Dust Clusters in Magnetized Plasma:

by

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A Thesis Submitted in Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy in the School of Physics,
The University of Sydney
March, 2005
Declaration of Originality

To the best of my knowledge, this thesis contains no copy or paraphrase of work published by another person, except where duly acknowledged in the text. This thesis contains no material which has been presented for a degree at the University of Sydney or another other university.

Felix Cheung
“For something of such simple structure to be so visually impressive and physically astounding, it must be one of nature’s masterpieces”

– the Author, 31\textsuperscript{st} of March, 2005
Abstract

This is the first detailed experimental investigations of dust clusters in magnetized rf discharge plasma. Our experiments performed in a weakly magnetized (up to 100 G) inductive rf discharge show that dust cluster rotation is dependent on the number of particles and the magnetic field. Comparison of our experimental results with current theoretical models demonstrated that dust rotation is largely due to momentum impact transferred from the partially magnetized ions in the plasma to the dust particles.

Particles in dust clusters were found to fluctuate predominantly in the azimuthal direction rather than in the radial direction. A strong correlation between the packing sequence probability of a cluster and the particle fluctuation was observed. A magnetic field has been shown to decrease the amount of particle fluctuations in a dust cluster.

In our experiments, it was found that the radial positions of the dust particles self-adjusted to accommodate for the change in the ambipolar electric field. It was also found that dust particles can be confined by a magnetically-induced electrostatic trap.

Furthermore, our experimental results demonstrated the possibility in manipulating dust particles with 3 degrees of freedom using a magnetic field as an external, controlled parameter. The implications of dust rotation due to a magnetic field in industrial applications, such as the removal of dust contamination in the manufacture of microelectronics and the fabrication of micro- and nano- devices in nanotechnology, are discussed.
Acknowledgements

I am deeply grateful to a group of people without which the completion of this thesis would not be possible. In particular, I would like to highlight three very important people. Firstly, I would like to thank my co-supervisor, Dr. Alex Samarian, for the vast amount of knowledge he taught me not only in Physics, but also in life. His innovative ideas led the project into a new direction. More importantly, his encouragement to express my visualization skills and artistic ideas during my scientific research had resulted in many illustrations and concepts seen throughout this thesis. Secondly, I would like to thank my former colleague, Dr. Nathan Prior, for his help in building my experimental apparatus and the mechanical skills he passed onto me. His unconditional contribution is a key element in the success of this project. Thirdly, I would like to thank my supervisor, Prof. Brian James, for his leadership in this project and his help in my transfer of PhD to the University of Sydney. His comments and suggestions are part of what constructed the final form of this thesis.

Thanks must go to three talented students: Cameron Ford and Stephen Barkby with their help in developing the computer routines in the simulation of planar cluster systems; and Christopher Brunner for his help in the analysis of cluster instabilities.

Thanks must go to the University of Sydney. This includes Dr. Nicole Bordes and the Sydney Regional Scientific Visualization Laboratory for their help in scientific visualization.

Thanks must go to the Flinders University of South Australia. This includes my former supervisor, Dr. Leon Mitchell, for his introduction into the field of dusty plasma; the technicians from the electrical and mechanical workshop, Bob Northeast, Mike Mellow, Peter Mariner at the School of Physics for the construction of the experimental apparatus; Dr. Michelle Hale and the School of Biological Sciences for the use of the image acquisition system; and Ms. Katie Green and the School of Chemistry for the use of the copper vapor deposition unit.

Thanks must go to the Australian Research Council for their financial support of this project over the last three and a half years.

Thanks must go to the State Key Laboratory of Materials Modifications by Beams at the Dalian University of Technology. This includes Jayson Ke Jiang for contacting me through our research website and thus initiated our collaborative researches and Lu-Jing Hou for repeating his computer simulations specifically for my experimental conditions.

Lastly, I would like to thank my family for their unconditional support both financially and emotionally in the last 25 years. Without their faith in me, I could not have endured this one year of painstaking thesis writing. In particular, I am in great debts to my beloved mother and grandmother as I will never be able to compensate the number of fine lines that have appeared on their faces over this period of time.
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Foreword

This thesis is intended for two types of readers – those with a background of plasma physics who wish to understand the influence of magnetic field on the behavior and the dynamics of dust particles in a plasma, and those with plasma processing background who wish to examine the possibility of employing a magnetic field as a particle manipulation tool in industrial applications.

Initially, this thesis concentrated on the topic of dust rotation in a magnetized plasma. Further analysis of the experimental data and computer simulations during the course of writing led to a number of new topics. These topics, which include the structural properties and fluctuations of dust clusters, have been added to the thesis for completeness. Due to the large amount of information, this thesis has been structured so that the topics are discussed as the experimental results are presented. In addition, there is frequent cross-referencing of topics (i.e. see Section … for …) to increase the coherence of the discussions.

The illustrations of this thesis are all produced by the author unless stated otherwise in the captions. Readers will find the illustrations helpful in visualizing the underlying physics. Due to the illustrative approach of this thesis, a chapter devoted to the possible implications in industrial applications has been included.

SI units are used, except where stated otherwise. A table which explains the constants, notations, and variables used throughout the thesis is available at the end for convenience. The notation (3.5) refers to Equation 5 of Chapter 3, which differs to the notation (3,5) for the packing sequence of dust clusters with 3 particles in the inner ring and 5 particles in the outer ring. And notation [24] refers to Reference 24 as listed in the Bibliography.
1. Introduction

The purpose of this introduction is to provide some general background information on dusty plasma. Section 1.1 begins with the explanation of dusty plasma, its origin and formation, and different examples of its existence in nature, in cosmic surroundings, and in industrial situations. And in order to understand the importance of the research in this scientific area, several key features of dusty plasma will be highlighted.

Section 1.2 discusses the possible implications from the study of dust dynamics and transport in industrial applications. Then by providing a chronology of conducted studies to date, the significance of our research relative to the international efforts in the study of dust dynamics under the influence of magnetic field will be illustrated.

Section 1.3 presents a summary of the existing models used in the explanation of dust rotation in a magnetized plasma. The assumptions and deficiencies associated with these models will also be discussed.

After getting a general picture of the problems encountered in our current understanding of dust dynamics in a magnetized plasma, the purposes of our research, the basic structure of this thesis, and a summary of our major findings will be outlined in Section 1.4.

1.1. Dusty Plasma

1.1.1. What is dusty plasma?

Plasma, the fourth state of matter after solid, liquid, and gas, consists of positively charged ions (denoted by symbol $i$) and negatively charged electrons (denoted by symbol $e$). There will also be a percentage of neutrals (denoted by symbol $n$) in the plasma if the background gas is not completely ionized. Dusty plasma (also known as complex plasma, colloidal plasma, or Coulomb plasma) is the result after introducing another constituent, namely dust (denoted by symbol $d$), into the plasma.

Dust, in this context, refers to microscopic particles with all shapes and sizes ranging from a few nanometers up to tens of microns. Typically, dust used in laboratory research is of uniform size in the dispersed phase (monodisperse), spherical, and made out of dielectric materials. Dust particles can be introduced into the plasma via homogeneous (purely gas phase) or heterogeneous (particle/catalyst induced) processes [1]. In the case of homogeneous processes, dust particles are formed as the product from combining chemically reactive gases in the plasma. In the case of heterogeneous processes, dust particles are released into the plasma from some external source, such as particle injection, fragments chipped off by etching, macroscopic particles due to sputtering, etc (see Figure 1-1). Agglomeration of dust particles can also occur resulting in the formation of irregular structures.

In the plasma, dust particles collect a high number of ions and electrons from their surroundings onto their surfaces. Since ions have a larger mass than electrons, and in typical discharge plasma condition the ion temperature is significantly lower than the electron
temperature, the mobility $\mu$ of the electrons can be hundreds of times higher than that of the ions (i.e. $\mu_e >> \mu_i$). So typically, the electrons dominate over the ions in the electric current flowing towards the particle surface until electrostatic equilibrium is established with dust negatively charged.

![Figure 1-1](image1.png)

**Figure 1-1** – Three methods in which dust particles can be introduced into the plasma – via chemical reaction of a mixture of gases (left), fragments due to etching or sputtering (center), and particle injection with dust shaker (right).

Under different plasma conditions, dust particles can undergo photoelectron emission, thermionic emission, and secondary electron emission (see Figure 1-2) (see Section 1.1.3 for Classification of dusty plasma) [1]-[10]. Therefore, depending on the combination of charging processes, dust particles can be positively or negatively charged. The number of electrons $Z$ on the dust surface can range from a few for nanometer sized particle up to thousands for micron-sized particle ($Z \propto a$, where $a$ is the dust radius).

![Figure 1-2](image2.png)

**Figure 1-2** – Dust particles in dusty plasmas are electrically charged primarily by collection of plasma electrons and ions (leftmost). But depending on the plasma conditions, dust particles can also undergo photoemission (left), thermionic emission (right), and secondary electron emission (rightmost).

In the gas discharge plasma, dust particles can levitate near the sheath between the bulk plasma and the boundary due to a balance in the vertical direction mainly between the gravitational force and the electric force (from the electric field within the sheath), that is:

$$m_d g = eZE_s \quad (1.1)$$

where $m_d$ is the mass of the dust, $g$ is the gravitational constant, $e$ is the elementary charge and $E_s$ is the sheath electric field.

The dust particles with like charges naturally repel against each other. Therefore an electrostatic trap with a radial confinement electric field $E_C$ is applied to inhibit these highly
charged dust particles from escaping. Here the electrostatic trap is normally created via the introduction of a shaped electrode (e.g. metal ring, metal slit, etc) or biased electrode. The dust particles then arrange themselves typically into single or multi-layered lattices analogous to atoms in a crystal. The existence of such crystals was first predicted in 1986 [11], later observed in the laboratory in 1994 [12]-[14]. They are now commonly known as dust crystals (or plasma crystals, Coulomb crystals).

In the horizontal direction, the spatial arrangement of the particles in a dust crystal is governed by two competing effects, namely, the interparticle Coulomb repulsion and the confinement electric field [13]-[15]. The interparticle repulsion favors the formation of hexagonal structures which is the ground state for an infinite two-dimensional single component system, while the confinement electric field imposes the shape of the potential onto the lattice boundary. The interparticle distance $d$, which characterizes the spacing between adjacent particles in the horizontal direction, is determined by the number of dust particles inserted into the plasma and the plasma conditions. Typically, the interparticle distance is in the sub-millimeter region.

In the vertical direction, ions accelerated in the sheath stream downwards onto a dust particle producing an ion wake cone (or ion focus) with periodic local potential maxima and minima below the particle [16]-[18]. Since the strongest potential minimum (or ion focus) for negative charge is directly beneath the dust particle, particles preferably align themselves vertically into a straight line (see Figure 1-3). Such influence of the ion wake cone on the dust particles can overcome the Coulomb repulsion depending on the plasma conditions (e.g. ion density). As a result, the structure of multi-layered dust crystals with a considerable number of dust particles $N$ are simple hexagonal if the influence of ion wake cone dominates or hexagonal close packed if the interparticle repulsion dominates. The interlayer distance $l$, which characterizes the spacing between adjacent particles in the vertical direction, is usually 1.5-2 times more than the interparticle distance.

The ion drag force, the thermophoretic force, the surface charge dipolar effect [19], and the shadow attraction due to anisotropic pressure of neutrals [20]-[22] can also play an important role in modifying the equilibrium position of the particles and the structure of a crystal depending on plasma conditions. The physics behind each of the forces can be understood easily by considering the scenario of two isolated dust particles (see Figure 1-4):
- Ion drag force - the ion flux flowing onto the upstream side of the particles pushes the particles downstream;
- Thermophoretic force - there can exist a net transfer of momentum from the neutrals onto the dust surface due to a temperature gradient;
- Surface charge dipolar effect - an electric dipole moment arises on the dielectric particle because of asymmetric charging by the greater ion flux density to the upstream side; and
- Shadow attraction due to anisotropic pressure of neutrals - because of the presence of another particle, the flux of captured and scattered neutrals around a particle is not symmetrical resulting in a shadow attraction.

Figure 1-4 – Particle position can also be influenced by the ion drag force (leftmost), the thermophoretic force (left), the dipolar effect (right), and the anisotropic pressure (rightmost) to varying extent depending on the plasma conditions.

Indeed, one of the bewildering properties of these crystals is that different structures can be formed due to the combination of these forces. As an example, coexisting body centered cubic and face centered cubic structures have been observed in single dust crystal as a result of inhomogeneity of the background plasma conditions [12]. The structure which is more energetically favored will more likely be the one observed.

Large dust crystals can have sizes corresponding to hundreds of interparticle distances in the horizontal plane, with up to ten or so layers in the vertical direction. Dust crystals with only a small number of particles \((N < 100)\) are called dust clusters (or Coulomb clusters). Since the number of particles is reduced, dust clusters exhibit many unique physical properties not found in large crystals. In contrast to the large crystals, the particles in dust clusters \((N < 40)\) tend to arrange themselves into a single layer with shell structures due to the cylindrical symmetry in a typical radial confinement potential. These individual shell structures are called cluster rings. For intermediate dust clusters \((N > 40)\), the particles on the outer rim of the cluster will form concentric rings, while the inner particles form hexagonal lattice structure [23]-[24] (see Figure 1-5).

The structural configurations (or packing sequences) of dust particles in a dust cluster can be arranged into a “periodic” table by the number of cluster rings [23]-[24]. Due to the relative ease in the motion analysis of the individual particle, dust clusters have been the subject of many theoretical and experimental studies. Some examples which have been reported include the investigations of particle ordering, phase transitions, and energy spectra [23]-[32]. Moreover, dust clusters are simple systems characterized by the strong electrostatic
interaction of highly charged particles within an external confinement potential analogous to other phenomena such as laser-cooled ions in a trap [33]-[34] and electrons in quantum dots [35].

Figure 1-5 – Images captured from the top of a small Planar-7 dust cluster (left) and an intermediate Planar-94 dust cluster (right).

1.1.2. Examples of dusty plasma

The existence of dusty plasma is not limited to the laboratory. Various forms of dusty plasmas can be found in nature, in our cosmic surroundings, and in industrial situations.

In nature, dusty plasmas are formed in the path of cloud-to-ground and ball lightning where the surrounding atmosphere will almost always contain water droplets, dust particulates and pollutants. In fact, it has been proposed that ball lightning is the result of oxidation of silicon nanoparticles suspended in air after a lightning strike on the ground [36]-[37]. Noctilucent clouds, which are high atmospheric suspensions ($\approx 80$ to $85$ km above sea level in the mesosphere where the temperature is below $\approx 155$ K) of small charged ice-coated particles [38], constitute yet another example of dusty plasma (see Figure 1-6).

In our cosmic surroundings, dust is abundant as evident from the formation of planets and stars. Different examples of dusty plasma appear in the forms of planetary rings, interstellar gases [39], space nebulae [40], cometary tails [41], and zodiacal lights [42]-[43] (see Figure 1-7). In fact, all matter at the beginning of our universe was in the form of dusty plasma. And dusty plasma remains the most common structural form of matter in the universe.

In industry, there are many situations where dusty plasma can occur. For example, in the fabrication of microelectronics, dust particles are produced during plasma etching of silicon wafer or in thin film deposition using reactive gases [44]. In the former case, the dust particles are considered to be undesirable contamination which obliterates the fabrication process. In the combustion of fossil fuel and rocket solid fuel, dust is produced as a byproduct which is then charged by the ions from the exhaust gases. In magneto-hydrodynamics (MHD) power generator, dust is formed from fossil fuel combustion plasma or introduced as additives to enhance the conductivity of plasma jet. In Tokamak systems, any solid material which comes into contact with the high energy plasma would normally become vaporized and ionized. However, as the temperature of the outer portion of the plasma is much cooler than the center, dust particles which originated from the plasma erosion of the walls and the electrodes can survive [45]. As a result, dust is always present in the plasma inside such systems (see Figure 1-8).
Figure 1-6 – Different examples of dusty plasma commonly found in nature: (Left) Cloud-to-ground lightning mostly occurs in thunderstorms. Other situations in which lightning occurs are dust storms, blizzards, volcanic eruptions and nuclear explosions where large volumes of particle clouds are present in the atmosphere [Courtesy of Charles Alison, Oklahoma Lightning]; (Center) only few real photos of ball lightning had ever been taken. This particular photo was taken by a student in Nagano, Japan in 1987. Ball lightning had been known to blast holes in roofs, blow out electrical outlets, and drift around in the air; (Rightmost) Noctilucent clouds are suspension of ice crystals/ice coated particles at extremely high altitude. And as the name suggests, noctilucent clouds literally shine at night. The clouds are formed in the very dry part of the atmosphere such as the polar mesosphere [Courtesy of Pekka Parviainen, Polar Image].
Figure 1-7 – Different examples of dusty plasma commonly found in our cosmic surroundings: (Leftmost) an ultraviolet image of the rings of Saturn taken during the Cassini spacecraft's orbital insertion revealed the detailed structures of the rings. The “Cassini Division” in faint red at left contains thin rings of dust whereas the A rings in turquoise contains a more icy composition [Courtesy of NASA/JPL/University of Colorado]; (Left) an image of part of the Eagle Nebula M16. Dark pillar-like structures are actually columns of cool interstellar hydrogen gas and dust which are also incubators for new stars [Courtesy of Jeff Hester & Paul Scowen, Arizona State University/NASA]; (Right) The Zodiacal light is visible in this image as a triangular-shaped cone of light rising on the left side of the photo. Zodiacal light is composed of fine particles of dust in orbit around the sun and is visible because of sunlight scattered from the particles. Since the band of dust completely circles the sun, the Zodiacal light actually stretches across the entire night sky. [Courtesy of Jerry Lodriguss, Astropix]; (Rightmost) the comet Hale-Bopp has two tails, an ion and a dust tail. The ion tail is comprised of carbon monoxide boiled off by the heat of the sun, then ionized by solar radiation. It is soft blue in color and streams directly away from the sun due to the magnetized solar wind. The dust tail is composed of dust particles liberated by evaporating gases. Like the ion tail, the yellowish-white dust tail behind the comet is initially pushed away from the sun, but is usually curved away as the comet swings around the sun, leaving the comet with the fork-tailed appearance [Courtesy of Bob Luffel, Alpine Astronomical].
Figure 1-8 – Different examples of dusty plasma commonly found in industry: (Leftmost) the U-25 MHD pilot plant was constructed and operated at the Institute for High Temperatures (IVTAN) of the Russian Academy of Sciences in Moscow during 1960s. The purpose of U-25 at that time was to produce electricity through direct MHD conversion from ionized exhaust from burning a mixture of hydrocarbons and pure oxygen. Alkaline particles are added as additives during the combustion of gases to increase conductivity of plasma [Courtesy of IVTAN, Russian Academy of Sciences]; (Left) dust impurities are well known to lower the performance of fusion magnetic confinement devices and pose safety hazards due to the accumulation of radioactive tritium. [Reproduced by permission of EFDA-JET]; (Right) an enlarged image of a grain of salt on a section of a microprocessor indicates the smallness and complexity of the electrical network. And so understandably, microscopic dust particles produced in plasma processing must be removed or the microprocessor will be contaminated and ultimately become unusable. [Courtesy of Intel]; (Rightmost) the space shuttle Atlantis utilized three solid propellant rocket engines to produce 375,000 pounds of thrust for take off in 1988. Dust and water vapor are produced during the combustion of the solid fuel. The water vapor then condensed into ice particles, which becomes negatively charged by the ionosphere electrons and acts as an enhanced backscatter target [Courtesy of NASA].
1.1.3. Classification of dusty plasma

Dusty plasmas can be divided into four groups according to the process by which the background plasma is generated, namely, ultraviolet-induced (UV-induced), nuclear-induced (or radioactivity-induced), thermal, and discharge (see Figure 1-9).

Figure 1-9 – Dusty plasmas can be categorized into UV-induced, nuclear-induced, thermal, and discharge dusty plasmas.

For UV-induced dusty plasma, the background plasma is the result of photoelectrons being emitted from the dust surface in the presence of strong ultraviolet (UV) radiation. Therefore, the dust particles in such plasma are positively charged. The source of the UV radiation can either be external or internal. In the external case, the dust particles are irradiated by some UV source placed outside the plasma volume (e.g. UV laser, UV lamp, solar radiation, etc). In the internal case, the UV source is generated by the self-radiation of the plasma itself (e.g. hollow cathode gas discharge, etc). Since the stars are a source of UV radiation, most UV-induced dusty plasmas are found in space or in the mesosphere of the
Earth. Planetary rings, noctilucent clouds, zodiacal lights, cometary tails are examples of UV induced dusty plasma.

For nuclear-induced dusty plasma, the background plasma is primarily produced as a result of alpha or beta particles traveling through the ionized gas. In addition, as the energy of these nuclear particles is sufficient to penetrate into the dust particle, the dust particle itself can become radioactive, causing secondary electron emission from the dust surface after nuclear conversion. The number of secondary electrons ejected from each original nuclear particle varies from several units up to hundreds. And as a result, the dust particles can acquire a positive charge in nuclear-induced plasma. Dust carrying plasmas in fusion devices is an example of nuclear-induced dusty plasma.

For thermal dusty plasma, the background plasma is produced from the thermal electrons released from the surface of the dust particles under high temperature. The high temperature can be achieved via laser heating or combustion of flammable gases in which the dust particles are introduced from the outside of the plasma, or incineration of solid fuel in which the dust particles are produced along with the formation of the plasma. A candle flame is an example of weakly ionized thermal dusty plasma containing soot particles, whereas the exhaust of a rocket engine is an example of thermal dusty plasma.

For discharge dusty plasma, the background plasma is formed by glow discharge in air or noble gases (i.e. He, Ar, Ne, Kr, or Xe). There are two types of discharge, namely, radio frequency (rf) and direct current (dc). Most dusty plasma experiments performed in laboratory and processing plasmas used in industries are in rf discharges. The rf discharge can be produced using either capacitive parallel plates or inductive coils. Typically, the electron temperature $T_e$ of such discharge ranges from 0.5 to 8 eV and the mean plasma density $n_e$ ranges from $10^7$ cm$^{-3}$ to $10^9$ cm$^{-3}$. For dc discharges, dust can be suspended in regions of high electric fields, such as the cathode sheath, striations, or artificial electric double layers (e.g. anode double layers in Q-machine, magnetic discharge, variable cross-section discharge tube, etc). The electron temperature of such discharge ranges from 1 to 4 eV and the mean plasma density is approximately $10^9$ cm$^{-3}$. The dust particles in rf and dc discharge dusty plasmas will always be charged negatively due to the higher mobility of electrons.

1.1.4. Dusty plasma research

There are many features in dusty plasmas different to the classical ion-electron plasmas. Firstly, as the dust radius $a$ is characteristically much smaller than the plasma Debye length $\lambda_D$ given by:

\[
\frac{1}{\lambda_D^D} = \frac{1}{\lambda_{D_i}^i} + \frac{1}{\lambda_{D_e}^e}
\]  

where \( \lambda_{D_i} = \left( \frac{\varepsilon_0 k T_i}{n_i e^2} \right) \) is the ion Debye length,
\[ \lambda_D = \left( \frac{e_k T_i}{n_e e^2} \right) \] is the electron Debye length,

\( k \) is the Boltzmann’s constant, \( T_i \) and \( T_e \) are the ion and electron temperatures respectively, and \( n_i \) and \( n_e \) are the number densities for ions and electrons respectively, the dust particles can be treated as point particles which hold multiple charges. Together with the singly charged ions and electrons, the multiply charged dust particles make dusty plasma a unique three component plasma system as opposed to its two component classical counterpart. More importantly, unlike classical plasma where the charge of each component remains constant, the charge on the dust particles is itself a dynamical variable, which introduces many new phenomena such as wave damping [46]-[47].

Secondly, because the dust particles may carry a large fraction of the charge from the plasma, the condition for the quasi-neutrality of the plasma must include the effect of the dust particles, that is:

\[ n_i - n_e = Z n_d \tag{1.3} \]

where \( n_d \) is the number density for dust particles. Here the establishment of the equilibrium of dust charges in dusty plasma can be viewed as an analogy to the establishment of the degree of ionization in steady state plasma. If the dust particles are densely packed, it is possible that a considerable fraction of electrons are lost to the dust particles such that \( Z n_d \gg n_e \). In other situations, for example in UV induced plasma, the dust particles can be charged positively by emitting electrons which cause the number of ions to be depleted such that \( Z n_d = -n_e \). So the presence of dust particles can essentially alter the local plasma potential profile by changing the local charge densities.

Thirdly, the charge-to-mass ratio of the dust particles is very small compared to that of the ions and electrons. Therefore, the dynamic response of the dust particles will introduce many new wave phenomena at very low frequencies such as dust-acoustic waves [48]-[52] and dust lattice waves [53].

Fourthly, the charged dust particles can become strongly coupled and the interaction energy of neighboring particles can exceed their thermal energy. This is different to most classical plasmas in space and laboratory which are weakly coupled. Consequently, from the solid state physics point of view, dusty plasma is a convenient system to study phase transitions and to model melting of solids in two- or three-dimensions [17]-[18] [54]-[68].

Lastly, though microscopic in nature, dust clusters and crystals can be easily illuminated with a laser and therefore observed with the naked eye (commonly aided with a microscope). The detailed structure of dust crystals and the motion of the individual particle are clearly visible. This makes dusty plasma an ideal system for the experimental study of particle dynamics and transport properties on the kinetic level.

Not to forget that classical plasma physics has its limitations in accurately describing many real life plasma situations because, commonly and unavoidably, dust particles are also present. Although previously neglected for simplicity, there is a necessity of incorporating dust to obtain a more complete understanding of plasma. And so the accuracy in modeling physical phenomena, the interplay between solid particles and plasma, the ease of production...
and visualization, along with being a perfect hybrid between plasma physics and solid state physics are just some of the useful features that makes the study of dusty plasma one of the faster growing fields of physics. The degree of interest in dusty plasma is evident from the number of published papers in scientific journals and conference proceedings [69]-[71]. The number of people studying dusty plasma is still growing with diverse topics spanning astrophysics, electrical engineering, environmental sciences, material sciences, and meteorology. Of the many groups that are actively participating in research on dusty plasma, the National Aeronautics and Space Administration (NASA) and the Russian Space Agency (RSA) are both major sponsors in the study of dusty plasma under microgravity conditions in space. The first experiment performed on the International Space Station (ISS) was on dusty plasma. Industry giants such as IBM, Sony, and Toshiba teamed together this year to investigate using dusty plasma to control dust in the manufacture of next generation 65nm and 45 nm semiconductor chips and the Playstation 3 graphic synthesizer (GS) chip [72]. Interestingly, dust particles in a plasma are now being implemented as a fabrication tool or a chemical catalyst in many industries, rather than being regarded as a contaminant which historically was the cause of the boom in dusty plasma physics.

1.2. Dust Dynamics and Transport

Dust particles are commonly produced during plasma processing as an undesirable byproduct which is notoriously detrimental to the manufacture of microelectronics. The challenge to remove these dust particles was one of the motivations for the study of dusty plasma. Early dusty plasma experiments had been concentrated on crystal formation, phase transitions, and particle charging mechanism [12]-[15][48][54][73]-[79]. Recently, the interest has shifted towards problems associated with the dynamical behavior of dust particles. In particular, much experimental, theoretical, and modeling effort has been devoted to understanding the transport of dust particles in a plasma. This includes the investigation of the establishment and evolution of various microscopic collective motions excited by external influences such as laser beams, biased electrodes [48]-[49][55][78]-[88]. Such knowledge is essential not only for the discovery of new dust removal techniques, but also for the development of fine manipulation tools for controlling individual or collective movement of micro- and nanoparticles for nanotechnological applications.

1.2.1. Innovations from dust dynamics and transport

Dust contamination is well known to be a common problem in the microelectronics industries, and in almost all surface processing technologies [8][72]. The phenomenon is prominent especially after long periods of plasma operation. Both etching and sputtering tools used in reactive ion etching and magnetron sputtering generate submicron dust particles or agglomerates in the processing plasma. Similarly, polymerizing plasma used in plasma enhanced chemical vacuum deposition (PEVCD) has a strong tendency to form dust particles within the plasma and accumulate flaky films on the walls. These particles or flakes can have detrimental effects on the quality, and therefore the yield of the devices being manufactured if
deposited onto the plasma treated surfaces. Understandably, dust contamination is an impediment that must be overcome in order to increase the quality and efficiency of production.

Electronic devices for which production can be affected by dust contamination include complementary metal oxide semiconductors (CMOS), charge coupled devices (CCD), image sensors, liquid crystal displays (LCD), plasma displays, and solar cells. By developing better dust removal techniques, a company can increase its production efficiency, product quality, and ultimately its competitiveness in the market.

At the moment, clean room technologies using highly sophisticated air filtering systems are used in reducing the amount of dust in the plasma processing environment. Dust contamination which is generated during plasma processing is removed only after they have been deposited onto the plasma treated surface. Wet cleaning is the most common dust removal technique in which dust contamination is dissolved in solvents (e.g. sulphuric, nitric, or hydrofluoric acids, ammonium hydroxide, hydrogen peroxide, etc) and washed away as slurries [89]. However, wet cleaning is undesirable due to cost of chemical purity, safety hazards, storage space constraints, and environmental considerations. Moreover, with future miniaturization in integrated circuit design and manufacturing, the size of dust particles which cause defects will decreases to an extent that wet cleaning becomes ineffective or even problematic (see Figure 1-10). Since the size of particles which adversely affect semiconductor device yields and quality is approximately half that of the circuit pattern size, alternative cleaning methods to remove dust particles of 100 nm or less for the future 65nm and beyond semiconductor devices will be needed. And in the semiconductor industry, such need is projected to occur before 2011.

Dry cleaning methods such as cryogenic aerosol-based cleaning, laser cleaning, and nanoprobe cleaning (see Figure 1-11) are currently being investigated as potential alternatives to wet cleaning [89]. Such techniques avoid many of the problems associated with wet chemical cleaning and are easily integrated into the existing plasma processing procedures. Furthermore, the removal of nanoscale particles without substrate damage or physical alteration can be achieved.

Irrespective of the type of dry cleaning method used, the dust removal procedures are performed only after the particles have been deposited onto the surface of the plasma treated surface. In reality, dust can also be removed during plasma operation when the particles are
suspended in the plasma or after plasma extinction when the particles are falling onto the plasma treated surface. During plasma operation, the dust particles can be removed using suction (e.g. electrostatic vacuum cleaner) or expulsion methods (e.g. gas injection, electrostatic probe, laser pulse, ion drag, etc). After plasma extinction, the dust particles can be circumvented away from the plasma treated surface using transportation methods (e.g. temperature gradient, gas flow). So in essence, the study of dust transports in a plasma provides potential for the discovery of dust removal techniques not yet known.

![Alternative dry cleaning methods](image)

Figure 1-11 – Some of the alternative dry cleaning methods being developed include cryogenic aerosol-based cleaning (left), laser cleaning (center), and nanoprobe cleaning (right) [Courtesy of Sony Semiconductor & LCD].

Apart from dust removal, the ability to maneuver particles in a plasma will allow precise positioning of particles on substrates. In micro- and nano-technology, the construction of novel devices using small components in a plasma can be made possible if the position and motion of individual particle can be accurately controlled. From our preliminary understanding of dusty plasma, the motion of fine particles in the plasma can be influenced using a number of methods which have been documented in literature. For instance, individual particles can be expelled from a chain or layer of particles or set into various types of complex motion using laser light pressure [90]-[91], electrostatic probes [92], electron or ion beams, etc. Particle position can be changed by modulating the applied voltage on the confinement electrodes [93]-[94], the gas pressure, the plasma input power, the temperature gradient, etc.

1.2.2. Dust dynamics in a magnetic field

Dust dynamics under the influence of a magnetic field in various types of dusty plasmas had long been studied in the last quarter of a century. For example, photos obtained by the spacecrafts Voyager I in 1980 and Voyager II in 1981 showed dark radial features moving in a perplexing manner on the B rings of Saturn (see Figure 1-12). These features which appeared both above and below the rings are now known as spokes and are made up of microscopic charged dust particles that have been levitated away from the larger bodies in the ring plane. Since the spokes orbit synchronously with the magnetosphere rather than in Keplerian motion about Saturn, the structure and the motion of the rings are believed to be influenced not exclusively by the gravitational field but also by the magnetic field [95]-[97].

Another intriguing feature about Saturn is the ever changing structure of the F rings.
Images obtained from the Voyager spacecrafts [96]-[98] showed a kinked and braided appearance and detected bright clumps in the F rings with temporal dependence ranging from days to months (see Figure 1-13). Previously, several studies used gravitational interactions with the moons or imbedded moonlets in the F rings to explain such observations. In recent times, the effect of the magnetic field of Saturn is being considered as a possible cause of the changing structure of the F rings [99]. For comparison purpose, the plasma parameters of the B rings, F rings, and spokes of Saturn are shown on Table 1-1.

<table>
<thead>
<tr>
<th></th>
<th>$n_e$ (cm$^{-3}$)</th>
<th>$T_e$ (eV)</th>
<th>Half Thickness of Ring (cm)</th>
<th>$a$ (µm)</th>
<th>$n_d$ (cm$^{-3}$)</th>
<th>$B$ (G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spokes</td>
<td>$\approx 0.1$ to $10^2$</td>
<td>$\approx 2$</td>
<td>$\approx 3 \times 10^6$</td>
<td>$\approx 100$</td>
<td>1</td>
<td>$\approx 0.2$</td>
</tr>
<tr>
<td>B rings</td>
<td>$\approx 0.1$</td>
<td>$\approx 2$</td>
<td>$\approx 3 \times 10^4$</td>
<td>$\approx 10$ to $10^3$</td>
<td>$6.3 \times 10^7$</td>
<td>$\approx 0.2$</td>
</tr>
<tr>
<td>F rings</td>
<td>$\approx 10$</td>
<td>$\approx 100$</td>
<td>$\approx 10^3$</td>
<td>$\approx 100$ to $10^6$</td>
<td>$10^{12}$</td>
<td>$\approx 0.2$</td>
</tr>
</tbody>
</table>

Table 1-1 – The electron density, the electron temperature, the half thickness, the dust size, the dust density, and the magnetic field strength at the spokes, B rings, and F rings of Saturn respectively [101]-[102].

Figure 1-12 – Dark radial features called spokes move in curious patterns on the B rings of Saturn. This is an image of a 35 minute sequence (top to bottom) of the change in spoke appearance. The marked arrow also shows the formation of a new, radially aligned spoke [Courtesy of Calvin J. Hamilton, Views of the Solar System].
Figure 1-13 – The F rings of Saturn have a complex structure of narrow, braided, bright rings along which knots are visible [Courtesy of Calvin J. Hamilton, Views of the Solar System].

Also, it is well known that significant amount of dust particles is present in fusion devices due to flaking, blistering, and violent arcing on chamber walls. These dust particles are a safety hazard as they contain toxic and radioactive materials such as Tritium. Recently, studies showed that dust particles can acquire very high speeds due to accelerated plasma flow in a magnetic field and contaminate the core plasma, which ultimately lowers the performance of fusion devices [100].

In the laboratory, dust dynamics under the influence of an applied magnetic field in various types of discharges and conditions have been reported (see Figure 1-14). In 1996, a numerical analysis to study the effects of the azimuthal ion flow induced by \( E \times B \) drift on single dust particle in a magnetized cylindrical electron cyclotron resonance (ECR) plasma was reported by Nunomura et al. [103]. The numerical analysis, with the support of preliminary experimental results of deposited distribution of silicon dioxide particles and microballoons in an axial magnetic field \( B = 870 \, \text{G} \), showed that the azimuthal ion drag force can cause the dust particle to spin out from the plasma at low neutral pressure or spiral inwards at high neutral pressure.

In 1998, the first observation of dust crystal rotation using an axial magnetic field of variable strength \( B \approx 0 \) to \( 400 \, \text{G} \) was made in a dc argon glow discharge by Sato et al. [104]-[106] (see Table 1-2 for details of experimental parameters). Three-dimensional conical dust crystals consisting of methyl methacrylate-polymer spheres rotated with an angular velocity \( \omega \approx 0.2 \, \text{rads}^{-1} \) in a magnetic field \( B = 400 \, \text{G} \). The direction of rotation was in the left-handed sense with respect to the magnetic field. An increase in either the magnetic field or the plasma density would result in an increase in the angular velocity. The particles in the upper layers were observed to rotate faster than the particles in the lower layers, resulting in a velocity shear in the vertical direction. More importantly, it was found that rotation would start only if the number density of the dust particles exceeded certain threshold.
Figure 1-14 – The chronological chart above shows the historical development in the studies of dust rotation in magnetized plasma. The blue markers show the date at which the manuscripts by various research groups were submitted to their respective scientific journals or conference proceedings. The purple crosses indicate the time when various crystals and clusters rotation experiments were performed by our group. The red arrows mark the four International Conferences on the Physics of Dusty Plasma ever held in the last decade.

<table>
<thead>
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<tbody>
<tr>
<td>$n_e$ (cm$^{-3}$)</td>
<td>$10^7$ to $10^8$</td>
<td></td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$T_i$ (eV)</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Applied Voltage on Electrode - $V_A$ (V)</td>
<td>-</td>
<td>$\pm 15$</td>
</tr>
<tr>
<td>Dust Type</td>
<td>Methyl methacrylate-polymer spheres</td>
<td></td>
</tr>
<tr>
<td>Dust Density (kgm$^{-3}$)</td>
<td>$1.2 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>$a$ (µm)</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$Z$</td>
<td>$-3 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td>$n_d$ (cm$^{-3}$)</td>
<td>$10^3$ to $10^7$</td>
<td>$10^3$ to $10^4$</td>
</tr>
<tr>
<td>$B$ (G)</td>
<td>0 to 400</td>
<td>0 to 3000</td>
</tr>
<tr>
<td>$\omega$ (rads$^{-1}$)</td>
<td>0.2</td>
<td>$\approx 0$ to $3$</td>
</tr>
<tr>
<td>Rotation Direction</td>
<td>Left-handed</td>
<td></td>
</tr>
</tbody>
</table>

Table 1-2 – The experimental conditions, the properties of dust used, the strength of the applied magnetic field, and the magnitude and the direction of crystal rotation in the experiments performed by Sato et al. [104][105][106][118]-[119].
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>n_e (cm$^{-3}$)</td>
<td>$10^9$</td>
<td>$5 \times 10^8$ to $5 \times 10^9$</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>2</td>
<td>2.7 to 3.7</td>
<td>1.5</td>
</tr>
<tr>
<td>n_i (cm$^{-3}$)</td>
<td>$2 \times 10^4$</td>
<td>$2 \times 10^9$</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>$T_i$ (eV)</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Dust Type</td>
<td>Polystyrene spheres</td>
<td>Melamine formaldehyde spheres</td>
<td>Silicon dioxide spheres</td>
</tr>
<tr>
<td>Dust Density (kgm$^{-3}$)</td>
<td>$1.5 \times 10^3$</td>
<td>$1.5 \times 10^3$</td>
<td>$2.5 \times 10^3$</td>
</tr>
<tr>
<td>a (µm)</td>
<td>2.5</td>
<td>4.5</td>
<td>9</td>
</tr>
<tr>
<td>Z</td>
<td>-25</td>
<td>-</td>
<td>$-5 \times 10^4$</td>
</tr>
<tr>
<td>n_d (cm$^{-3}$)</td>
<td>$6 \times 10^2$</td>
<td>$1.3 \times 10^3$</td>
<td>$1.3 \times 10^2$</td>
</tr>
<tr>
<td>B (G)</td>
<td>30</td>
<td>140</td>
<td>2</td>
</tr>
<tr>
<td>$\omega$ (rads$^{-1}$)</td>
<td>0.06</td>
<td>0.3</td>
<td>0.9</td>
</tr>
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<td>Rotation Direction</td>
<td>Left-handed</td>
<td>Left-handed</td>
<td>Left-handed</td>
</tr>
</tbody>
</table>

Table 1-3 – The experimental conditions, the properties of dust used, the strength of the applied magnetic field, and the magnitude and the direction of crystal rotation in the experiments performed by other experimental groups [12][24][111][124]-[125].

Later in the year, Lin I et al. [12][24] made a brief mention of the slow rotation ($\omega \approx 0.06$ rads$^{-1}$) of an intermediate dust cluster using polystyrene particles under a weak magnetic field $B = 30$ G in a capacitive rf discharge (see Table 1-3 for details of experimental parameters). Lin I et al. promised to elaborate more on their experimental observations in future publications, but unfortunately never did.

Subsequently, other groups have reported more cases of dust crystal rotation in the presence of magnetic field. At the beginning of 1999, our group began the experimental investigation in the rotation of large crystals ($\omega \approx 0.1$ rads$^{-1}$ at $B = 130$ G) and three-dimensional clusters ($\omega \approx 0.1$ rads$^{-1}$ at $B \approx 20$ G) using melamine formaldehyde spheres in an inductive rf argon discharge. Although the results obtained were preliminary, this is the first time small cluster systems were observed experimentally to exhibit rotational behavior [107]-[109].

And not long after, Yokota rotated a UV-induced annular crystal consisting of fine aluminum particles about the magnetic dipole axis of a spinning magnetized miniature sphere with field $B \approx 0$ to 50 G in order to mimic some of the mechanisms in planetary ring formation [101][110]. However, unlike in previous experiments, the driving mechanism behind the dust rotation is expected to be mostly due to the neutral friction of the spinning sphere rather than the magnetic field.

Also in the same year, Konopka et al. observed rotation ($\omega \approx 0.3$ rads$^{-1}$) of monolayer dust crystals formed with melamine formaldehyde spheres induced with a cylindrical permanent magnet of strength $B = 140$ G in a capacitive rf helium discharge [111] (see Table 1-3 for details of experimental parameters). Rigid-body rotation in the left-handed sense with respect to the magnetic field was observed for large dust crystals and sheared rotation was observed for annular dust crystals. Similar to the ideas used by Nunomura et al., a model based on the azimuthal component of ion drag force caused by ion drift in the $E \times B$ field was
proposed by Konopka et al. to explain the observed rotation. This model can explain most qualitative features observed in dust rotation experiments. In addition, the model showed that the angular velocity of the dust particles is dependent on the radial profile of the confinement electric field. As a result, Konopka’s model soon gained popularity and became the prominent candidate for explaining dust rotation in a magnetic field. However, the model was not without flaws. Most importantly, despite many attempts at improvement [112]-[115], Konopka’s model still failed to provide a quantitative description of the dust rotation. To be more specific, the calculated value of the ion drag force was sometimes one to two orders smaller than the driving force required for the cluster rotation [109][116].

In 2000, our group began to investigate planar dust clusters under the influence of magnetic field. Because planar dust clusters are two-dimensional and have a smaller number of particles, their simplicity allows for easier analysis and hence a better understanding in the driving mechanisms for dust rotation. Later in the year, Ishihara et al. studied the precession of single dust particle placed in the plasma with the presence of an external magnetic field. In their model, the dust particles rotate about its axis due to the ion drift in the $E \times B$ field, thus generating a magnetic dipole moment, causing it to precess in the magnetic field [117].

At about the same time, Sato et al. extended their dust rotation experiment to not only dc but also rf argon glow discharges with much stronger variable magnetic field up to $B \approx 10000$ G [106][118]-[119] (see Table 1-2 for details of experimental parameters). The motivation about the use of stronger magnetic field is that the dust particles become partially magnetized when $B > \approx 4000$ G. As well, their group managed to rotate dust crystals with particles of different diameters ($a \approx 0.1$ to a few tens $\mu$m), but qualitatively did not observe any effect of particle size on the rotation. Interestingly, Sato et al. also observed the phenomenon of “angular velocity saturation” at $B \approx 4000$ G, the value predicted in the following year by Kaw et al. [118] using collisional fluid theory for magnetized ions and Ishihara et al. [120] who considered the coupling between harmonic oscillations and the Lorentz force on the dust particles.

In 2001, our group performed further experiments and detailed analysis in the rotation of planar dust clusters using melamine formaldehyde spheres of smaller sizes in an inductive rf argon discharge. Moreover, the phenomenon of “periodic pauses” and “angular velocity saturation” using a very low magnetic field strength $B \approx 30$ G were reported [108]-[109][116][121].

In 2002, Shukla developed a model capable of describing dust dynamics in a partially-ionized nonuniform magnetoplasma, taking into account collisional interactions between the plasma particles, pressure gradient and electromagnetic forces [122].

In 2003, Hua et al. [123] observed the rotation and contraction of dust particles in the presence of magnetic field. Later in the year, Kersten et al. [124]-[125] reported their experiments and analysis of the rotation ($\omega \approx 0.9$ rads$^{-1}$) of annular crystals consisting of silicon dioxide spherical particles using a permanent magnet ($B = 2$ G) in a rf discharge coupled with a dc magnetron sputtering device (see Table 1-3 for details of experimental parameters). Also, our group initiated the investigation of intermediate cluster rotation in a magnetic field around this time.

Very recently in 2005, Hou et al. [126] performed some computer simulation on the formation and rotation of planar dust clusters in an axial magnetic field. The cluster rotation was found to be dependent on the particle number, the structural configuration, the discharge pressure, and the magnetic field strength.
Further experimental investigations of dust dynamics under the influence of magnetic field are either undergoing or will be expected in the future. For example, Koepke et al. [127] are preparing to study the dynamics of small diameter dust particles in their large volume “Gold-Tank” plasma using variable magnetic strength up to $B \approx 1000$ G. In such experimental configurations, where the dust mass is minimized, and the dust charge, plasma column diameter, and the Lorentz force are maximized, the dust particles are expected to be completely magnetized.

1.3. Current Theoretical Models on Dust Rotation

The reason the magnetic field causes rotation in dust clusters and crystals is still not properly understood. There are many models in the literature which have attempted to explain this. However, before attempting to identify the cause of the dust rotation, the magnitude of the driving force acting on a particle in the azimuthal direction for rigid-body rotation, $F_d^{\theta}$, can simply be estimated based on Epstein’s well known neutral-dust frictional force equation (under the assumption of complete accommodation of neutrals) provided that the gas is at rest (see Section 4.4.1 for Neutral gas flow) [128]:

$$F_d^{\theta} = -F_{nd} = \frac{4}{3} \delta m_n n_n v_{r_n} \pi a^2 \omega \rho$$

(1.4)

where $\delta \approx 1$ is the coefficient dependent on the type of scattering, $m_n$ is the mass of the neutrals,

$$n_n = \frac{p}{kT_n}$$

is the number density of neutrals,

$$v_{r_n} = \sqrt{\frac{8kT_n}{\pi m_n}}$$

is the average thermal velocity of neutrals,

$p$ and $T_n$ are the pressure and temperature of neutrals respectively, $k$ is the Boltzmann’s constant, $a$ is the dust particle radius, $\omega$ is the angular velocity of the crystal/cluster rotation, and $\rho$ is the radius of the crystal/cluster.

In the following sections, the existing models used in the explanation of dust rotation in a magnetized plasma will be presented.

1.3.1. Konopka’s model

Nunomura et al. [103] first proposed that the azimuthal component of the ion drag force caused by drift of the ions in the $E \times B$ field might influence the transport of dust particles in a magnetic field. The ion drag force model was later adopted by Konopka et al. to explain dust rotation [111] and has since become the predominant explanation for dust rotation.

In Konopka’s model, the cluster rotation is attributed to the momentum transferred from
the azimuthal component of ion drag force on the dust particles (see Figure 1-15 and Figure 1-16). The magnitude of the ion drag force in the azimuthal direction can be estimated using the expression from Barnes et al. [129] which had been widely used in literature:

$$F_{\theta d} = m_i n_i u_z \pi \left( b_C^2 + 4 b^2 \right) u_\theta$$  \hspace{1cm} (1.5)

The definition of the variables in (1.5) are listed as follows: $m_i$ and $n_i$ are the mass and number density of the ions respectively,

$$u_z \approx \sqrt{v_{ti}^2 + u_{z, dist}^2}$$ is the mean velocity of ions,

$$v_{ti} = \frac{8kT_i}{\pi m_i}$$ is the average thermal velocity of ions,

$$u_{z, dist} = -\frac{(\mu_0 / p) E_s}{\sqrt{1 + (\alpha_0 / p) E_s}}$$ is the axial component of the ion drift velocity,

$T_i$ is the temperature of ions, $\mu_0$ is the zero field mobility and $\alpha_0$ is a gas parameter for argon [130], $E_s$ is the sheath electric field,

$$b_C = a \left( 1 - \frac{2e\Phi_s}{m_iu_z^2} \right)$$ is the collection impact parameter,

$$\Phi_s = \frac{1}{4\pi\varepsilon_0} \frac{eZ}{a(1 + a/\lambda_{D_i})}$$ is the dust floating potential relative to the plasma,

$e$ is the elementary charge, $Z$ is the number of electrons on the dust surface, $\varepsilon_0$ is the permittivity of free space, $\lambda_{D_i}$ is the ion Debye length,

$$b_z = \frac{1}{4\pi\varepsilon_0} \frac{e^2Z}{m_iu_z^2}$$ is the orbital impact parameter for scattering angle $\frac{\pi}{2}$ rad,

$$u_{\theta} = \frac{1}{c} \left( \frac{\mu_0 / p}{1 + (\alpha_0 / p) E_s} \right)$$ is the azimuthal component of ion drift velocity,

$$\Gamma = \frac{1}{2} \ln \left( \frac{\lambda_{D_i}^2 + b_z^2}{b_C^2 + b_z^2} \right)$$ is the standard Coulomb logarithm integrated from $b_C$ to $\lambda_{D_i}$,

$c$ is the speed of light, $B$ is the magnetic field strength, and $E_C$ is the confinement electric field.

Note that the ion drag force in (1.5) consists of two components: collection and orbital. The collection component represents the momentum transfer due to ions hitting the surface of the dust particle while the orbital component represents the momentum exchange due to Coulomb collisions of the streaming ions with the dust.
Figure 1-15 – Without magnetic field, the dust particles will experience only the vertical component of the ion drag force.

Figure 1-16 – If an axial magnetic field is applied, the ions will be partially magnetized. As a result, the dust particles will experience an azimuthal component in addition to the vertical component of the ion drag force. According to the prevalent theory, this azimuthal component is believed to be the major driving mechanism for the rotation of dust clusters and crystals.
In Konopka’s model, the collection impact parameter is based on OML approximation [131] and the orbital impact parameter is assumed to have asymptotic orbit angle of $\frac{\pi}{2}$ rad. And the calculation of the Coulomb logarithm for the orbital component, the interval of integration starts from $b_{\min} = b_c$ to $b_{\max} = \lambda_{D_i}$ which means that ions with impact parameter larger than the ion Debye length are excluded from the contribution of the momentum exchange with the dust particle.

### 1.3.2. The choice of Coulomb logarithm in ion drag force estimation

Several theoretical groups [112]-[115] have suggested that (1.5) might be inadequate in providing an accurate quantitative estimation of the ion drag force. This is because unlike electron-ion plasma where the interaction between species is weak and Coulomb collision outside $\lambda_{D_i}$ can be neglected, particles in dusty plasma are highly charged. And a considerable amount of ions is deflected strongly even outside the Debye sphere. Therefore the Coulomb logarithm, which determines the orbital component of the ion drag force, is usually underestimated.

There has been debate about how the estimated value of the Coulomb logarithm could be enlarged [112]-[113]. From one side of the debate, Khrapak et al. [114]-[115] proposed to additionally consider the small angle ion dust collisions with maximum cut-off larger than the plasma Debye length, which can be achieved by the substitution of the standard Coulomb logarithm with a modified Coulomb logarithm $\Lambda$ given by:

$$\Lambda = 2\int_0^{\infty} e^{-\frac{x}{\lambda_{D_i}}} \ln \left( \frac{2\lambda_{D_i}^2 + b^2}{2b_c^2 + b^2} \right) dx$$

in (1.5). Such substitution can lead to a dramatic increase of the orbital force by a factor of up to 40 especially in the case of subthermal ions (i.e. $u_i << v_i$). From the other side of the debate, Piel et al. [132]-[133] argued that the electron Debye length $\lambda_{D_e}$ can be used as the maximum cut-off in the standard Coulomb logarithm as justified by their analysis.

In both approaches, the Coulomb logarithm is enlarged which will in turn increase the calculated value of the ion drag force. For our purposes, the question of which approach should be used can be avoided by taking the Coulomb logarithm $\Gamma \approx 10$ as an upper limit (i.e. assuming that the maximum cut-off is much larger than the orbital impact parameter as in the case of electron-ion plasma). This will in turn give us an upper limit estimation of the ion drag force using the expression from Barnes et al. [129].
1.3.3. Ishihara’s model

Ishihara et al. [120] attempted to use coupling between the Lorentz force and harmonic oscillations as the explanation for the cluster rotation, predicting an angular velocity for cluster/crystal rotation in a weak magnetic field given by:

\[ \omega = \frac{1}{2} (a^2 - 1) \Omega_{L_d} \quad \text{where} \quad \Omega_{L_d} < < \sqrt{\frac{eZ e E}{m_d} \frac{\partial E_c}{\partial \rho}} \]  

(1.7)

where \( a \) is the normalized initial radial displacement, 
\[ \Omega_{L_d} = \frac{e Z B}{m_d} \] is the dust Larmor angular frequency (or dust cyclotron angular frequency), 
\[ \frac{\partial E_c}{\partial \rho} \] is the confinement electric field gradient, 
and \( m_d \) is the mass of the dust particle. Note that according to Ishihara’s model, the angular velocity shows linearly dependency for weak magnetic field.

1.3.4. Kaw’s model

Based on the ion drag force model, Kaw et al. [118] also made predictions of the angular velocity of dust rotation in magnetic field using collisional fluid theory. In their model, the dust rotation is generated by the ion drag force which is balanced by ion momentum loss to neutrals in a steady state. The resulting angular velocity for rigid-body cluster/crystal rotation in a magnetic field is given by:

\[ \omega = \frac{v_d c}{v_{dn} m_i \rho} \left( \frac{\Omega_{L_d}}{\nu_i^2 + \Omega_{L_d}^2} \right) \]  

(1.8)

where 
\[ v_d \approx \frac{m_i}{m_d} n_i \pi \left( b_C^2 + 4 b_{ei}^2 \Gamma \right) u_x \] is the dust-ion collisional frequency [134], 
\[ v_{dn} \approx \frac{m_n}{m_d} n_n \pi a^2 v_{rho} \] is the dust-neutral collisional frequency, 
\[ v_{in} = n_i \sigma_{in} u_x \] is the ion-neutral collisional frequency, 
\( \sigma_{in} \) is the ion-neutral collisional cross-section,
and \( \Omega_{i} = \frac{eB}{m_i} \) is the ion dust Larmor angular frequency (or ion cyclotron angular frequency).

For comparison purpose, (1.8) can be rewritten in the form:

\[
\omega = \frac{\xi_{\text{Kaw}}}{1 + \xi_{\text{Kaw}}^2} \frac{V_{\text{di}} eE_{\text{c}}}{n_{\text{i}} \rho \nu_{\text{in}}} \frac{1}{v_{\text{dn}}} \frac{1}{v_{\text{dr}}} \frac{1}{v_{\text{in}}}
\]

(1.9)

where \( \xi_{\text{Kaw}} = \frac{\Omega_{i}}{\nu_{\text{in}}} \) is the dimensionless Kaw’s angular velocity saturation coefficient. For \( \xi_{\text{Kaw}} < 1 \), the angular velocity of the cluster rotation is linearly dependent on the magnetic field strength. Angular velocity saturation is expected to occur when \( \xi_{\text{Kaw}} = 1 \). And as the magnetic field increases further, that is \( \xi_{\text{Kaw}} >> 1 \), the angular velocity is expected to be inversely proportional to the magnetic field strength.

1.3.5. Shukla’s model

Shukla [122] later developed a dust rotation model based on ion drag force taking into account the coupling of ions with electrons and charged dust particle. Pressure and magnetic field gradients were also incorporated into his model. For comparison purpose with our experimental data, only the expression under the special condition of uniform pressure and magnetic field will be shown here. The angular frequency for rigid-body cluster/crystal rotation in a plasma with uniform pressure and magnetic field is given by:

\[
\omega = \frac{\xi_{\text{Shukla}}}{1 + \xi_{\text{Shukla}}^2} \frac{Zn_{\text{d}}}{n_{\text{i}}} \frac{v_{\text{dr}}}{v_{\text{dn}}} \frac{eE_{\text{c}}}{m_{\text{i}} \rho \nu_{\text{in}}} \frac{1}{v_{\text{in}}}
\]

(1.10)

where \( \xi_{\text{Shukla}} = \frac{Zn_{\text{d}}}{n_{\text{i}}} \frac{v_{\text{dr}}}{v_{\text{dn}}} \frac{\Omega_{i}}{\nu_{\text{in}}} \) is the dimensionless Shukla’s angular velocity saturation coefficient and \( n_{\text{d}} \) is the number density of dust particles. Using Shukla’s model, the angular velocity saturation is expected to occur when \( \xi_{\text{Shukla}} = 1 \). Indeed, (1.10) in Shukla’s model has slight resemblance to (1.9) in Kaw’s model. A major difference is that the angular velocity of the cluster/crystal rotation is expected to show dependencies on the number density and the charge of the dust particles.
1.4. Aim, Outlines, and Outcomes of this Thesis

There is a considerable interest in the understanding of dust dynamics in a plasma under the influence of magnetic field across a number of scientific disciplines. And the growth of this interest is likely to continue due to the possible relevance of dust dynamics in a variety of important industrial applications.

With respect to the dynamics of dust in a magnetic field, dust rotation is probably the most visually impressive phenomenon yet least understood. At the beginning of this research, there were a minimal number of experiments performed on dust rotation in magnetized plasma. And experiments performed were focused on the rotation of large crystal. Thus the main aim of our research is to investigate in details the behavior of dust clusters in magnetized plasma, in particular, it dependencies on the magnetic field strength, number of particles, and size of particles. Dust clusters have been chosen because they consist of small number of particles which provides a simple system for the analysis and identification of the driving mechanisms for dust rotation.

The second aim of our research is to study the packing sequences and fluctuations of dust clusters with and without the presence of a magnetic field, which will provide further insights into the dynamical behavior of dust in magnetized plasma.

The third aim for our research arises from the possibility of particle manipulation in a plasma using a magnetic field which has potential industrial applications. A special chapter at the end of this thesis is devoted to explain several ideas for using magnetic fields in such applications as dust removal, plasma diagnostics, and particle coating.

This thesis has been structured into six major chapters. Apart from the first and second chapters which give an introduction to dust dynamics in a magnetized plasma and a description of our experimental apparatus, the third, fourth, and fifth chapters have been devoted to experimental observations of the formation, the rotation, and the fluctuation of dust particles in a magnetized plasma. Each of these three topics is of special interest in the field of dusty plasma, and the significance of our findings will be illustrated by the sixth chapter which explains the possible innovations in industrial applications.

At the end, this thesis draws out several important points. First and foremost, dust clusters rotate in an inductive rf argon plasma under the influence of an axial magnetic field. The rotational direction of dust clusters in our experiment is in the left-handed sense with the angular velocity dependent on the number of the particles and magnetic field. And the angular velocity does not exhibit the linear dependency with magnetic field as observed in the rotation experiments of large dust crystals. Also, dust clusters exhibit features such as threshold magnetic field, angular velocity saturation, and periodic pauses which have not been observed previously.

Secondly, large dust crystals rotate in an inductive rf argon plasma under the influence of an axial magnetic field. And the rotational direction of large dust crystals in our experiment is in the left-handed sense with the angular velocity directly proportional to the magnetic field strength.

Thirdly, we compare the current theoretical models with our experimental results and show that the models based on ion drag forces provide the best agreement. However, under most experimental conditions, the models are inadequate in providing an accurate quantitative description of the dust rotation, especially for dust clusters.
Fourthly, the packing sequences and the fluctuations of dust clusters are studied both experimentally and by computer simulations. The packing sequences of dust cluster are determined by the interparticle interaction and the confinement potential well. And the packing sequence probability is correlated to the stability of dust clusters.

Fifthly, the magnetic field has an influence on the cluster radius and its configuration due to a modification in the spatial profile of the number densities of electrons and ions, producing a change in the confinement potential profile in plasma. This opens the opportunity of establishing an electrostatic confinement induced by magnetic field. In our experiment, dust particles can be confined over a flat unbiased electrode using magnetic field only.

And lastly, based on our understanding of dust dynamics in magnetized plasma, magnetic fields can be used as a tool to control finely the position and motion of particles in a plasma.
2. The Experimental Apparatus

This chapter provides a detailed description of our experimental apparatus: the plasma chamber and dust particles used to form dust clusters and crystals; the magnetic coil configurations used to rotate the dust clusters and crystals; and the imaging technique used to motion-capture the trajectories of the dust particles.

The plasma chamber used in our experiments was initially built for the investigation of dust crystal solidification and laser-induced oscillations by Prior et al. [90]-[91]. Except for the different components inside the chamber, including the electrode and the magnetic coil, no changes were made to the basic structure of the chamber for our experiments.

2.1. The Plasma Chamber

The experiments were carried out in a stainless steel cylindrical plasma chamber with an internal diameter of 19.5 cm and internal height of 19.5 cm. There were four evenly spaced rectangular ports 5.5 cm wide and 10 cm high on each side (see Figure 2-1). Of the four, two adjacent ports had windows designed for observational purposes. The dust crystals were
observed through one port using a Panasonic WV-BP554 video camera attached to an Olympus SZ-series microscope and illuminated from the other observational port using an 8mW Helium-Neon (He-Ne) laser. A 110 Ls⁻¹ turbo-molecular vacuum pump was connected to one port, and a magnetically coupled dust dispenser was mounted at another. A Penning ionization gauge was used to measure the pressure near the pump inlet.

The top of the chamber was sealed with a quartz plate on top of which was a planar spiral rf coil consisting of 8 turns of 6mm diameter copper tubing potted in araldite. A 6 cm diameter hole in the center of the coil allowed observation of the dust crystal from above using another video camera attached to a microscope similar to the one on the side window. Mesh wire on top of the chamber provided shielding.

The base of the chamber had a feed-through consisting of multiple tungsten pins through a ceramic disc, which provided connections to the electrode and the magnetic coil above it (see Figure 2-2). A height adjustable stainless steel mount, on top of which the confinement electrode and magnetic coil were placed, was attached to the base of the chamber.

![Figure 2-2](image.png)

Figure 2-2 – The magnetic coil was positioned directly underneath the electrode mount. All electrical connections to the electrode and to the magnetic coil were hidden inside the stainless steel mount.

Before each experiment, the chamber was evacuated to a pressure of approximately $10^{-6}$ Torr. A high vacuum valve between the pump and the chamber allowed the chamber to be isolated from the pump during experiments.

The gas was admitted into the vacuum chamber via an inlet port at the top of the chamber. The operating pressure typically ranges from 100 to 1000 mTorr. The rf coil was powered by a Hewlett Packard 3325A synthesizer/function generator and an A-300-RF power amplifier with impedance matching network. A peak-to-peak (p-p) rf voltage was applied to the rf coil.
to produce the background plasma. Using a Langmuir probe, the plasma density $n_e$ and electron temperature $T_e$ were measured to be $\approx 10^{15} \text{ m}^{-3}$ and $\approx 3 \text{ eV}$ respectively.

2.2. The Dust Particles and The Shaker

The dust particles used in the experiments were monodisperse melamine formaldehyde polymer spheres with density of $1.5 \times 10^3 \text{ kg m}^{-3}$ (see Figure 2-3). Two different sizes of dust particles were used in the experiments. The diameter of the larger dust particles was $6.21 \pm 0.09 \mu\text{m}$ and the diameter of the smaller dust particles was $2.71 \pm 0.06 \mu\text{m}$ as specified by the manufacturer [135]. The dust was stored inside the dust shaker.

![Image](Image 2)

Figure 2-3 – Monodisperse melamine formaldehyde particles of diameter $6.21 \pm 0.01 \mu\text{m}$ viewed under a microscope.

The purpose of the dust shaker was to disperse the desired amount of dust particles uniformly into the plasma. The body of the shaker was made from a 1 cm long by 1 cm diameter stainless steel rod with a 6 mm diameter hole bored out at the center (see Figure 2-4). The base of the shaker was also made from stainless steel with a 400 $\mu\text{m}$ hole drilled at the center. In addition, a copper disc with 40 $\mu\text{m}$ diameter hole provided finer control of dust dispersion. The base could be conveniently screwed on and off for dust refills.

![Image](Image 3)

Figure 2-4 – Melamine formaldehyde particles were stored inside a shaker. The shaker was mounted onto a teflon rod which was rotated to disperse the dust particles.
The dust shaker was connected by teflon rod to a magnetically coupled mechanical arm which allowed rotational and horizontal movement of the shaker. A gentle rotation of the magnetically coupled mechanical arm would cause the teflon rod to rotate and the shaker to sprinkle dust into the desired region. Afterwards, the shaker and the rod would be retracted to avoid obstruction to the view of the dust crystal.

2.3. The Making of Dust Clusters and Crystals

The purpose of the confinement electrode was to create an electrostatic potential to trap the dust particles. The electrodes were made from conventional printed circuit board (PCB). Photolithography or mechanical fabrication was used to produce the different patterns of electrically isolated regions on the electrode.

In the case of photolithography (see Figure 2-5), the surface of the PCB was originally coated with a layer of photoresist which becomes soluble when exposed to UV light. Parts of the PCB were covered by a patterned stencil of the electrode (usually a photographic negative) such that the copper area to be removed was not protected from the UV light. The exposed areas of photoresist were then completely dissolved by a solvent and the copper underneath the photoresist was revealed. The PCB was then submerged into an etching tank with strong acidic solution. At the end, copper on parts exposed to UV light would be etched away while the original pattern of the electrode remained on the PCB. The electrical connections to the various regions on the PCB were by pins through the insulating sheet. Electrodes used in both the large and the annular dust crystal experiments were made by such methods.

![Figure 2-5](image)

Figure 2-5 – An electrode made using photolithography: (a) PCB covered with patterned mask and exposed to UV light, (b) Exposed photoresist dissolved by solvent, (c) Revealed copper parts were etched by strong acidic solution, (d) A pattern of the original mask remained on the PCB, (e) The residual photoresist was removed with alcohol.

In the case of mechanical fabrication, circular grooves were milled on the PCB using a computer-controlled milling machine with a 0.5 mm diameter drill. The electrical connections to the various regions on the PCB were also by pins through the insulating sheet. Vapor deposition method was then used to provide a smooth conductive coating over the surface of the PCB (see Figure 2-6). For good surface adhesion and resistance to oxidation, a gold and copper mixture was used. Vapor deposits in between the conductive regions of the PCB board were removed using a sharp surgical blade under a microscope. Electrodes made by this method were of higher quality and were used in dust cluster experiments (see Figure 2-7).
Figure 2-6 – The electrode was coated with a mixture of gold and copper which was evaporated from the high temperature molybdenum boat.

Figure 2-7 – A groove of an electrode viewed under a microscope. Because the pattern of the electrode was milled using a computer-controlled milling machine, the edge of the pattern was sharper than in the case of electrodes made using photolithography.

To make different size dust crystals, different confinement electrode designs and plasma conditions were required. For large dust crystals, the electrode design consisted of a PCB disc of diameter 25 mm which was etched with a circular groove of diameter 9.8 mm. A positive voltage of +6 V was applied to the center while the outer ring was grounded. As the plasma is always positively biased relative to the electrode, applying the positive voltage on the electrode will decrease the negative bias at the center. This in turn will setup a vertical electric field which levitates and a radial electric field which confines the dust particles. The pressure in the chamber was 350 mTorr and a 17.5 MHz, 400 mV p-p signal was applied to the rf coil.

For annular dust crystals, the electrode design consisted of a PCB disc of diameter 25 mm which was etched with two concentric circular grooves of diameter 7.1 mm and 12.3 mm. A positive voltage of +10 V was applied to the intermediate annular region while the center and the outer annular regions were grounded. The pressure in the chamber was 250 mTorr of
For small, intermediate, and three-dimensional dust clusters, the electrode design consisted of a 1 inch square of PCB which was etched with four concentric circular grooves of diameter 5 mm, 9.8 mm, 14.6 mm, and 19.4 mm etched. To create a confinement well, the center was powered while all other regions were grounded. The center ring was used to confine the dust particles in this experiment while the outer rings were made for future purposes. The optimum experimental condition for dust cluster formation is dependent on the structural configuration. Typically, a positive voltage of +10 V would be applied to the center, the pressure in the chamber would be approximately 100 mTorr, and a 17.5 MHz, 500 mV p-p signal was applied to the rf coil.

2.4. The Magnetic Coil

The magnetic coil was designed to reside inside rather than outside of the chamber in order to avoid obstruction of the four ports for the camera, the laser, the magnetically coupled dust dispenser, and the pump.

In early large dust crystal experiments, the magnetic coil consisted of a 5.8 cm diameter by 3.7 cm thick mild steel cylinder with 350 turns of 0.8 mm diameter copper windings around it (see Figure 2-8). Mild steel was chosen initially as the material for the coil core due to its high permeability. The coil could generate a magnetic field strength of up to 120 G with a maximum current of 3 A. The magnetic field was measured with a magnetometer at about the height where crystals formed (see Figure 2-9). And the uniformity of the magnetic field was about ± 0.5 G cm⁻². However, the surface of the magnetic core gradually oxidized which led to outgassing (i.e. the release of gases and vapors from the surface of material which introduces contamination into the system and limits the ultimate pressure achievable) in the vacuum system. Secondly, a considerable amount of heat was dissipated in the coil which created an unnecessary temperature gradient within the system. Even though the stainless steel coil mount would transfer most of the heat away, a better magnet design was needed.

![Diagram of magnetic coil](image)

Figure 2-8 – The core of the magnetic coil used in large dust crystal experiments was made out of mild steel. Electrical connections to the electrode were hidden from the plasma by running through the core center.
Figure 2-9 – The uniformity of the magnetic field produced by the original coil at maximum current of 3 A was measured with a magnetometer 5mm above the mild steel core (near the height where crystals were formed).

In later dust cluster experiments, the magnetic coil consisted of a 15 cm diameter by 1.5 cm thick nickel plated steel core with 4200 turns of 0.5 mm copper windings around it (see Figure 2-10). The coil could generate a magnetic field strength of up to 100 G with a maximum current of 3 A. Although the resistance of the coil was due to both an increase in the wire length and a decrease in the wire thickness, the temperature rise was halved due to the greater size of the core. In addition, the magnetic field was uniform over a larger area $\approx \pm 0.12$ G cm$^{-2}$.due to the larger diameter of the coil (see Figure 2-11).

Figure 2-10 – The core of the magnetic coil used in dust cluster experiments was made out of nickel plated steel. Electrical connections to the electrode were hidden from the plasma by running through the core center.
Figure 2-11 – The uniformity of the magnetic field produced by the larger coil at 2 A was measured with a magnetometer 5mm above the nickel plated steel core (near the height where clusters were formed).

2.5. The Particle Imaging and Tracking system

For imaging purpose, the dust crystal formed above the confinement electrode was illuminated by a height adjustable polarized 8 mW He-Ne laser. Due to the narrow width of the laser beam (up to several times the interparticle distance), only a limited number of particles could be illuminated. So for large crystal experiments, a half-cylindrical perspex lens was placed in front of the laser to spread the beam into a horizontal sheet (see Figure 2-12).

Figure 2-12 – The laser beam was expanded into a horizontal light sheet in order to illuminate large dust crystals.

The motion of the particles in dust crystals can be observed through a microscope by the naked eye or with the use of a video camera operating at 50 frames per second (fps), with an exposure time of 0.008 s. A disadvantage in increasing the beam width of the laser was the reduction in its illumination intensity. The intensity was sufficient for observation purpose,
but the level of contrast created difficulty for computer analysis. The video output was, therefore, passed through a sinc-function signal filter to increase the contrast of the images. The processed images were then recorded on videotapes.

For analysis, the recorded images were transferred to a computer using a video capture card (see Figure 2-13). For large dust crystal and some early dust cluster experiments, a National Instruments PCI-1408 IMAQ acquisition board was used to capture the images from the recorded videos. For later dust cluster experiments, a Pinnacle Studio DC10plus 32-bit PCI board was used.

The motion of the dust particles was tracked using available computer software, which determined the $x$- and $y$-coordinates of the dust particles and stored them as individual text files (in .dat format) for each frame of the video. A macro (single computer instruction that results in a series of instructions in machine language) was written in Microsoft Excel™ which then processed the text files into a single data file (in .xls format) (see Section 8.1 for Appendix I – Macro Used in Data Reorganization).

Figure 2-13 – The dust particles were illuminated by a He-Ne laser and observed through a microscope by the naked eye or with the use of a video camera. The recorded images were transferred to a computer using a video capture card.

Typically in the Microsoft Excel™ data (see Figure 2-14), the first column is time. Then every two columns after the first column are the $x$- and $y$-coordinates of the individual dust particle in a cluster. The number of columns depends on the number of particles in the cluster (e.g. $6 \times 2 = 12$ columns for Planar-6). The number of rows depends on the number of frames captured (e.g. $6 \times 60 \times 2 = 720$ rows for 6 minutes of image sequence captured at 2 fps).
Figure 2-14 – The data file of the x- and y-coordinates of 6 particles in a dust clusters as a function of time. In this case, the motion of the particles was tracked at 2 fps. Knowing the diameter of the confinement electrode, calibration was done to convert the x- and y-coordinates measured in pixels to meters.
3. Structures of Dust Clusters and Crystals

The formation of dust clusters and crystals is one of the most general and basic phenomena in dusty plasmas. Two-dimensional clusters consisting of a finite number of charged particles confined by an external potential are of particular interest as they are classical analog to ion traps [33]-[34] and quantum dots [35]. This chapter is separated into four sections. Section 3.1 and Section 3.2 describes the experiments and the computer simulation on the formation, structural configurations and properties of planar dust clusters. Two different sizes of dust particles were used in the experiment. Section 3.3 describes the formation of particle strings, three-dimensional clusters, and large dust crystals. Section 3.4 describes the confinement of dust particles in a plasma using a magnetic field solely.

3.1. Planar Dust Clusters in Experiments

3.1.1. Summary of experimental conditions

Dust clusters with different numbers of particles in a horizontal plane were formed. In particular, Planar-2 (two particles in a horizontal plane) to Planar-16 dust clusters were formed using large dust particles, hereafter referred to as L-Planar, and Planar-2 to Planar-12 dust clusters were formed using small dust particles, hereafter referred to as S-Planar. For convenience, the experimental conditions used in the formation of L-Planar and S-Planar dust clusters had been summarized in Table 3-1.

<table>
<thead>
<tr>
<th>Planar Dust Clusters</th>
<th>L-Planar Clusters</th>
<th>S-Planar Clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge Type</td>
<td>Inductive rf argon discharge</td>
<td></td>
</tr>
<tr>
<td>$n_e$ (cm$^{-3}$)</td>
<td>$10^9$</td>
<td></td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$T_i$ (eV)</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>Applied Voltage on Electrode - $V_A$ (V)</td>
<td>+ 10</td>
<td></td>
</tr>
<tr>
<td>Dust Type</td>
<td>Melamine formaldehyde polymer spheres</td>
<td></td>
</tr>
<tr>
<td>Dust Density (kgm$^{-3}$)</td>
<td>$1.2 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>$a$ (µm)</td>
<td>3.105</td>
<td>1.395</td>
</tr>
<tr>
<td>$Z$</td>
<td>$\approx 7 \times 10^4$</td>
<td>$\approx 3 \times 10^4$</td>
</tr>
<tr>
<td>$n_d$ (m$^{-3}$)</td>
<td>$\approx 2 \times 10^{10}$ to $3 \times 10^{10}$</td>
<td>$\approx 3 \times 10^{10}$ to $1 \times 10^{11}$</td>
</tr>
<tr>
<td>$p$ (mTorr)</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Applied Voltage on rf Coil</td>
<td>500 mV at 17.5 MHz</td>
<td></td>
</tr>
<tr>
<td>$B$ (G)</td>
<td>0 to 90</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-1 – Summary of the experimental conditions used in formation of planar dust clusters.
3.1.2. L-Planar and S-Planar clusters

**L-Planar Clusters**

**Structural Configuration:**

**Cluster Radius:**

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar-2</td>
<td>199 µm</td>
</tr>
<tr>
<td>Planar-3</td>
<td>242 µm</td>
</tr>
<tr>
<td>Planar-4</td>
<td>289 µm</td>
</tr>
<tr>
<td>Planar-5</td>
<td>392 µm</td>
</tr>
</tbody>
</table>

**S-Planar Clusters**

**Structural Configuration:**

**Cluster Radius:**

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar-2</td>
<td>159 µm</td>
</tr>
<tr>
<td>Planar-3</td>
<td>170 µm</td>
</tr>
<tr>
<td>Planar-4</td>
<td>252 µm</td>
</tr>
<tr>
<td>Planar-5</td>
<td>(5)</td>
</tr>
</tbody>
</table>

Figure 3.1 – Structural configurations and cluster radius of dust clusters formed in our rotation experiments. The diameter of the dust particles is 6.21 µm for L-Planar clusters and 2.79 µm for S-Planar dust clusters.
The structural configurations and the cluster radii (see Section 3.1.3 for *Cluster radius and radial distance*) for the planar dust clusters investigated in the experiments are shown in Figure 3-1. The packing sequences (the number of particles in each ring, starting with the innermost ring) of the dust cluster approximately agree with that observed and predicted by I et al. [24], Bedanov et al. [23], and Schweigert et al. [30].

Planar dust clusters are two-dimensional systems with small number of particles. Due to their simplicity, much detailed analysis can be performed on such systems. Considering the dust particles as point source of mass, the coordinates of the geometrical centre \((x_{cm}, y_{cm})\) of the planar dust clusters at time \(t\) is given by:

\[
x_{cm}(t) = \frac{\sum_{j=1}^{N} x_j(t)}{N}, \quad y_{cm}(t) = \frac{\sum_{j=1}^{N} y_j(t)}{N}
\]

(3.1)

where \(x_j\) and \(y_j\) are the geometric coordinates of the \(j\)th dust particle, and \(N\) is the number of dust particles in the clusters. Incidentally, the geometrical center is also the center of mass for planar dust clusters.

3.1.3. Cluster radius and radial distance

The cluster radius \(\rho\) at time \(t\) is given by:

\[
\rho(t) = \frac{\sum_{j=1}^{N'} R_j(t)}{N'}
\]

(3.2)

where \(R_j = \sqrt{(x_j - x_{cm})^2 + (y_j - y_{cm})^2}\) is the radial distance (the distance between the \(j\)th dust particles in the outer ring and the center of mass) and \(N'\) is the number of particles in the outer ring (see Figure 3-2).

Figure 3-3 shows the time evolution of cluster radius for different L-Planar and S-Planar cluster systems as the magnetic field was increased stepwise by 15 G for every 60 seconds up to 90 G. An increase in magnetic field would lead to an immediate decrease in the cluster radius for both L-Planar and S-Planar clusters (some small delay due to manual adjustment of applied current to the magnetic coil). Also, S-Planar clusters have higher amount of fluctuations in the radius than L-Planar clusters (see Chapter 5 for Fluctuations of Planar Dust Clusters). Note that the magnetic field might not have necessarily changed precisely every 60 seconds. Therefore, the time at which the magnetic field changed was found by looking at the change of steps in Figure 3-3.

Figure 3-4 and Figure 3-5 shows the time averaged cluster radius dependence on the magnetic field strength for L-Planar and S-Planar clusters respectively. As shown, the time averaged cluster radius decreased as the magnetic field strength increased for both L-Planar and S-Planar clusters. The change in cluster radius is due to the modification of confinement...
electric field by magnetic field (see Section 3.4.2 for *The effect of axial magnetic field on confinement electric field*).

Figure 3-2 – The cluster radius is the average distance between the dust particles in the outer ring and the center of mass.

Figure 3-3 – Radius of L-Planar (left) and S-Planar (right) clusters plotted against time. Here the magnetic field was increased stepwise by 15 G every 60 seconds. Due to the larger fluctuations of the S-Planar clusters, the step response of the cluster radius is not as apparent.
Figure 3-4 – The time averaged cluster radius of L-Planar clusters decreased when the magnetic field strength was increased.

Figure 3-5 – The time averaged cluster radius of S-Planar clusters decreased when the magnetic field strength was increased.

Figure 3-6 shows the time averaged cluster radius dependence on the number of particles for L-Planar clusters. The increase in the radius of single ring clusters like L-Planar-2, -3, and -4 was large and apparent. “One-and-a-half ring” clusters such as L-Planar-6, -7 and -8, which consisted of a single ring with one dust particle at the centre, experienced only a slight increase in the cluster radius as the number of particles increased. Double ring clusters such as L-Planar-10, -11 and -12 experienced almost no change in the cluster radius. And as more
particles were introduced, the size (and the cluster radius) of the dust clusters became more
dependent on the number of rings rather than the number of particles. Similar trend in the
time averaged cluster radius dependence on the number of particles is expected for S-Planar
clusters but was not apparent in Figure 3-7 due to the lack of data.

Figure 3-6 – The time averaged cluster radius of L-Planar clusters increased as the number
of particles increased. But as more particles are introduced, the cluster radius becomes more
dependent on the number of rings rather than the number of particles.

Figure 3-7 – The time averaged cluster radius of S-Planar clusters increased as the number
of particles increased.
3.1.4. Metastable states

Dust clusters are recognized as achieving consistent packing sequences after sufficient settling time. Such a packing sequence characterized by the approximate cylindrical symmetry of the confinement potential as the particles establish themselves in concentric rings is called a ground state which corresponds to the global minimum cluster energy (see Section 3.2.2 for Cluster energy). Dust clusters could also alternate between different packing sequences with the movement (or structural transition) of an inner particle to the outer ring or vice-versa. Such packing sequences are called excited states and are normally short-lived as the cluster will quickly switch back to its ground state.

However, there are instances where some clusters seemed more open to structural changes. That is, some clusters can remain in two or more stable configurations. Such stable configurations are called metastable states and had been observed previously in other numerical experiments [23][30].

In our experiments, only one stable configuration was observed for dust clusters with 2 to 4 particles. The dust particles in other L-Planar and S-Planar clusters arranged themselves into one particular ground state configuration especially when the magnetic field was high. However, when the magnetic field was absent or low, some L-Planar and S-Planar have metastable states. For example, in the case of S-Planar-5, two stable configurations had been observed with packing sequences (5) and (1,4) (see Figure 3-8). Similarly, L-Planar-12, S-Planar-8, -9, -10, and -12 each have two metastable states.

![Figure 3-8](image-url)

Figure 3-8 – The packing sequences of metastable states in some S-Planar cluster systems are shown.

To gain further insight into cluster systems with metastable states, the radial distances can be monitored over a period of time. Figure 3-9 shows the radial distances for S-Planar-12 over a 50-seconds interval when the magnetic field was at 15G. Initially, the S-Planar-12 (3,9) system had three particles in the inner ring and nine particles in the outer ring. However, at 26th second, a particle in the outer ring moved into the inner ring and changed the cluster into

![Figure 3-9](image-url)
a L-Planar-12 (4,8) configuration (see Figure 3-10). Later at 34th second, a particle from the inner ring moved into the outer ring while another particle moved from the outer ring into the inner ring. Similar structural transitions in L-Planar-12 occurred at several other times during the 60 seconds of rotation at 15G.

Figure 3-9 – During the rotation of L-Planar-12 at 15G, structural transformation between the metastable states (3,9) and (4,8) was observed. At times, a particle from the inner ring will move to the outer ring or vice-versa (as highlighted by the red arrows).

Figure 3-10 – Metastable states of L-Planar-12 observed at low magnetic field strength of 15G. (Left) At 26th second, L-Planar-12 arranged itself into packing sequence (3,9). (Right) At 34th second, L-Planar-12 arranged itself into packing sequence (4,8).
3.2. Planar Dust Clusters in Computer Simulation

A computer simulation was setup to model the formation of planar dust clusters with different number of dust particles (see Section 8.2 for Appendix II – Routines for Computer Simulations). The computer model we use represents each particle as point sources of mass and charge. The particles start randomly arranged around the potential well. Four forces are then employed to govern the movement of each particle. They are:

- Coulomb (Debye) repulsion,
- Potential well confinement,
- Small stochastic force, and
- Frictional force

Our model used discrete time steps to determine the evolution of the dust particles. At each step in time, the new position and velocity of each particle is calculated based on the force vector at its current position and velocity. Depending on the choice of the time step and the dissipation constant, an appropriate time was set to end the simulation once the particles settle down into their equilibrium position.

The trajectory of the particles was recorded and stored in multi-dimensional matrix format. The eccentricity, inter-ring twist, packing sequence probability, and cluster energy were then analyzed. All results derived are from computer simulations of coulomb clusters. All the coding was done in Matlab and has been included in the appendix.

Three different algorithms were tested to see which algorithm performed best. These were:

- Euler method,
- Beeman method, and
- Verlet method

All three methods were used to run a trial simulation, each starting with the same initial conditions. Two factors were taken into account in determining the best algorithm. They are:

- Run time
- Accuracy

We saw extremely close agreement between all three methods which indicates their approximate equal accuracy of calculation. Comparing the run times of each method we found the following Euler algorithm is 2 orders faster than those of Beeman and Verlet. Therefore the Euler algorithm was chosen to perform our simulations.

Note that the units of measurement used have been scaled up (see Section 3.2.2 for Cluster energy and Section 8.2.7 for Subroutine – rescale) to prevent the calculation of extremely small values. Once the analysis has been completed the values are rescaled to SI units using a scaling law that we have derived.

3.2.1. Cluster eccentricity and inter-ring twist

The outer ring of a planar dust cluster can be approximated by an ellipse with semi-minor axis \(a\), and semi-major axis \(b\) (see Figure 3-11). We define the cluster eccentricity \(e_C\) as:
For a perfectly circular cluster, the cluster eccentricity is zero. The cluster eccentricity provides a macroscopic description of the particle geometry in a cluster.

\[
e_c = \sqrt{1 - \left(\frac{b}{a}\right)^2}, \quad 0 \leq e \leq 1
\]  

(3.3)

Figure 3.11 – For a perfectly circular dust cluster, its cluster eccentricity is zero. For an elliptical dust cluster, its eccentricity can have value in between zero to one.

Next, we consider the inter-ring rotation in a planar dust cluster. We define the primary inter-ring twist \(\Theta\) of a planar dust cluster as the average of the differences in angular positions between every particle on the innermost ring with its closest anti-clockwise neighboring particle in outermost ring (see Figure 3.12), that is:

\[
\Theta = \frac{1}{\text{Number of Innermost Ring Particles}} \sum_{i} |\theta_i - \theta_\text{Closest Anti-clockwise Neighboring Particle in Outermost Ring}|
\]  

(3.4)

In a similar manner, the definition of the inter-ring twist can be extended to other cluster system with multiple rings (e.g. secondary inter-ring twist of the second outermost ring, etc). In contrast with cluster eccentricity, the inter-ring twist provides a microscopic description of the particle geometry in a cluster.

Figure 3.12 – The above diagram shows Planar-8 cluster with (right) and without inter-ring twist (left).
In our computer simulation, only the primary inter-ring twist of double ring clusters was considered (see Section 8.2.9 for Subroutine – inter-ring twist). So for simplicity, we will refer this as simply inter-ring twist.

3.2.2. Cluster energy

Planar clusters are two-dimensional system. Therefore all dust particles can be considered as carrying equal negative charge (due to the balance of gravitational and electrostatic force in the vertical direction \(mg = eZ\). For a two dimensional cluster system with \(N\) particles of equal charges, the \(j\)th particle will have Coulomb potential energy due to its neighboring particles:

\[
U_{ip_j} = \frac{e^2Z^2}{4\pi\varepsilon_0} \sum_{i=1,i\neq j}^{N} \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}
\]  

(3.5)

In addition, while sitting on a parabolic potential well, the \(j\)th particle will have confinement potential energy:

\[
U_{C_j} = \alpha(x_j^2 + y_j^2)
\]  

(3.6)

where \(\alpha\) is an arbitrary constant dependent on the gradient of the potential well (the potential energy at the origin is taken as zero). Therefore, the total potential energy of each particle in a dust cluster or the cluster energy is given by:

\[
U_j = U_{C_j} + U_{ip_j} = \alpha(x_j^2 + y_j^2) + \frac{e^2Z^2}{4\pi\varepsilon_0} \sum_{i=1,i\neq j}^{N} \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}
\]  

(3.7)

For convenience, we introduce the following units: \(x_0 = \left(\frac{e^2Z^2}{4\pi\varepsilon_0}\right)^{\frac{1}{3}} \alpha^{\frac{1}{3}}\) and \(y_0 = \left(\frac{e^2Z^2}{4\pi\varepsilon_0}\right)^{\frac{1}{3}} \alpha^{\frac{1}{3}}\)

for length and \(U_0 = \left(\frac{e^2Z^2}{4\pi\varepsilon_0}\right)^{\frac{1}{2}} \alpha^{\frac{1}{3}}\) for the energy. Our results will be given in these units from now on and (3.7) can be rewritten in the simple form:

\[
U_j = (x_j^2 + y_j^2) + \sum_{i=1,i\neq j}^{N} \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}
\]  

(3.8)

which no longer depends explicitly on the value of \(Z\) or \(\alpha\).

If the potential well is not circular but has some eccentricity \(e_P\), then cluster energy is given by:
Note that although the eccentricity of the confinement potential $e_p$ is different to the cluster eccentricity $e_C$. A perfectly circular potential well can just as well produce elliptical dust clusters (see Figure 3-13). Although, the eccentricity of the confinement potential is expected to have some influence on the cluster eccentricity (see Section 3.2.4 for Packing sequences).

Figure 3-13 – Even in a perfectly circular potential well, the Planar-10 (2, 8) cluster is elliptical.

3.2.3. Our model for computer simulations

At the start of the simulation, each of the $N$ particles will be positioned randomly around the potential well using a Gaussian distribution for generating both the $x$- and $y$-coordinates. This simulates the random placement of particles around a target area by particle injection. Depending on the position of the particle, each will experience a different force vector. This force vector is calculated for each particle from four different contributing forces, namely: confinement force $F_C$, interparticle force $F_{IP}$, neutral drag force $F_{nd}$, and stochastic force $F_S$.

The interparticle force which the $j$th particle experience is given by:

$$F_{IP} = \frac{e^2 Z^2}{4 \pi \varepsilon_0} \sum_{i=1, i \neq j}^{N} \frac{1}{\left( x_i - x_j \right)^2 + \left( y_i - y_j \right)^2} \quad (3.10)$$

Moreover, the confinement force that the parabolic potential well exerts on the $j$th particle in the $x$- and $y$-direction is given by:

$$F_{C_{xj}} = -\frac{\partial U_{C_j}}{\partial x} = -2\alpha x_j \quad (3.11)$$

$$F_{C_{yj}} = -\frac{\partial U_{C_j}}{\partial y} = \frac{2\alpha y_j}{1 - e_p^2} \quad (3.12)$$
And the neutral frictional force is given by:

\[ F_{nd} = -\kappa u_d \]

(3.13)

where \( \kappa = (4/3)m_p n sv_T \pi a^2 \) is the dissipation coefficient and \( u_d \) is the dust drift velocity. The neutral drag force models the frictional dissipation of the dust particles in the system, which is necessary in order for the particles to settle down into their equilibrium position. In our simulations, the dissipation coefficient was varied from 0.1 to 10 sNm\(^{-1}\).

In addition, a stochastic force that fluctuates randomly (with Gaussian statistical distribution) in its magnitude of projections along both \( x \)- and \( y \)-direction was implemented into the model. A scalar coefficient was used to control the distribution width. In polar coordinates system, the occurrences of stochastic force are expected to have no preference in direction but otherwise in magnitude. Figure 3-14 shows the frequency of occurrence of stochastic force of specific magnitude and direction tabulated over many simulations. And as expected, the occurrence of stochastic force is about the same for all angles. The magnitude of stochastic force follow the Poisson distribution \( P(n_b) \) given by:

\[ P(n_b) = \frac{\lambda^{n_b} e^{-\lambda}}{n_b !} \]

(3.14)

where \( n_b \) is the bin number and \( \lambda \) is the data mean. The Poisson distribution is expected for the occurrences of data which are discrete and random.

Figure 3-14 – Frequency of occurrence of stochastic force was plotted as a function of magnitude (blue solid line) and as a function of direction (red solid line). The range was divided into 20 equal bins, where the frequency of occurrence was calculated within each bin. As expected, the occurrences of stochastic force have no preference in the direction. The magnitude of stochastic force appeared to have a Poisson’s distribution (dashed line).
The code for our modeling was written in MATLAB which computes the time steps in the evolution of the dust particles based on the force vector at its current position and velocity (see Section 8.2.1 for Main routine – planar cluster simulation). Depending on the choice of the time step and the dissipation constant, an appropriate time was set to end the simulation once the particles to settle down into their equilibrium position.

3.2.4. Packing sequences

By running thousands of simulations using different numbers of particles (see Section 8.2.5 for Subroutine – packing sequence), the frequency of occurrence in the packing sequence was obtained for various planar cluster systems. The packing sequence probability was then calculated by normalizing the frequency of occurrence to 1. Figure 3-15 shows the packing sequence probability for planar cluster systems with 2 to 15 particles. A plot of the packing sequence probability versus the number of particles is also available later in the comparison of total instability coefficient (see Section 5.2.4 for Total instability coefficient versus number of particles).

Our results show that planar cluster systems with less than 5 particles can have one and only one possible packing sequence which corresponds to its ground state. For planar cluster systems with 6 to 17 particles, the trend in packing sequence probability with the number of particles seems to be periodic. That is, for planar cluster systems with odd number of particles, there exists a predominant configuration with packing sequence probability greater than 94%. For planar cluster systems with even number of particles, there exist two metastable states with packing sequence probability greater than 10%. In particular, the two metastable states of Planar-10, -12, and -14 have very close packing sequence probability. Planar cluster systems with more than 17 particles have an additional ring in their configuration which allows them to form multiple metastable states. And the trend in packing sequence probability with the number of particles becomes more complicated.

The eccentricity of the confinement potential is expected to have some influence on the shape of the dust clusters and hence their packing sequence probability. Figure 3-16, Figure 3-17, and Figure 3-18 shows the packing sequence probability of Planar-5, -8, and -11 clusters respectively as a function of eccentricity of confinement potential ranging from 0 (circular) to 0.4 (elliptical). Note that higher values of confinement potential eccentricity will cause our code to incorrectly identify different packing sequences, thereby producing erroneous data. As shown, an increase in the confinement potential eccentricity will increase the packing sequence probability in favor of cluster configurations with lower degree of symmetry (i.e. elliptical configuration).

Taking Planar-8 as an example, the packing sequence probability of (2,6), which is more elliptical than (1,7) due to the two inner particles stretching the outer ring in one direction, increases as the eccentricity of the confinement potential increases. And for Planar-5, the packing sequence probability of (5), which is easier to fit in an elliptical potential well than (1,4), also increases as the eccentricity of the confinement potential increases. As a matter of fact, judging from the metastable states observed in our experiment particularly with Planar-5 (1,4), our confinement potential can be qualitatively deduced to be quite circular.
<table>
<thead>
<tr>
<th>Cluster System</th>
<th>Number of Particles at the Center of the Cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>None</td>
</tr>
<tr>
<td>Planar-2</td>
<td></td>
</tr>
<tr>
<td>Planar-3</td>
<td></td>
</tr>
<tr>
<td>Planar-4</td>
<td></td>
</tr>
<tr>
<td>Planar-5</td>
<td></td>
</tr>
<tr>
<td>Planar-6</td>
<td></td>
</tr>
<tr>
<td>Planar-7</td>
<td></td>
</tr>
<tr>
<td>Planar-8</td>
<td></td>
</tr>
<tr>
<td>Planar-9</td>
<td></td>
</tr>
<tr>
<td>Planar-10</td>
<td></td>
</tr>
<tr>
<td>Planar-11</td>
<td></td>
</tr>
<tr>
<td>Planar-12</td>
<td></td>
</tr>
<tr>
<td>Planar-13</td>
<td></td>
</tr>
<tr>
<td>Planar-14</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-15 – The diagram above shows some of the packing sequences obtained from our computer simulations for different planar cluster systems. The number on the bottom of the packing sequence is the packing sequence probability. Also the more probable a particular packing sequence is, the whiter its diagram will be.
Figure 3-16 – Packing sequence probability versus potential eccentricity for Planar-5 cluster.

Figure 3-17 – Packing sequence probability versus potential eccentricity for Planar-8 cluster.

Figure 3-18 – Packing sequence probability versus potential eccentricity for Planar-11 cluster.
In addition to the packing sequence probability, the cluster radius of the different planar clusters was also calculated. Figure 3-19 shows the cluster radius dependence on the number of particles for planar cluster systems. The trend obtained from our simulations is in good agreement with the values measured in experiment.

![Figure 3-19](image)

Figure 3-19 – Cluster radius versus number of particles obtained from experiment and simulation. The cluster radii from simulation have been normalized against the cluster radius of Planar-2 measured from experiment.

3.2.5. Inter-ring twist spectrum

Although computer simulations of planar cluster systems had been performed previously by other research groups [23][30][31][136], only little attention [137] had been given to the inter-ring twist of these structures. In this section, the inter-ring twist spectrum of two packing sequences in a Planar-10 cluster system namely, (3,7) and (2,8), are investigated. Note that for the packing sequence (3,7), the twist angle can range from 2π/21 to 4π/21 rad. For packing sequence (2,8), the twist angle can range from 0 to π/4 rad.

Firstly, we consider a perfectly circular Planar-10 cluster system in our model. The particles in such cluster system are distributed into the inner and outer ring with eccentricity \( e_c = 0 \). The relationship between the cluster energy of such system as a function of inter-ring twist was derived for packing sequence (3,7) and (2,8) (see Section 8.2.6 for Subroutine – cluster energy) and plotted in Figure 3-20 and Figure 3-21 respectively. For packing sequence (3,7), the minimum cluster energy occurs at \( \Theta = 2\pi/21 \) rad. For packing sequence (2,8), the minimum cluster energy occurs at \( \Theta = \pi/8 \) rad. Theoretically, these twist angles are favored most as the particles which are dynamical will continue to twist from higher cluster energies until they reach a stable equilibrium.
Figure 3-20 – The cluster energy versus inter-ring twist for packing sequence (3,7) ($e_C = 0$). The structural configurations of the packing sequence with inter-ring twist $2\pi/21$ and $3\pi/21$ are drawn along with the plot.

Figure 3-21 – The cluster energy versus inter-ring twist for packing sequence (2,8) ($e_C = 0$). The structural configurations of the packing sequence with inter-ring twist 0 and $\pi/8$ are drawn along with the plot.

Next, from our computer simulations of planar-10 (see Section 8.2.5 for Subroutine – packing sequence), the frequency of occurrence in the inter-ring twist (see Section 8.2.9 for Subroutine – inter-ring twist) was obtained for different values of dissipation coefficient $\kappa$. After normalization, Figure 3-22 and Figure 3-23 shows the inter-ring twist spectrum for packing sequences (3,7) and (2,8) respectively.
Figure 3-22 – One predominant peak was observed at $2\pi/21$ rad in the inter-ring twist spectrum of Planar-10 with packing sequence (3,7). The red line corresponds to $\kappa = 1.0$ kgs$^{-1}$, the green line corresponds to $\kappa = 0.1$ kgs$^{-1}$, and the blue line corresponds to all values of $\kappa$.

Figure 3-23 – Two peaks were observed at 0 and $\pi/8$ rad in the inter-ring twist spectrum of Planar-10 with packing sequence (2,8). The red line corresponds to $\kappa = 1.0$ kgs$^{-1}$, the green line corresponds to $\kappa = 0.1$ kgs$^{-1}$, and the blue line corresponds to all values of $\kappa$. 
For packing sequence (3,7), the inter-ring twist spectrum shows a very distinct peak at $\Theta = 2\pi/21$ rad as expected from our model for all values of dissipation coefficients. The peak $\Theta = 2\pi/21$ rad is sharper and more symmetrical as the dissipation coefficient becomes smaller (i.e. lower neutral pressure). A small peak appeared at $\Theta = 3\pi/21$ rad as well. We believe the small peak at $\Theta = 3\pi/21$ is a metastable state which appears when dissipation coefficient is high. For packing sequences (2,8), the inter-ring twist spectrum showed two distinct peaks at $\Theta = 0$ and $\pi/8$ rad within a broad spectrum. Moreover, the maximum peak of the spectrum is at $\Theta = 0$ especially when the dissipation coefficient becomes smaller.

The occurrence of 2 stable states for packing sequence (2, 8) with one of them at $\Theta = 0$ contradicts our energy-twist relationship obtained in Figure 3-21. Apparently, the contradiction arises from the assumption of zero eccentricity in the Planar-10 cluster system which appears to provide an inaccurate description of the interparticle interaction and cluster energy (see Section 3.2.7 for Energy and inter-ring twist relation). Therefore the effect of eccentricity on the inter-ring twist will be investigated (see Section 3.2.7 for Influence of cluster eccentricity on inter-ring twist and energy).

3.2.6. Energy and inter-ring twist relation

![Cluster Energy Dependence on Inter-Ring Twist](image)

Figure 3-24 – The cluster energy dependence on the inter-ring twist obtained from simulations (top) was opposite to that expected if the Planar-10 (2,8) cluster was assumed to be perfectly circular (bottom).
Based on the energy-twist relation that was derived for a perfectly circular planar-10 cluster system, and the inter-ring twist spectrum, one can see the favorable twist angle corresponds to maximum cluster energy, which seems to be in contradiction with the fact that system seeks the minimum energy state. Figure 3-24 shows the energy (see Section 8.2.6 for Subroutine – cluster energy) of cluster with packing sequence (2,8) obtained from our simulations as a function of inter-ring twist. The resultant energy-twist relation is completely opposite to the trend obtained for a perfectly circular planar-10 cluster system. In fact, $\Theta = 0$ corresponds to the minimum cluster energy which explains the peak found in the packing sequence (2,8) inter-ring twist spectrum. This is shown in the next section that the cluster adjusts its twist angle to accommodate for the change in eccentricity.

3.2.7. Influence of cluster eccentricity on inter-ring twist and energy

It is indisputable that the simplistic model based on a perfectly circular Planar-10 cluster system does not describe the interparticle interaction and cluster energy accurately. The only way to minimize energy for a given configuration without compromising the inter-ring twist is by changing the cluster eccentricity. Through this change of cluster eccentricity, the particles are able to compensate for the additional energy caused by the inter-shell rotation by further reducing their radial distance as much as possible. Therefore the favorable inter-ring twist changes from $\pi/8$ rad for a circular cluster to 0 for an elliptical cluster. Since the particles in a planar cluster system are dynamical, they will seek inter-ring twist corresponding to the minimum cluster energy.

This explanation is further confirmed by Figure 3-25 which shows the cluster eccentricity as a function of inter-ring twist for the packing sequence (2,8). An increase in the inter-ring twist corresponds to a decrease in the cluster eccentricity. The cluster eccentricity dependence on inter-ring twist can be approximated by the empirical relation:

$$e_c = -0.72\Theta^2 + 0.05\Theta + 0.64$$  \hspace{1cm} (3.15)
As shown in Figure 3-26, the cluster energy decreases with cluster eccentricity. This is in agreement with the fact that clusters with some eccentricity is more favorable ($e_c \approx 0.64$). Supplement to Figure 3-25, the zero twist angle corresponds to the preferred cluster eccentricity, which explains the $\Theta = 0$ peak in the inter-ring twist spectrum for packing sequence (2,8).

![Figure 3-26](image)

*Figure 3-26 – In general, the cluster energy decreases as the cluster eccentricity increases.*
### 3.3. Other Types of Dust Clusters and Crystals

#### 3.3.1. Summary of experimental conditions

The experimental conditions used in the formation of other types of dust clusters, circular crystals, and annular crystals had been summarized in Table 3-2 and Table 3-3.

<table>
<thead>
<tr>
<th>Other Types of Dust Clusters</th>
<th>Particle Strings &amp; Three-dimensional Clusters</th>
<th>Intermediate Clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Discharge Type</strong></td>
<td>Inductive rf argon discharge</td>
<td></td>
</tr>
<tr>
<td>$n_e$ (cm$^{-3}$)</td>
<td>$10^9$</td>
<td></td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$T_i$ (eV)</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td><strong>Applied Voltage on Electrode - $V_A$ (V)</strong></td>
<td>+ 8 to 13</td>
<td>+ 0 to 25</td>
</tr>
<tr>
<td><strong>Dust Type</strong></td>
<td>Melamine formaldehyde polymer spheres</td>
<td></td>
</tr>
<tr>
<td><strong>Dust Density (kgm$^{-3}$)</strong></td>
<td>$1.2 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>$a$ ($\mu$m)</td>
<td>3.105</td>
<td></td>
</tr>
<tr>
<td>$Z$</td>
<td>$\approx 7 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td>$n_d$ (m$^{-3}$)</td>
<td>$\approx 2 \times 10^{10}$ to $3 \times 10^{10}$</td>
<td>$\approx 1 \times 10^{10}$</td>
</tr>
<tr>
<td>$p$ (mTorr)</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td><strong>Applied Voltage on rf Coil</strong></td>
<td>500 mV at 17.5 MHz</td>
<td></td>
</tr>
<tr>
<td>$B$ (G)</td>
<td>0 to 90</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-2 – Summary of experimental conditions used in the formation of other clusters.

<table>
<thead>
<tr>
<th>Dust Crystals</th>
<th>Large Crystals</th>
<th>Annular Crystals</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Discharge Type</strong></td>
<td>Inductive rf argon discharge</td>
<td></td>
</tr>
<tr>
<td>$n_e$ (cm$^{-3}$)</td>
<td>$10^9$</td>
<td></td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$T_i$ (eV)</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td><strong>Applied Voltage on Electrode - $V_A$ (V)</strong></td>
<td>+ 6</td>
<td>+ 10</td>
</tr>
<tr>
<td><strong>Dust Type</strong></td>
<td>Melamine formaldehyde polymer spheres</td>
<td></td>
</tr>
<tr>
<td><strong>Dust Density (kgm$^{-3}$)</strong></td>
<td>$1.2 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>$a$ ($\mu$m)</td>
<td>3.105</td>
<td></td>
</tr>
<tr>
<td>$Z$</td>
<td>$\approx 7 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td>$n_d$ (m$^{-3}$)</td>
<td>$\approx 5 \times 10^{10}$</td>
<td>$\approx 1 \times 10^{11}$</td>
</tr>
<tr>
<td>$p$ (mTorr)</td>
<td>350</td>
<td></td>
</tr>
<tr>
<td><strong>Applied Voltage on rf Coil</strong></td>
<td>400 mV at 17.5 MHz</td>
<td>250</td>
</tr>
<tr>
<td>$B$ (G)</td>
<td>130</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-3 – Summary of the experimental conditions used in the formation of dust crystals.
3.3.2. Particle strings and three-dimensional clusters

The applied voltage on the confinement electrode was set at $+10$ V. Large dust particles were dispersed into the plasma sheath above the circular electrode. One-dimensional particle strings and three-dimensional clusters were formed $\approx 5$ mm above the confinement electrode. The structural configurations of particle strings and three-dimensional clusters investigated in our experiments are drawn in Figure 3-27.

![Diagram of particle strings and clusters](image)

Figure 3-27 – Drawn structural configurations of the particle strings and three-dimensional dust clusters formed in our rotation experiments using 6.21 $\mu$m diameter melamine formaldehyde spheres.

3.3.3. Large crystals and annular crystals

Approximately a thousand dust particles were dispersed into the plasma sheath, forming a large crystal with two layers $\approx 5$ mm above the circular electrode (see Figure 3-28). The diameter of the crystal was about the same size as that of the confinement electrode, which was $\approx 10$ mm in diameter. The top layer of the crystal was consisted of a combination of hexagonal and pentagonal structures. The interparticle distance was about 0.27 mm. Within the large crystal, the dust particles display small random fluctuations around its position. At times, dust particle might exchange position with their neighboring particles.

In the case of annular crystal, similarly about a thousand dust particles were dispersed into the plasma sheath. However, due to the annular shape of the confinement potential, an annular crystal with two layers of particles was formed $\approx 5$ mm above the electrode (see Figure 3-29). The inner and outer diameters of the annular crystal were $\approx 6$ mm and $\approx 12$ mm respectively. In other words, the width of the annular crystal was $\approx 3$ mm with an interparticle separation of 0.22 mm. The top layer of the crystal was consisted of a combination of...
hexagonal and pentagonal structures. When the magnetic field was off, the particles in the annular crystal exhibited random fluctuations similar to those observed in large crystals.

Figure 3-28 – Looking over the top, the large crystal consisted of more than a thousand particles dispersed into two layers.

Figure 3-29 – Due to the shape of the annular region of electrostatic confinement, an annular crystal with a central void was formed.
3.4. Magnetically Induced Confinement of Particles

3.4.1. Confinement electric field

Dust particles are at equilibrium position due to the balance of the interparticle repulsion and the radial confinement electric force in the horizontal direction. If the charge of the particles is assumed to be approximately constant in such low magnetic field regime, then a decrease in the cluster radius is an evidence of an increase in the magnitude of the confinement electric field.

Figure 3-30 – Knowing the cluster radius and dust charge, the confinement electric field can be calculated using simple geometry.

At equilibrium, the force acting on the charged particle due to the confinement potential is balanced by the net force due to the neighboring particles. Utilizing the simple geometry of single ring cluster (see Figure 3-30), the confinement electric field $E_C$ at cluster radius $\rho$ is given by:

$$E_C(\rho) = \frac{eZ}{4\pi\varepsilon_0} \sum_{i=1}^{N-1} \frac{\sin\left(\frac{i\pi}{N}\right)}{2\rho \sin\left(\frac{i\pi}{N}\right)} = \frac{eZ}{16\pi\varepsilon_0 \rho^2} \sum_{i=1}^{N-1} \frac{1}{\sin\left(\frac{i\pi}{N}\right)}$$

(3.16)

where $N$ is the number of particles in the cluster. Similarly, utilizing the simple geometry of one-and-a-half ring cluster, the confinement electric field is given by:

$$E_C(\rho) = \frac{eZ}{4\pi\varepsilon_0 \rho^2} \left[ 1 + \sum_{i=1}^{N-1} \frac{1}{4\sin\left(\frac{i\pi}{N}\right)} \right]$$

(3.17)
where $N'$ is the number of particles in the outer ring. For multi-ring clusters, similar formulae could be obtained in principle, but would be considerably more complicated due to inter-ring twist (see Section 3.2.1 for Cluster eccentricity and inter-ring twist).

Knowing the cluster radius and estimating the dust charge as $Z \approx 7 \times 10^3$ for large particles and $Z \approx 3.1 \times 10^3$ for small particles using plasma conditions from probe measurements and the orbit motion limited (OML) approximation to calculate the dust charge [90], the confinement electric field was calculated and plotted as a function of magnetic field (see Figure 3-31 and Figure 3-32).

Figure 3-31 – The confinement electric field obtained from L-Planar clusters increased with magnetic field.

Figure 3-32 – The confinement electric field obtained from S-Planar clusters increased in magnitude with increasing magnetic field strength.
From the figures, evidently the confinement electric field is modified by the magnetic field. Moreover, the magnitude of the confinement electric field increased monotonically with the magnetic field in the manner:

\[ E_C(B, \rho) \propto B^{k_1}, \quad B \geq 0 \]  

(3.18)

where \( k_1 \) is some arbitrary constant. From the figures, \( k_1 \approx 0.5 \) to 1.

Visualizing the same data from another point of view, Figure 3-33 and Figure 3-34 shows the confinement electric field plotted as a function of cluster radius at different magnetic field settings. Assuming the center of mass of the cluster is aligned with the center of confinement potential, the two figures will map out the radial profile of the electric field.

From first look, the confinement electric fields experienced by L-Planar and S-Planar clusters are quite similar. This result is expected since the confinement electric field is inherent to the experimental apparatus and independent of the particle size. However, the slight difference between the levitation height of L-Planar and S-Planar cluster might give small discrepancies in the radial profiles of confinement electric field. Since the electric confinement potential \( \Phi_C \) is given by:

\[ \Phi_C(B, \rho) = \int_0^\rho E_C(B, \rho')d\rho' \]  

(3.19)

the linear dependency of confinement electric field over its radial profile (i.e. \( E_C \propto \rho \)) corresponds to a parabolic electric potential (i.e. \( \Phi_C \propto \rho^2 \)). Using the boundary condition \( E_C(B, 0) = 0 \) and (3.18), the confinement electric field is expected to take the form:

\[ E_C(B, \rho) = (k_2B^{k_1} + k_3)\rho, \quad B \geq 0 \]  

(3.20)

where \( k_2 \) and \( k_3 \) are some arbitrary constants, \( k_1 \approx 0.5 \) to 1.

It is not possible to find the arbitrary constants \( k_2 \) and \( k_3 \) analytically without knowing \( k_1 \) exactly. But as an example, if we assume \( k_1 = 1 \) (i.e. \( E_C \propto B \)), then by plotting the slopes of Figure 3-33 and Figure 3-34 (i.e. confinement electric field gradient \( \frac{dE_C}{d\rho} \)) as a function of magnetic field (see Figure 3-35), the confinement electric field dependence could be approximated by the empirical relation:

\[ E_C(B, \rho) = -(4190B + 174589)\rho, \quad B \geq 0 \]  

(3.21)

\[ E_C(B, \rho) = -(2754B + 342043)\rho, \quad B \geq 0 \]  

(3.22)

at the levitation height of L-Planar and S-Planar clusters respectively.
Figure 3-33 – The radial profile of the confinement electric field mapped out using L-Planar clusters is linear. The lines of linear regression are shown along with the experimental data.

Figure 3-34 – The radial profile of the confinement electric field mapped out using S-Planar clusters is expected to be linear but the experimental data showed otherwise. The lines of linear regression are shown along with the experimental data.
The arbitrary constants $k_2$ and $k_3$ can be obtained from the $\frac{\delta E_c}{\delta \rho}$ versus $B$ graph. The equations and the lines of linear regression are shown along with the experimental data.

3.4.2. The effect of axial magnetic field on confinement electric field

In our experiments, the increase in the magnitude of the confinement electric field due to the presence of magnetic field is demonstrated by the decrease in the cluster radius. As a matter of fact, the height of the dust particles has also been observed to self-adjust to accommodate for the change in the sheath electric field in the presence of magnetic field. Apparently, the modification in the electric field by the magnetic field is not unique to our system only. This effect is common in any magnetized discharges [138][139] and had also been observed in the experiments performed by Konopka et al. [111], Hua et al. [123], and Kersten et al. [124]-[125]. The modification of the electric fields by the magnetic field is presumably due to the magnetization of electrons in the plasma which affects the ambipolar diffusion, thus causes a change in the spatial profile of the number density of electrons and ions.

To understand this effect, we can estimate the Larmor frequency (or cyclotron frequency) $f_L$ and the Larmor radius (or cyclotron radius) $R_L$ of a charged particle in a magnetic field:

$$f_L = \frac{\Omega_L}{2\pi} = \frac{eZB}{2\pi m}$$  \hspace{1cm} (3.23)

$$R_L = \frac{u^\bot}{\Omega_L} = \frac{mu^\bot}{eZB}$$  \hspace{1cm} (3.24)
where $u^\perp$ is the drift velocity orthogonal to the magnetic field. For our experimental conditions ($B \approx 0.01$ T, $T_d \approx T_i \approx 0.025$ eV and $T_e \approx 3$ eV, $v_{f_d} \approx 2.4 \times 10^{-4}$ ms$^{-1}$ for large dust particles and $v_{f_d} \approx 7.8 \times 10^{-4}$ ms$^{-1}$ for small dust particles, $v_{f_i} \approx 400$ ms$^{-1}$, $v_{f_e} \approx 1.2 \times 10^6$ ms$^{-1}$), the Larmor frequency and Larmor radius for large dust particles, small dust particles, ions, and electrons are shown in Table 3-4. Considering the fact that the diameter of our magnetic coil is approximately 10 cm, the dust particles are unmagnetized, the ions are partially magnetized, and the electrons are highly magnetized in our system.

<table>
<thead>
<tr>
<th></th>
<th>Larmor Frequency</th>
<th>Larmor Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Dust (Z ≈ $7.0 \times 10^3$)</td>
<td>$\approx 9 \times 10^{-6}$ s$^{-1}$</td>
<td>4 m</td>
</tr>
<tr>
<td>Small Dust (Z ≈ $3.1 \times 10^3$)</td>
<td>$\approx 5 \times 10^{-5}$ s$^{-1}$</td>
<td>3 m</td>
</tr>
<tr>
<td>Ion</td>
<td>$4 \times 10^3$ s$^{-1}$</td>
<td>$2 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>Electron</td>
<td>$3 \times 10^8$ s$^{-1}$</td>
<td>$7 \times 10^{-4}$ m</td>
</tr>
</tbody>
</table>

Table 3-4 – The Larmor frequency and Larmor radius of large dust particle, small dust particle, ions, and electrons in a magnetic field of 0.01 T are tabulated above.

Because the electrons are magnetized, the diffusion coefficient of the electrons is expected to decrease in a magnetic field. Since the ratio of electron Larmor frequency to electron-neutral collisional frequency ($n_e \approx 10^{21}$ m$^{-3}$, $\sigma_{ne} = 8 \times 10^{-20}$ m$^2$ [143]) is about 3.75, therefore the motion of the electrons is restrained near the center of the coil by the magnetic field. And for ions ($\sigma_{ni} = 8 \times 10^{-19}$ m$^2$ [143] and $u_{Z_i} \approx 850$ ms$^{-1}$), this ratio is about 0.006. Therefore, the diffusion of ions is not affected by the magnetic field. In other words, with the introduction of magnetic field, the plasma will change from ion-limited ambipolar diffusion to electron-limited ambipolar diffusion [142], which will in turn change the ambipolar radial electric field.

This conclusion can further be confirmed by considering the continuity and mobility-diffusion equations of steady state gas discharge [139] in cylindrical coordinates, assuming cylindrical symmetry (i.e. $\frac{\partial}{\partial \vartheta} = 0$):

$$
\nabla \cdot \Psi = \frac{\partial \Psi^\rho}{\partial \rho} + \frac{\Psi^\rho}{\rho} + \frac{\partial \Psi^z}{\partial z} = S
$$

(3.25)

$$
\Psi^\rho = -D \frac{\partial n}{\partial \rho} + \mu neE_c
$$

(3.26)

$$
\Psi^z = -D \frac{\partial n}{\partial z} + \mu neE_s
$$

(3.27)

where $\Psi$ is the flux, $n$ is the number density, $S$ is the source term, $D$ is the diffusion coefficient, and $\mu$ is the mobility. The minus sign in the equation corresponds to electrons whereas the plus sign corresponds to ions. As an approximation, we assume the sheath electric field is in the form:
and the confinement electric field is in the form (see Section 3.4.1 for Confinement electric field):

\[ E_c = (k_2B + k_3) \rho \]  \hspace{1cm} (3.29)

where \( k_1, k_2, \) and \( k_3 \) are arbitrary constants. Substituting (3.29) into (3.26) and (3.28) into (3.27), we get:

\[ \Psi' = -D \frac{\partial n}{\partial \rho} + \mu ne(k_2B + k_3) \rho \]  \hspace{1cm} (3.30)

\[ \Psi' = -D \frac{\partial n}{\partial z} + \mu ne k_1 \]  \hspace{1cm} (3.31)

And substituting (3.30) and (3.31) into (3.25), we get:

\[ -D \left( \frac{\partial^2 n}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial n}{\partial \rho} + \frac{\partial^2 n}{\partial z^2} \right) + \frac{\mu}{kT} ne(k_2B + 2k_3 + k_1) = S \]  \hspace{1cm} (3.32)

Using the Einstein relations \( D = \mu kT \), we get from (3.32):

\[ \left( \frac{\partial^2 n}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial n}{\partial \rho} + \frac{\partial^2 n}{\partial z^2} \right) = \frac{n_{ef}(B)}{kT} - \frac{S}{\mu kT} \]  \hspace{1cm} (3.33)

where \( f(B) = 2k_2B + 2k_3 + k_1 \) is a linear function of \( B \) for \( B \geq 0 \). Note that (3.33) is actually separable, that is:

\[ n(\rho, z) = N(\rho)M(z) = \frac{S}{\mu kT} \]  \hspace{1cm} (3.34)

where \( N(\rho) \) and \( M(z) \) are arbitrary functions which satisfy:

\[ \frac{\partial^2 N}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial N}{\partial \rho} + \left( k_4 \pm \frac{e(B)}{kT} \right) N = 0 \]  \hspace{1cm} (3.35)

\[ \frac{\partial^2 M}{\partial z^2} - k_4 M = 0 \]  \hspace{1cm} (3.36)
where $k_4$ is an arbitrary constant. The radial symmetry of the number density of electrons and ions gives the boundary condition:

$$\frac{\partial n}{\partial \rho_{\rho=0}} = 0 \quad (3.37)$$

to solve (3.35). For $k_4 > 0$ (i.e. $k_4 = \lambda^2$, $\lambda > 0$), we get:

$$n(\rho, z) = J_0(\sqrt{\frac{\lambda^2 \pm ef(B)}{kT}} \rho)(k_5 \sin(\lambda z) + k_6 \cos(\lambda z)) \mp \frac{S}{\mu_{ef}(B)} \quad (3.38)$$

for $k_4 = 0$, we get:

$$n(\rho, z) = J_0(\sqrt{\frac{ef(B)}{kT}} \rho)(k_5 \rho + k_6) \mp \frac{S}{\mu_{ef}(B)} \quad (3.39)$$

for $k_4 < 0$ (i.e. $k_4 = -\lambda^2$, $\lambda > 0$), we get:

$$n(\rho, z) = J_0(\sqrt{-\frac{\lambda^2 \pm ef(B)}{kT}} \rho)(k_5 e^{\lambda z} + k_6 e^{-\lambda z}) \mp \frac{S}{\mu_{ef}(B)} \quad (3.40)$$

where $k_5$, $k_6$ and $\lambda$ are arbitrary constants and $J_0$ is the Bessel function of the first kind and the zeroth order. Also the boundary condition:

$$\frac{\partial n}{\partial z_{z=\infty}} = 0 \quad (3.41)$$

will give us nontrivial solution from (3.39) and (3.40) only and can be summarized as:

$$n(\rho, z) = k_6 J_0(\sqrt{-\frac{\lambda^2 \pm ef(B)}{kT}} \rho)e^{-\lambda z} \mp \frac{S}{\mu_{ef}(B)} \quad (3.42)$$

Most interestingly, due to the Bessel function, the maximum number density of electrons will always occur at the center of the system (i.e. $\rho = 0$) provided that $ef(B) > \lambda^2 kT_e$. And as the magnetic field increases (i.e. $f(B)$ increases), the number density of the electrons will further increase and be restricted near the center of the system (see Figure 3-36). As the ions are almost unmagnetized under our range of magnetic fields, the radial profile of the number density will remain unchanged. Therefore, an increase in the number density of electrons will locally decrease the confinement potential at the center of the system (see Figure 3-37). This is expected from the Poisson’s equation:
\[ \nabla^2 \Phi = -\frac{e}{\varepsilon_0} (n_i - n_e) \]  
(3.43)

Figure 3-36 – Sketch of the radial profile of the electron number density. The maximum will always occur at the center of the system.

Figure 3-37 – Due to magnetic field, the number density of the electrons increases near the center of the system. As a result, the confinement potential will have a local minimum at the center of the coil.
3.4.3. Intermediate clusters and magnetic confinement of dust particles

The magnetic field modifies the spatial profiles of electrons and ions, producing a change in the confinement potential profile in plasma. This effect suggests the possibility of dust confinement using magnetic field only.

In our experiment, two particular intermediate clusters L-Planar-100 and L-Planar-50 were studied. For L-Planar-100, dust particles were confined in an electrostatic potential setup by a biased confinement electrode. For L-Planar-50, the confinement of dust particles was achieved over an unbiased electrode using magnetic field only.

In the case of L-Planar-100, large dust particles were dispersed into the plasma sheath above the circular electrode with applied voltage +15 V. A monolayer intermediate cluster with approximately a hundred particles was formed ≈ 5 mm above the confinement electrode. The diameter of L-Planar-100 was ≈ 4 mm with interparticle distance ≈ 0.45 mm (see Figure 3-38). The dust particles arranged themselves into a combination of hexagonal and pentagonal structures near the center of the cluster, while maintaining concentric rings near the outer rim.

![Figure 3-38](image)

Figure 3-38 – Looking from the top, the images of L-Planar-100 (left) and L-Planar-50 (right) intermediate dust clusters were captured. The shift of L-Planar-50 away from the center of electrode is due to the misalignment of magnetic field.

When the magnetic field was switched on, L-Planar-100 was observed to undergo rotation in the left-handed sense with respect to the magnetic field. The details of the rotational behavior of L-Planar-100 will be discussed later (see Section 4.2.1 for Rotation of intermediate clusters). Similar to small dust clusters, the cluster radius of L-Planar-100 decreased as magnetic field strength increased from 0 to 100 G (see Figure 3-39). The decrease in cluster radius of L-Planar-100 is due to the increase in the magnitude of the ambipolar radial electric field induced by magnetic field (see Section 3.4.2 for The effect of axial magnetic field on confinement electric field).

Afterwards, the applied voltage on the confinement electrode was gradually decreased to +10 V while the magnetic field strength was maintained at 90 G. And correspondingly, the cluster radius of L-Planar-100, which was still rotating in the presence of magnetic field, increased. Figure 3-40 shows the time averaged cluster radius increases as the applied voltage on the confinement electrode decreases, which is due to the decrease in the magnitude of the confinement electric field.
Figure 3-39 – The time averaged cluster radius of L-Planar-100 ($V_A = +15 \text{ V}$) and L-Planar-50 ($V_A = 0 \text{ V}$) clusters decreases when the magnetic field strength was increased.

Figure 3-40 - The time averaged cluster radius of L-Planar-100 cluster increased as the applied voltage on the confinement electrode decreased ($B = 90 \text{ G}$).

As the applied voltage on the confinement electrode was further reduced below +10 V, dust particles began to escape from the electrostatic potential well. And by the time the applied voltage on the confinement electrode reached 0 V, about half of the particles have escaped and Planar-50 was formed (see Figure 3-38). To our knowledge, this is the first time dust particles have ever been confined in the laboratory over a flat, unbiased electrode using magnetic field only.

The diameter of L-Planar-50 was $\approx 4 \text{ mm}$ with interparticle distance $\approx 0.52 \text{ mm}$. The magnetic field strength could be reduced up to 30 G after which all particles of L-Planar-50
would escape. The time averaged cluster radius dependence on the magnetic field strength for L-Planar-50 at 0 V is shown on Figure 3-39. The time averaged cluster radius decreased as the magnetic field strength increased.

Note that in Figure 3-38, L-Planar-50 was not positioned at the center of the electrode. The shift of L-Planar-50 away from the center is due to a slight misalignment of the biased electrode and the magnetic coil which will be discussed in the next section (see Section 3.4.4 for Superposition of electrostatic potentials and translational force).

3.4.4. Superposition of electrostatic potentials and translational force

The confinement potential within our magnetized plasma is the result of the superposition of the electrostatic potentials induced by the biased electrode and the magnetic coil. Therefore the symmetry of the confinement potential is largely affected by the positions of the electrode and magnet relative to the plasma production source. In our experiment, care was taken in the axial alignment of the plasma chamber (diameter = 19.5 cm), the biased electrode (diameter = 5 mm), and the magnetic coil (diameter = 15 cm). But due to the relative small scale of the dust cluster (diameter = 5 mm), a slight 0.1% shift (or 0.15 mm) of magnetic coil away from the center of the system will translate to a 3% shift of the dust cluster from the center of the biased electrode. Understandably, dust clusters are highly sensitive to the symmetry of the electrostatic potential in which they are confined in.

Figure 3-41 – The position of the particles in the outermost ring of L-Planar-100 with respect to the center of the biased electrode ($V_{A} = +15V$) at different magnetic field strength.
Figure 3-41 and Figure 3-42 shows the positions of the particles in the outermost ring of L-Planar-100 relative to the center of the confinement electrode. The boundary formed by the particles in the outermost ring will allow us to visualize the eccentricity, dimension, and amount of shift in the confinement potential at different magnetic fields.

In Figure 3-41, since the biased voltage was kept at a constant value, the center of the electrostatic potential due to the confinement electrode was fixed in space. In the absence of magnetic field, the confinement potential is expected to be centered above the confinement electrode. However, a slight shift in the cluster away from the center of the confinement electrode was observed, which suggests the presence of a translational force such as thermophoretic force acting on the particles (see Section 4.4.2 for Dust charge gradient). With increasing magnetic fields, the cluster radius reduced in dimension and shifted closer to the center of the electrode proximate (if not aligned) to the center of the magnetic coil. Both observations are expected from the increase in magnitude of the ambipolar radial electric field induced by the magnetic field, resulting in a dominance of the confinement force over the translational force. With respect to the figures, the translational force can be deduced to be acting in the positive y-direction.

Figure 3-42 further confirms the dominance of confinement force as the cluster shifts closer to the center of the electrode as more voltage is applied to the biased electrode. Notice that, when the confinement force dominates over the translational force, the cluster eccentricity remained at 0.

Figure 3-42 – The position of the particles in the outermost ring of L-Planar-100 with respect to the center of the biased electrode applied with different voltages ($B = 90$ G).
Figure 3-43 shows the positions of the particles in the outermost ring of L-Planar-50 relative to the center of the confinement electrode. In this case, the cluster in the confinement potential which is due solely to the ambipolar electric potential induced by the magnetic field shifted largely away from the center of the electrode towards the positive y-direction as magnetic field decreases. Moreover, the cluster eccentricity increases as magnetic field (and the magnitude of the ambipolar radial electric field) decreases. Such change in cluster eccentricity occurs in the presence of a translational force that is of similar magnitude to the confinement force as later shown by our computer simulations (see Section 4.1.7 for *Translational force*).

![Figure 3-43 – The position of the particles in the outermost ring of L-Planar-50 with respect to the center of an unbiased electrode at different magnetic field strength.](image)

### 3.5. Summary

In this chapter, the structural configuration and properties of planar dust clusters, dust strings, three dimensional dust clusters, intermediate dust clusters, large dust crystals, and annular dust crystals have been studied.

By using computer simulation, dust cluster systems were modeled by considering classical Coulomb repulsion, electrostatic confinement, and friction force. Calculations were made for the interaction of these forces resulting in stable and metastable structures with a high degree of symmetry. The effect of inter-ring rotation on the energy of the system had
been taken into consideration in the hyperfine structure of these stable structures. A statistical distribution of inter-ring rotations and energies was found and compared to those predicted by theory. Subsequently, a more accurate model for inter-ring rotation which incorporated the curvature of rings was developed. Using this information and the known distribution of rotations, the energy spectrum for the Planar 10 cluster was studied. The energy bands within the spectrum increased and broadened as plasma pressure increased.

In particular, we learned that:

- The radius of dust clusters and crystals decreases with magnetic field strength and increases with biased voltage on the confinement electrode.
- The radius of dust clusters and crystals increases with number of particles. For the number of particles gets larger, the rate of increase becomes smaller.
- The fluctuation in the cluster radius of S-Planar clusters is higher than L-Planar clusters.
- Some planar clusters have metastable states.
- The cluster radius is dependent on the confinement electric field. Thus the confinement electric field in unmagnetized plasma and magnetized plasma can be measured using clusters with small number of particles.
- In the presence of a magnetic field, the electrons can become magnetized.
- The confinement electric field (and the electric field) is modified by the magnetic field due to the magnetization of electrons.
- Dust confinement can be achieved over an unbiased, flat electrode by using an ambipolar radial electric field induced solely by magnetic field.
- The confinement potential in a magnetized plasma system is due to the superposition of electrostatic potentials induced the biased electrode and the magnetic field.
4. Rotation of Dust Clusters and Crystals

This chapter describes the experimental results obtained from the rotation of dust clusters and crystals under the influence of magnetic field. These results are important not only because they are the first experimental record of dust rotation in an inductive rf plasma (the system commonly used in plasma processing), but also because they are the first set of detailed analysis in the rotation of planar clusters.

Due to the small number of particles, planar dust clusters provide a simple system for the investigation of the driving mechanisms for dust rotation. Much effort was devoted to the analysis of the rotation of planar dust clusters and will be presented in Section 4.1. Two different sizes of dust particles were used in our experiments of planar cluster rotation. Section 4.2 describes the experimental results obtained from the rotation of vertical strings, three-dimensional clusters, intermediate dust clusters, large crystals, and annular crystals.

It should be mentioned that in contrast to experiments in dc discharges and capacitive rf-discharges [84]-[88][103]-[104][106][111][118]-[119], our experiments only needed small magnetic field strength (≈ tens of gauss) to rotate the dust clusters and crystals. Our data obtained at these low range magnetic fields will be verified with the existing models on dust rotation in Section 4.3. Further remarks on dust rotation experiments are given in Section 4.4.

4.1. Planar Cluster Rotation

4.1.1. Properties of the planar cluster rotation

When the magnetic field $B$ was switched off, planar dust clusters exhibited small random fluctuations but always remained around their equilibrium position. However, when the magnetic field was switched on, the planar dust clusters were observed to go undergo rotation. The direction of rotation was in the left-handed sense with respect to the magnetic field. A change in the direction of the magnetic field caused the rotation to reverse. This is the first time such small cluster systems were observed experimentally to exhibit rotational behavior.

Figure 4-1 shows the particle trajectories of the L-Planar-10 cluster under rotation when the magnetic field strength was 30G, 60G, and 90G. The trajectories during rotation were circular for all three magnetic field settings. Figure 4-2 shows the particle trajectories of the S-Planar-10 cluster under rotation when the magnetic field strength was 30G, 60G, and 90G. The trajectories during rotation were slightly elliptical for lower magnetic field strength, but became more circular with increasing magnetic field.

The angular positions $\theta_j$ of the $j$th dust particle with respect to an x-axis passing through the center of mass at time $t$ is given by (see Figure 4-3):

$$\theta_j(t) = \tan^{-1}\left(\frac{y_j(t) - y_{cm}(t)}{x_j(t) - x_{cm}(t)}\right)$$

(4.1)
Figure 4-1 – Particles in the L-Planar-10 cluster followed a circular trajectory during rotation when the magnetic field strength was 30G (red circles), 60G (blue circles) and 90G (green circles). The motion (displayed at 2 fps for 6 s) of the dust particles is indicated by the increase in size of the circle marker size.

Figure 4-2 – Particles in the S-Planar-10 cluster followed a slightly elliptical trajectory during rotation when the magnetic field strength was 30G (red circles), but changed to a more circular trajectory when increased to 60G (blue circles) and 90G (green circles). The motion (displayed at 5 fps for 2.4 s) of the dust particles is indicated by the increase in size of the circle marker size.
Figure 4-3 – The angular position of the dust particle is measured with respect to the x-axis.

As an example, the angular positions of the dust particles are shown as a function of time in Figure 4-4 for L-Planar-10 and in Figure 4-5 for S-Planar-10. Apart from the natural fluctuations, both results show that the dust particles were in phase with each other during rotation and the angular positions changed at a constant rate, indicating that the Planar-10 clusters rotated uniformly as a rigid-body. In general, if the number of particles is large ($\approx 5$ or more for L-Planar, $\approx 3$ or more for S-Planar) or if the magnetic field strength is strong, then the dust cluster is likely to experience uniform rotation. On the other hand, if the number of particles is small, then the dust cluster displays quite different motion.

One of the notable differences between L-Planar and S-Planar cluster rotation is that the particles in the latter case seems to have higher random fluctuations (see Chapter 5 for *Fluctuations of Planar Dust Clusters*). Moreover, some systems seemed more open to structural changes. That is, some clusters could alternate between metastable states (see Section 3.1.4 for *Metastable states*) with the movement of an inner particle to the outer ring or vice-versa.

Figure 4-4 – As shown by the angular position of the individual dust particles over time, the L-Planar-10 cluster underwent relatively uniform rotation ($B = 30$G). The grey dotted line marks the angle $\pi$ rad and $-\pi$ rad.
Figure 4-5 – As shown by the angular position of the individual dust particles over time, S-Planar-10 cluster underwent relatively uniform rotation ($B = 30G$). The grey dotted line marks the angle $\pi$ rad and $-\pi$ rad.

The angular positions of the individual particles for L-Planar-2 cluster are shown as a function of time in Figure 4-6. While the two particles remained diametrically opposite each other at all times during rotation, the angular rotation of the two particles went through “periodic pauses” (see Section 4.1.6. for Phase diagram and period pause) at a particular angle. Here we define periodic pause as rotation with some deceleration at some particular angle. The extreme case of period pause will be a complete stop in rotation which can degenerate to oscillations.

Figure 4-6 – As shown by the angular position of the individual dust particles over time, the L-Planar-2 cluster exhibits “periodic pauses” in its rotation ($B = 30G$). The grey dotted line marks the angle $\pi$ rad and $-\pi$ rad.
4.1.2. Angular velocity

From our experimental data of angular position versus time, the angular velocity of a dust particle was estimated from the slope of the linear regression line. In other words, the angular velocity $\omega$ of the $j$th particle in the dust cluster at time $t$ is given by:

$$\omega_j(t) = \frac{d\theta_j(t)}{dt} \approx \frac{\sum_{i=1}^{n} (t_i - t)(\theta_j(t_i) - \bar{\theta}_j(t))}{\sum_{i=1}^{n} (t_i - t)^2} = \frac{\sum_{i=1}^{n} (i - \frac{n}{2})(\theta_j(t_i) - \bar{\theta}_j(t))}{\Delta t \sum_{i=1}^{n} (i - \frac{n}{2})^2}, \quad \sum_{i=1}^{n} (t_i - t) = 0 \quad (4.2)$$

where $n$ is the number, $\bar{\theta}_j(t)$ is the mean, and $\Delta t$ is the time increment of the sample data of angular positions used in the linear regression.

The angular velocity of the dust cluster was then obtained from the average angular velocity of all dust particles:

$$\omega(t) = \frac{\sum_{j=1}^{N} \omega_j(t)}{N} \quad (4.3)$$

Figure 4-7 shows the angular velocity dependence on the magnetic field strength for L-Planar clusters. In general, the angular velocity of the L-Planar clusters increased as the magnetic field strength increased. Single ring clusters like L-Planar-3 and -4 required higher magnetic field strength ($B \approx 45$ G for L-Planar-3 and $15$ G for L-Planar-4) to initiate the rotation. And when rotation did initiate, the angular velocity increased linearly with increasing magnetic field strength. L-Planar-2 exhibited the highest threshold (see Section 4.1.5 for Threshold magnetic field) for steady rotation with oscillations only at lower field values and rotation with periodic pauses at higher field values. One-and-a-half ring clusters such as L-Planar-6, -7 and -8 were much easier to rotate. The angular velocity also increased linearly with increasing magnetic field strength. It was found that for double ring clusters such as L-Planar-10, -11 and -12, the angular velocity increased with increasing magnetic field at lower field values, but then began to saturate at higher field values (see Section 4.1.3 for Angular velocity saturation).

Figure 4-8 shows the angular velocity dependence on the magnetic field strength for the S-Planar clusters. Similar to L-Planar clusters, the angular velocity of the S-Planar clusters increased as the magnetic field strength increased. Moreover, the angular velocity saturated at higher field values. However, unlike L-Planar clusters, the trends of the angular velocity dependence were similar for all cluster systems.

Using the data from Figure 4-7 and Figure 4-8, the angular velocity dependence on the magnetic field $B$ for L-Planar and S-Planar clusters with different numbers of particles $N$ could be approximated respectively by the empirical relations (see Section 8.2 for Appendix II – Routines for Computer Simulations)
Figure 4-7 – In general, the angular velocity of L-Planar clusters increased with increasing magnetic field strength. L-Planar-3 and -4 required a higher magnetic field strength to initiate rotation.

Figure 4-8 – In general, the angular velocity of S-Planar clusters increased with increasing magnetic field strength.
Figure 4-9 plots (4.4) and Figure 4-10 plots (4.5) for the range of magnetic field strength from 0 to 100 G.

\[ \omega = \exp\left(-\frac{23}{N}\right)B^{\frac{8.3}{N^{1/5}}} \]  
(4.4)

\[ \omega = \exp\left(-\frac{3.63}{N^{1/5}}\right)B^{0.7} \]  
(4.5)

Figure 4-9 – Using the empirical relation indicated, the above plot shows the angular velocity dependence on the magnetic field for L-Planar clusters with different numbers of particles.

Figure 4-10 – Using the empirical relation indicated, the above plot shows the angular velocity dependence on the magnetic field for S-Planar clusters with different numbers of particles.
Conversely, using (4.4) and (4.5), Figure 4-11 and Figure 4-12 reveals the angular velocity dependence on the number of particles for L-Planar and S-Planar clusters at different settings of magnetic field strength. For L-Planar clusters, the angular velocity decreased initially with small number of particles, but then gradually increased with larger number of particles. Such change seemed to occur at the transition point between single ring and multi-ring clusters. However, for S-Planar clusters, the angular velocity did not depend significantly on the number of particles.

Figure 4-11 – Using the empirical relation, the above plot shows the angular velocity dependence on the number of particles for L-Planar clusters at different settings of magnetic field strength. There seems to be a transition point in the trend between single ring and multi-ring clusters.

Figure 4-12 – Using the empirical relation, the above plot shows the angular velocity dependence on the number of particles for S-Planar clusters at different settings of magnetic field strength. The angular velocity was not strongly dependent on the number of particles.
Very recently, Hou et al. [126] obtained the angular velocity dependence on the number of particles at different settings of magnetic field strength (see Figure 4-13) from computer simulations using our experimental conditions as specified on Table 3-1. The angular velocity was found to decrease as the number of particles increased. Although a non-flat electrode was used in their simulation while a flat biased electrode was used in our experiments, comparison of the results obtained by the two groups show promising agreement.

![Figure 4-13](image)

**Figure 4-13** – Using our experimental conditions, Hou et al obtained the following “angular velocity versus number of particles” plots for L-Planar clusters from simulation. Note that a non-flat electrode was used in their simulation while a flat biased electrode was used in our experiments. [Courtesy of Lu-Jing Hou, State Key Laboratory of Materials Modifications by Beams at the Dalian University of Technology].

4.1.3. Angular velocity saturation

As demonstrated in the previous section, the angular velocity of double ring L-Planar cluster increases with magnetic field at lower field values, but saturates at higher field values ($B \approx 30 \text{ G}$). Apparently, such angular velocity saturation is not unique to our system only. Similar findings had also been reported by Sato et al. [106][118]-[119]. However, the magnetic field strength required to observe the angular velocity saturation in our experiments is two orders lower than Sato’s experiments ($B \approx 5000 \text{ G}$) and that predicted by Kaw et al. [118] and Ishihara et al. [120]. Furthermore, the angular velocity saturation is dependent on the number of particles in our experiments, but not for Sato’s experiment [140].

Here, we attribute the magnetization of electrons as a possible explanation for the angular velocity saturation observed in double-ring clusters. As the electrons are magnetized, the number density of electrons will increase in the center of the system (see Section 3.4.2 for The effect of axial magnetic field on confinement electric field), which will decrease or even change the direction of the ambipolar radial electric field at the center. Sato et al. mentioned the difficulty in producing symmetrically stable crystal in their experiments for $B > 1 \text{ T}$ [106][118]-[119] which gives further support in the reversal of direction of the ambipolar radial electric field when electrons are further magnetized.
Due to magnetization of electrons, the driving force acting on the particles in the inner ring of the cluster will decrease or rotate in the opposite direction to those particles in the outer ring.

Consequently, the driving force acting on the particles in the inner ring of the cluster will decrease or even attempt to rotate in the opposite direction to the particles in the outer ring (see Figure 4-14). Since the cluster remains as a rigid-body during rotation due to the strong interaction between the particles, the moment of inertia does not change. And the net effect will be a decrease in the total torque on the dust cluster system, which in turn will decrease the angular velocity of the dust cluster.

A special comparison between single ring Planar-8 cluster and double ring Planar-10 cluster can be used to demonstrate our model (see Figure 4-15). Planar-8 has seven particles in the outer ring and one particle at the centre. From (1.4), the magnitude of the driving force acting on each outer particle of Planar-8 in the azimuthal direction for rigid-body rotation is given by:

$$ F^0_d = \kappa \omega_b R_8 $$

(4.6)

where $\kappa = \frac{4}{3} \delta m_n n \nu r_a \pi a^2$ is the dissipation coefficient, $\omega_b$ and $R_8$ are the angular velocity and the cluster radius of Planar-8 cluster respectively. For large dust particles ($a \approx 3.1 \mu m$), $\kappa \approx 3.43 \times 10^{-12} \text{ kgs}^{-1}$. And for small dust particles ($a \approx 1.4 \mu m$) $\kappa \approx 6.92 \times 10^{-13} \text{ kgs}^{-1}$. 
Based on the prevalent ion drag force models, the azimuthal ion drag force acting on the particles in the outer ring of Planar-8 cluster might to have dependency on the confinement electric field and magnetic field. In the simplest model based on our experimental data, we assume this dependency to be linear in the form:

$$F_{id}^\theta = \alpha E_{C_i} B_8 + \gamma$$

(4.7)

where $\alpha$ and $\gamma$ are arbitrary constants, and $E_{C_i}$ and $B_8$ is the confinement electric field and magnetic field strength used to rotate Planar-8 cluster. Note that in (4.7), the driving force is not zero in the absence of the magnetic field, in conflict with the observation that the clusters do not rotate when the magnetic field is zero. This is because the ion drag force is expected to show nonlinear dependence on $E_C B$ at low magnetic fields (see 4.1.4 for Angular momentum). And as such, the linear form assumed in (4.7) does not hold in the absence of magnetic field.

Therefore, by equating (4.6) with (4.7) and get:

$$\kappa \omega_s R_8 = \alpha E_{C_i} B_8 + \gamma$$

(4.8)

The values of $\alpha$ and $\gamma$ for Planar-8 can be obtained by plotting different values of $\kappa \omega_s R_8$ obtained from experiments against $E_{C_i} B_8$ (see Section 8.5 for Appendix V – Determination of $\alpha$ and $\gamma$). Now Planar-10 has structural configuration consisting of seven particles in the outer ring and 3 particles in the inner ring. We assumed that the same nature of ion drag force, which acted in the on the particles in the outer ring of Planar-8, acts on the particles in the outer ring of Planar-10. In addition, there will be a driving force $f_{id}$ acting on the particles in
the inner ring of Planar-10. So the equation of rotational motion is given by:

\[
7(\alpha \xi E_c B_{10} + \gamma)R_{10} + 3f^\theta_{d10}r_{10} = 7\kappa \omega_{10} R_{10}^2 + 3\kappa \omega_{10} r_{10}^2
\]  

(4.9)

where \( \xi = R_{10} / R_8 \) is the scaling factor (since \( E_C \propto \rho \)), and \( \omega_{10}, R_{10} \) and \( r_{10} \) are the angular velocity, the outer radius, and the inner radius of the Planar-10 cluster respectively. And hence the driving force acting on the particles in the inner ring is given by:

\[
f^\theta_{d10} = \kappa \omega_{10} \left( \frac{7R_{10}^2}{3r_{10}} + r_{10} \right) - \frac{7(\alpha \xi E_c B_{10} + \gamma)R_{10}}{3r_{10}}
\]  

(4.10)

We can substitute the experimental values from Planar-10 into (4.10) and obtain the values of driving force on the dust particles in the inner ring as a function of magnetic field.

Figure 4-16 shows the driving force on the particles in the outer ring and inner ring of L-Planar-10 as a function of magnetic field \( (\alpha \approx 1 \times 10^{-16} \text{ kgm}^2\text{s}^{-2}\text{V}^{-1}\text{T}^{-1} \text{ and } \gamma \approx 3 \times 10^{-16} \text{ kgm}^2\text{s}^{-2}) \). As we can see, the driving force acting on the particles in the inner ring increased initially at low field values, peaked at \( B = 30 \text{ G} \), and then decreased gradually at high field values. For magnetic field \( B > 80 \text{ G} \), the driving force acting on particles in the inner ring reversed its direction which confirms our model. It should be mentioned that since the diameter of L-Planar-3 was 17-24% larger than the inner ring of L-Planar-10, the nature of this driving force did not violate the linear dependency we observed in the angular velocity for L-Planar-3.

Similarly, Figure 4-17 shows the driving force on the particles in the outer ring and inner ring of S-Planar-10 cluster as a function of magnetic field \( (\alpha \approx 5 \times 10^{-17} \text{ kgm}^2\text{s}^{-2}\text{V}^{-1}\text{T}^{-1} \text{ and } \gamma \approx 8 \times 10^{-17} \text{ kgm}^2\text{s}^{-2}) \). However, the driving force acting on the particles in the inner ring was of similar magnitude as that in the outer ring. This is in good agreement with the fact that no angular velocity saturation was observed for S-Planar-10 within our range of magnetic fields.

In theory, the ambipolar radial electric field is expected to impose the same effect on the inner ring of both L-Planar and S-Planar clusters, resulting in angular velocity saturation in both cases. One possible explanation for such discrepancy is the higher levitation heights for the smaller dust particles in S-Planar, which place them into region of ambipolar electric field with different dependency on axial magnetic field. This assumption is supported by the fact that magnetic field does not change the cluster radius of the S-Planar clusters much compared to L-Planar clusters (see Section 3.1.3 for Cluster radius and radial distance). Our model is also consistent with the experimental findings of Sato et al. [103][106] because a larger confinement potential was used in their experiment. Hence a larger magnetic field would be necessary for them to observe the angular velocity saturation.
Figure 4-16 – The driving force acting on the particles in the inner ring of L-Planar-10 increases initially at low magnetic field strength, but then gradually decreases at high magnetic field strength.

Figure 4-17 – The driving force acting on the particles in the inner ring of S-Planar-10 was of similar magnitude as those particles in the outer ring.
4.1.4. Angular momentum

Since an increase in magnetic field strength resulted in a decrease in the cluster radius (see Section 3.1.3 for Cluster radius and radial distance) and an increase in the angular velocity of the cluster rotation (see Section 4.1.2 for Angular velocity), the aim in the following analysis is to investigate the possibility of conservation of angular momentum of the dust cluster. The total angular momentum $L$ of the dust cluster at time $t$ is given by:

$$L(t) = \sum_{j=1}^{N} m_j R_j(t) \omega_j(t)^2$$

(4.11)

Figure 4-18 and Figure 4-19 show total angular momentum dependence on the magnetic field with different numbers of particles for L-Planar and S-Planar clusters respectively. In general, at relatively low magnetic fields $B \approx 0$ to $30$ G, the total angular momentum for all planar clusters increases with magnetic field. At higher fields $B > 30$ G, the total angular momentum remained approximately unchanged.

The result has two implications. Firstly, the increase observed in total angular momentum at low magnetic fields could mean the azimuthal ion drag force have nonlinear dependence on $E_C \times B$ field. Secondly, the conservation of total angular momentum at higher magnetic fields indicates that the increase in angular velocity is a direct compensation for its decrease in cluster radius as the electric field is being modified by the magnetic field. Thus the conservation of momentum is a special feature which characterizes the rotation of dust clusters at higher magnetic fields. And we believe that for this range of magnetic fields, the azimuthal ion drag force is linearly proportional to $E_C \times B$ field. Thus our conclusion is that the angular momentum of dust cluster is conserved when this linear dependency holds.

Next, we investigated the total angular momentum of the clusters averaged over all magnetic field settings and obtained its dependence on the number of particles in the cluster. And as shown on Figure 4-20 and Figure 4-21, the total angular momentum averaged over all magnetic field settings is linearly dependent on the number of particles in the cluster for all planar clusters.
Figure 4-18 – For $B > 30$ G, total angular momentum of L-Planar cluster remained approximately unchanged with increasing magnetic field strength.

Figure 4-19 – Except for S-Planar-8 and S-Planar-10, the total angular momentum of S-Planar cluster remained approximately unchanged with increasing magnetic field strength. (Note the scale of the above plot is 20 times smaller than Figure 4-18).
Figure 4-20 – For L-Planar clusters, the total angular momentum averaged over all magnetic field settings showed linear dependency with the number of particles. The equation and the line of linear regression are displayed along with the experimental data.

Figure 4-21 – For S-Planar clusters, the total angular momentum averaged over all magnetic field settings showed linear dependency with the number of particles. The equation and the line of linear regression are displayed along with the experimental data.
4.1.5. Threshold magnetic field

From Figure 4-7 and Figure 4-8, it is possible to estimate the value of the threshold magnetic field $B_{th}$ at which cluster rotation began (see Section 8.4 for Appendix IV – Determination of Threshold Magnetic Field).

Figure 4-22 shows the threshold magnetic field required to initiate rotation for L-Planar and S-Planar dust clusters. The threshold magnetic field seems to be inversely proportional to the number of particles, that is, for L-Planar clusters:

$$B_{th} = \frac{90}{N} - 8$$

(4.12)

and for S-Planar clusters:

$$B_{th} = \frac{24}{N}$$

(4.13)

Figure 4-22 - From our experimental data, the threshold magnetic field required to initiate rotation for L-Planar clusters was inversely proportional to the number of particles. The equations and the lines of linear regression are displayed along with the experimental data.
4.1.6. Phase diagram and period pause

At low magnetic field, clusters with smaller number of particles ($N < 5$ or less for L-Planar clusters, $N < 3$ for S-Planar clusters) appear to exhibit "periodic pauses" (see Section 4.1.1 for Properties of the planar cluster rotation). In this section, the phase diagrams obtained for some of the L-Planar clusters will be shown.

Phase diagrams are cylindrical plots of angular velocity against angular position of the individual dust particles about the axis of rotation (see Figure 4-23). The axial component represents the magnetic field strength used in rotating the particles. The radial and the angular component of the plot represent the angular velocity and the angular position of the dust cluster respectively. Consequently, for a particular magnetic field strength, a circular plot in the phase diagram can be interpreted as uniform dust cluster rotation. Whereas a linear plot in the phase diagram can be interpreted as dust cluster oscillations about an angle. The advantage of using phase diagram is that the occurrence of "periodic pause" becomes apparent.

Figure 4-24 shows the phase diagrams for single ring cluster L-Planar-2, -3, one-and-a-half ring cluster L-Planar-7, and double ring L-Planar-10 clusters at different magnetic field strengths. The hue of the color in the vertical direction was used to indicate the different magnetic field strength settings (blue = 15 G through to red = 90 G).

![Phase Diagrams](image)

Figure 4-23 – If the dust particles in the cluster undergo uniform angular motion, a circular phase diagram is expected. If the particles exhibit "periodic pauses", then the phase diagram would be deformed. And if the particles oscillate about a particular angle, then a straight line is expected.
Figure 4-24 – Three-dimensional phase diagrams of L-Planar clusters. The axial component of the plot represents the magnetic field strength used for the cluster rotation (pink – 90G, yellow – 75G, green – 60G, lilac – 45G, light blue – 30G, blue – 15G). The angular velocity has actually been scaled up 1000 times the actual value (except Planar-2).
From the phase diagram, the particles in L-Planar-2 are shown to oscillate at a particular angle for a particular magnetic field strength. The angle at which the particles prefer to oscillate in is an indication of the asymmetry in the system. Interestingly, an increase in the magnetic field strength changed the angle about which the L-Planar-2 oscillated. For L-Planar-3, the rotation was not uniform for low magnetic field (≈45 G or less), with particles oscillating or slowing at a particular angle. However, the rotation became increasingly uniform as the magnetic field was increased. For L-Planar-7, and -10, the rotation was uniform for relatively high magnetic field (≈30G or more).

4.1.7. Translational force

Here a simple model is proposed to explain the oscillations and periodic pauses of Planar-2 clusters (see Section 4.1.6 for Phase diagram and period pause). If there is a force in our system acting across the dust cluster plane, we refer to such force as translational force. An example of such translational force can be thermothoretic force (See section 4.4.2 for Dust charge gradient), which appears due to temperature gradient in our chamber (see Section 2.4 for The Magnetic Coil). If this gradient is nonuniform, then such force can have variation in magnitude along the x-direction (see Figure 4-25) where \( F_1 \) and \( F_2 \) are forces that act on each of the two particles.

![Figure 4-25 – Translational forces F1 and F2 act on the particles across the dust cluster plane in the positive y-direction.](image)

If this variation in magnitude is \( \Delta F \), then the cluster would experience a net torque:

\[
\tau = \rho \Delta F \cos^2 \theta \quad (4.14)
\]

which will provide an additional azimuthal component to drive the cluster particles into rotation. In other words, the total driving force for the dust cluster rotation is a summation of an azimuthal driving force \( F_d^\theta \) and the translational force \( F_t \). In the absence of a
translational force, the dust cluster would go into uniform rotation. In the presence of a translational force, then depending on the magnitude and gradient of $F_t$, the cluster rotation will degenerate into either periodic pauses or oscillations as shown in Figure 4-26.

**Periodic Pause**

At $\theta = \pi$, if $F_d^\theta + F_t > \Delta F$

**Oscillation**

At $\theta = \pi$, if $F_d^\theta + F_t \leq \Delta F$

Figure 4-26 – Depending on the magnitude of translational force, the cluster rotation will degenerate into either oscillations or periodic pauses.

In order to verify the validity of this simple approach, we use our model for computer simulations (see Section 3.2.3 for Our model for computer simulations), which incorporates an additional translational force $F_t$ acting in the $y$-direction (see Section 8.2.10 for Subroutine – translational force) along with an azimuthal driving force $F_d^\theta$ (see Section 8.2.11 for Subroutine – azimuthal force). As a simple model, we modeled the translational force with the functional form:

$$F_t(x) = \Delta F(x + k_2)$$

(4.15)

where $k_2$ is an arbitrary constants. The effect of this force on the structural configuration of Planar-10 cluster from our simulation is shown in Figure 4-27. The translational force distorted the cluster along the $y$-direction and increased its eccentricity. A shift of the cluster in the $y$-direction was also observed. In our experiment, we have shown that the change in cluster eccentricity and large shift away from the center of the system occurs when the translational force is significant relative to the confinement force (see Section 3.4.4 for Superposition of electrostatic potentials and translational force).

Figure 4-28 shows the angular position of a particle in the outer ring of Planar-10 over time. When $\Delta F = 0.1 F_d^\theta$ was chosen in our simulations, the particle goes under oscillations. When $\Delta F = 0.01 F_d^\theta$, the particle undergoes non-uniform rotation. More specifically, the
particle will decelerate and reverse its direction momentarily at particular angles. This result from simulations coincides with our experimental observations as shown previously in Figure 4-6. At the moment, although our model is at its simplest since we assumed $F_t$ to be a constant function, our simulations show the validity of our model. The exact functional form $F_t$ will require further investigations in the future.

![Figure 4-27](image)

**Figure 4-27** – The addition of a translational gradient force into our computer simulation causes an elongation of the shape and a shift in the center of the cluster.

![Figure 4-28](image)

**Figure 4-28** – The plot above shows the angular position of a particle in the outer ring of Planar-10 cluster over time. When the $\Delta F = 0.1F_0^d$, the particle goes under oscillations (red line). When $\Delta F = 0.01F_0^d$, the particle undergoes non-uniform rotation (blue line). The grey dotted line marks the angle $\pi$ rad and $-\pi$ rad. Note that the jaggedness of the graph is due to the resolution of the data.
4.1.8. Cylindrical asymmetry in confinement potential

Another reason for periodic pause could be due to the asymmetry in the confinement potential. Ideally, in a confinement potential with perfect eccentricity, the particles in the dust clusters are expected to undergo uniform rotation. (see Figure 4-29). However, if there is a potential hill along the azimuthal direction of the confinement well, then the particles is expected to decelerate during rotation (see Figure 4-30). And if the particles are unable to overcome this potential hill, the cluster is expected to oscillate about the potential hill.

Figure 4-29 – For cylindrically symmetric confinement well, dust undergo uniform rotation.

Figure 4-30 – However, if there is a potential hill along the confinement well in the azimuthal direction, then dust will decelerate during rotation.
Such potential hill could arise due to the asymmetry of the system, such as some eccentricity in the confinement well. In fact, a fixed angle at which L-Planar-2 oscillates about suggests that there is a preferred symmetry in our system (see Section 4.1.6 for Phase diagram and period pause). When magnetic field is low, the particles do not have enough driving force to overcome the potential hill. Therefore oscillations at a particular angle were observed. In fact, looking at the phase diagram for L-Planar-2, the angle at which the cluster oscillated increases as magnetic field strength increases. This can be visualized as an attempt of the dust particles to overcome the potential hill.

The torque acting on the particles in a cluster is given by:

$$\tau = I\alpha$$  \hspace{1cm} (4.16)

where $I$ is the moment of inertia and $\alpha$ is the angular acceleration, which can be obtained from the slope of the linear regression line through our experimental data of angular velocity versus time, given by:

$$\alpha_j(t) = \frac{d}{dt} \omega_j(t) \approx \frac{\sum_{i=1}^{n} (t_i - t)(\omega_j(t_i) - \overline{\omega_j(t)})}{\sum_{i=1}^{n} (t_i - t)^2} = \frac{\sum_{i=1}^{n} \frac{2}{n}(\omega_j(t_i) - \overline{\omega_j(t)})}{\Delta t \sum_{i=1}^{n} (i - \frac{n}{2})^2}, \quad \sum_{i=1}^{n} (t_i - t) = 0 \hspace{1cm} (4.17)$$

where $n$ is the number, $\overline{\omega_j(t)}$ is the mean, and $\Delta t$ is the time increment of the sample data of angular velocities in the linear regression. Since the moment of inertia of L-Planar-2 does not change during periodic pause, the torque acting on the particles along the azimuthal direction can be visualized by a polar plot of angular acceleration versus angular position. Figure 4-31 shows a polar plot of angular acceleration versus its angular position which essentially maps out the angular profile of the effective confinement potential.
4.2. Rotation of Other Types of Dust Clusters and Crystals

4.2.1. Rotation of intermediate clusters

When the magnetic field was switched on, L-Planar-100 was observed to undergo rotation in the left-handed sense with respect to the magnetic field. Apart from the natural fluctuations and occasional inter-ring movements, L-Planar-100 maintained its overall integrity during rotation.

Figure 4-32 shows the angular velocity dependence on the magnetic field strength for L-Planar-100 and L-Planar-50. Similar to planar dust clusters, the angular velocity of both intermediate clusters increased as the magnetic field strength increased. However, unlike planar dust clusters, the angular velocity of both intermediate clusters increased linearly with no sign of saturation for magnetic field up to 100 G. We believe intermediate clusters do not exhibit angular velocity saturation at low magnetic fields as small planar clusters would because only a minority of particles at the center of the cluster is influenced by the change of electric field induced by magnetic field (see Section 4.1.3 for Angular velocity saturation).
Figure 4-32 - The angular velocity of L-Planar-100 \((V_A = 15 \text{ V})\) and L-Planar-50 \((V_A = 0 \text{ V})\) clusters is directly proportional to the magnetic field strength.

Note that the angular velocity of L-Planar-50 \((V_A = 0 \text{ V})\) is slower than L-Planar-100 \((V_A = 15 \text{ V})\) at all magnetic field strengths. Such observation is expected because, in the absence of a biased electrode, L-Planar-50 experiences a weaker confinement electric field as predicted by ion drag force models. From (1.9) and (1.10), the rate of change in the angular velocity of intermediate cluster rotation with respect to weak magnetic field (i.e. \(\xi < 1\)) was derived to be:

\[
\frac{\partial \omega}{\partial \xi} \propto \frac{E_C}{\rho}
\]  

(4.18)

For L-Planar-50, the confinement electric field is induced solely by the magnetic field, which we can call by \(E_B\). For L-Planar-100, the confinement electric field is due to both the biased electrode and the magnetic field, given by \((E_E + E_B)\). And so using (4.18), the slopes of Figure 4-32, and the fact that \(\rho = 1.94 \pm 0.18 \times 10^{-3} \text{ m}\) for L-Planar-50 and \(\rho = 1.92 \pm 0.23 \times 10^{-3} \text{ m}\) for L-Planar-100 over our range of magnetic fields, we have:

\[
\frac{E_E + E_B}{E_B} = \frac{1.92 \pm 0.23 \times 10^{-3} \times 0.0022}{1.94 \pm 0.18 \times 10^{-3} \times 0.0013} = 1.67 \pm 0.41
\]

(4.19)

or

\[
E_E = (0.7 \pm 0.4)E_B
\]

(4.20)
And so the electric fields induced by the biased electrode and the magnetic coil are of same order of magnitude.

Figure 4-33 shows the angular velocity of Planar-100 ($B = 90$ G) as a function of the applied voltage on the biased electrode. The angular velocity of Planar-100 increased as the applied voltage (and the confinement electric field) increased in magnitude.

![Graph showing the relationship between voltage and angular velocity](image)

Figure 4-33 – When the magnetic field was at 90 G, the angular velocity of L-Planar-100 increased as the applied voltage on the confinement electrode increased.

4.2.2. Rotation of particle strings and three-dimensional clusters

For a single particle or vertical strings of particles, no noticeable particle motion was observed in the vertical or horizontal directions. However, due to the single dimension of the particle configurations, the question of whether rotational motion was exhibited by the particles about their own axis is yet to be determined.

The properties observed in three-dimensional dust clusters were similar to those observed in planar dust clusters. Firstly, the dust particles of 2-on-1 (two particles in upper layer and one in lower), 3-on-1, 4-on-1, 5-on-1, and tetrahedral clusters followed a circular trajectory in their rotation. Secondly, 3-on-1, 4-on-1, 5-on-1, and tetrahedral clusters all underwent uniform rigid-body rotation in the presence of a magnetic field. The direction of the rotation is in the left-handed sense. As an example, Figure 4-34 and Figure 4-35 shows the angular position of the dust particles (except for the stationary particle at the center) and the angular velocity of 5-on-1 cluster as a function of time. The angular velocity of 5-on-1 dust clusters increased as the magnetic field strength increased. Thirdly, 2-on-1 cluster rotated with periodic pauses similar to that of L-Planar-2 cluster. Thus it appeared more difficult to rotate L-Planar-2 and the 2-on-1 clusters compared to the others.
Figure 4-34 – As shown by the angular position of the individual dust particles over time, 5-on-1 cluster underwent relatively uniform rigid-body rotation at all magnetic field strength.

Figure 4-35 – The angular velocity of 5-on-1 cluster increased as the magnetic field strength increased.

4.2.3. Rotation of large crystals and annular crystals

Initially when the magnetic field was off, the large crystal was stationary with small
random fluctuations of the individual particles. When the magnetic field was switched on, the crystal instantaneously contracted down to a diameter of 7.38 mm (see Figure 4-36). Moreover, the crystal rotated collectively in a left-handed sense with respect to the direction of the magnetic field. The angular velocity of the crystals increased with increasing magnetic field strength. At a magnetic field strength of 130 G, the angular velocity of the crystal was measured to be approximately 1 revolution per minute (approximately 0.1 rad/s).

No measurable difference in the angular velocity was observed at different radial distances from the center of the crystal, and there was no velocity shear between the layers of the crystal. Except for the random fluctuations, the integrity of the large plasma crystal was preserved during rotation.

When the magnetic field was off, the particles in the annular crystal exhibited random fluctuations similar to those observed in large crystals. When the magnetic field was at its maximum value, the annular crystal rotated collectively, again in the left-handed sense. The angular velocity of the annular crystals increased with increasing magnetic field strength. At magnetic field strength of 130 G, the angular velocity was measured to be approximately 1 revolution per minute (or 0.1 rad/s). The particles within the inner region of the ring were moving at the same angular velocity as the particles in the outer region of the ring.

![Magnetic Field Out](image)

Figure 4-36 – The diameter of the large crystal decreased from ≈ 10 mm to ≈ 7 mm after the magnetic field was switched on. The crystal rotated collectively in a left-handed sense with respect to the direction of the magnetic field.

4.3. Comparison of Experimental Results with Current Theories

Several models have attempted to explain the driving mechanisms for the rotation of dust
crystals and clusters in a magnetic field, with most of them based on the ion drag force. These models have been described in the previous section. In all cases, the experimental results seemed to agree well with the respective proposed models. Nevertheless, whether these models can explain experiments performed under other conditions is an open question. In the following sections, we will compare our experimental data with the current theories on dust rotation both qualitatively and quantitatively.

4.3.1. With reference to Konopka’s model

Under the assumption of Konopka’s model, by equating (1.5) and (1.4), the angular velocity of cluster/crystal rotation has dependency:

$$\omega \propto \frac{E_c B}{\rho}$$  \hspace{1cm} (4.21)

For parabolic confinement potential $E_c \propto \rho$, therefore:

$$\omega \propto B$$  \hspace{1cm} (4.22)

This qualitative description agrees with many experimental observations. For example, in experiments performed by Sato et al. [106][118]-[119], the linear dependency of angular velocity on magnetic field for large dust crystals has been observed for $B < 4000$ G. However, at higher magnetic fields $B > 4000$ G, nonlinear dependence was observed.

In relation to our experiments, the linear dependency of angular velocity is in good agreement for intermediate dust clusters L-Planar-100 and L-Planar-50. For planar dust clusters and three-dimensional clusters, the angular velocity was not linearly dependent on the magnetic field. Moreover, the angular velocity of double ring L-Planar clusters saturates at magnetic fields $B > 30$ G. Since the confinement electric fields have been measured and the relationship $E_c \propto \rho$ verified using one-and-a-half ring L-Planar clusters, it seems that Konopka’s model cannot provide an accurate qualitative description of the rotation of small clusters.

Also under the assumption of Konopka’s model, since:

$$u_i^0 \propto E_c B$$  \hspace{1cm} (4.23)

then from (1.5), the driving force is expected to be dependent on both confinement electric field and magnetic field, that is:

$$F_d = F_{id}^0 \propto E_c B$$  \hspace{1cm} (4.24)
Figure 4-37 and Figure 4-38 show the estimated value of driving force (by substituting our experimental conditions into (1.4)) plotted as a function of $E_C \times B$ field (obtained from Figure 3-31 and Figure 3-32) for L-Planar and S-Planar clusters respectively. Both plots seem to agree with the linear dependency predicted by Konopka’s model.

Note that in Figure 4-37 and Figure 4-38, the driving force is not zero when magnetic field is zero. There are three reasons for this. The first reason is the linear dependency of the ion drag force does not hold for low values of $E_C B$, which was shown in previous section (see Section 4.1.4 for Angular momentum). Secondly, the driving force was calculated quite accurately (error less then 10%), but the estimated error of electric field determination is about 60% which certainly would affect the presented data. Thirdly, we have shown that the driving force consists of two terms, namely, ion drag force and translational force (see Section 4.1.7 for Translational force). And we believe that the linear approximation of driven force dependence on $E_C \times B$ is not valid especially for low values of $E_C B$ where the ion drag force becomes less predominant.
Figure 4-38 – The linear dependency of driving force on $E_C \times B$ field is not as strong for S-Planar clusters, especially for S-Planar-8.

For the parameters from our experimental conditions ($m_n \approx 1.9 \times 10^{-13}$ kg, $T_n \approx 300$ K, and $v_{r_n} \approx 400$ ms$^{-1}$), the estimated values of the driving forces using (1.4) for L-Planar clusters, S-Planar clusters, intermediate clusters, large crystal, and annular crystal are shown on Table 4-1. By substituting our experimental conditions ($m_i \approx m_n \approx 1.9 \times 10^{-13}$ kg, $n_i \approx 10^{15}$ m$^3$, $T_i \approx T_n$, $u_\Sigma \approx 850$ ms$^{-1}$, $\mu_0 = 1.9 \times 10^3$ mbar-cm$^2$V$^{-1}$s$^{-1}$ [130], $\alpha_0 = 3.5 \times 10^{-2}$ mbar-cmV$^{-1}$ [130], $\Gamma \approx 10$, and $\lambda_D \approx 2 \times 10^{-5}$ m) into (1.5), Table 4-2 shows the estimated values of the azimuthal ion drag forces for L-Planar clusters, S-Planar clusters, intermediate clusters, large crystal, and annular crystal. Note that the estimated value here is an upper limit estimation of the azimuthal ion drag force because the Coulomb logarithm have been taken as $\Gamma \approx 10$ (see Section 1.3.2 for The choice of Coulomb logarithm in ion drag force estimation). In comparison with Table 4-1, the upper estimation of the azimuthal ion drag force is one to three orders lower in magnitude than the driving force. Therefore, Konopka’s model cannot provide a complete quantitative description for the rotation of dust clusters and crystals in our experiments.

<table>
<thead>
<tr>
<th></th>
<th>$n_n$ (m$^3$)</th>
<th>$B$ (G)</th>
<th>$\omega$ (rads$^{-1}$)</th>
<th>$\rho$ (m)</th>
<th>$a$ (m)</th>
<th>$F_{\theta}^d$ (N)</th>
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<tr>
<td>L-Planar Clusters</td>
<td>$\approx 3.2 \times 10^{21}$</td>
<td>$\approx 90$</td>
<td>$\approx 0.3$ (see Figure 4-9)</td>
<td>$\approx 4 \times 10^{-4}$</td>
<td>$3.1 \times 10^{-6}$</td>
<td>$4 \times 10^{-16}$</td>
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<tr>
<td>S-Planar Clusters</td>
<td>$\approx 3.2 \times 10^{21}$</td>
<td>$\approx 90$</td>
<td>$\approx 0.6$ (see Figure 4-10)</td>
<td>$\approx 3 \times 10^{-4}$</td>
<td>$1.4 \times 10^{-6}$</td>
<td>$1 \times 10^{-16}$</td>
</tr>
<tr>
<td>L-Planar-100</td>
<td>$\approx 3.2$</td>
<td>$\approx 90$</td>
<td>$\approx 0.2$</td>
<td>$\approx 4 \times 10^{-4}$</td>
<td>$3.1 \times 10^{-6}$</td>
<td>$3 \times 10^{-15}$</td>
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Table 4-1 – The estimated values of driving forces for planar clusters, intermediate clusters, large crystal, and annular crystals are tabulated on the rightmost column.

<table>
<thead>
<tr>
<th></th>
<th>$p$ (mTorr)</th>
<th>$Z$</th>
<th>$B$ (G)</th>
<th>$E_C$ (Vcm$^{-1}$)</th>
<th>$E_S$ (Vcm$^{-1}$)</th>
<th>$a$ (m)</th>
<th>$F_{id}$ (N)</th>
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<tr>
<td>L-Planar Clusters</td>
<td>100</td>
<td>≈ 7.0 $\times$ 10$^3$</td>
<td>≈ 90</td>
<td>≈ 1</td>
<td>≈ 10</td>
<td>3.1 $\times$ 10$^{-6}$</td>
<td>1 $\times$ 10$^{-17}$</td>
</tr>
<tr>
<td>S-Planar Clusters</td>
<td>100</td>
<td>≈ 3.1 $\times$ 10$^3$</td>
<td>≈ 90</td>
<td>≈ 1</td>
<td>≈ 10</td>
<td>1.4 $\times$ 10$^{-6}$</td>
<td>3 $\times$ 10$^{-18}$</td>
</tr>
<tr>
<td>L-Planar-100</td>
<td>100</td>
<td>≈ 7.0 $\times$ 10$^3$</td>
<td>≈ 90</td>
<td>≈ 1</td>
<td>≈ 10</td>
<td>3.1 $\times$ 10$^{-6}$</td>
<td>1 $\times$ 10$^{-17}$</td>
</tr>
<tr>
<td>Large Crystal</td>
<td>350</td>
<td>≈ 7.0 $\times$ 10$^3$</td>
<td>≈ 130</td>
<td>≈ 1</td>
<td>≈ 10</td>
<td>3.1 $\times$ 10$^{-6}$</td>
<td>2 $\times$ 10$^{-17}$</td>
</tr>
<tr>
<td>Annular Crystal</td>
<td>250</td>
<td>≈ 7.0 $\times$ 10$^3$</td>
<td>≈ 130</td>
<td>≈ 1</td>
<td>≈ 10</td>
<td>3.1 $\times$ 10$^{-6}$</td>
<td>2 $\times$ 10$^{-17}$</td>
</tr>
</tbody>
</table>

Table 4-2 – The estimated values of ion drag forces for planar clusters, intermediate clusters, large crystal, and annular crystal are tabulated on the rightmost column.

4.3.2. With reference to Ishihara’s model

Angular velocity estimations of cluster/crystal rotation using (1.7) from Ishihara’s model are shown on Table 4-3. Here, the normalized initial radial displacement is assumed to be $a = 1.2$. This assumption is certainly appropriate for the purpose of upper limit estimation as later we will show that the radial fluctuation is at most 4% only (see Section 5.2.1 for Radial instability coefficient versus number of particles).

In comparison, angular velocity estimations using Ishihara’s model are at least four orders less than our experimentally measured values. Despite our choice of $a = 1.2$, this model provides the weakest quantitative description for cluster/crystal rotation in magnetic field.
Table 4-3 – In comparison, the angular velocities of L-Planar clusters, S-Planar clusters, intermediate clusters, large crystal, and annular crystal estimated using Ishihara’s model are systematically 4 orders lower than those measured in our rotation experiments.

### 4.3.3. With reference to Kaw’s model

By substituting our experimental conditions ($\sigma_n = 8 \times 10^{19} \text{m}^2$ [143] and $E_c \approx 100 \text{Vm}^{-1}$) into (1.8) from Kaw’s model, Table 4-4 shows the angular velocity estimations for L-Planar clusters, S-Planar clusters, intermediate clusters, large crystal, and annular crystal. In all cases, Kaw’s model was able to provide the most satisfactory quantitative description for the angular velocity of cluster/crystal rotation compared to other models, especially for the case of large crystals, annular crystals, and intermediate clusters. Although in the case of planar clusters, the angular velocity prediction using Kaw’s model was 1 to 2 orders higher than our measured values.

Also for L-Planar clusters, the magnetic field at which angular velocity saturation occurs is expected to be $B \approx 9000 \text{G}$ (when $\xi_{Kaw} = 1$). On the contrary, angular velocity saturation was observed in L-Planar cluster at magnetic field as low as 30 G. Moreover, Kaw’s model did not take into account of the amount of dust, and therefore does not show any angular velocity dependency on the number of particles, which was observed in our experiment.

We believe that another reason Kaw’s model was unable to provide an accurate description for planar clusters is that, the ions under our experimental conditions are in the nonlocal collisionless fluid stage. Since the ion-neutral collisional mean free path:

$$\lambda_n = \frac{u_e}{\nu_n}$$

in our system is approximately $4 \times 10^{-4} \text{m}$, it is of comparable size to planar dust clusters. Therefore the collisional fluid description of the ions used in Kaw’s model is not appropriate.
Table 4-4 – Using Kaw’s model, the estimated values of angular velocities of large crystal and annular crystal are in close agreement with those measured in our rotation experiments. However, for L-Planar clusters, S-Planar clusters, and intermediate clusters, the estimated values are 1 to 2 orders larger than our experimental values.

4.3.4. With reference to Shukla’s model

By substituting our experimental conditions \( \sigma_i = 8 \times 10^{-19} \text{m}^2 \) [143] and \( E_c \approx 100 \text{ Vm}^{-1} \) into (1.10) from Shukla’s model Table 4-5 shows the estimated values of the angular velocities for L-Planar clusters, S-Planar clusters, intermediate clusters, large crystal, and annular crystal. In comparison with our experimental data, the estimated values of the angular velocities are 3 to 4 orders lower.

For L-Planar clusters, the magnetic field at which angular velocity saturation occurs is expected to be \( B \approx 10^3 \text{ T} \) (when \( \xi_{\text{Shukla}} = 1 \)). This value is far beyond what we have observed in our experiments (\( B \approx 30 \text{ G} \)). From Shukla’s model, the magnetic field of angular velocity saturation of the cluster rotation is expected to decrease as the number density of dust increases. On the contrary, angular velocity saturation was only observed for L-Planar clusters which have relatively lower number density of dust than large crystal and annular crystal.

### Table 4-5

<table>
<thead>
<tr>
<th>C &amp; References</th>
<th>( n_i (\text{m}^{-3}) )</th>
<th>( \rho (\text{m}) )</th>
<th>( \Omega_i (\text{rads}^{-1}) )</th>
<th>( \omega (\text{rads}^{-1}) ) [estimated]</th>
<th>( \omega (\text{rads}^{-1}) ) [experimental]</th>
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<tr>
<td>L-Planar Clusters</td>
<td>( 3.2 \times 10^{21} )</td>
<td>( 2.0 \times 10^{10} )</td>
<td>( 1.9 \times 10^{13} )</td>
<td>( 5 \times 10^{4} ) to 1 ( \times 10^{-3} )</td>
<td>( 0.3 )</td>
</tr>
<tr>
<td>S-Planar Clusters</td>
<td>( 3.2 \times 10^{21} )</td>
<td>( 3.0 \times 10^{10} ) to ( 1 \times 10^{11} )</td>
<td>( 1.7 \times 10^{14} )</td>
<td>( 4 \times 10^{4} ) to 4 ( \times 10^{-3} )</td>
<td>( 0.6 )</td>
</tr>
<tr>
<td>L-Planar-100</td>
<td>( 7.0 \times 10^{3} )</td>
<td>( 4 \times 10^{4} )</td>
<td>( 2.2 \times 10^{4} )</td>
<td>( 1 \times 10^{-5} )</td>
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</tr>
<tr>
<td>Large Crystal</td>
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<td>( 2 \times 10^{-4} )</td>
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</table>
An annular crystal $\approx 7.0 \times 10^3 \approx 10^{-2} \approx 3.1 \times 10^9 \approx 8 \times 10^{-4} \approx 0.1$

Table 4-5 - In comparison, the angular velocities of L-Planar clusters, S-Planar clusters, intermediate clusters, large crystal, and annular crystal estimated using Shukla's model are systematically 2 to 4 orders lower than those measured in our rotation experiments.

4.4. Further Remarks on Dust Rotation

In this section, we discuss the possibility in other factors which might influence the dust rotation in magnetic field.

4.4.1. Neutral gas flow

A possible mechanism which can cause dust rotation is the neutral gas flow. However, the velocity of possible neutral gas rotation can be shown to be very small by estimating the velocity acquired by the neutrals due to collisions with the ions. In this case, the neutrals will acquire an azimuthal velocity due to the ion drift given by [116]:

$$u_n^\theta = \frac{n_n \nu_n}{n_n \nu_m} \frac{1}{c} \left( \frac{\mu_0}{p} \right)^2 B E_C \left[ 1 + \left( \frac{\alpha_0}{p} \right) E_S \right]$$

(4.26)

where $\nu_{nn} = n_n \sigma_{nn} v_r$, is the neutral-neutral collisional frequency and $\sigma_{nn}$ is the neutral-neutral collisional cross-sections.

Using the parameters from our experiments ($n_n \approx 10^{21} \text{ m}^3$, $n_i \approx 3 \times 10^{15} \text{ m}^3$, $\sigma_{ni} = 8 \times 10^{-19} \text{ m}^2$ and $\sigma_{nn} = 3 \times 10^{-19} \text{ m}^2$ [143], $u_\Sigma \approx 850 \text{ ms}^{-1}$, $v_r \approx 400 \text{ ms}^{-1}$, $\mu_0 = 1.9 \times 10^3 \text{ mbar-cm}^2\text{V}^{-1}\text{s}^{-1}$ and $\alpha_0 = 3.5 \times 10^{-2} \text{ mbar-cm} \text{V}^{-1}$ [130], $p \approx 1.3 \times 10^{-1} \text{ mbar}$, $B \approx 100 \text{ G}$, $E_C \approx 1 \text{ Vcm}^{-1}$, and $E_S \approx 10 \text{ Vcm}^{-1}$), our calculation showed that the azimuthal drift velocity of neutrals is $7 \times 10^{-10} \text{ ms}^{-1}$ which corresponds to an angular velocity of $7 \times 10^{-7} \text{ rads}^{-1}$ (for $\rho \approx 10^{-3} \text{ m}$). Therefore the neutral gas can be considered as stationary ($u_n^\theta \approx 2 \text{ rads}^{-1}$).

4.4.2. Dust charge gradient

Another possible mechanism for explaining dust rotation is dust charge gradient across the horizontal plane in the presence of a translational non-electrostatic force [109][144]. A dust charge gradient is present if there are inhomogeneities in the plasma, such as temperature gradient or number density gradient of electrons and ions. In such case, the angular velocity of rotation is given by [144]:

$$\theta_{\text{rot}} = \frac{1}{2} \frac{e \omega R_{\Sigma}}{m}$$

(4.50)

where $\omega$ is the angular velocity of the plasma, $R_{\Sigma}$ is the radius of the dust charge gradient, and $m$ is the mass of the dust particle.

Using the parameters from our experiments ($\omega \approx 2 \times 10^{-3} \text{ rad} \text{s}^{-1}$, $R_{\Sigma} \approx 10^{-3} \text{ m}$, and $m \approx 10^{-18} \text{ kg}$), our calculation showed that the angular velocity of rotation is $10^{-5} \text{ rads}^{-1}$, which is much smaller than the dust rotation measured in our experiments. Therefore the dust charge gradient can be considered as negligible in our experiments.
\[
\omega = F_{\text{non}} \frac{\partial Z(\rho)}{2m_{\nu}Z_{\text{eq}}} \frac{\partial \rho}{\partial \rho}
\]

(4.27)

where \( \frac{\partial Z(\rho)}{\partial \rho} \) is the dust charge spatial gradient and \( F_{\text{non}} \) is the translational non-electrostatic force.

In our experiments, the presence of a translational non-electrostatic force was demonstrated by the slight shift in clusters away from the center of the confinement electrode (see Section 3.4.4 for Superposition of electrostatic potentials and translational force). This translational non-electrostatic force can be considered, for example, thermophoretic force given by \([145][146]\):

\[
F_{\text{th}} = \frac{16\sqrt{\pi}}{15} \frac{a^2 \chi}{v_{\nu}} \frac{\partial T}{\partial \rho}
\]

(4.28)

where \( \chi \) is the heat conductivity of the neutrals and \( \frac{\partial T}{\partial \rho} \) is the temperature gradient. As an estimation, for argon gas at 300 K, \( \chi \approx 0.0173 \text{ mkgs}^{-3} \) \([147]\), \( \frac{\partial T}{\partial \rho} \approx 50 \text{ Km}^{-1} \) in the sheath, \( \frac{\partial Z(\rho)}{\partial \rho} \approx 500 \text{ m}^{-1} \), we get \( \omega \approx 5 \times 10^{-4} \text{ rads}^{-1} \) for large dust particles and \( \omega \approx 1 \times 10^{-3} \text{ rads}^{-1} \) for small dust particles. Such small values show us that the dust charge gradient with thermophoretic force is not responsible for cluster/crystal rotation under our experimental conditions.

### 4.4.3. Divergence of magnetic field

Interestingly, all of the current theories on dust rotation do not take into account the divergence of the axial magnetic field in real experiments. Since the sheath electric field is at least ten times the magnitude to the confinement electric field, only a small tilt in the angle of the magnetic field away from the vertical axis will increase the magnitude of the azimuthal component of ion drag force (see Figure 4-39). For a magnetic field divergence of 11.5 degrees, the \( E_C \times B^\parallel \) component and the \( E_S \times B^\rho \) component will be equal. Thus the estimation in the ion drag force can increase twice in the divergence of magnetic field.
Figure 4-39 – Since the sheath electric field is at least ten times the magnitude of the radial electric field, only a small tilt in the angle of the magnetic field away from vertical will increase the magnitude of the azimuthal component of ion drag force.

4.5. Summary

Planar dust clusters, dust strings, three dimensional dust clusters, intermediate dust clusters, large dust crystals, and annular dust crystals have been rotated collectively as a rigid-body in the left handed direction with respect to the magnetic field in an inductive rf plasma. The magnetic field used in our experiments was 2 to 3 orders less than that used in other experiments.

In particular, we learned that:

■ In general, angular velocity of the cluster/crystal rotation increases with magnetic field strength. The angular velocity is linearly dependent on the magnetic field for large crystals and intermediate dust clusters.
■ The angular velocity of dust clusters exhibit nonlinear dependency on magnetic field and dependence on number of particles in the cluster.
■ Single ring L-Planar clusters requires a certain threshold magnetic field strength over which the rotation will initiate. In general, the threshold magnetic field decreases as the number of particles increases.
■ Double ring L-Planar clusters exhibits angular velocity saturation.
■ L-Planar-2 cluster exhibit periodic pauses during rotation for magnetic field strength up to 100 G.
■ Theories based on the ion drag force have the best agreement with experimental results. In most case, the models provide a good qualitative description of dust rotation in magnetized plasma. Under some condition, the models give adequate...
quantitative predictions.
- For proper description of planar cluster rotation, future theoretical development is needed.
5. Fluctuations of Planar Dust Clusters

The purpose of the following study is to analyze the structural fluctuations (or instabilities) of planar dust clusters under the influence of a magnetic field. As we have learnt, the cluster radii of L-Planar and S-Planar clusters fluctuated with time. From the application point of view, such fluctuations are undesirable as the ultimate goal of particle manipulation lies in the precision in controlling the position of the dust particles. For most applications, the fluctuations of the outer particles are expected to be the most important. As a result, we made it a standard procedure in our analysis that only particles in the outer ring of the dust clusters would be used in the determination of instabilities.

Using the same experimental data obtained from the planar dust cluster rotation experiments, the instabilities of planar dust clusters from 3 to 12 particles with two different dust sizes were studied (actually some periods of data which were used in rotation analysis could not be used in instability analysis due to structural changes between metastable states). As the motion of the dust particles are strongly confined in the axial direction, the instabilities are expected to be important in the azimuthal and the radial direction only. Thus only the radial and the azimuthal instabilities of these clusters were analyzed.

5.1. Cluster Instabilities

5.1.1. Radial instability

We define the radial instability as a measure of how widely the outer particles in a dust cluster moved stochastically away from their mean position in the radial direction. Experimentally, the radial instability is calculated from the first standard deviation of a population of cluster radii $\rho_1, \rho_2, ..., \rho_M$, measured at different times $t$ (see Figure 3-3):

$$\sigma^\rho = \sqrt{\frac{\sum_{i=1}^{M} (\rho_i - \bar{\rho})^2}{M}}$$  \hspace{1cm} (5.1)

where $\bar{\rho}$ is the mean and $M$ is the number of cluster radii in the sample. If the range of the cluster radii varies widely from their mean value, then the magnitude of the radial instability will be higher. For comparison purposes, we also define the radial instability coefficient (a dimensionless quantity) as:

$$S^\rho = \frac{\sigma^\rho}{\rho}$$  \hspace{1cm} (5.2)
5.1.2. Azimuthal instability

Similar to radial instability, we define the azimuthal instability as a measure of how widely the outer particles in a dust cluster moved stochastically away from their mean position in the angular direction. Since the dust clusters were rotating in the plasma with varying angular velocity, therefore a rotating coordinate system was devised to measure the angular position of the particles (see Figure 5-1). Here, the angular position of each particle in the outer ring was measured relative to a baseline through the centre of mass and the \( j \)th particle in the outer ring. Since the particles in the cluster rotated on average as a rigid-body and fluctuated only slightly around their equilibrium position, this baseline served as a suitable coordinate system as it rotated with the cluster at all times.

![Figure 5-1 – A rotating axis based on a particle in the spinning cluster was used to determine azimuthal instability coefficients.](image)

Using this coordinate system, the azimuthal instability coefficient based on the \( j \)th particle is calculated from the average of the first standard deviation of a population of angular positions \( \theta_1, \theta_2, \ldots, \theta_M \), measured at different times \( t \) for particle 1, 2, \ldots, \( j-1, j+1, \ldots, N' \) (\( N' \) is the number of particles in the outer ring):

\[
S^\theta (\text{based on the } j\text{th particle}) = \frac{\sum_{k=1,k\neq j}^{N'} S^\theta_k}{N'-1} \tag{5.3}
\]

where \( S^\theta_i = \sqrt{\frac{\sum_{i=1}^{M} (\theta_i - \bar{\theta})^2}{M}} \),
θ is the mean and \( M \) is the number of angular positions in the sample.

Table 5-1 shows the azimuthal instability coefficients based on each of the particle in S-Planar-6 system. And as shown from the table, the calculated values of the azimuthal instability coefficients are dependent weakly on the particular particle chosen. Consequently, the azimuthal instability coefficient of the dust cluster was obtained by averaging over all dust particles:

\[
S^\theta = \frac{1}{N^\prime} \sum_{j=1}^{N^\prime} S^\theta_j \text{ (based on the } j\text{th particle)}
\]

(5.4)

The unit for azimuthal instability coefficient is radians. The azimuthal instability (with units in meters) is given by:

\[
\sigma^\theta = \rho S^\theta
\]

(5.5)

5.1.3. Total instability and instability ratio

Since the sheath electric field is much larger than the confinement electric field, the motion of the dust particles are strongly confined in the axial direction. Therefore the total instability and the total instability coefficient are given by:

\[
\sigma = \sqrt{\sigma^\rho^2 + \sigma^\theta^2}
\]

(5.6)

\[
S = \frac{\sigma}{\rho} = \sqrt{S^\rho^2 + S^\theta^2}
\]

(5.7)

We define the instability ratio as:

\[
\Theta = \frac{\sigma^\theta}{\sigma^\rho} = \frac{S^\theta}{S^\rho}
\]

(5.8)
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Table 5-1 - Azimuthal instability coefficients based on different outer particles in S-Planar-6 (1,5) at different magnetic field strength.
5.2. Cluster Instabilities versus Number of Particles

5.2.1. Radial instability coefficient versus number of particles

Figure 5-2 shows the radial instability coefficient dependence on the number of particles for L-Planar and S-Planar clusters at different settings of magnetic field strength. The radial instability coefficients of L-Planar and S-Planar clusters have been measured to be approximately 1% and 2 to 3% respectively. In general, the radial instability coefficient of S-Planar clusters is approximately 2 to 3 times larger than L-Planar clusters. Also, at low magnetic field strength, for both large and small particles, the radial instability coefficient for clusters with 6 or 8 particles tends to be higher compared to other configurations.

![Figure 5-2](image)

Figure 5-2 – Radial instability coefficient plotted as a function of the number of particles for L-Planar clusters (left) and S-Planar clusters (right) at different magnetic field strength. The grey dotted line highlights the higher instability observed in particular cluster systems.

5.2.2. Azimuthal instability coefficient versus number of particles

Figure 5-3 shows the azimuthal instability coefficient dependence on the number of particles for L-Planar and S-Planar clusters at different settings of magnetic field strength. In general, the azimuthal instability coefficient of S-Planar clusters is approximately 2 to 3 times larger than L-Planar clusters. In relation to Figure 5-2, the radial and azimuthal instability coefficients of clusters with 6 or 8 particles were more unstable than clusters with 7 or 9 particles for both L-Planar and S-Planar clusters. The azimuthal instability coefficients for L-Planar-10 and L-Planar-12 were also relatively high, which was not observed in their radial counterparts.
5.2.3. Dominance of azimuthal component in cluster instability

Next, the absolute values of the radial and the azimuthal instabilities were compared to determine the major component of cluster instability. Figure 5-4 and Figure 5-5 shows the azimuthal and radial instabilities dependence on the number of particles at different magnetic field strength for L-Planar and S-Planar clusters respectively. In comparison, the fluctuations of particles in the azimuthal direction are approximately 10 times larger than in the radial direction for both L-Planar and S-Planar clusters.

Figure 5-4 – Azimuthal instability (left) and radial instability (right) plotted as a function of the number of particles for L-Planar clusters at different magnetic field strength. The grey dotted line highlights the higher instability observed in particular cluster systems.
A more detailed comparison can be made via the use of the instability ratio. Figure 5-6 shows the instability ratio dependence on the number of particles at different magnetic field strength for L-Planar and S-Planar clusters respectively. Except for L-Planar-10, the instability ratio is approximately between 2 to 12. That is, the azimuthal instability of most cluster systems is 2 to 12 times greater than the radial instability.

The differences in magnitude of cluster instabilities observed in the radial and the
azimuthal direction are expected for the following reasons. Firstly, the amplitude of the particle fluctuation is determined by the spatial gradient of the potential well. Since the motion of the dust particles is strongly confined by the sheath electric potential in the vertical direction, particle fluctuations will predominately be in the azimuthal and the radial direction. In the radial direction, the particles oscillate in the superposition of the confinement potential and the interparticle potential. In the azimuthal direction, the particles are simply in the potential well due to its neighboring particles. This potential well is shallower compared to the radial case, therefore particles are expected to a higher fluctuation in the azimuthal direction. Secondly, as dust particles fluctuate, the position of the potential well will shift accordingly. This will further enlarge the amplitude of particle fluctuation in the azimuthal direction.

5.2.4. Total instability coefficient versus number of particles

Figure 5-7 shows the total instability coefficient as a function of the number of particles at different magnetic field strength for L-Planar and S-Planar cluster respectively. Since the fluctuations of particle are dominant in the azimuthal direction, therefore as expected, the total instability coefficient dependence on the number of particles resembles that observed in azimuthal instability. In general, the total instability coefficient of S-Planar clusters is approximately 2 to 3 times larger than L-Planar clusters.

Interestingly, for planar cluster systems with 6 to 12 particles, the total instability coefficient of the clusters with respect to the particle number appears to be higher for clusters with even numbers of particles and lower for clusters with odd numbers of particles. If we
compare the total instability coefficient with the packing sequence probability of the clusters (see Section 3.2.4 for Packing sequences), apparently, there is strong correlation between the two. Figure 5-8 shows both the packing sequence probability and the total instability coefficient against the number of particles for one-and-a-half and double ring planar clusters.

On one hand, because Planar-6, -8, -10, and -12 have two metastable states, therefore the particles in these cluster systems are less dedicated to their equilibrium position and tend to move a lot more as the cluster attempts to undergo structural transitions between metastable states. On the other hand, because Planar-7, -9, and -11 have a predominant configuration with packing sequence probability greater than 94%, therefore the particles in these cluster systems are more “loyal” and tend to fluctuate less and hence reduce the cluster instability. In other words, large particle displacement in these clusters is not favored as the difference in the energy of the allowed packing sequences is high.

Figure 5-8 – Packing sequence probability (top) and total instability coefficient (bottom) versus number of particles for one-and-a-half and double ring planar clusters. The number on the left of the black line marker indicates the packing sequence probability while the number on the right of the black line marker indicates the number of particles in the inner ring of the cluster system. Cluster systems with a high probability (> 94 %) for a particular configuration are more stable than others with more than one particular probable configuration.
Consequently, the total instability coefficient is expected to be low if cluster system has a predominant configuration with high packing sequence probability and high if metastable states are present. This is further validated with the observation that planar cluster systems with 2 to 4 particles have the lowest total instability coefficients since they can have one and only one possible packing sequence corresponding to its ground state.

5.3. Cluster Instabilities and Magnetic Field

5.3.1. Radial and azimuthal instabilities versus magnetic field

Figure 5-9 and Figure 5-10 shows the azimuthal and radial instabilities dependence on the magnetic field strength for L-Planar and S-Planar clusters respectively. For L-Planar cluster systems, in general, an increase in magnetic field corresponds to a decrease in the azimuthal and radial instabilities. On the other hand, for S-Planar cluster systems, the azimuthal and radial instabilities remained approximately constant and appeared to be unaffected by the magnetic field. The result is expected as the magnetic field significantly reduced the size of L-Planar clusters but not so much for S-Planar clusters (see Section 3.1.3 for Cluster radius and radial distance). This in turn reduced the radial and azimuthal instabilities of L-Planar clusters.

Figure 5-11 shows the instability ratio of azimuthal instability dependence on the magnetic field strength for L-Planar and S-Planar systems. Apart from L-Planar-10 and L-Planar-11, the instability ratio remained approximately the same over all magnetic fields for both L-Planar and S-Planar systems.

![Figure 5-9](image_url) – Azimuthal instability (left) and radial instability (right) plotted as a function of the magnetic field strength for L-Planar clusters.
5.3.2. Total instability coefficient versus magnetic field

Figure 5-12 shows the total instability coefficient as a function of the magnetic field strength for L-Planar and S-Planar cluster respectively. As the magnetic field strength increased, the total instability coefficient increased for L-Planar cluster systems but remained approximately the same for S-Planar cluster systems.
Previously, cluster systems with only one ground state configuration have been shown to have lower total instability coefficient than those cluster systems with a number of metastable states (see Section 5.2.4 for Total instability coefficient versus number of particles). Therefore, the increase in the magnetic field strength which decreases the total instability coefficient is expected to reduce the number of metastable states to one configuration of high packing sequence probability. In fact, this coincides with our experimental observation that metastable states of dust clusters were observed only at low but not high magnetic field strength (see Section 3.1.4 for Metastable states). And so it can be generalized that an increase in magnetic field will increase the packing sequence probability of the preferred configuration and inhibits structural transitions between metastable states.

![Figure 5-12](image)

**Figure 5-12 –** Total instability coefficient plotted as a function of the magnetic field strength for L-Planar clusters (left) and S-Planar clusters (right).

### 5.4. Further Remarks on Dust Fluctuations

The cluster eccentricity might also have an influence on the cluster instability. Figure 5-13 shows snapshots of the particle position relative to the center of mass for S-Planar-7 and S-Planar-8.

As shown, S-Planar-7 was quite circular as the particles in this cluster system were uniformly distributed azimuthally and at a consistent radial distances from the center of mass. On the contrary, at least one of the particles in the outer ring in S-Planar-8 was positioned on average a larger distance from the centre of mass than the other particles. The fluctuation of such misfit particle is expected to have an effect on its neighboring particles and will require further investigation in the future.
5.5. Summary

In this chapter, both the radial and the azimuthal cluster instabilities of planar dust clusters under the influence of a magnetic field have been analyzed. In particular, we learned that:

- Clusters with smaller particles will have both radial and azimuthal instabilities larger than clusters with larger particles.
- The instability ratio for our cluster systems has been measured to be approximately 2 to 12. That is, the cluster instability in the azimuthal direction is 2 to 12 times larger than that in the radial direction.
- The total instability coefficient of the clusters is strongly correlated with their packing sequence probability. That is, clusters with two or more metastable states have higher total instability coefficient than those with one configuration.
- Planar clusters with one and only one possible packing sequence corresponding to its ground state have the lowest total instability coefficient.
- An increase in magnetic field will reduce the size of the cluster, which will in turn decrease the cluster instabilities.
- The instability ratio remained approximately the same over all magnetic fields.
- An increase in magnetic field will increase the packing sequence probability of the ground state and inhibit structural transitions between metastable states.
6. Future Applications

The ability to manipulate and stabilize particles can provide radical innovations in a wide range of industrial applications. This chapter aims to suggest possible innovations arising from our understanding of dusty plasma in a magnetic field. The chapter is separated into five sections. Each section will illustrate the connection between our basic understandings of dust motion, especially rotation in a magnetic field, with a particular application.

6.1. Dust Removal

Dust contamination is a common problem in the production of many types of electronic devices (e.g. CMOS, CCD, image sensors, LCD, plasma displays, and solar cells, etc) that requires the use of etching, sputtering, or polymerizing plasma [8][72]. Dust deposited onto the plasma treated surfaces can destroy or lower the quality of the electronic devices. At the moment, the most common dust removal method used by the industry is wet cleaning (WC). Dry cleaning methods such as cryogenic cleaning (CC), laser cleaning (LC), and nanoprobe cleaning (NC) might also be used in dust removal in the very near future [89].

Here, two dust cleaning methods which utilize an axial magnetic field to remove dust contamination are proposed (see Figure 6-1):

- Dust confinement method – dust contamination suspended in the plasma is confined and transported away from substrate using a magnetic field generated by a movable magnetic coil placed inside or outside the plasma chamber;
- Dust rotation method – dust contamination suspended in the plasma is rotated by a magnetic field to a location where it can be removed using laser or other expulsion tools (e.g. gas jet, electrostatic probe, etc).

Since these two methods achieve dust removal during plasma operation, they do not interfere and can be used in conjunction with conventional wet and dry cleaning methods. Moreover, a major innovative point in the second method is that the expulsion tool is not required to locate the individual dust particle for removal. The magnetic field simply assists the expulsion tool by rotating the dust particles into the path of expulsion and thus the procedure can be automated much more easily.

There are many advantages of utilizing magnetic field to remove dust contamination compared to conventional wet and dry cleaning methods. These include:

- Environmentally friendliness – compared to the use of toxic solvents (WC) or greenhouse gas (CC);
- Low development cost – compared to nanotechnology (NC);
- Low installation and maintenance cost – compared to lasers (LC) or nanoprobe (NC);
- Minimal running cost – compared to the expensive solvents used in wet cleaning;
- Compactness – compared to special storage space necessary for high purity solvents (HC) or dry ice (CC);
- The position of magnet is not limited by the internal geometry of the operation unit; and
- No safety hazards with low magnetic field (≈ 100 G) – compared to highly acidic or basic solution (WC), laser burns (LC), or asbestos-like nanotube components (NC).

**Dust Confinement Method**

**Dust Rotation Method**

Figure 6-1 – By applying an axial magnetic field, dust contamination can either be removed via dust confinement (top) or dust rotation method (bottom)

### 6.2. Plasma Diagnostics

The interest in the research community and the demand of using plasma in various industrial applications has initiated the development of numerous diagnostic methods. Specific properties of plasma which commonly need to be determined include the local electric potential, the plasma boundary layer, potential difference, and the number density profiles of electrons and ions.
In most laboratory situations, the plasma source can be specifically designed and built to incorporate diagnostic tools necessary to determine these properties. Unfortunately in most industrial situations, for example in dust-free plasma processing facilities, the operational environment cannot be easily modified and typically offers very limited access for diagnostic tools. Moreover, many of the conventional diagnostic methods require a substantial installation and maintenance cost or takes very long to provide the necessary information. In contrast, dust particles, which are micron-sized, inexpensive, highly responsive, and in most cases inherent in the plasma processing unit could be used as a new form of diagnostic tool. In fact, dust particles have already being used as a diagnostic tool to determine the plasma sheath edge [148].

Our experimental results with planar dust clusters have expanded plasma diagnostic possibilities. As shown on Table 6-1, a variety of classical diagnostic methods can be used to measure the number density, size, shape, mass, chemical composition, charge, temperature, and velocity distribution of the dust particles [8]. The fact that magnetic fields can rotate dust particles in a controlled manner means it can play a supportive role during visualization (e.g. particle counter, etc) and optical analysis (e.g. light scattering, infrared spectroscopy, etc) of the dust particles in a plasma. More details on how magnetic fields can be used to manipulate particles will be given later (see Section 6.3 for Magnetic Manipulation Device).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Diagnostic Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number density</td>
<td>Particle counters, light scattering, laser heating, plasma monitoring</td>
</tr>
<tr>
<td>Size, shape, density, mass</td>
<td>Rayleigh/Mie scattering, laser heating, scanning electron microscopy, transmission electron microscopy, mass spectrometry</td>
</tr>
<tr>
<td>Chemical composition</td>
<td>Infrared absorption, Raman spectroscopy, elemental analysis, mass spectrometry</td>
</tr>
<tr>
<td>Charge</td>
<td>Mass spectrometry, photodetachment, resonant oscillations in electric field</td>
</tr>
<tr>
<td>Temperature</td>
<td>Phosphorescence, black body radiation</td>
</tr>
<tr>
<td>Kinetic Temperature, Velocity distribution</td>
<td>Light scattering, visualization</td>
</tr>
</tbody>
</table>

Table 6-1 – The above table shows an overview of the dust-related parameters and their corresponding diagnostics techniques [8].

Single ring and one-and-a-half ring planar dust clusters have been demonstrated to be a convenient diagnostic tool for:

- Characterization of local electric field and electrostatic potential
- Symmetry, dimension, and eccentricity of electrostatic potential
- Temperature gradient

In particular, by knowing the levitation height and the interparticle distance, the sheath electric field (i.e. $mg = eZe_s$) and the confinement electric field can be determined respectively (see Section 3.4.1 for Confinement electric field). The phase diagram (see Section 4.1.6 for Phase diagram and period pause) and packing sequences (see Section 3.2.4...
for Packing sequences) can be used in the qualitative evaluation of the symmetry and eccentricity of the plasma potential.

Also, intermediate clusters have been demonstrated in its sensitivity in diagnosing the symmetry, dimension, and shift of the electrostatic potential as biased voltage on the electrode or magnetic field are changed (see Section 3.4.4 for Superposition of electrostatic potentials and translational force).

Using planar clusters with small numbers of particles to diagnose local electric fields not only minimizes the perturbation of the plasma caused by electrostatic probe. By observing the position and movement of the particles, information can be obtained about the electric field and the potential distribution near electrodes and substrate surfaces where other plasma diagnostic methods fail. The dust particles are effectively electrostatic micro-probes on a scale not achievable even with the finest conventional probes. Techniques allowing the particles to be monitored during the discharge operation provide a simple means for monitoring the plasma in industrial processing.

There are still further developments required for these proposed diagnostic methods. For example, in theory, a vertical profile of the confinement and sheath electric fields can be mapped out using dust particles of different sizes or masses. However, the accuracy of the method is largely dependent on our knowledge about the charge of the dust particle (the mass of the dust particle is usually known as specified by the manufacturer). Currently, the charge of the particle can be determined in situ by resonance method [149] or transient motion method [150].

Also, large crystals will be needed in the characterization of larger electric potentials. However, more sophisticated imaging hardware and software to detect the positions of the particles and sophisticated models to calculate the interparticle forces are necessary for large crystals to be a viable diagnostic tool.

### 6.3. Magnetic Manipulation Device

At present, the abilities to stabilize the position and accurately control the trajectory of particles are highly sought in micro- and nano-technological applications. Specially coated powders (see Section 6.4 for Thin Film Coating with Magnetron Sputtering), novel materials, and micro- and nano-devices which have been plasma treated or crafted within a plasma needs to be transported in one form or another along the production line.

From our existing knowledge about dusty plasma, methods of manipulation which can be applied to maneuver individual particle or a collection of particles in a plasma include (see Figure 6-2):

- Optical method – laser can be used not only to thrust selective particles from a particle string or monolayer crystal, but also oscillate particles in the vertical or horizontal directions [90]-[91];
- Electron and ion method – both electron and ion beams can be used to adjust vertical and horizontal position of particles via drag force and/or charging of dust;
- Electrical method – charge gradients established in a plasma via electrostatic probe [92], external electrode [93], or introduction of system asymmetry [156] can be used...
to set a collection of particles into rotation or vortex motion. The motion can be highly organized if the particles are strongly correlated. Also by voltage variation on the probe or the electrode, individual particle can be expelled, maneuvered vertically and horizontally, set into oscillations [90]-[91], or orbital motion [94]; and

- Magnetic method – external application of magnetic field normal to the dust plane can cause both dust clusters and crystals to rotate [24][103]-[106][108]-[109][111][116][118]-[119][121][124]-[125][127].

Obviously the position and trajectory of the particles can also be influenced using other methods such as neutral gas flow, temperature gradient, and variation in neutral gas pressure or plasma input power. However, these methods are nonselective, have significantly slower response time, and are usually intrusive to the processing plasma. So instead, these methods should be regarded as implemental tools in particle manipulation.

Figure 6-2 – Particles can be selectively set into oscillations or pushed in the horizontal and vertical direction using laser, ion beam, electrostatic probe, etc (left). The position of the particles can move in the radial/horizontal and axial/vertical direction using confinement electrode (center), and in the azimuthal direction using magnetic field, introduction of system asymmetry, etc (right).

Also, it is well known that the particles can be individually oscillated, expelled (in order to remove undesired particles), or set into rotation using a laser beam (inter-ring rotation). Interestingly, experimental work had been performed by Klindworth et al. [87] which demonstrated that both inter-ring rotation and rigid cluster rotation are possible depending on the “magic particle number” utilizing multiple laser beams. Intuitively, magnetic fields can be used as the “third helpful hand” in rotating the cluster. However, what the combined effect of the laser and magnetic field will have on the cluster motion will need further investigation.

Figure 6-3 shows a conceptual depiction of how magnetic fields can be introduced into the production line of a typical plasma processing unit to manipulate particles.

- Production unit – dust particles which have been plasma treated with tailored properties or special coatings begin their journey here inside the production chamber. An axial magnetic field is introduced using a magnetic coil sitting outside the chamber underneath the particles. Electrode initially used to confine the particles is switched off while the magnetic coil is switched on. The dust particles levitating in the plasma is confined using magnetic field (see Section 3.4.3 for Intermediate clusters and magnetic confinement of dust particles);
Figure 6-3 – A conceptual depiction showing the use of magnetic field in particle manipulation along the production line.
Transportation unit – by moving the magnetic coil along the rail, dust particles can be transported along the plasma from unit to unit while maintaining the isolation of the vacuum chamber.

Quality control unit – when dust particles arrive in view of observation window, they can be examined for quality control with the aid of microscope and illumination laser. Defective particles can be individually selected and removed using a high power pulsed laser as they are rotated in a magnetic field (see Section 6.1 for Dust Removal). To increase accuracy, an additional electrode can be used to fine tune the radial and vertical position of the dust particles; and

Collection unit – finally the dust particles are collected using another confinement electrode or simply deposited onto substrate. The coil is then returned to its original position and the whole process can be repeated.

Essentially the use of magnetic fields can be implemented at all stages of production line for various purposes. And the advantages of utilizing magnetic fields to manipulate particles include:

- Multifunction (e.g. rotate, radial and vertical movement, stabilize),
- No modification of access (e.g. optical window, port for electrostatic probes) to the operation unit is necessary,
- Low installation and maintenance cost compared to lasers, electron and ion guns,
- Minimal running costs especially if permanent magnet or low strength magnetic fields are employed,
- Easy operation and reproduction,
- Compactness,
- Interchangeability,
- Maintain isolation of vacuum system,
- Slow motion of particle can be achieved without increasing neutral damping,
- Increases stability of particles,
- Path of magnet is not limited by the internal geometry of the operation unit, and
- Potential and severity of safety hazards related to magnetic field (e.g. cardiac pacemaker, ferromagnetic biological implants) is much lower than lasers (e.g. eye, skin), electron and ion guns (e.g. hot cathode, high voltage, x-rays leakage, etc).

### 6.4. Thin Film Coating with Magnetron Sputtering

In industry, coating thin films (in the scale of few hundred layers of atoms) of metals or insulators on substrates is commonly performed for the purpose of surface modification. Particularly in micro- and nanotechnology, thin film coating on very small particles is currently being developed to manufacture powders with tailored functionalities (e.g. geometrical shape, chemical, electrical, and optical properties). Gas injection, chemical catalysis, and magnetron sputtering are some of the possible methods used to coat powders in plasma. However, only a few papers have addressed the particle dynamics in plasmas designed for coating powders [153]-[155]. Especially, particle dynamics in a magnetized plasma designed to coat powders has not been described so far.
In magnetron sputtering, conventionally, target electrode of a single type of material is bombarded by plasma ions, allowing free atoms of the target metal to be released and to deposit onto particles levitating in the plasma. A magnetic field is employed in magnetron sputtering to confine the plasma electrons near the target in order to increase the ionization rate and hence the sputtering rate needed for industrial production. To increases target life, a rotating magnetic field is often used to avoid set erosion paths by sweeping the erosion region across a wide area of the target [151].

In relation to our findings, obviously a magnetic field can be used in the transportation of coated powders (see Section 6.3 for Magnetic Manipulation Device). In addition to particle transportation, however a secondary magnetic field can also be introduced into a magnetron sputtering device to coat multi-layers of films onto particles. Figure 6-4 shows a magnetron sputtering device with two different types of metals used as targets. Small clusters or large crystals in the plasma can be rotated while ions of the two different metals coat alternatively onto the particle surface.

The coating of powders using magnetron sputtering has the advantage of achieving the thinnest, most uniform, and the most bonded layers of films possible onto any type of particles (metals, ceramics, and heat-sensitive plastics) that can be levitated in the plasma. As a matter of fact, the concept is not far from the reality. Silicon dioxide particles have been shown to rotate in a rf discharge coupled with a dc magnetron sputtering device using a permanent magnet [124]-[125].

Figure 6-4 – Particles can be rotated while the ions of two different metals coat alternatively onto the particle surface via magnetron sputtering.
6.5. Particle Assisted Ion Etching

Apart from coating dust with special materials, dust particles themselves can be arranged into ordered structure and be used as a masking tool to assist in ion etching. In this concept, ordered structures of dust particles are levitated and above the substrate. Fluxes of reactive ions are focused onto the wafer, creating an ordered array of micro- and nanometer-sized craters as a result of surface etching. Therefore, ordered structure of dust particles serve as a lithographic mask which particle position and interparticle distance can be adjusted using electric and magnetic field (see Figure 6-5).

Figure 6-5 – Without particles, plasma ions would stream uniformly towards the target electrode. If ordered structures of dust particles were levitated in the plasma, plasma ions would stream towards the charged particles and be focused onto the substrate electrode.

With the application of a magnetic field, the ordered array of particles can be rotated to etch out circular patterns on the wafer. The quality of the etching patterns will be largely determined by the stability of the dust particles. Our analysis in cluster instabilities is very important for minimizing the random motion of dust particles. Dust particles are required to have instabilities in the micro- or nanometer scales ($S^\theta$ and $S^\rho$ were in the range of $10^{-5}$ m for our dust particles). Note that as the size of the dust particle decreases, our experimental results show that the cluster instabilities will increase. So the minimization of instabilities will become more challenging especially if particles are to be used in nanotechnological applications.

From our analysis, the application of an axial magnetic field had been demonstrated to be capable of reducing the cluster instabilities along with the size of the clusters and inhibiting structural transitions between metastable states (see Section 5.3.2 for Total instability coefficient versus magnetic field). The latter capability can be used in controlling the packing sequence of the clusters while in the plasma and during the deposition process.

In situations where dust clusters are required to form defined ring structures, our analysis will help to identify the appropriate packing sequence with minimal instabilities. Using one-and-a-half ring planar cluster as an analogy, particle arrangement in Planar-7 configuration will be have less total instability compared to other form of arrangement such as Planar-6 and -8 (see Section 5.2.4 for Total instability coefficient versus number of particles).
7. Conclusion

This thesis has provided a detailed experimental record of the behavior of dust particles in magnetized plasma. The experiments were performed in inductive rf discharge plasma under pressure range 100 to 500 mTorr, power 500 mV at 17.5 MHz. The dust particles used were melamine formaldehyde with radius 3.105 µm and 1.395 µm. Dust clusters with different numbers of particles in a horizontal plane were formed. In particular, Planar-2 (two particles in a horizontal plane) to Planar-16 dust clusters were formed. Also, experiments with particle strings, three-dimensional clusters, and large and annular dust crystals were conducted.

It has been demonstrated that the dust clusters rotate with application of an axial magnetic field. The direction of rotation is in the left-handed sense. And the angular velocity is dependent on the number of particles and the magnetic field.

Comparisons were made between the theoretical and experimental values of angular velocity. It was demonstrated that models based on ion drag force provide the best agreement with the experimental data. But future development in the driving mechanism for dust cluster rotation will be necessary to accurately predict the magnitude of driving force measured in our experiments. Possible explanations were proposed for discovered features in cluster rotation such as angular velocity saturation of double ring clusters, oscillations and periodic pause of Planar-2 clusters. It was shown that the divergence of magnetic field and translational force might need to be taken into account along with the ion drag force to adequately describe observed phenomena. The cluster instabilities were analysed and the fluctuations was found to be higher for S-Planar clusters than for L-Planar clusters. The cluster instability in the azimuthal direction is 2 to 12 times larger than that in the radial direction. It was established that an increase in magnetic field will reduce the size of the cluster, which will in turn decrease the cluster instabilities. The instability ratio (or the ratio between azimuthal and radial instabilities) remained approximately the same over all magnetic fields.

We achieved dust particles confinement by magnetically induced electrostatic trap. The confinement was carried out over an unbiased, flat electrode by using an ambipolar radial electric field induced by magnetic field. The confinement electric field is created due to magnetization of electrons which changes the ambipolar diffusion coefficient of electrons.

The experiments were complemented with computer simulations of the structural configuration and stabilities of planar dust clusters. Dust cluster systems were modeled by considering Coulomb (Debye) repulsion, electrostatic confinement, and friction force. A statistical distribution of inter-ring rotations and energies was found and compared to those predicted by theory. Subsequently, a more accurate model for inter-ring rotation which incorporated the curvature of rings was proposed.

The packing sequences of dust clusters with different number of particles were obtained. It was found that the total instability coefficient of the clusters is strongly correlated with their packing sequence probability. That is, clusters with two or more metastable states have higher total instability coefficient than those with one configuration. And planar clusters with one and only one possible packing sequence corresponding to its ground state have the lowest total instability coefficient.
8. Appendix

The appendices in this chapter describe the procedures used in obtaining some of the information essential in this thesis.

8.1. Appendix I – Macro Used in Data Reorganization

The computer software available was only capable of determining the x- and y-coordinates of the dust particles. The data were stored as individual text files for each frame of the video. Therefore the following program was written in the macros of Microsoft Excel to merge the separate text files and reorganize the data into one continuous spreadsheet.

```vba
Sub InsertFile2()
    ' This macro merges the text files of the x and y coordinates
    ' of the dust particles from designated directory
    ' Dim Check, Counter, Counter1
    Check = True: Counter = -1   ' Initialize variables.
    Do ' Outer loop.
        Do While Counter < 10      ' Inner loop.
            Counter = Counter + 1   ' Increment Counter.
            Counter1 = Counter * 20 + 1 ' Increment Counter1.
            With ActiveSheet.QueryTables.Add(Connection:= _
                "TEXT;C:\WINDOWS\Desktop\SParticles\Planar-6000" & Counter & ".txt", Destination _
                :=Cells(Counter1, 1))
                .Name = "Planar-6000" & Counter
                .FieldNames = True
                .RowNumbers = False
                .FillAdjacentFormulas = False
                .PreserveFormatting = True
                .RefreshOnFileOpen = False
                .RefreshStyle = xlInsertDeleteCells
                .SavePassword = False
                .SaveData = True
                .AdjustColumnWidth = True
                .RefreshPeriod = 0
                .TextFilePromptOnRefresh = False
                .TextFilePlatform = xlWindows
                .TextFileStartRow = 1
                .TextFileParseType = xlFixedWidth
                .TextFileTextQualifier = xlTextQualifierDoubleQuote
            End With
        Loop
    Loop
End Sub
```
Dim Check1, Counter2, Counter3
Check1 = True: Counter2 = 9  ' Initialize variables.
Do  ' Outer loop.
  Do While Counter2 < 100  ' Inner loop.
    Counter2 = Counter2 + 1  ' Increment Counter2.
    Counter3 = Counter2 * 20 + 1  ' Increment Counter3.
  With ActiveSheet.QueryTables.Add(Connection:="TEXT;C:\\WINDOWS\\Desktop\\SParticles\\Planar-600" & Counter2 & ".txt", Destination :=Cells(Counter3, 1))
    .Name = "Planar-600" & Counter2
    .FieldNames = True
    .RowNumbers = False
    .FillAdjacentFormulas = False
    .PreserveFormatting = True
    .RefreshOnFileOpen = False
    .RefreshStyle = xllInsertDeleteCells
    .SavePassword = False
    .SaveData = True
    .AdjustColumnWidth = True
    .RefreshPeriod = 0
    .TextFilePromptOnRefresh = False
    .TextFilePlatform = xlWindows
    .TextFileStartRow = 1
    .TextFileParseType = xlFixedWidth
    .TextFileTextQualifier = xlTextQualifierDoubleQuote
    .TextFileConsecutiveDelimiter = False
    .TextFileTabDelimiter = True
    .TextFileSemicolonDelimiter = False
    .TextFileSpaceDelimiter = False
    .TextFileColumnDataTypes = Array(1, 1, 1)
    .TextFileConsecutiveDelimiters = Array(1)
    .Refresh BackgroundQuery:=False
  End With
  Cells(1, 6).Select
  If Counter = 9 Then  ' If condition is True.
    Check = False  ' Set value of flag to False.
    Exit Do  ' Exit inner loop.
  End If
  Loop
Loop Until Check = False  ' Exit outer loop immediately

End With

.TextFileSpaceDelimiter = False
.TextFileColumnDataTypes = Array(1, 1, 1)
.TextFileFixedColumnWidths = Array(4, 2)
.Refresh BackgroundQuery:=False
End With
.Cells(1, 6).Select
If Counter2 = 99 Then   ' If condition is True.
    Check1 = False    ' Set value of flag to False.
    Exit Do   ' Exit inner loop.
End If
Loop
Loop Until Check1 = False   ' Exit outer loop immediately

Dim Check2, Counter4, Counter5
Check2 = True: Counter4 = 99   ' Initialize variables.
Do   ' Outer loop.
    Do While Counter4 < 1000   ' Inner loop.
        Counter4 = Counter4 + 1   ' Increment Counter4.
        Counter5 = Counter4 * 20 + 1   ' Increment Counter5.
    With ActiveSheet.QueryTables.Add(Connection:=_ 
        "TEXT;C:\WINDOWS\Desktop\SParticles\Planar-60" & Counter4 & ".txt", Destination := 
        Cells(Counter5, 1))
        .Name = "Planar-60" & Counter4
        .FieldNames = True
        .RowNumbers = False
        .FillAdjacentFormulas = False
        .PreserveFormatting = True
        .RefreshOnFileOpen = False
        .RefreshStyle = xlInsertDeleteCells
        .SavePassword = False
        .SaveData = True
        .AdjustColumnWidth = True
        .RefreshPeriod = 0
        .TextFilePromptOnRefresh = False
        .TextFilePlatform = xlWindows
        .TextFileStartRow = 1
        .TextFileParseType = xlFixedWidth
        .TextFileTextQualifier = xlTextQualifierDoubleQuote
        .TextFileConsecutiveDelimiter = False
        .TextFileTabDelimiter = True
        .TextFileSemicolonDelimiter = False
        .TextFileCommaDelimiter = False
        .TextFileSpaceDelimiter = False
        .TextFileColumnDataTypes = Array(1, 1, 1)
        .TextFileFixedColumnWidths = Array(4, 2)
        .Refresh BackgroundQuery:=False
    End With
    Cells(1, 6).Select
    Check1 = False    ' Set value of flag to False.
    Exit Do   ' Exit inner loop.
End If
Loop
Loop Until Check1 = False   ' Exit outer loop immediately
End With
Cells(1, 6).Select
If Counter4 = 999 Then ' If condition is True.
    Check2 = False ' Set value of flag to False.
    Exit Do ' Exit inner loop.
End If
Loop
Loop Until Check2 = False ' Exit outer loop immediately

Dim Check3, Counter6, Counter7
Check3 = True: Counter6 = 999 ' Initialize variables.
Do ' Outer loop.
    Do While Counter6 < 10000 ' Inner loop.
        Counter6 = Counter6 + 1 ' Increment Counter6.
        Counter7 = Counter6 * 20 + 1 ' Increment Counter7.
    With ActiveSheet.QueryTables.Add(Connection:= _
        "TEXT:C:\WINDOWS\Desktop\SParticles\Planar-6" & Counter6 & ".txt", Destination _
        :=Cells(Counter7, 1))
        .Name = "Planar-6" & Counter6
        .FieldNames = True
        .RowNumbers = False
        .FillAdjacentFormulas = False
        .PreserveFormatting = True
        .RefreshOnFileOpen = False
        .RefreshStyle = xlInsertDeleteCells
        .SavePassword = False
        .SaveData = True
        .AdjustColumnWidth = True
        .RefreshPeriod = 0
        .TextFilePromptOnRefresh = False
        .TextFilePlatform = xlWindows
        .TextFileStartRow = 1
        .TextFileParseType = xlFixedWidth
        .TextFileTextQualifier = xlTextQualifierDoubleQuote
        .TextFileConsecutiveDelimiter = False
        .TextFileTabDelimiter = True
        .TextFileSemicolonDelimiter = False
        .TextFileCommaDelimiter = False
        .TextFileSpaceDelimiter = False
        .TextFileColumnDataTypes = Array(1, 1, 1)
        .TextFileFixedColumnWidths = Array(4, 2)
        .Refresh BackgroundQuery:=False
    End With
Cells(1, 6).Select
If Counter6 = 1648 Then ' If condition is True.
    Check3 = False ' Set value of flag to False.
Exit Do  ' Exit inner loop.
End If
Loop
Loop Until Check3 = False  ' Exit outer loop immediately
Range("D1").Select
ActiveCell.FormulaR1C1 = "1"
Range("D2").Select
ActiveCell.FormulaR1C1 = "2"
Range("D3").Select
ActiveCell.FormulaR1C1 = "3"
Range("D4").Select
ActiveCell.FormulaR1C1 = "4"
Range("D5").Select
ActiveCell.FormulaR1C1 = "5"
Range("D6").Select
ActiveCell.FormulaR1C1 = "6"
Range("D7").Select
ActiveCell.FormulaR1C1 = "7"
Range("D8").Select
ActiveCell.FormulaR1C1 = "8"
Range("D9").Select
ActiveCell.FormulaR1C1 = "9"
Range("D10").Select
ActiveCell.FormulaR1C1 = "10"
Range("D11").Select
ActiveCell.FormulaR1C1 = "11"
Range("D12").Select
ActiveCell.FormulaR1C1 = "12"
Range("D13").Select
ActiveCell.FormulaR1C1 = "13"
Range("D14").Select
ActiveCell.FormulaR1C1 = "14"
Range("D15").Select
ActiveCell.FormulaR1C1 = "15"
Range("D16").Select
ActiveCell.FormulaR1C1 = "16"
Range("D17").Select
ActiveCell.FormulaR1C1 = "17"
Range("D18").Select
ActiveCell.FormulaR1C1 = "18"
Range("D19").Select
ActiveCell.FormulaR1C1 = "19"
Range("D20").Select
ActiveCell.FormulaR1C1 = "20"
Range("D21").Select
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D22").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D23").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D24").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D25").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D26").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D27").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D28").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D29").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D30").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D31").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D32").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D33").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D34").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D35").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D36").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D37").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D38").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D39").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D40").Select  
ActiveCell.FormulaR1C1 = "R[-20]C"  
Range("D21:D40").Select  
Selection.AutoFill Destination:=Range("D21:D33000"), Type:=xlFillDefault  
Range("D21:D33000").Select  
Range("E1").Select  
ActiveCell.FormulaR1C1 = "x1"  
Range("F1").Select  
ActiveCell.FormulaR1C1 = "y1"
Sub Main
    Range("G1").Select
    ActiveCell.FormulaR1C1 = "x2"
    Range("H1").Select
    ActiveCell.FormulaR1C1 = "y2"
    Range("I1").Select
    ActiveCell.FormulaR1C1 = "x3"
    Range("J1").Select
    ActiveCell.FormulaR1C1 = "y3"
    Range("K1").Select
    ActiveCell.FormulaR1C1 = "x4"
    Range("L1").Select
    ActiveCell.FormulaR1C1 = "y4"
    Range("M1").Select
    ActiveCell.FormulaR1C1 = "x5"
    Range("N1").Select
    ActiveCell.FormulaR1C1 = "y5"
    Range("O1").Select
    ActiveCell.FormulaR1C1 = "x6"
    Range("P1").Select
    ActiveCell.FormulaR1C1 = "y6"
    Range("Q1").Select
    ActiveCell.FormulaR1C1 = "x7"
    Range("R1").Select
    ActiveCell.FormulaR1C1 = "y7"
    Range("S1").Select
    ActiveCell.FormulaR1C1 = "x8"
    Range("T1").Select
    ActiveCell.FormulaR1C1 = "y8"
    Range("U1").Select
    ActiveCell.FormulaR1C1 = "x9"
    Range("V1").Select
    ActiveCell.FormulaR1C1 = "y9"
    Range("W1").Select
    ActiveCell.FormulaR1C1 = "x10"
    Range("X1").Select
    ActiveCell.FormulaR1C1 = "y10"
    Range("Y1").Select
    ActiveCell.FormulaR1C1 = "x11"
    Range("Z1").Select
    ActiveCell.FormulaR1C1 = "y11"
    Range("AA1").Select
    ActiveCell.FormulaR1C1 = "x12"
    Range("AB1").Select
    ActiveCell.FormulaR1C1 = "y12"
    Range("AC1").Select
End Sub
8.2. Appendix II – Routines for Computer Simulations

8.2.1. Main routine – planar cluster simulation

The following is the main routine written in MatLab for our computer simulations of planar cluster system. The routine calls for other subroutines which will be explained in the following subsections.

%Start with clean state

clear all;
clear track;
tic

%Specifying variables used in multiple functions

global n friction deltat eps0 e alpha beta gamma outshell
alpha = 1;    %Confinement force parameter
beta = 1;     %Azimuthal driving force parameter
gamma = 0.1;    %Linear force parameter
friction = 1;
deltat = 0.001;
es0 = 8.8542e-12;
e = -1.602E-19;
% e = e./2;
rscale = 0.14;
escale=0;
for n = 11,
fprintf('Simulating type %d-planar cluster, (program will terminate after planar 16)\n',n);

%spectra = zeros(500,2);
%equilibrium = zeros(n,2500);

twist = zeros(35000,8);
omega = zeros(35000,1);
omega_out = zeros(35000,1);
omega_in = zeros(35000,1);

%stdevy = zeros(50,1);
%positions = zeros(n,2,15000);

for w = 1
%remainder = rem(w,10);
%if (remainder == 0)
    fprintf('Commencing %d of 500 simulations\n', w);
    %end
% Initialise

x = 0; % y posn of particle
y = 0; % y posn of particle
p = zeros(n,2); % position of all particles, of the form p(x,y), ith particle is the ith row
rce = zeros(n,2); % Current velocity of each particle. 'u' in s = t + (1/2)*a*t^2

% Length of the simulation, altered to provide a period of stability for analysis purposes

t = (5/deltat)/friction^2.5*exp(friction/1.5);
t = round(t);
t = t + 20000;
P= zeros(35000,1);
K= zeros(35000,1);
B= zeros(35000,1);
vidrad = 3e-1;
vid = 'y';
tracking = 'n';
map = 'n';
progress = 'n';
condition = 'u';
saving = 'n';

% Timestamp video file

time = strrep(datestr(now),':','_');

% Generate the particles (random configuration around center)

for i = 1:n,
    x = (1e-1)*randn(1);
    y = (1e-1)*randn(1);
    for j = 1:3,
        p(i,1) = x;
        p(i,2) = y;
    end
end
if (vid == 'y'),
    aviobj = avifile([num2str(n),'.',time,'.',num2str(friction),'.',num2str(deltat),'.avi']);
end
if (progress == 'y'),
    h = waitbar(0,'Please wait...');
end

% Pre: Everything is ready for the simulation including particles in position

for j = 2:t, % For each time increment
    debyef = p.*0;%debye(p);
    interparticlef = eforce2(p);  % interparticle force on the particle
    confinementf = wforce3(p,escale);  % confinement force on the particle
    for k = 1:n,
        ranx = rscale*randn(1);  %This loop generates random force with normal dist'bn
        rany = rscale*randn(1);  %with magnitude determined by scale factor.
        randomf(k,1) = ranx;
        randomf(k,2) = rany;
    end
    %friction = 0;
    randomf = randomf.*0;
    % net force on the particle
    force = (interparticlef+debyef).*1e17 + confinementf - friction*v + randomf;
    if (j>8e3)
        azimuthalf = aforce(p,v,1);
        force = force + azimuthalf;
    end
    if(abs(confinementf + (interparticlef+debyef).*1e17 + randomf) < abs(friction*v))
        force= 0;
    end
end

%EULER METHOD
%---------------------------------------------------------------
% Post: All particles have been acted upon by the potential gradient
% and the interparticle force and random force
% move the particle
p = p + v.*deltat + 0.5*force.*deltat^2;
% stores current velocity
v = v + force.*deltat;

if ((j>0.9e4)&(j<= 4.4e4))
    %ENERGY CALCULATIONS
    %---------------------------------
    %P(j-0.9e4) = energy(p);
    %temp = sum(0.5.*v.^2);
%K(j-0.9e4) = sum(temp);
%B(j-0.9e4) = P(j-0.9e4) + K(j-0.9e4);
twist(j-9e3,:) = findtwist(n,p);
omega = findanglev(p,v);
omega_out(j-9e3,w)=omega(1);
omega_in(j-9e3,w)=omega(2);
%---------------------------------
end
if (tracking == 'y'),
    track(j,:,:) = p(:,:);
end
if (vid == 'y'),
    % This all makes a movie -- round(t/500)
    hold on;
    if (mod(j, 0.1/deltat) == 0),
        plot(p(:,1), p(:,2),'o','MarkerSize',6,'MarkerFaceColor',[0 0 0], 'MarkerEdgeColor', 'k');
        set(gca,'YLim',[-vidrad,vidrad], 'NextPlot','replace','XLim',[-vidrad,vidrad]);
        text(0.25, -0.29, num2str(j));
        frame = getframe(gca);
        aviobj = addframe(aviobj,frame);
    end
    cla;
    hold off;
end;
if (progress == 'y'),
    if (mod(j,round(t/500)) == 0),
        waitbar(j/t,h);
    end
end
type = hulls(p);
inner = type(1);

%spectra(w,1) = mean(B);
%spectra(w,2) = inner;
%equilibrium(:,:,w) = p;
%stdevy(w) = std(twist(:,w),0,1);
%----------------------------------------------------------------------------------------------------
%CLOSING DOWN CODE

if (progress == 'y'),
    close(h);
end
if (vid == 'y'),
    aviobj = close(aviobj);
% These are different plots displaying the simulation

if (tracking == 'y'),

    % This plots a 2D map of particle propagation
    figure;
    set(gcf,'Visible','off');
    hold on;  % Don't auto-refresh plot
    plot(track(:,1), track(:,2));
    plot(track(t,:), track(t,:),'k*');
    title(['Simulation map, n = ', num2str(n), ', friction = ', num2str(friction), ', deltat = ', num2str(deltat)]);

    % print('-depsc','-r600',['Map n',num2str(n),' f',num2str(friction),' ',strrep(datestr(now),':','_'),'.eps'])
    print('-djpeg50','-r100',[time,' Map Propogation ',num2str(n),'.jpg']);

    hold off;
    close;

    % This plots each particles x and y motion
    figure;
    set(gcf,'Visible','off');
    plot([1:t], track(:,:,1));
    title(['Particle x motion, n = ', num2str(n), ', friction = ', num2str(friction), ', deltat = ', num2str(deltat)]);
    set(gca,'YLim',[-1,1]);

    % print('-depsc','-r600',[X n',num2str(n),' f',num2str(friction),' ',strrep(datestr(now),':','_'),'.eps'])
    print('-djpeg50','-r100',[time,' X Propogation ',num2str(n),'.jpg']);

    close;

    figure;
    set(gcf,'Visible','off');
    plot([1:t], track(:,:,2));
    title(['Particle y motion, n = ', num2str(n), ', friction = ', num2str(friction), ', deltat = ', num2str(deltat)]);
    set(gca,'YLim',[-0.03,0.03]);

    % print('-depsc','-r600',[Y n',num2str(n),' f',num2str(friction),' ',strrep(datestr(now),':','_'),'.eps'])
    print('-djpeg50','-r100',[time,' Y Propogation ',num2str(n),'.jpg']);

    close;
if (map == 'y'),

    % This plots a 2D map of particle final position

    cla;
    figure;
    set(gcf,'Visible','off');
    plot(pavg(:,1), pavg(:,2), 'k*');
    title(['Simulation map, n = ', num2str(n), ', friction = ', num2str(friction), ', deltat = ', num2str(deltat)]);
    %print('-depsc','-r600',[time,' Map ',num2str(n),'.eps'])
    print('-djpeg50','-r100',[time,' Map ',num2str(n),'.jpg'])

    map = 'n';
end
close all;
close all hidden;
end
if(saving == 'y')
    if(n==2),
        filename = 'two';
    elseif(n==3),
        filename = 'three';
    elseif(n==4),
        filename = 'four';
    elseif(n==5),
        filename = 'five';
    elseif(n==6),
        filename = 'six';
    elseif(n==7),
        filename = 'seven';
    elseif(n==8),
        filename = 'eight';
    elseif(n==9),
        filename = 'nine';
    elseif(n==10),
        filename = 'ten';
    elseif(n==11),
        filename = 'eleven';
    elseif(n==12),
        filename = 'twelve';
    elseif(n==13),
        filename = 'thirteen';
    elseif(n==14),
        filename = 'fourteen';
    else
        error('Invalid value for n.');
    end
end
The following is a subroutine which determines the interparticle force exerted by the neighbouring particles.

function [F] = eforce2(p)

% Calculates the interparticle force on a particle i, exerted by (n-1) other particles, with parameters p.

global eps0 e
q1 = 10000*e;
q2 = 10000*e;
theta = 0;
fX = 0;
fY = 0;
Fx = 0;
Fy = 0;
F = [0,0];
sign = 1;
x = p(:, 1);
y = p(:, 2);
r = 9e10;
j = 1;
% retrieves size of particle matrix

[n,rubbish] = size(p);

% fx & fy are the x & y components of the interparticle force

for j = 1:n
    dx = (x - p(j,1));
    dy = (y - p(j,2));
    r = sqrt(dx.^2 + dy.^2);
dxe0 = (dx == 0);
dxl0 = (dx < 0);
dye0 = (dy == 0);
dyl0 = (dy < 0);
dyg0 = (dy > 0);
dx(j) = NaN;
r(j) = NaN;
if (~isempty(find(r==0))),
    fprintf('Error (eforce2): particles co-incide\n');
    return;
end
sign = -1*(dx>0) + (dy>0);
sign = ones(n,1);
sign((dx > 0)||(dx == 0)&(dy < 0))) = -1;
theta = atan(dy./dx);
fx = sign.*((q1*q2)./(r.^2*4*pi*eps0)).*cos(theta);
fy = sign.*((q1*q2)./(r.^2*4*pi*eps0)).*sin(theta);
fx(~isfinite(fx)) = 0;
fy(~isfinite(fy)) = 0;
Fx = sum(fx);
Fy = sum(fy);
F(j,:) = [Fx, Fy];
end

8.2.3. Subroutine – confinement force

The following is a subroutine which determines the force exerted onto the particle by the confinement well.

function[F] = wforce3a(posn,ecc)

% Gives the force in it's components '(fx, fy)' exerted by a confinement well at position 'posn(x, y)'

global alpha
x = posn(:,1);
y = posn(:,2);

% theta = 0;
% sign = 1;
% F = [0,0];

% 'confinement force' - F = -dU/dr force = -2*sqrt(x.^2 + y.^2);

fx = -x.*alpha.*2;
fy = -y/(1-ecc.^2).*alpha.*2;
F = [fx, fy];
return;

8.2.4. Subroutine – Debye force

The following is a subroutine which determines the Debye force on a particle.

function[F] = debye(p)

% Calculates the debye force on a particle.

global eps0 e
q1 = 10000*e;
q2 = 10000*e;
theta = 0;
fx = 0;
fy = 0;
Fx = 0;
Fy = 0;
F = [0,0];
sign = 1;
x = p(:, 1);
y = p(:, 2);
r = 9e10;
j = 1;

% retrieves size of particle matrix
[n,rubbish] = size(p);

% fx & fy are the x & y components of the interparticle force

for j = 1:n
    dx = (x - p(j,1));
dy = (y - p(j,2));
r = sqrt(dx.^2 + dy.^2);
dxe0 = (dx == 0);
dxl0 = (dx < 0);
dye0 = (dy == 0);
dyl0 = (dy < 0);
dyg0 = (dy > 0);
dx(j) = NaN;
r(j) = NaN;
if (~isempty(find(r==0)));
    fprintf('Error (eforce2): particles co-incide\n');
    return;
end
sign = -1*(dx>0) + (dy>0);
sign = ones(n,1);
sign((dx > 0)|(dx == 0)&(dy < 0))) = -1;
theta = atan(dy./dx);
kappa = 1./r;
kappa(~isfinite(kappa)) = 0;
fx = sign.*cos(theta).*((q1*q2)./(r.*4*pi*eps0)).*exp(-kappa.*r).*(1./r + kappa);
fy = sign.*sin(theta).*((q1*q2)./(r.*4*pi*eps0)).*exp(-kappa.*r).*(1./r + kappa);
fx(~isfinite(fx)) = 0;
fy(~isfinite(fy)) = 0;
Fx = sum(fx);
Fy = sum(fy);
F(j,:) = [Fx, Fy];
end
return;

8.2.5. Subroutine – packing sequence

The following is a subroutine which determines the packing sequence of a cluster system after the particles have settled into their equilibrium positions.

function[struct] = hulls(p);

% Calculates the packing sequence of a cluster system.

struct = [];
while (length(p)>2),
    temp = convhull(p(:,1), p(:,2));
    struct(end + 1) = length(unique(p(temp,:),'rows'));
p(temp,:) = 0;
p = [p(:,1)==0,1), p((p(:,2)==0),2)];
end
8.2.6. Subroutine – cluster energy

The following is a subroutine which determines the cluster energy.

```matlab
function[U] = energy(p)
%
% Calculates the cluster energy.
%
global eps0 e alpha
q1 = 10000*e;
q2 = 10000*e;
U = 0;
D= 0;
x = p(:, 1);
y = p(:, 2);

% retrieves size of particle matrix
[n,rubbish] = size(p);

% fx & fy are the x & y components of the interparticle force
for j = 1:n
    dx = (x - p(j,1));
    dy = (y - p(j,2));
    r = sqrt(dx.^2 + dy.^2);
    for i = 1:n
        if(i==j)
            u(i) = 0;
        else
            kappa = 1./r(i);
            u(i) = (q1*q2)./(r(i).*4*pi*eps0).*1e17;
            %d(i) = (q1*q2)./(r(i).*4*pi*eps0).*1e17.*exp(-kappa.*r(i));
        end
    end
    U = U + sum(u);
    %D = D + sum(d);
end
r = sqrt(x.^2 +y.^2);
confinement_energy = alpha.*r.^2;
```
\[ W = \text{sum(confinement\_energy)}; \]
\[ U = U + W + D; \]
\[ \text{return;} \]

8.2.7. Subroutine – rescale

The following is a subroutine which rescales the values of energies obtained from simulations to our experimental values.

\[
\text{oldalpha} = 1; \\
\text{oldbeta} = 1e17; \\
\text{newalpha} = 1e5; \\
\text{newbeta} = 1; \\
\text{scale} = ((\text{newalpha}/\text{oldalpha})^\text{*}(\text{newbeta}/\text{oldbeta})^2)^{(1/3)}; \\
\text{newspectra(:,1)} = \text{spectra(:,1)}^\text{*scale}; \\
\text{newspectra(:,2)} = \text{spectra(:,2)};
\]

8.2.8. Subroutine – energy spectra

The following is a subroutine which produces a histogram of the cluster energys.

\[
\text{rescale;} \\
\text{clear bin;} \\
\text{maxi} = \text{max(newspectra(:,1));} \\
\text{mini} = \text{min(newspectra(:,1));} \\
\text{delta} = 2e-14; \\
\text{low} = \text{mini} - 10^\text{delta}; \\
\text{high} = \text{maxi} + 10^\text{delta}; \\
\text{range} = \text{low:delta:high}; \\
\text{binnum} = \text{size(range,2)}; \\
\text{bin} = \text{zeros(1,binnum)}; \\
\text{for i} = 1:500 \\
\quad \text{if(newspectra(i,2)==1)} \\
\quad \quad \text{energy} = \text{newspectra(i,1)}; \\
\quad \quad \text{energy} = \text{energy} - \text{low}; \\
\quad \quad \text{energy} = \text{energy}/\text{delta}; \\
\quad \quad \text{energy} = \text{ceil(energy)}; \\
\quad \quad \text{energy} = \text{energy} + 1; \\
\quad \quad \text{bin(energy)} = \text{bin(energy)}+1; \\
\quad \text{end} \\
\text{end} \\
\text{[trash,index]} = \text{max(bin)}; \\
\text{middle} = \text{range(index)}; \\
\text{lhs} = \text{middle} - 15^\text{delta};
8.2.9. Subroutine – inter-ring twist

The following is a subroutine which determines the inter-ring twist of the cluster system.

```
function [twist] = findtwist(n,p)

% Calculates the inter-ring twist of the cluster system.

global m_centre
twist = 0;
type = hulls(p);
innernum = type(1);
outernum = n - innernum;
outerarg = zeros(1,outernum);
twists = zeros(1,innernum);
rp = [sqrt(p(:,1).^2 + p(:,2).^2),p];
rp = sortrows(rp,1);
p = [rp(:,2) rp(:,3)];
p = p - m_centre;
[args,rubbish] = cart2pol(p(:,1),p(:,2));

%for j=1:innernum,
%innerarg = args(j);

for i=1:outernum,
outerarg(i) = args(i+innernum);

%diff(i) = abs(outerarg(i)-innerarg);
%if(diff(i)>pi)
%    diff(i) = 2.*pi - abs(outerarg(i))-abs(innerarg);
%end

End

%twists(j) = min(diff);
%if(twists(j) ==0)
%    diff
%    fprintf('activated');
%end
```
%end

twist = outerarg;%min(twists);
return;

8.2.10. Subroutine – translational force

The following is a subroutine calculates a translational force field of increasing gradient across the horizontal plane.

function[F] = lforce(posn)

% Calculates translational force field of increasing gradient across the horizontal plane.

global gamma
fy = gamma.*(posn(:,1)+0.3);
fx = fy.*0;
F = [fx,fy];
return;

8.2.11. Subroutine – azimuthal force

The following is a subroutine which calculates an azimuthal force acting on the rings of planar cluster system.

function[F] = aforce(posn,vel,val)

% Calculates the azimuthal force on cluster acting on the rings of planar cluster system.

global beta friction outshell n outnum m_centre
m_centre_x = mean(posn(:,1)).*ones(n,1);
m_centre_y = mean(posn(:,2)).*ones(n,1);
m_centre = [m_centre_x,m_centre_y];
pos = posn - m_centre;
[theta,radius] = cart2pol(Pos(:,1),Pos(:,2));
[vthet,vmag] = cart2pol(vel(:,1),vel(:,2));
thetdiff = abs(theta - vthet);
friction.*vmag.*cos(thetdiff);
theta = theta + pi./2;
k=1;
mag = k.*radius.*beta.^2 - drag;
[fX,fY] = pol2cart(theta,mag);
type = hulls(posn);
outnum = type(end);
polarp = [theta, radius];
outshell = findshell(polarp);
for i = outnum+1:n
    fx(outshell(i)) = fx(outshell(i)).*val;
    fy(outshell(i)) = fy(outshell(i)).*val;
end
F = [fx, fy];
return;

8.2.12. Subroutine – $n^{th}$ ring of cluster

The following is a subroutine which index the ring of the cluster system.

function [outshells] = findshell(p)
    [trash, i] = sortrows(p, 2);
    outshells = flipud(i);
    return;

8.2.13. Subroutine – angular velocity

The following is a subroutine which determines the angular velocity of the planar cluster once it is set into rotation by the azimuthal force.

function [omega] = findanglev(posn, vel)

global outshell outnum n

[theta, radius] = cart2pol(posn(:,1), posn(:,2));
[vthet, vmag] = cart2pol(vel(:,1), vel(:,2));
thetdiff = abs(theta - vthet);
if (thetdiff > pi)
    thetdiff = 2.*pi - abs(theta) - abs(vthet);
end
vperp = vmag.*sin(thetdiff);

%omega = vperp./radius;
%omega1 = mean(abs(omega));
%omega2 = std(abs(omega), 0, 1);

for i = 1:outnum
    omegaout(i) = vperp(outshell(i))./radius(outshell(i));
end
for i = outnum+1:n
    omegain(i - outnum) = vperp(outshell(i))./radius(outshell(i));

8.3. Appendix III – Approximation Model of Angular Velocity

Based on the current theoretical models of dust rotation in a magnetized plasma (see Section 1.3 for *Current Theoretical Models on Dust Rotation*), it is expected that \( \omega \propto E_c B^k \) where \( k \) is an arbitrary constants. Since the confinement electric field is modified by the magnetic field (see Section 3.4.1 for *Confinement electric field*), we tried to approximate angular velocity with the functional form:

\[
\ln \omega = k_1 \ln B + k_2
\]

![Figure 8-1](image_url) – Using the plot of \( \ln \omega \) versus \( \ln B \) for L-Planar dust clusters, the arbitrary constants \( k_1 \) and \( k_2 \) were obtained from the slopes and the x-intercepts of the linear regression. The equations for the linear regression are shown at the top left corner.
where $k_2$ is an arbitrary constant. In order to find the arbitrary constants $k_1$ and $k_2$, we took the natural logarithm on both sides of (9.1) so that:

$$\ln \omega = k_1 \ln B + k_2$$

(9.2)

where $k_1$ and $k_2$ can be obtained from the slope and the x-intercept the linear plot of $\ln \omega$ against $\ln B$ respectively (see Figure 8-1). However, from the plot, the values of $k_1$ and $k_2$ seem to vary with different numbers of particles $N$.

By plotting the values of $k_1$ and $k_2$ against $N$ (see Figure 8-2), we get that for L-Planar clusters:

$$k_1 \approx \frac{8.3}{N^{3/2}}, \quad k_2 \approx \exp\left(-\frac{23}{N}\right)$$

(9.3)

Figure 8-2 – The dependency of the arbitrary constants $k_1$ and $k_2$ on the number of particles $N$ was obtained for L-Planar clusters.
and similarly for S-Planar clusters, we get:

\[ k_1 \approx \frac{0.7}{N^{1/4}}, \quad k_2 \approx \exp\left(-\frac{3.63}{N^{1/3}}\right) \]  \hspace{1cm} (9.4)

which allowed us to obtain (4.4) and (4.5).

### 8.4. Appendix IV – Determination of Threshold Magnetic Field

The threshold magnetic field was obtained from the x-intercept of the logarithmic regression line through angular velocity over magnetic field.

![Figure 8-3](image.png)

**Figure 8-3** – The threshold magnetic field was determined from the x-intercept of the logarithmic trend of angular velocity versus magnetic field strength data.
8.5. Appendix V – Determination of $\alpha$ and $\gamma$

The values of $\alpha$ and $\gamma$ for Planar-8 were obtained by plotting different values (see Table 8-1) of $\kappa_8 R_8$ against $E_{c8} B_8$ obtained from experiments (see Figure 8-4).

<table>
<thead>
<tr>
<th></th>
<th>S-Planar-8</th>
<th>L-Planar-8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.395 $\mu$m</td>
<td>3.105 $\mu$m</td>
</tr>
<tr>
<td>$\kappa = -(4/3)m_s n_s \nu \gamma \pi a^2$</td>
<td>$6.92 \times 10^{-13}$ kgs$^{-1}$</td>
<td>$3.43 \times 10^{-12}$ kgs$^{-1}$</td>
</tr>
<tr>
<td>$R_8$</td>
<td>obtained from Figure 3-5</td>
<td>obtained from Figure 3-4</td>
</tr>
<tr>
<td>$\omega_8$</td>
<td>obtained from Figure 4-8</td>
<td>obtained from Figure 4-7</td>
</tr>
<tr>
<td>$E_{c8}$</td>
<td>obtained from Figure 3-32</td>
<td>obtained from Figure 3-31</td>
</tr>
</tbody>
</table>

Table 8-1 – The experimental parameters from L- and S-Planar-8 cluster used in obtaining the plot in Figure 8-4.

Figure 8-4 – The above plot shows the $\kappa_8 \omega_8 R_8$ dependence on $E_{c8} B_8$. The equations and the lines of linear regression are shown along the data for L- and S-Planar clusters.
8.6. Appendix VI – Scientific Visualization of Phase Diagrams

The mathematical software package Maple 7.0 was used in generating the scientific visualization of phase diagrams (see Section 4.1.6 for Phase diagram and period pause).

In Maple 7.0, the coordinates of a point in 3-D is understood if it is written in the Cartesian form with the syntax \([x, y, z]\). Maple can also accept cylindrical coordinates. However, this only works for well defined continuous functions in 3-D or scattered points in 2-D. Scattered points in 3-D cannot be plotted if the data are in cylindrical coordinates.

Since our original datasets are in cylindrical coordinates, each of the components was transformed into Cartesian coordinates using Excel. Moreover, the magnitude of the angular velocity was scaled 1000 times. This was done or else the value of the angular velocity \(\omega\) will be too small to be seen on the graph.

Also columns of \([ , , ], [ and ]\), were inserted into the dataset to satisfy Maple syntax (see Figure 8-5). The data was then saved in comma delimited (.csv) format. The file was then opened in Microsoft Notepad where the menu “Edit>Replace” was used to clean up the syntax (see Figure 8-6). After clean up, the dataset was pasted into Maple.

Figure 8-5 – Columns of syntax were inserted into the data so that the dataset would be compliant with Maple 7.0. The comma delimited format will separate the data with commas.
Figure 8-6 – The comma delimited format must be cleaned up before it could be fully understood by Maple 7.0.

The following script was used to generate a 3-dimensional animation of the phase diagrams:

```maple
> with(plots):
> display([seq(PLOT3D(POINTS(["data"], AXESSTYLE(NORMAL), STYLE(POINT), AXESLABELS(Ang_Velocity, Ang_Velocity, Mag_Field), SYMBOL(CIRCLE, 5), COLOR(ZHUE), SCALING(CONSTRAINED), VIEW(-500..500, -500..500, 0..90), ORIENTATION(45, 3*n)), n=0..120)], insequence=true);
```

The word "data" was replaced with the actual data points from the dataset. The camera view of the plot was rotated using the calling sequence Orientation in Maple.
## Table of Constants

Constants and variables that have been used are listed below

### Fundamental constants

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Speed of light ( c )</td>
<td>( 3 \times 10^8 \text{ ms}^{-1} )</td>
</tr>
<tr>
<td>Charge of an electron ( e )</td>
<td>( 1.6 \times 10^{-19} \text{ C} )</td>
</tr>
<tr>
<td>Permittivity of free space ( \varepsilon_0 )</td>
<td>( 8.85 \times 10^{-12} \text{ Fm}^{-1} )</td>
</tr>
<tr>
<td>Gravitational constant ( g )</td>
<td>( 9.8 \text{ ms}^{-2} )</td>
</tr>
<tr>
<td>Boltzmann’s constant ( k )</td>
<td>( 1.38 \times 10^{-23} \text{ JK}^{-1} )</td>
</tr>
<tr>
<td>Mass of an electron ( m_e )</td>
<td>( 9.1 \times 10^{-31} \text{ kg} )</td>
</tr>
</tbody>
</table>

### Subscript indices

- Dust \( d \)
- Electron \( e \)
- Ion \( i \)
- Neutral \( n \)

### Superscript indices

- Radial \( \rho \)
- Azimuthal \( \theta \)
- Axial \( z \)
- Orthogonal \( \perp \)

### Parameters for dust particles

- Radius of dust particle \( a \)
- Angular position \( \theta \)
- Geometrical coordinates \( (x, y) \)
- Number of electrons on dust surface \( Z \)
Parameters for dust clusters/crystals

<table>
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<tr>
<td>Angular acceleration</td>
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<tr>
<td>Interparticle distance</td>
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<td>Cluster eccentricity</td>
<td>$e_C$</td>
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<td>Energy</td>
<td>$E$</td>
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<tr>
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<td>Interlayer distance</td>
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Other parameters

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Curriculum Vitae

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Conference Proceedings:


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2. 12th Gaseous Electronics Meeting, Batesman Bay (Australia), 3-6 February 2002.
4. 3rd Complex Plasma Laboratory Seminar, Sydney (Australia), 7 November 2002.
5. 13th Gaseous Electronics Meeting, Batesman Bay (Australia), 1-5 February 2004.
6. 4th Conference on the Physics of Dusty Plasmas, Orléans (France), 13-17 June 2005 [accepted].
Poster Presentations:
4. 4th Conference on the Physics of Dusty Plasmas, Orléans (France), 13-17 June 2005 [accepted].

Official Complex Plasma Laboratory Website: