Measurement of court speed and bounce

Rod Cross, Physics Department, Sydney University

2nd July 2006

The speed of a surface refers to the change in ball speed in a direction parallel to the surface. The bounce of a surface refers to the change in ball speed in a direction perpendicular to the surface. Slow courts like clay tend to be fast in the vertical direction (ie high bouncing) and fast courts such as grass tend to be slow in the vertical direction (ie low bouncing). To make matters even more confusing, clay courts are slow when a tennis ball hits the surface but are fast (ie slippery) under foot. The difference here is that clay particles get caught up in the ball cloth, so the cloth acts like sandpaper, while the particles roll under foot like ball bearings.

The speed and bounce of a tennis court, or any other playing surface, can be measured by filming the bounce of a ball using a digital video camera. That way, selected clips can be downloaded to a computer to determine the change in ball speed in both the horizontal and vertical directions. If a line is drawn across a diameter of the ball, its change in spin can also be measured. The same technique can be used to measure speed and bounce off the strings of a racquet or off a bat or club. The measurement procedure is relatively simple, but there are some important precautions to take to get good results. The most important of these are:

1. The ball must be incident at a small angle on the surface. The angle between the path of the ball and the surface itself must be less than about 18° and preferably about 15°. The reason for this is that the ball must slide along the surface throughout the bounce, without gripping the surface. That way, the change in horizontal speed is determined by the coefficient of sliding friction (COF) between the ball and the surface. The speed of a court depends on how smooth or rough it is. If the surface is smooth, it will have a low COF, meaning that the friction force on the ball is small and the ball will slow down by about 20 to 30% in the horizontal direction. If the surface is rough then the COF is large and the ball will slow down by about 40 or 50%. If the ball is incident at an angle greater than about 20° then the ball will grip the surface during the bounce and the ball will slow down by around 40 or 50% regardless of whether the surface is smooth or rough and regardless of the angle of incidence.

2. It is difficult to align a camera so that it is not tilted when viewing the ball and the court. The camera might appear to be set up properly but when you examine the film on

the computer you might find that the camera was tilted by one or two degrees. You can still work out the change in speed of the ball but a correction has to be made for the tilt of the camera. A spirit level on the tripod can assist here to reduce the tilt. An error of one or two degrees may not sound like much, but if the angles of incidence and reflection are only about 15 degrees then small errors in the measured angles make a big difference to the final results. The differences in speed or bounce between different courts are relatively small so it is important to reduce the measurement errors to pick these small differences reliably. The COF varies from about 0.6 for a fast court to about 0.8 for a slow court while the COR (Coefficient Of Restitution, the factor describing the vertical bounce) varies over an even smaller range, from about 0.75 to about 0.85.

3. Most video cameras stretch the image in the vertical direction by about 10% compared to the scale in the horizontal direction. That means you need to film objects separated by known distances in both the horizontal and vertical directions in order to calibrate the horizontal and vertical distances on the film. Those objects must be in the same plane as the ball path so you need to film them before you start firing firing balls on the court and then remove the objects when you are ready to fire balls. If the objects are behind the path of the ball rather than in the same plane then they will be closer together on the film and the distance calibration will be incorrect.

4. All balls slow down through the air, especially if they are large or light or both. Consequently, to measure the speed of the ball just before and after it hits the surface, you need to zoom in rather than zoom out so that you film the ball as close as possible to the impact point. If you zoom in too much then you may miss filming the ball completely. A suitable technique is to place two ball cans 2 m apart and zoom in so the space between the cans fills up about 2/3 of the width of the screen. That way you will get 3 or 4 images of the ball before it bounces and 5 or 6 images after it bounces. Also make sure you can see a horizontal line on the court or a vertical fence post so you can determine when you examine the film the angle at which the camera was tilted.

5. All camera lenses suffer from a problem known as barrel distortion. The result is that perfectly straight lines on a tennis court appear to be curved on the film, especially near the edges of the film. Straight lines remain straight in the middle of the film, but not at the edges. The solution is to move the camera well away from the impact point and then zoom in. That way, the image passes through the middle part of the lens and not through the edge of the lens. Barrel distortion arises because the focal point near the edge of a lens is not the

same as the focal point of the centre part of the lens. You should locate the camera outside the court about 6 or 8 m away from the impact point. The camera needs to be mounted on a small tripod, about 50 cm above the court so that the camera is at about the same height as the ball.

6. A standard ball launcher works fine although I also use a home–made launcher since it is more portable and doesn't require any batteries or a power point. I made it from an old fashioned door closer with two curved prongs on the end to hold the ball and with the piston removed to make sure the closer swings around rapidly. It launches the ball at about 10 m/s. The spring mechanism can be adjusted to alter the ball speed. A small amount of backspin is imparted to the ball with the home–made launcher but that is OK since it helps to ensure that the ball slides throughout the bounce. Topspin should be avoided unless you specifically want to examine the effect of topspin.

7. Depending on the ball speed, you will need an exposure time of around 1/500 sec or 1/1000 sec to get a reasonably sharp image of the ball. A ball travelling at 10 m/s = 10 mm/ms travels 10 mm in 1 ms, so even at 1/1000 sec you will get a slightly blurred or streaked image. If recording indoors you will need a bit of extra light. A few desk lamps with 100 W bulbs will be sufficient.

8. There is no point zooming out to get as many ball images as possible, unless you want to plot out the trajectory of a ball over a large distance. Because the ball slows down through the air, it is better to work with only 2 or 3 images before and after the bounce. Two is simpler and probably better than three since you can zoom in a bit closer and get each position of the ball more accurately.

9. The horizontal speed of the ball is easy to measure. If the ball travels a horizontal distance of say 0.40 m between each frame, and if the time between each frame is $T = 0.04$ seconds (ie 25 frames/sec) then the horizontal speed is $v_x = 0.4/0.04 = 10$ m/s. Over a distance of only 0.4 or 0.5 m, a tennis ball slows down by only about 1% so there is no need to correct for the slowing down effect. Heavier balls slow down even less, although balls with a large surface area such as soccer and basketballs could slow down by as much as 2 or 3% over this distance. The vertical speed is more tricky to measure because the ball accelerates in the vertical direction due to gravity. The vertical speed at the time of impact will therefore be larger than the vertical speed before impact.

CALCULATIONS

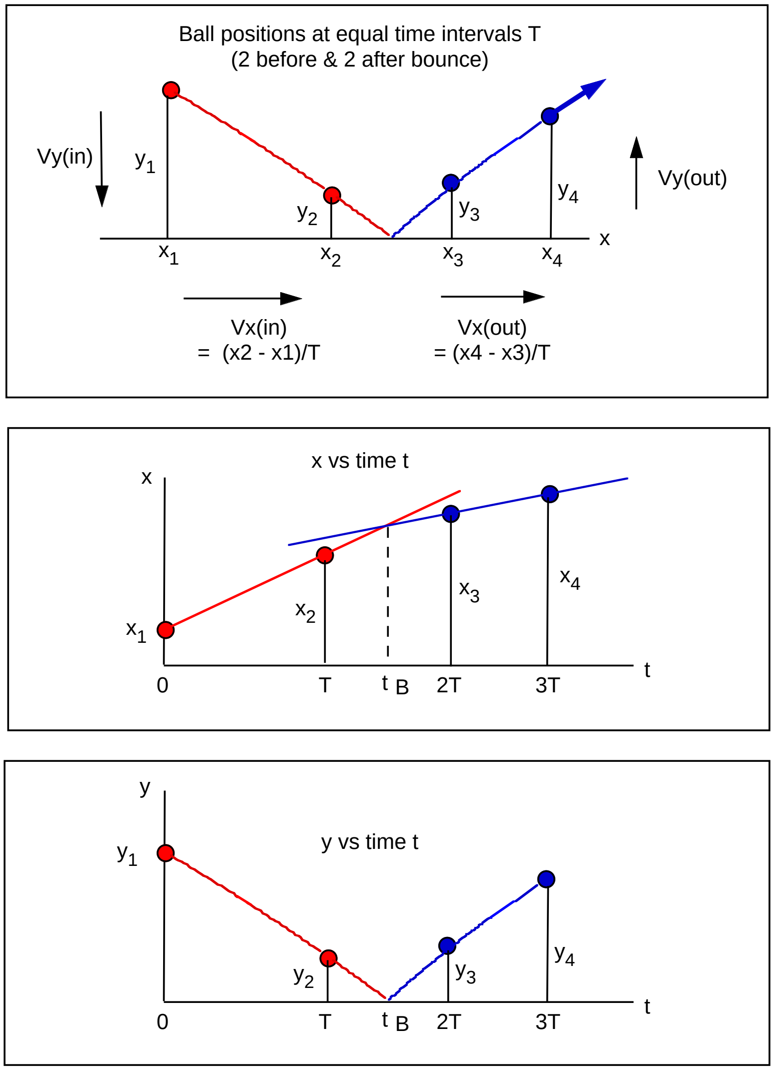

Figure 1: Ball positions in 4 successive video frames showing y vs x, x vs t and y vs t

Suppose that the first recorded position of the ball before impact is at a vertical height y_1 and the next recorded position before impact is at height y_2. An approximate value of the the vertical speed just before the bounce is $Vy(in) \approx (y_2 - y_1)/T$. After the bounce, $Vy(out) \approx (y_4 - y_3)/T$. More accurate values can be obtained by taking into account the acceleration due to gravity, as follows:

Assuming that the first frame corresponds to zero time, then at any time t before the bounce the vertical position of the ball will be given by

$$y = y_1 - v_{y1}t - 4.9t^2 \tag{1}$$

where v_{y1} is the vertical speed at zero time. 4.9 is half of 9.8, the acceleration due to gravity.

At time T, where the height of the ball is y_2, we have

$$y_2 = y_1 - v_{y1}T - 4.9T^2 \tag{2}$$

which can be rearranged to show that

$$v_{y1} = \frac{(y_1 - y_2)}{T} - 4.9T \tag{3}$$

The vertical speed at time t = 0 is v_{y1} and the vertical speed at time T is $v_{y2} = v_{y1} + 9.8T$. If the ball bounces at time t_B then the vertical speed on impact is $Vy(in) = v_{y1} + 9.8t_B$. The bounce time is not known but you can work it out knowing that the ball is at the same height just before and just after impact. Alternatively, if you plot a graph of the horizontal position x vs time, then you can fit two straight lines to the data points, one having a slope $Vx(in)$ and one having a slope $Vx(out)$. These two lines intersect at the time of impact. The first straight line is given by $x = x_1 + (x_2 - x_1)t/T$ and the second straight line is given by $x = 3x_3 - 2x_4 + (x_4 - x_3)t/T$. Since the x values are the same (say $x = x_B$) at $t = t_B$ we have

$$x_B = x_1 + (x_2 - x_1)t_B/T = 3x_3 - 2x_4 + (x_4 - x_3)t_B/T \tag{4}$$

so

$$t_B = \frac{(3x_3 - 2x_4 - x_1)T}{(x_1 - x_2 - x_3 + x_4)} \tag{5}$$

A similar procedure can be used to determine the vertical speed $Vy(out)$ at the time when the ball bounces off the court. In this case the vertical position of the ball is given by

$$y = y_o + v_{yo}t - 4.9t^2 \tag{6}$$

where y_o is the vertical position of the ball at $t = 0$ and v_{yo} is the vertical speed of the ball at $t = 0$. At $t = 0$ the ball is actually travelling down onto the surface but after the ball bounces we need to assume it travels upwards, starting at zero time from an imaginary point below the court surface. Assuming that the camera captures 2 images before the bounce and two after the bounce then the first image after the bounce is recorded at time $2T$ when the ball is at height y_3 and the second image after the bounce is recorded at time $3T$ when the ball is at height y_4. Hence

$$y_3 = y_o + 2Tv_{yo} - 4.9 \times (2T)^2 \tag{7}$$

and

$$y_4 = y_o + 3Tv_{yo} - 4.9 \times (3T)^2 \tag{8}$$

Subtracting Eq. (7) from Eq. (8) gives

$$y_4 - y_3 = Tv_{yo} - 4.9 \times 5T^2 \tag{9}$$

so

$$v_{yo} = \frac{(y_4 - y_3)}{T} + 24.5T \tag{10}$$

and

$$y_o = y_3 - 2Tv_{yo} + 4.9 \times (2T)^2 = 3y_3 - 2y_4 - 29.4T^2 \tag{11}$$

The vertical speed of the ball at any time after it bounces is

$$v_y = \frac{dy}{dt} = v_{yo} - 9.8t \tag{12}$$

from which we can calculate the speed $Vy(out)$ at the bounce time $t = t_B$. A useful check on the accuracy of t_B is to calculate it from the two y vs t curves. These two curves intersect at time $t = t_B$ when the ball is in contact with the court.

COR and COF

The bounce of a court (or more correctly, the bounce of a particular ball on that court) is specified by the coefficient of restitution (COR) defined by

$$\text{COR} = \frac{Vy(out)}{Vy(in)} \tag{13}$$

The speed of the court is specified by the coefficient of sliding friction (COF) between the ball and the court. If F is the horizontal friction force and N is the normal reaction force then COF $= F/N$. Since $F = mdV_x/dt$ and $N = mdV_y/dt$, where m is the mass of the ball,

$$\text{COF} = \frac{dV_x}{dV_y} = \frac{Vx(in) - Vx(out)}{Vy(in) + Vy(out)} \tag{14}$$

Note that $Vy(in)$ and $Vy(out)$ are both positive quantities here but the change in velocity is obtained by adding rather than subtracting the two speeds since the ball velocities are in opposite directions. The International Tennis Federation (ITF) defines court speed in terms of an alternative quantity called Pace, where Pace = 100(1 - COF). Official measurements of Pace are made using a sophisticated device known as the Sestee machine which employs arrays of infrared beams to measure incident and rebound speeds and angles.