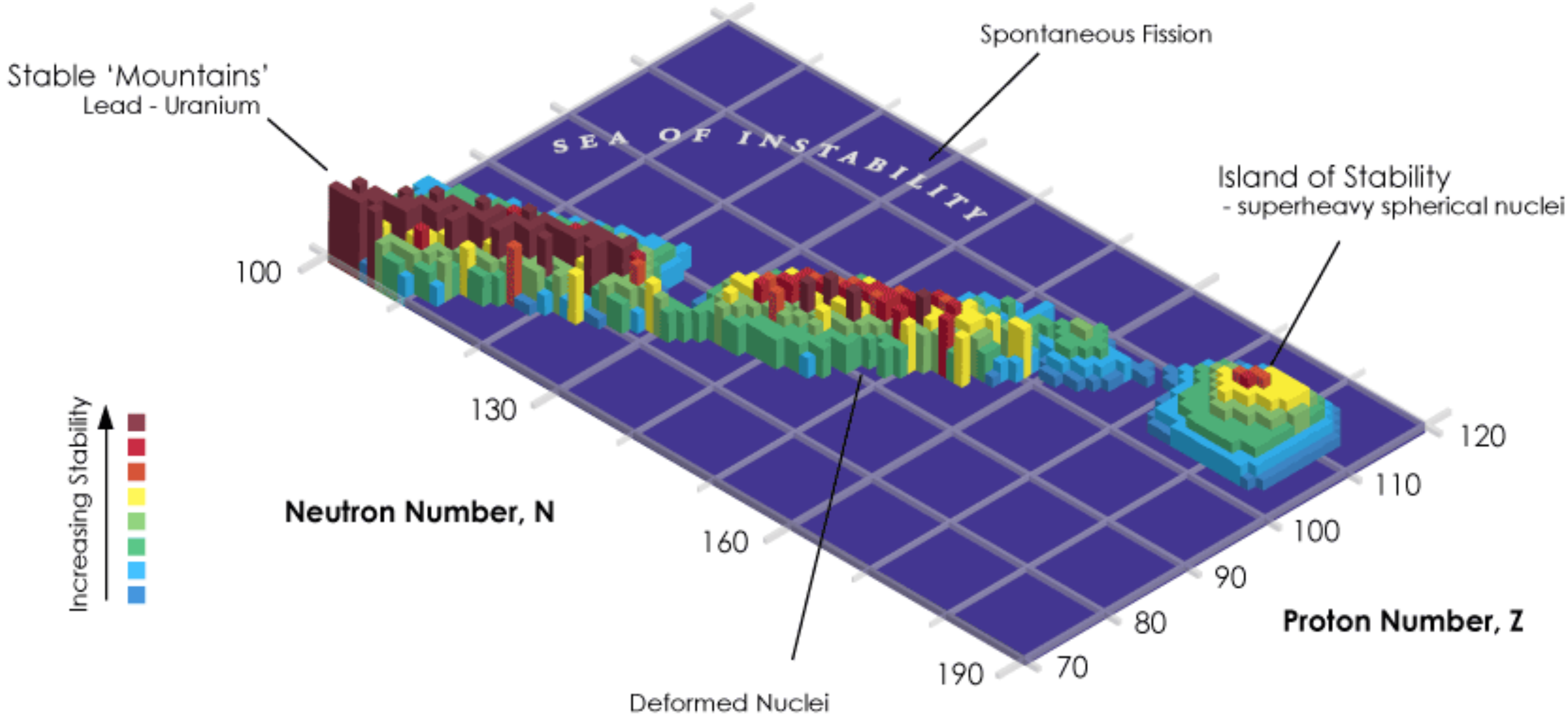


Not all combinations of protons and neutrons are allowed. For elements with small numbers of protons, the number of neutrons is roughly equal to the number of protons. Hence the most common isotope of oxygen is $^{16}_{8}O$, with eight protons and eight neutrons. 0.2% of oxygen atoms, however, are $^{18}_{8}O$, with eight protons and ten neutrons.

For elements with higher atomic number, there tend to be more and more neutrons per proton; so the most common isotope of uranium (atomic number 92) is $^{238}_{92}U$, with has 92 protons and 146 neutrons.



The nuclide chart shows all known isotopes. Black squares are stable isotopes; they fall further and further below the Z=N line as the atomic number increases.


Stable 'Mountains'
Lead - Uranium
Spontaneous Fission
SEA OF INSTABILITY
Island of Stability
- superheavy spherical nuclei
100
130
160
190
Neutron Number, N
70
80
90
100
110
120
Proton Number, Z
Increasing Stability
Deformed Nuclei

Large nuclei are unstable because the nuclear force is so short-range. In a nucleus, all the protons are repelling each other (the EM force), while all the protons and neutrons are attracting each other (the strong force). For light nuclei the strong force wins, but for heavy nuclei, the nucleus becomes so big that each nucleon only feels the attraction of nearby particles. Adding neutrons helps, because that adds strong force without EM force; but with enough protons, no number of neutrons will hold the nucleus together.

We'll see what happens next in a bit.

Neutrons are slightly more massive than protons. Left to themselves (outside atoms) they are unstable, and decay to a proton and electron

$$n \rightarrow p + e^{-} + \nu$$

The last particle is a *neutrino*, and was predicted theoretically long before it was detected, because experiments showed that the energy and momentum didn't balance without it. It is a very curious particle, which has no charge, no mass (**), and doesn't react to the strong nuclear force. It turns out to be almost undetectable, but is enormously important in determining the fate of stars. We'll find out about that in later lectures.

Binding energy

Since nuclei stick together, they must be more stable than their separate constituents. Physicists describe this by the concept of *binding energy*.

For instance, a proton and a neutron can combine to form *heavy hydrogen*, also known as *deuterium*:

$$p + n \rightarrow \text{deuterium}$$

$$^{1}_{1}\text{H} + n \rightarrow {}^{2}_{1}\text{H}$$

If you weigh these, you find that deuterium weighs less than a proton plus a neutron!

$$^{1}H + n \rightarrow {}^{2}H$$

1.00728 u   1.00866 u

sum: 2.01594 u

mass: 2.01355 u

The difference is 0.00239 u.

Where has this mass gone? The energy equivalent to this mass (Einstein's famous $E = mc^2$) is the *binding energy* of the nucleus.

It is the amount of energy required to break the nucleus apart.

Alternatively, it is the amount of energy released if we could put a proton and neutron together to form a deuterium nucleus.

This is where stars get their energy from.

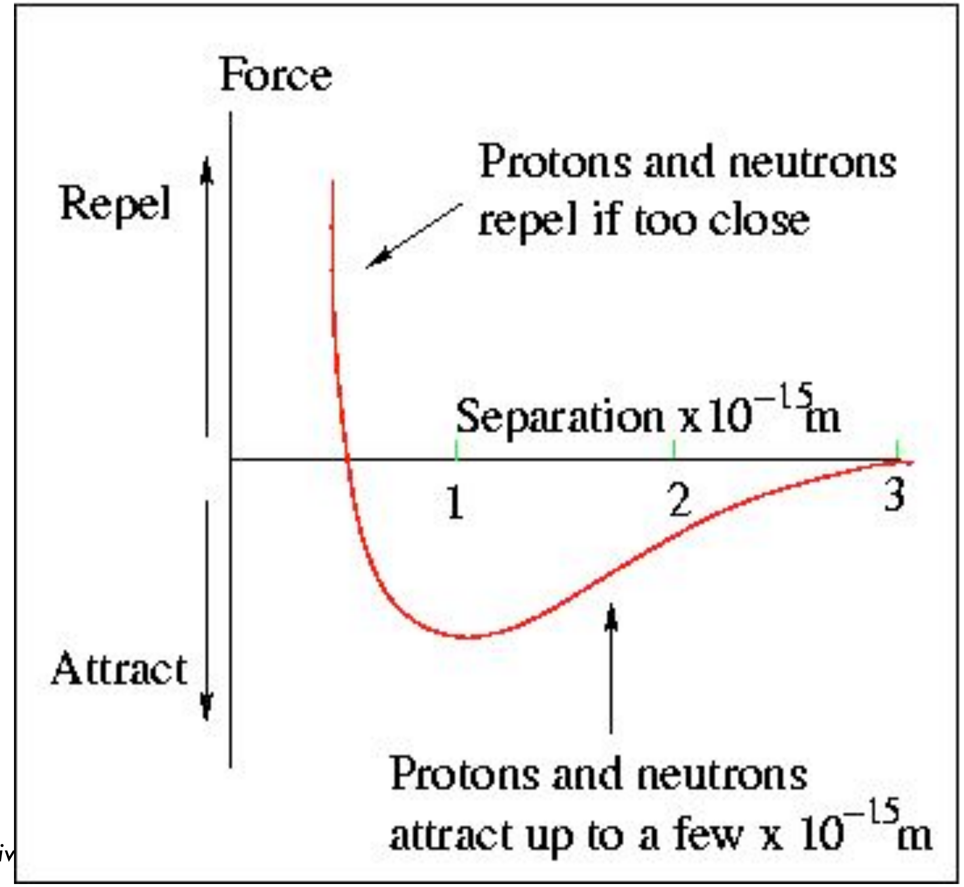
Aside: Why don't all the protons and neutrons around us rush together and release this energy in one stupendous explosion?

1 u = 1.66×10^{-27} kg, and $c = 3 \times 10^{8}$ m/s,

so $\qquad E = 3.6 \times 10^{-13}$ J.

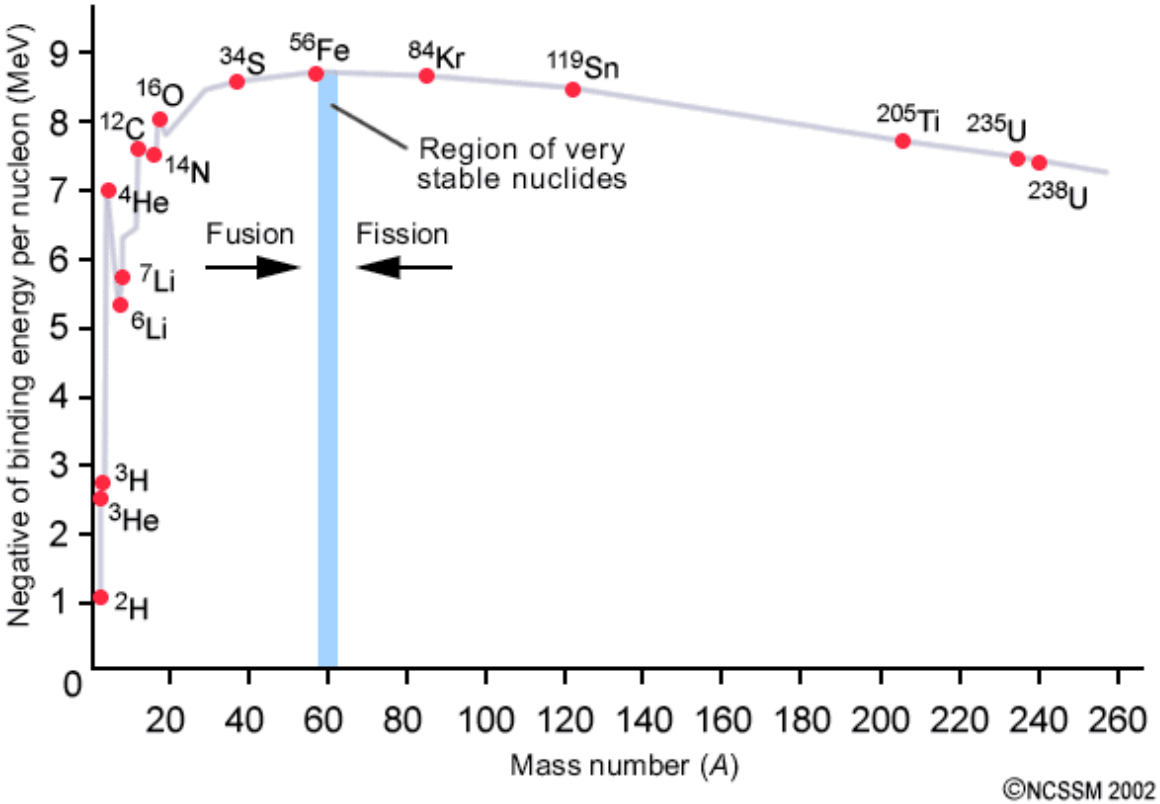
Which doesn't sound very much, until you consider this is the energy released per atom, and there are 3×10^{26} atoms of deuterium per kg. So forming one kg of deuterium would release 10^{14} J of energy, or the equivalent of 26 kilotons of TNT.

Aside: Why don't all the protons and neutrons around us rush together and release this energy in one stupendous explosion?



Because the nuclear force is so short-range that they have to be very close together for it to happen.

Different nuclei have different binding energies. We can measure the binding energy for every atom:



Iron has the most binding energy per nucleon of any element – about 8.8 MeV per nucleon, and after that it falls.

This means that there are two ways of releasing energy from a nucleus.

For elements lighter than iron, if they *combine* they release energy

→ **fusion**

For elements heavier than iron, however, adding mass costs energy. But if you can *split* a very heavy nucleus into two lighter ones, energy is released

→ **fission**

Aside: Radioactivity

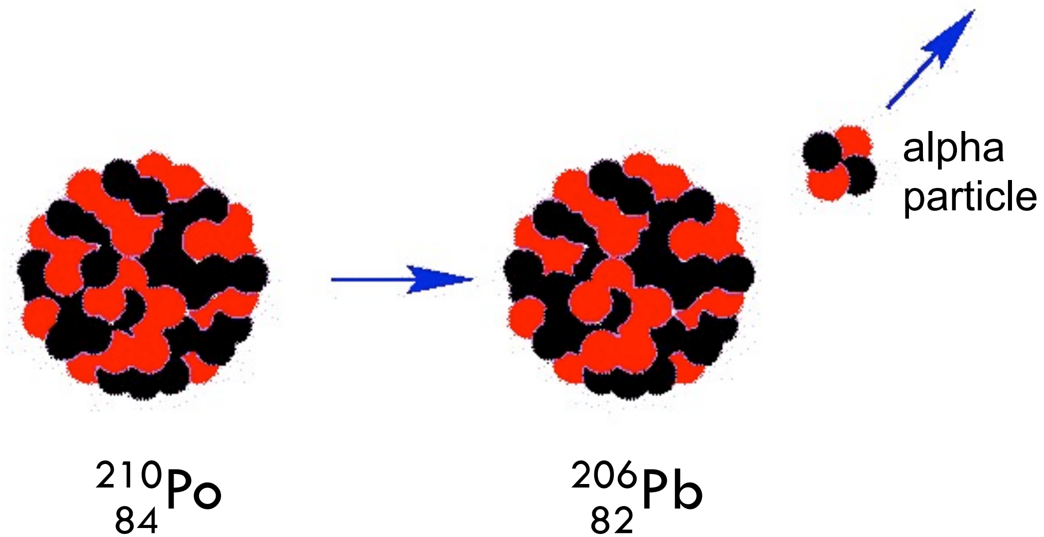
Radioactivity is where an atom with Z protons and N neutrons can turn itself into one with different Z and N.

There are several different types of radioactivity.

- A nucleus can emit an *alpha particle*, which is just a helium nucleus (two protons and two neutrons)

  e.g. $\quad {}^{210}_{84}\mathrm{Po} \rightarrow {}^{206}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$

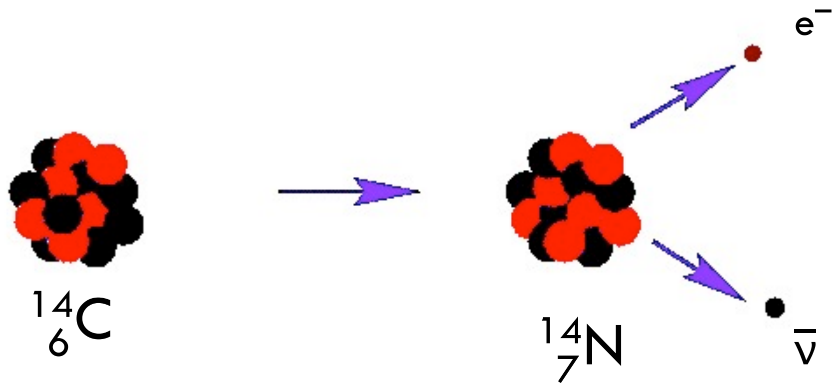
  The lead nucleus is more stable (= has more binding energy) so the He atom carries the extra energy off as kinetic energy.



$\boxed{\textit{alpha decay}}$

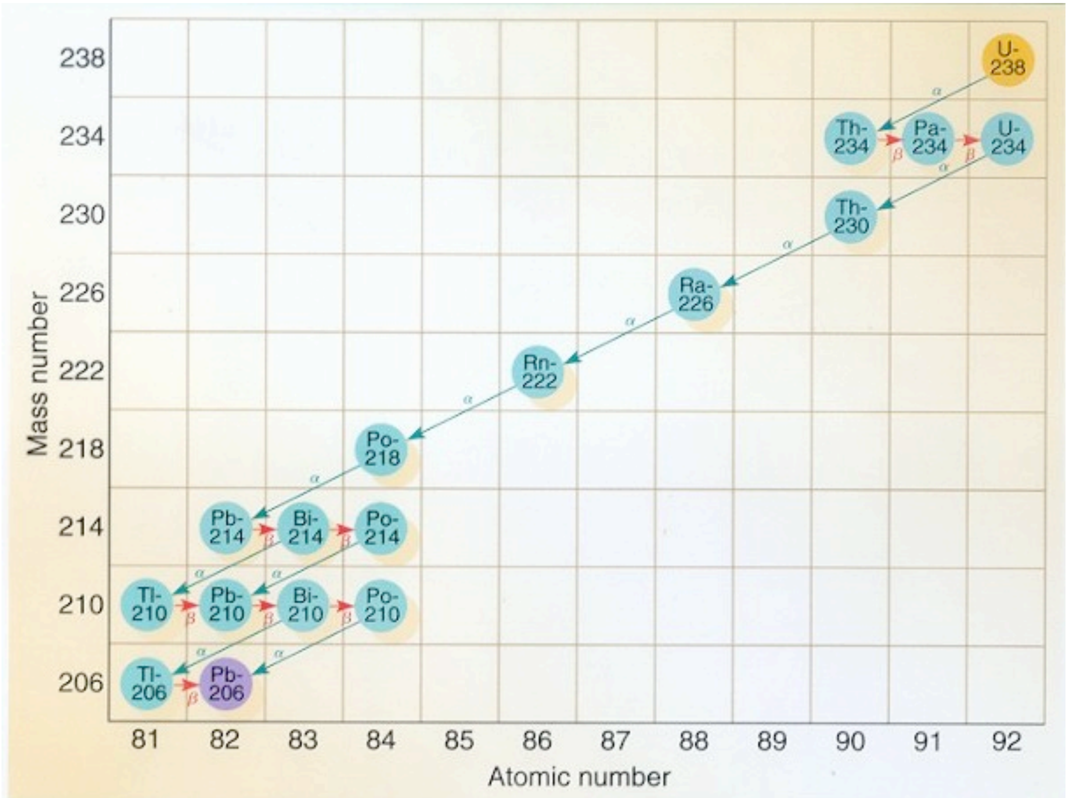
- A neutron can transform into a proton, or vice versa: the number of nucleons is unchanged but the number of protons goes up or down by one.

e.g. ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^{-}$ (+ neutrino)



beta decay

Naturally occurring radioactive materials often decay through complex chains of these processes.

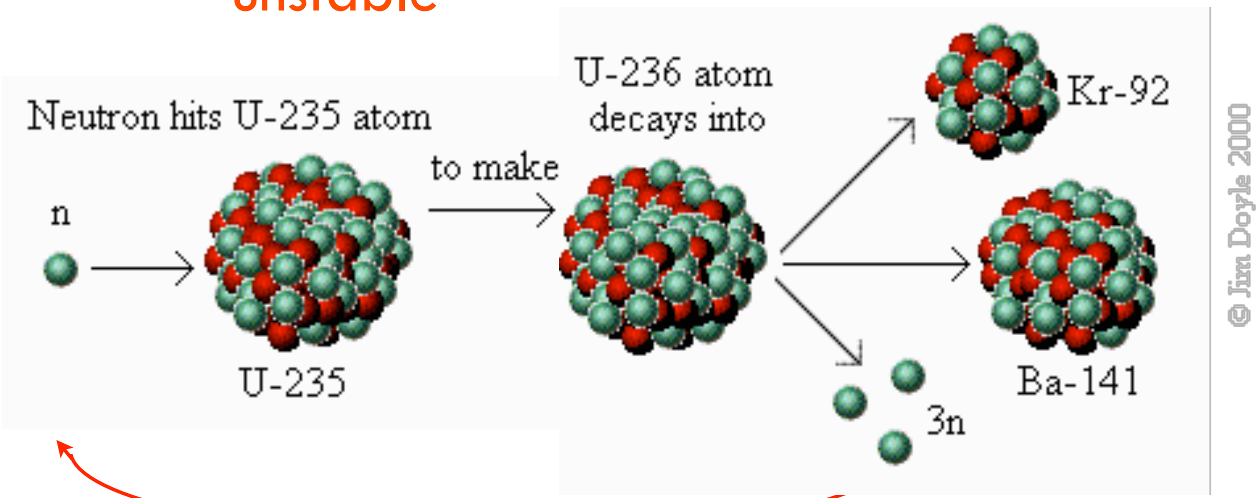


The diagonal lines are alpha decays (changing A and Z), while the horizontal lines are beta decays (keeping A constant and increasing Z by one).

- If an unstable nucleus forms, e.g. by neutron capture, then it can sometimes split into two separate pieces

e.g. $$^{235}_{92}U + n \rightarrow {}^{236}_{92}U \rightarrow {}^{92}_{38}Kr + {}^{141}_{56}Ba + 3n$$

unstable

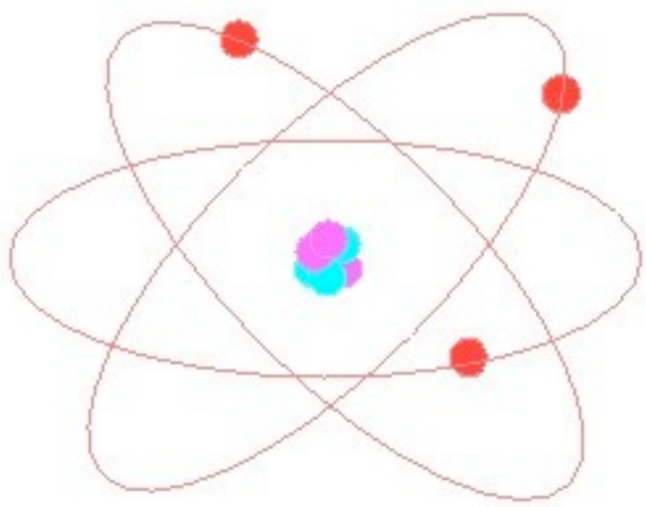


These extra neutrons can then react with other uranium nuclei, causing a chain reaction.

Electrons

What are the electrons doing during all of this?

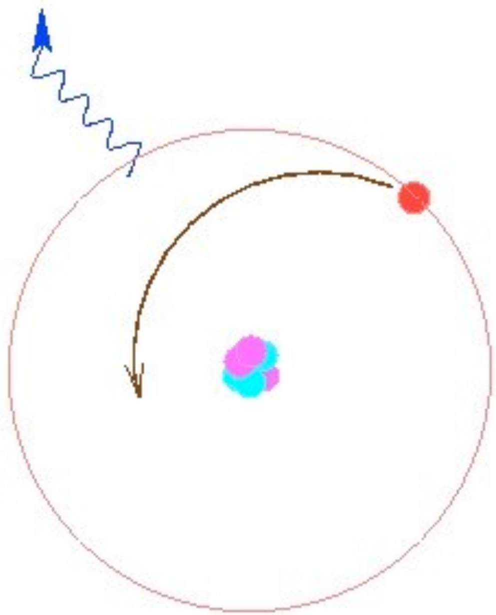
Ernest Rutherford (discoverer of the nucleus) pictured the electrons orbiting the nucleus, rather like a mini-solar system.



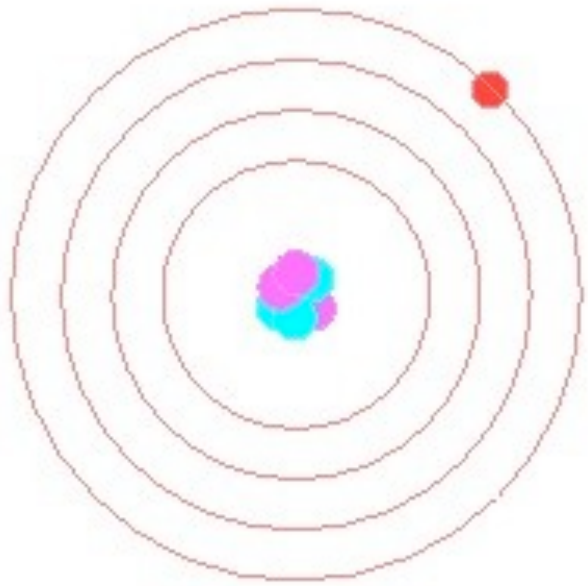
But there's a problem with this model. When an electron is accelerated, it radiates energy.

This means that, very quickly, every electron should lose its energy and spiral into the nucleus.

Atoms should be unstable.

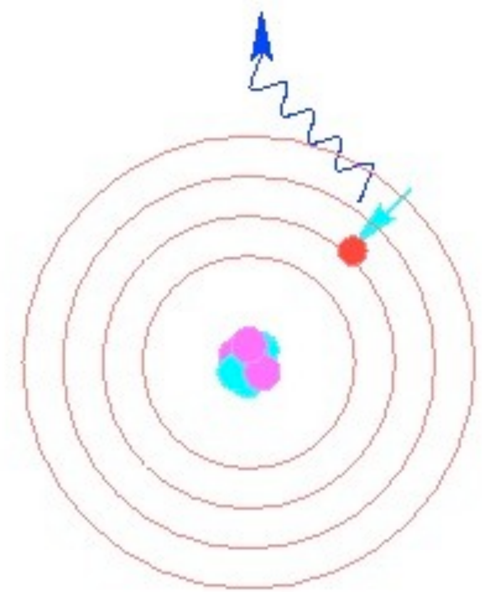


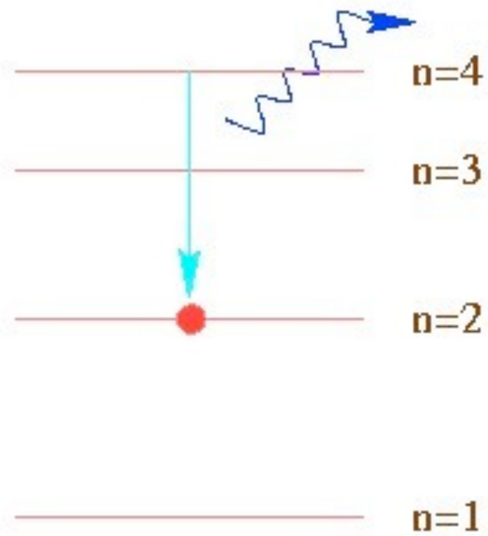
Niels Bohr proposed, ad hoc, that electrons had only certain allowed orbits: the orbits are *quantised*. Each orbit corresponds to a certain energy.



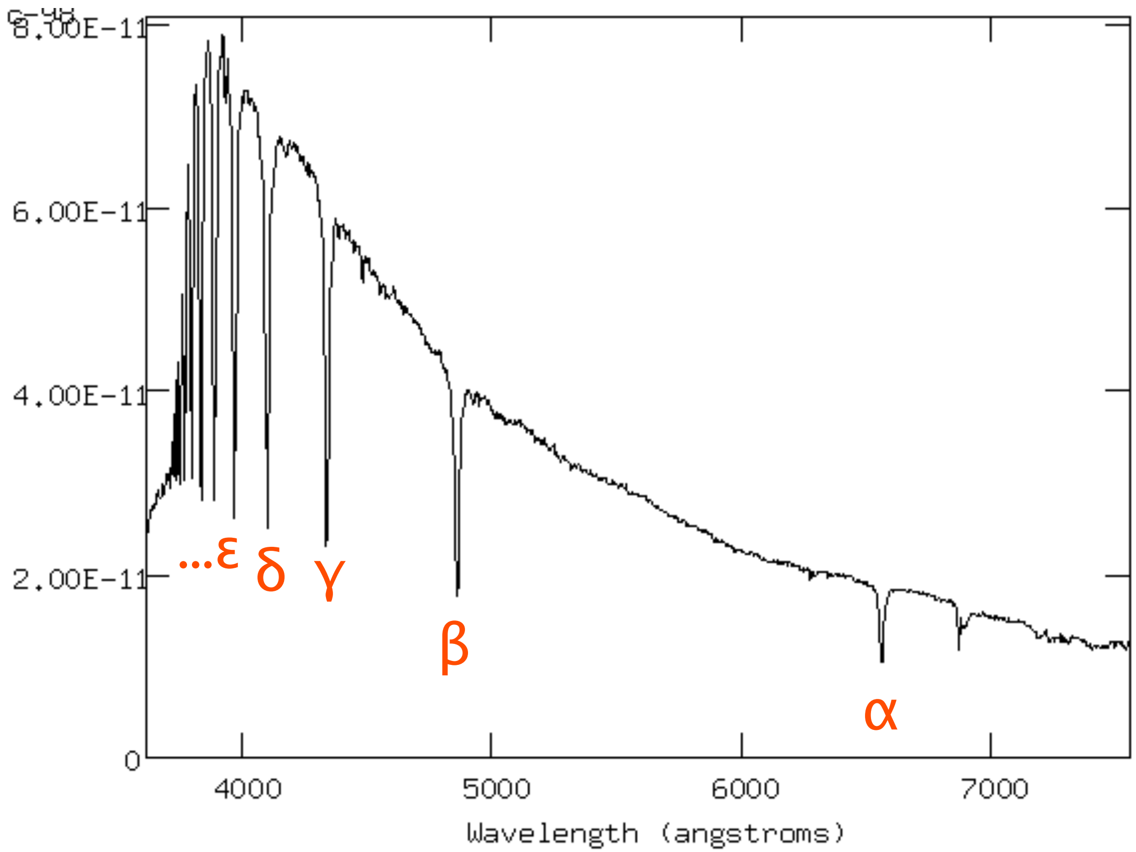
In order to move from one energy level to another, an electron has to absorb or emit exactly the right energy, corresponding to the *difference* in energy between the levels.

This energy appears as a *photon*, the particle of energy whose existence Einstein had deduced.





Bohr's theory immediately explained the observed spectra of the elements: the lines correspond to the discrete jumps between energy levels. In particular, it explained the spectrum of hydrogen.



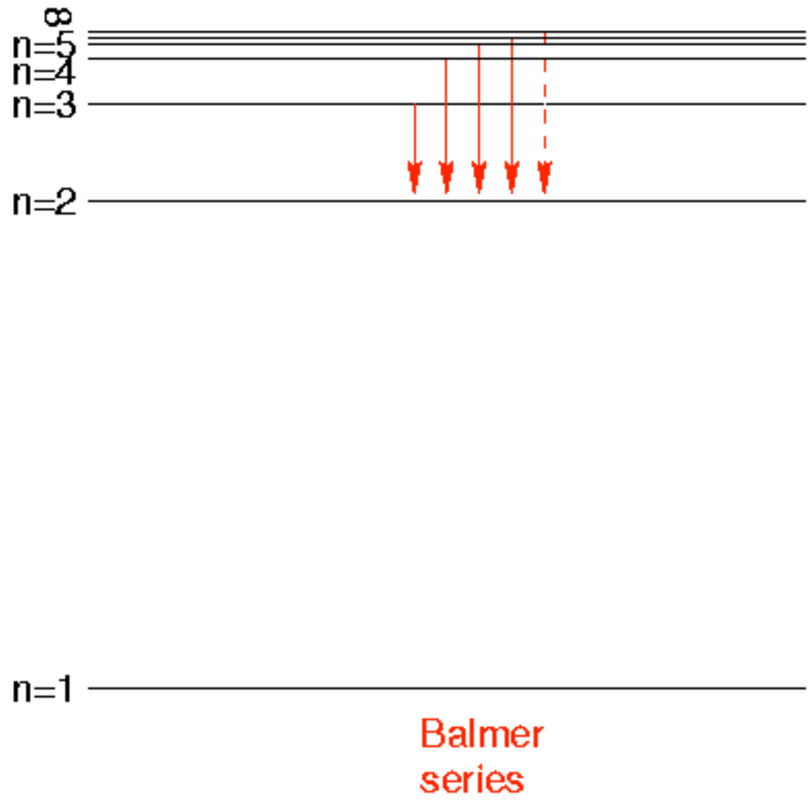
Spectrum of an A star, showing the Balmer lines.

Johann Balmer had devised a formula to fit the observed wavelengths of the lines in the hydrogen spectrum:

$$\lambda = \frac{364.5n^2}{n^2 - 4}$$

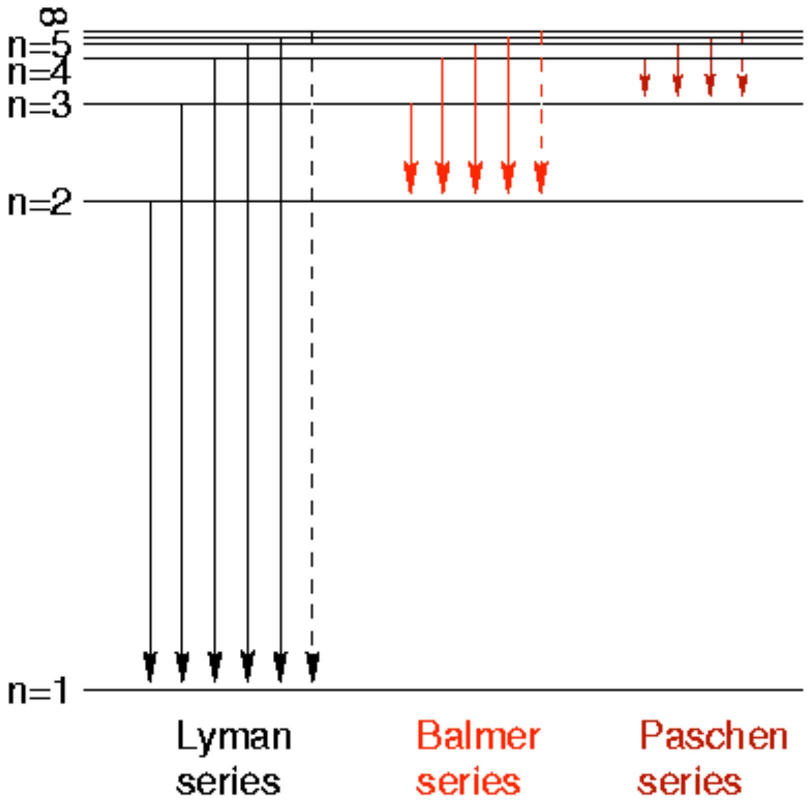
The formula worked with amazing precision, but was just a curiosity, until Bohr saw it.

Bohr’s model explained this formula as a consequence of his allowed orbits: the Balmer series consists of transitions to the n=2 level.

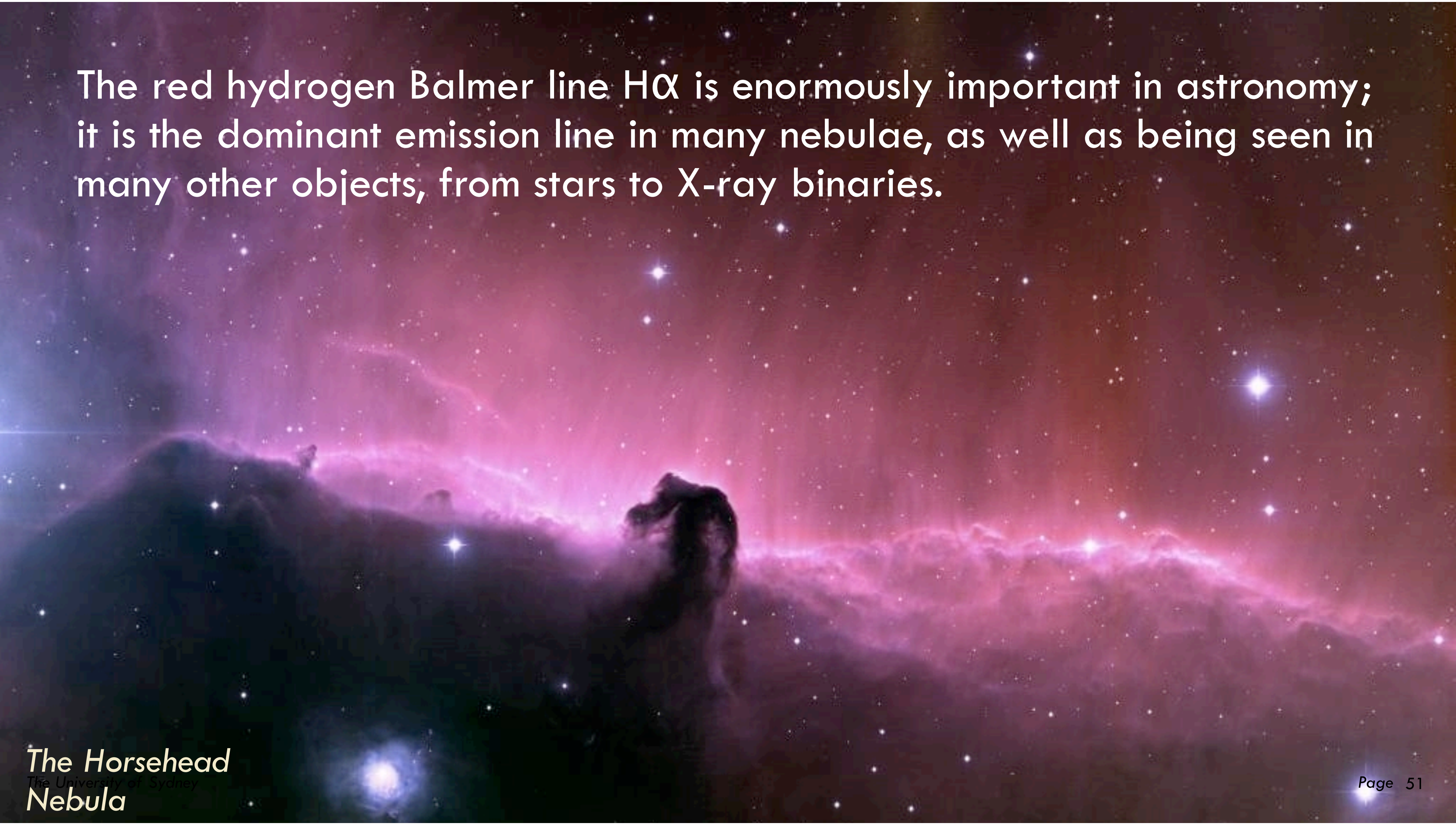


Bohr's model explained this formula as a consequence of his allowed orbits: the Balmer series consists of transitions to the n=2 level.

What's more, he predicted new sets of lines at different wavelengths, corresponding to transitions to the n=1 and n=3 levels.

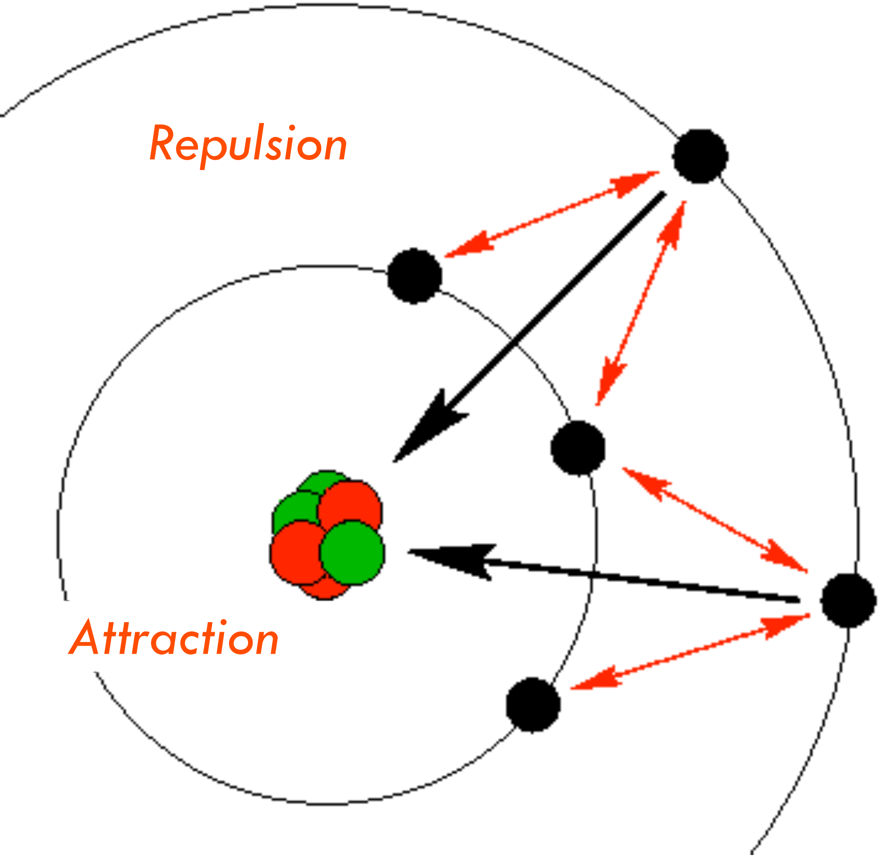


The red hydrogen Balmer line Hα is enormously important in astronomy; it is the dominant emission line in many nebulae, as well as being seen in many other objects, from stars to X-ray binaries.

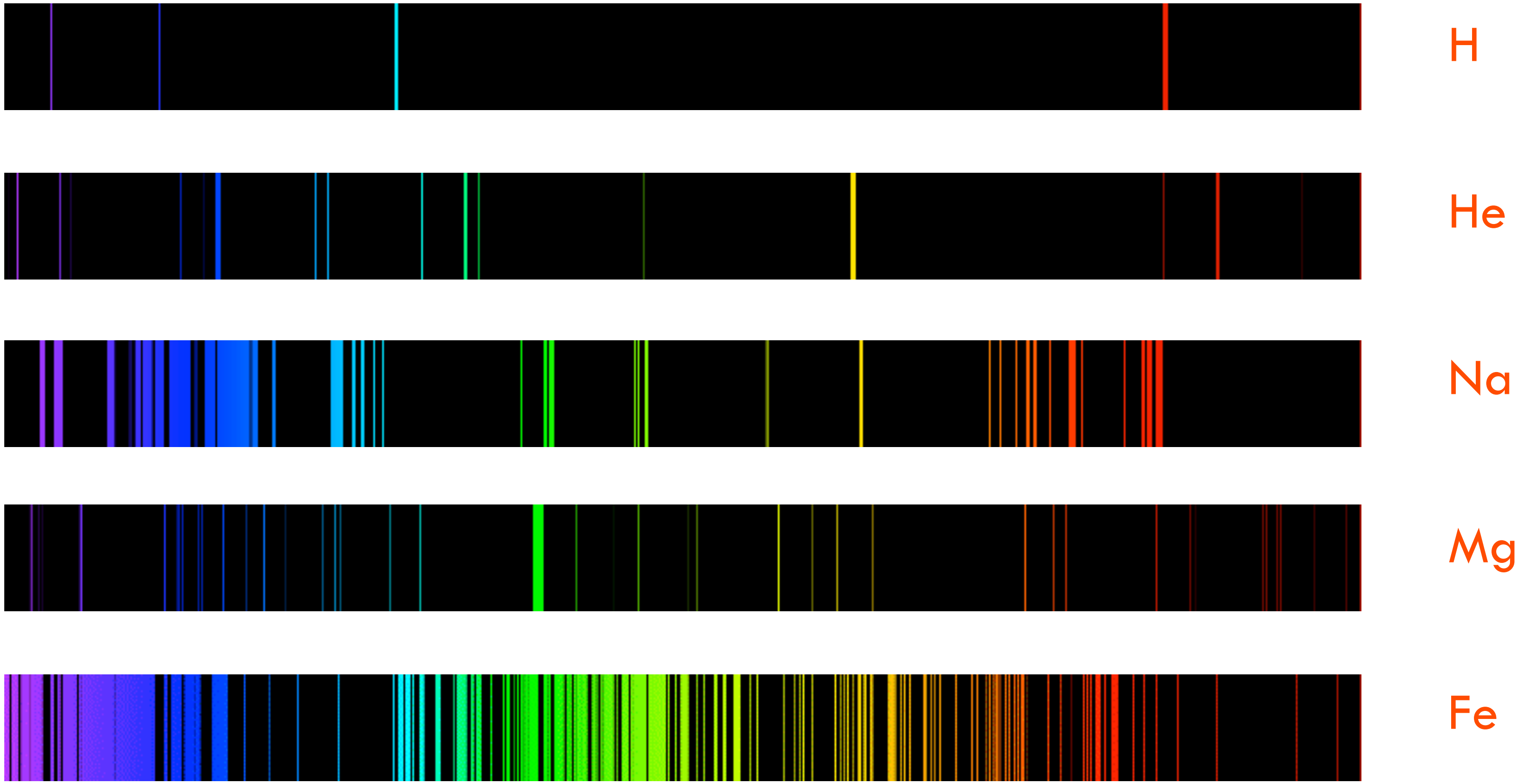


The Horsehead Nebula

Other elements have much more complicated spectra, because with multiple electrons the inner electrons "shield" some of the charge from the outer electrons, plus the electrons repel each other, so the energy spectrum isn't so neat.



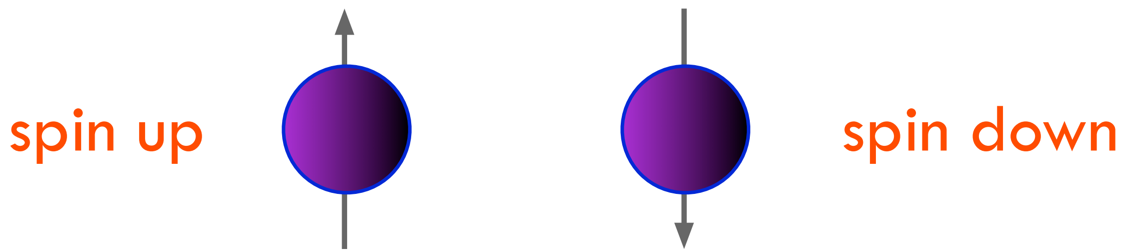
But it can be calculated: each set of energy levels is a unique fingerprint of the atom.



Why don't all the electrons just sit in the lowest energy state? Then all the elements would behave just like hydrogen!

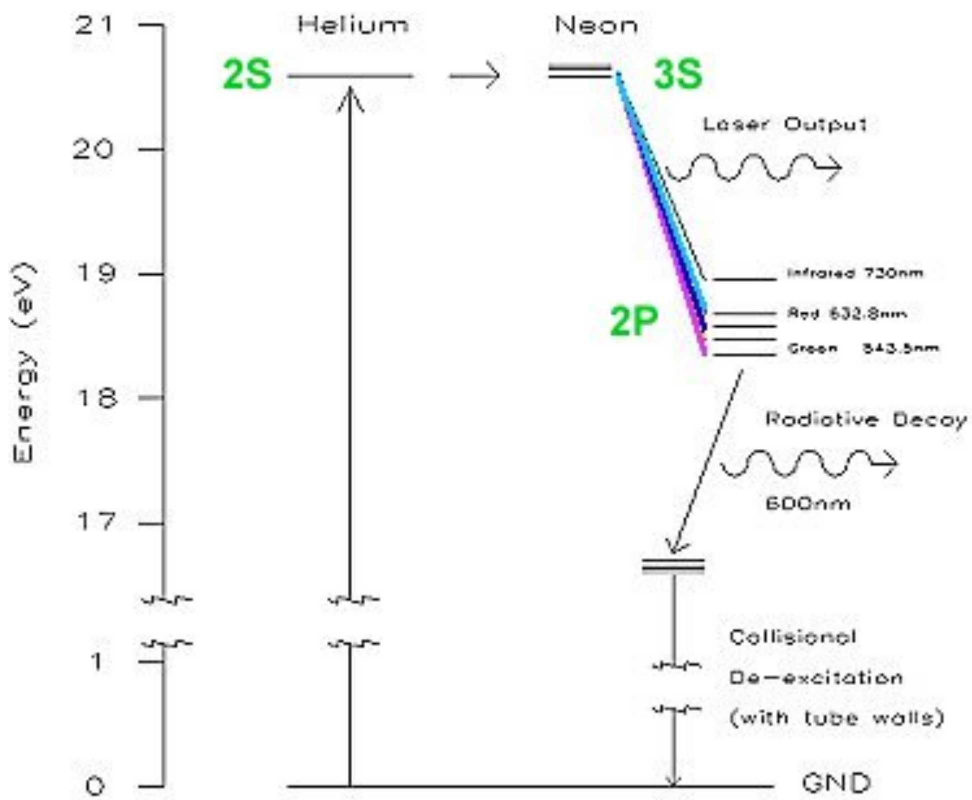
The "*Pauli exclusion principle*" states that only one electron is allowed in each quantum state.

Electrons have *spin*, so two electrons are allowed in each energy level, provided they have opposite spin. When one energy level is full, then electrons have no choice but to go into a higher energy level.



Aside 2: Lasers

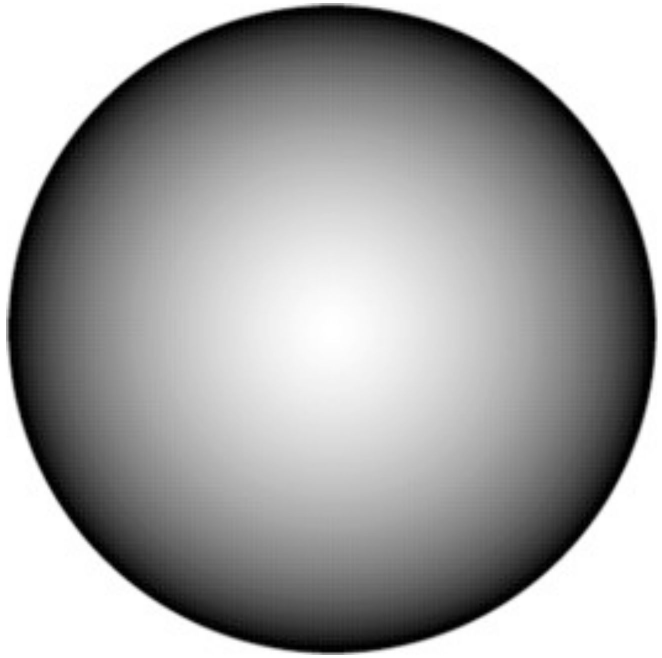
Lasers depend on electrons changing between energy levels; but get their special properties from the stimulated emission of photons. The laser requires a substance with more electrons in an excited level than in the lower energy state. When a photon of the correct energy is shone on the excited atoms, they make the transition to the lower energy state. The emitted photons are exactly in phase with the inducing photon, and travelling in the same direction.



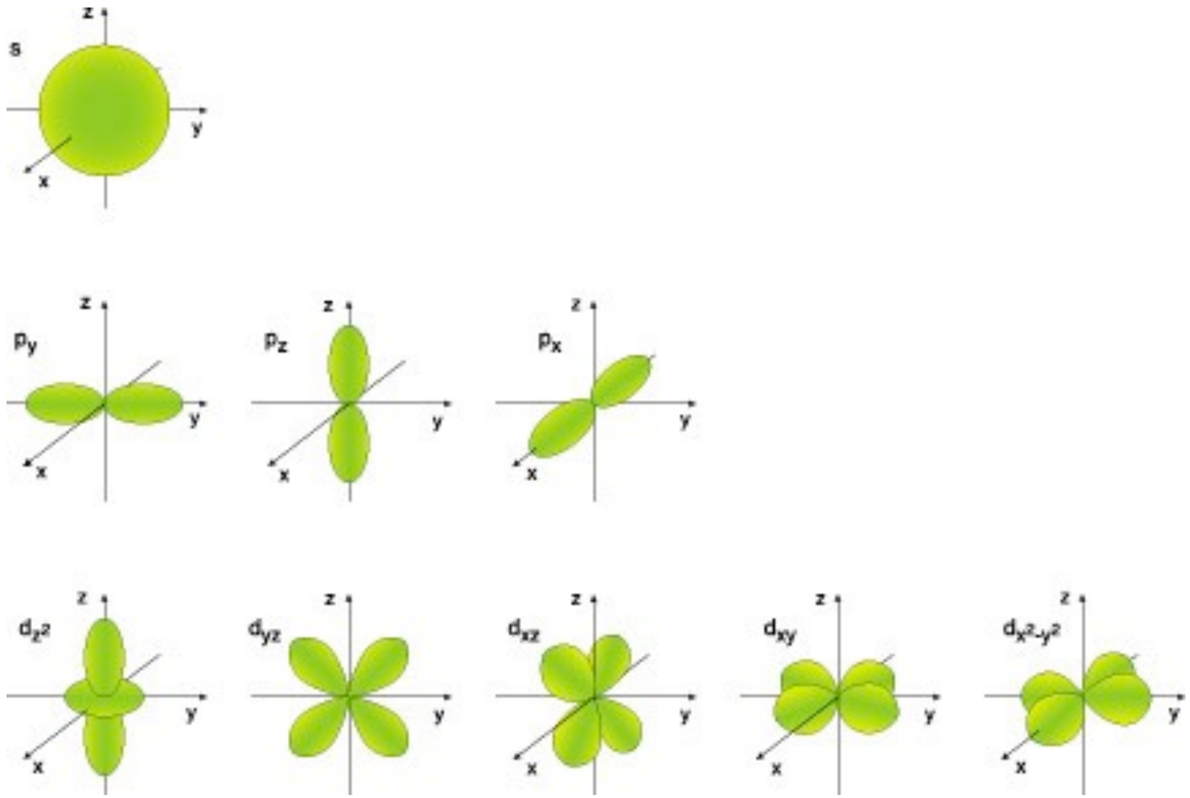
Now, this picture of electrons orbiting the nucleus is a useful picture, but it's not a very good description of what's actually happening.

Recall the Uncertainty Principle: we can't know where the electron actually is. Instead, we can define regions where the electrons *probably* are in each energy level.

In the lowest energy level, the probability distribution is spherically symmetric around the nucleus.

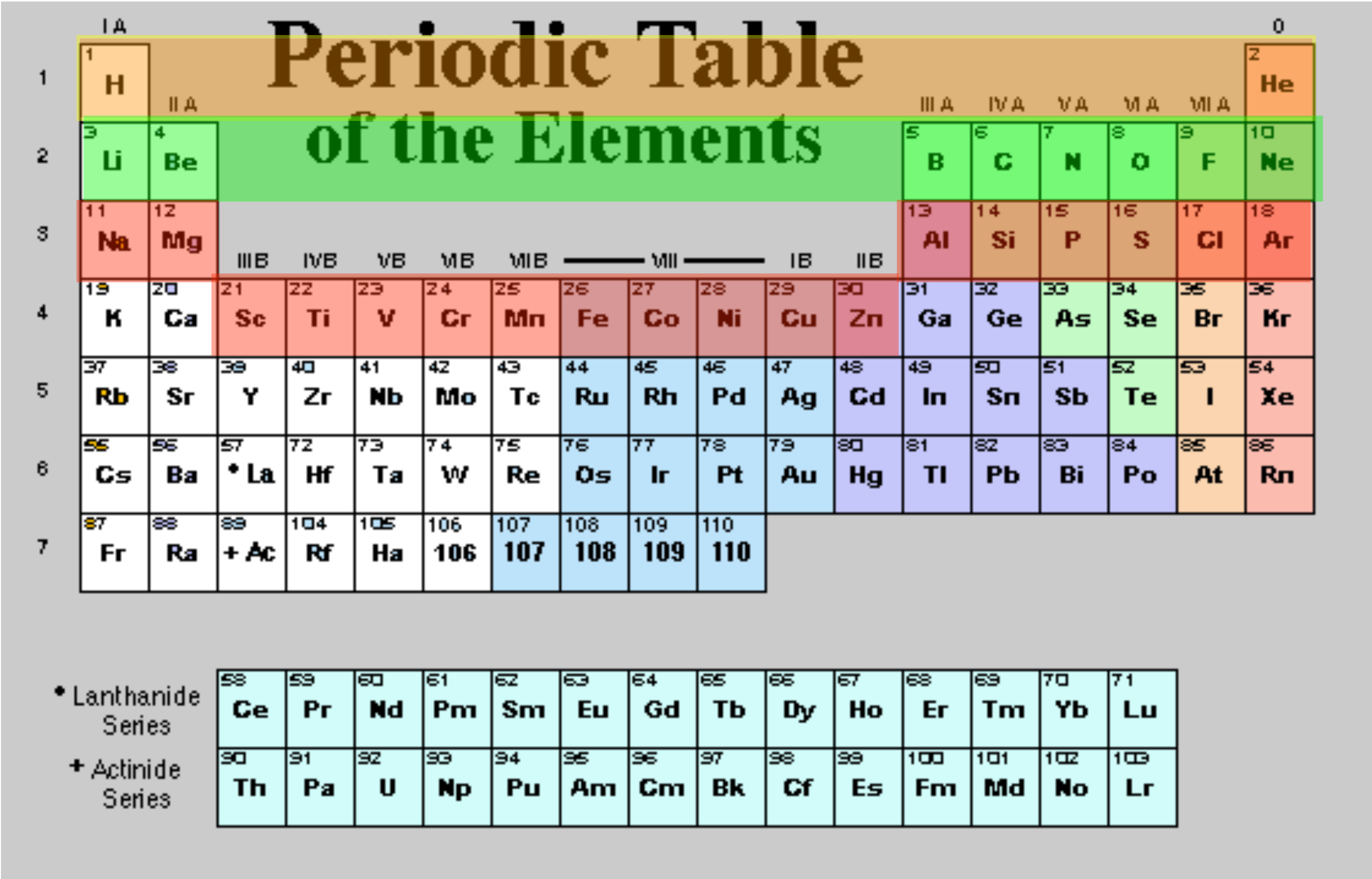


In higher energy levels, these probability patterns get more complicated. The patterns have more possible arrangements, so more electrons are allowed in higher energy levels in different sub-levels.



It turns out that there is one sub-level in the first energy level (hence two electrons), 1+3 in the second energy level (eight electrons), 1+3+5 in the third (eighteen electrons), and so on.

This observation explains the entire periodic table, and hence the properties of all elements!



n=1: 2 electrons

n=2: 8 electrons

n=3: 18 electrons

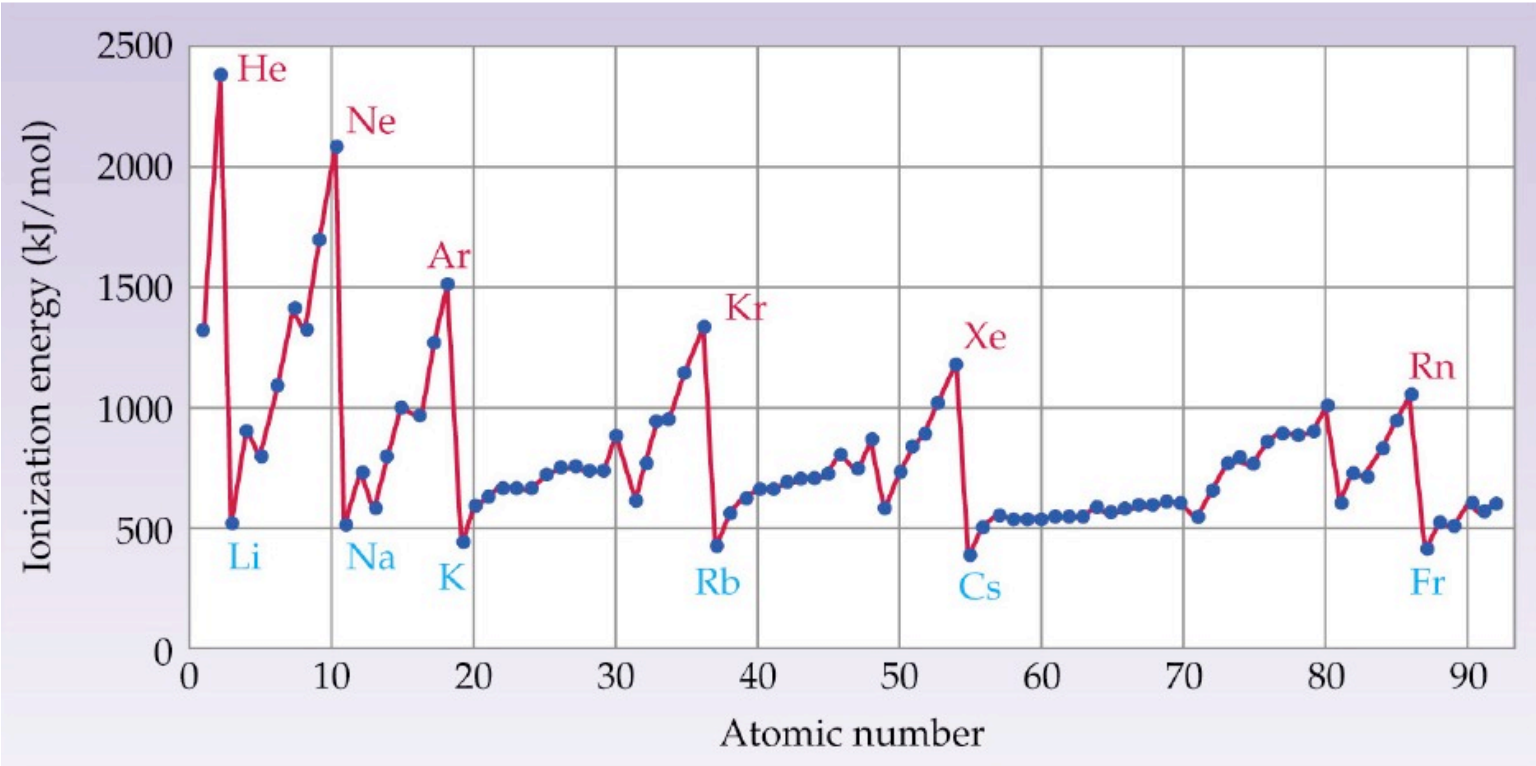
et cetera

All we have to do to explain the periodic table is to fill up the energy levels according to “Pauli’s housing plan”.

Here are the electron configurations of the first few elements.

	1s	2s	2p			
H	↑					$1s^1$
He	↑↓					$1s^2$
Li	↑↓	↑				$1s^22s^1$
Be	↑↓	↑↓				$1s^22s^2$
B	↑↓	↑↓	↑			$1s^22s^22p^1$
C	↑↓	↑↓	↑	↑		$1s^22s^22p^2$
N	↑↓	↑↓	↑	↑	↑	$1s^22s^22p^3$
O	↑↓	↑↓	↑↓	↑	↑	$1s^22s^22p^4$
F	↑↓	↑↓	↑↓	↑↓	↑	$1s^22s^22p^5$
Ne	↑↓	↑↓	↑↓	↑↓	↑↓	$1s^22s^22p^6$

These electron configurations explain all the chemical properties of the elements: how and with what other elements they form bonds etc. It also dictates how easy it is to remove electrons: the *ionisation energy*.



(insert all of chemistry here)

Next week

So that's quantum mechanics, at least enough to explain most of what we need to know about stars.

Next lecture, we'll look at what makes a star a star.

Further reading

- The best book I've found which covers all aspects of quantum mechanics is "**The New Quantum Universe**" by Tony Hey and Patrick Walters (Cambridge UP, 2003). It's still challenging (that's definitely the nature of the subject!) but it's very thorough, and includes all sorts of really up-to-date applications like quantum computing. It's not really a read-in-bed kind of book; if that's what you're after, you may prefer "**In Search of Schrodinger's Cat**" by John Gribbin (Corgi, 1984).

- Actually one of the best general introductions to quantum mechanics can be found in Bill Bryson's "**A Short History of Nearly Everything**" (Doubleday, 2003). It's a wonderfully readable overview of astronomy and physics (following up with geology and biology). He does an extremely good job of explaining difficult concepts like the wave nature of light and particle physics in terms non-scientists can grasp. A great read.

- "**E=mc2: A biography of the World's Most Famous Equation**" by David Bodanis (Pan, 2001) is a wonderful read, explaining the background to Einstein's work. It also has a very good section on the discovery of how the atom and the nucleus work.

- There's a little program which shows you the spectrum for any element at http://jersey.uoregon.edu/elements/Elements.html

- A good site for the nucleus in particular is "**Guide to the Nuclear Wallchart**" at http://www.lbl.gov/abc/wallchart/guide.html.

- We didn't have time to get into particle physics: quarks, gluons, anti-matter and more. If you'd like to find out more about these, try "**Understanding the Universe: from Quarks to the Cosmos**" by Don Lincoln (World Scientific, 2004). It's written by a particle physicist, and is an excellent introduction to the Standard Model of particle physics, how we know what we know, and where we go from here.

- The "Standard Model" of particle physics is summarised at http://particleadventure.org/standard-model.html

- I was given a wonderful coffee-table book called "**The Elements: a visual exploration of every known atom in the Universe**" by Theodore Gray (Black Dog & Levanthal, 2009). It includes all sorts of fascinating details about every element, including their electron structure and spectrum (which is how I justify its inclusion here).

Sources for images used:

- Young's double slit experiment: from Wikipedia: Double-slit experiment http://en.wikipedia.org/wiki/Double-slit_experiment
- Photoelectric effect: from "The Quantised World: Waves or particles?" http://nobelprize.org/physics/educational/quantised_world/waves-particles-1.html; and "Science and Technology", June 2005, "Shedding Light on Quantum Physics", http://www.llnl.gov/str/June05/Aufderheide.html
- Wavelength scales: from IPAC's "Infrared Astronomy" page, http://sirtf.caltech.edu/EPO/Field/emspec.html
- Electron double slit experiment: from Hitachi Research & Development: Double slit experiment http://www.hqrd.hitachi.co.jp/em/doubleslit.cfm
- "Use both exits": from The Double-Slit Garage Experiment http://improbable.com/airchives/paperair/volume7/v7i6/doubleslit.html
- Nucleus: from "Chemistry" by McMurry & Fay, http://wps.prenhall.com/wps/media/objects/602/616516/Chapter_02.html
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- Nuclide chart: from Subatomic research in Lund http://fys246.kosufy.lu.se/topics.asp
- Island of stability: from Wikipedia http://en.wikipedia.org/wiki/Island_of_stability
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- Horsehead nebula: from the NOAO Image Gallery, image by T.A.Rector (NOAO/AURA/NSF) and Hubble Heritage Team (STScI/AURA/NASA), http://www.noao.edu/image_gallery/html/im0661.html
- Spectra of different elements: from University of Oregon Physics Applets http://jersey.uoregon.edu/elements/Elements.html
- Atomic orbitals: from http://www.physto.se/~larsa/Larsas%20LicThesis.html
- Ionisation energy: from "Chemistry" by McMurry and Fay, Fig. 6.3, http://wps.prenhall.com/wps/media/objects/602/616516/Chapter_06.html