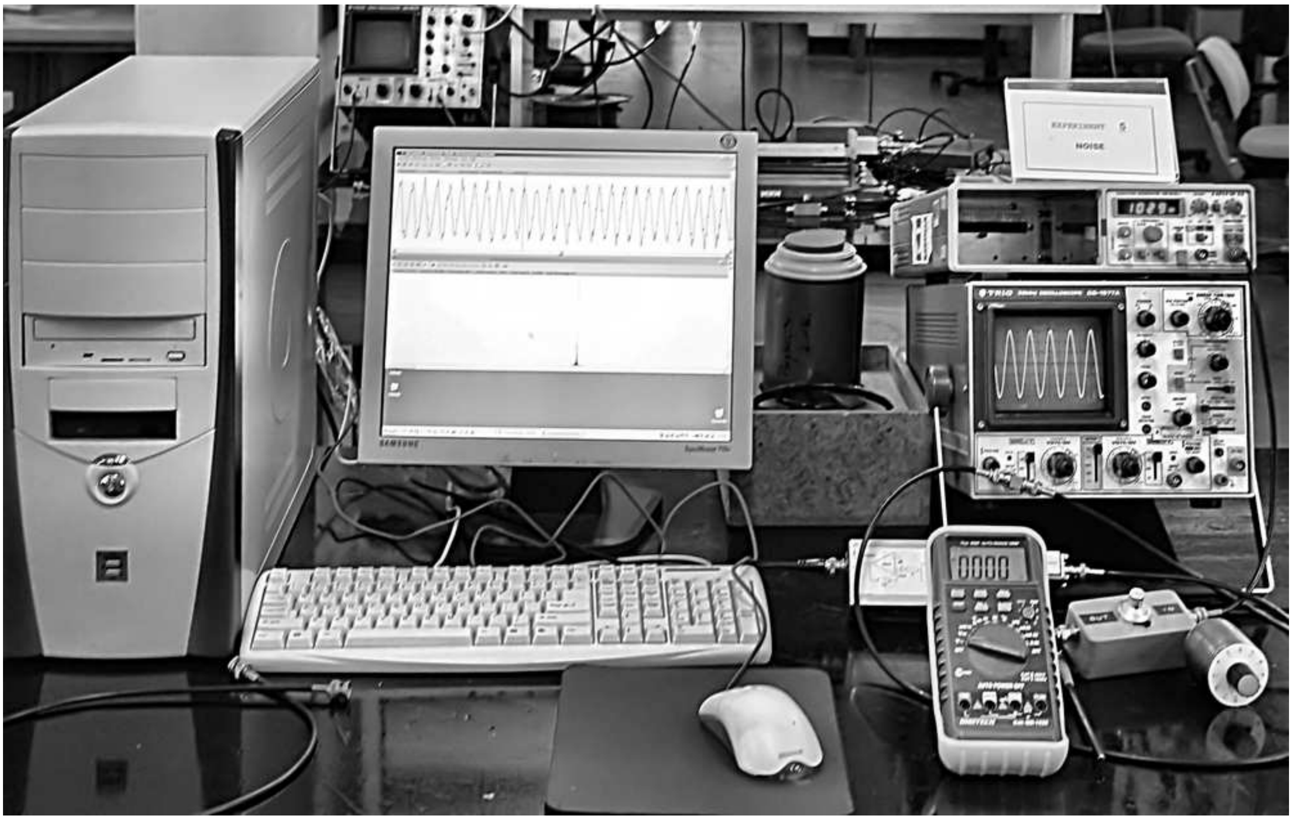
Experiment 5. Noise



Equipment list:-

1. Switched resistor set (0 to 470 kΩ) to measure the dependence of noise voltage on resistance.
2. 470 kΩ (nominal) resistor in a thin tube to measure dependence of noise voltage on temperature.
3. High gain ($\sim$10000), high input impedance amplifier to amplify the noise from the resistors.
4. 1000:1 attenuator/voltage divider to be connected to the amplifier during calibration.
5. Digital multimeter for resistance measurements.
6. Digital multimeter with thermocouple for temperature measurement.
7. Oscilloscope.
8. Function generator.
9. 10-turn precision 100 Ω potentiometer to adjust input voltage during calibration procedure.
10. Computer with sound card running *Intune* and *Origin* programs.
11. Small oven to heat the 470 kΩ resistor.
12. 3 coaxial cables with BNC plugs at each end.
13. 1 coaxial cable with BNC plug at one end and small jack at the other end.
14. 1 coaxial cable with BNC plug at one end and banana plugs at the other end.
15. 1 BNC tee piece connected to the oscilloscope.
16. 1 BNC male to male connector to connect attenuator or resistors to the amplifier input.

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1 Objectives

In this experiment you will study the small, random voltage fluctuations generated due to the thermal agitation of the electrons in a resistor, known as **Johnson noise** after J. B. Johnson who first measured this phenomenon at Bell Labs in 1928. Remarkably, the noise voltage dependence on resistance will be used to determine Boltzmann's constant! The dependence of noise on temperature will also be examined. In addition, you will determine the amplitude distribution function of the noise and compare this with the distribution function of other well known electrical signals.

2 Introduction

The random motion of electrons and holes in electrical circuits gives rise to voltages and currents that are called "noise" (more accurately **fundamental noise**). Although very small in magnitude, these voltages and currents are always present and set limits on the smallest signal that can be measured. In any amplifying system the noise can be considered as arising from the thermal noise in the resistance(s) at the amplifier input and from the electronic processes in the amplifying stages. Noise from the first amplifying stage is clearly of greater importance than that from later stages. Noise from later stages is present but is not amplified as much and is therefore less important. Noise generated outside the equipment (**environmental noise**) is not considered here but can be important in actual working systems.

2.1 Nyquist theory

Harry Nyquist (1889–1976) showed that the noise measured by Johnson is due to the thermal agitation of the electrons in a resistor and, using thermodynamics, developed the expression

$$\overline{V^2} = 4kT \int_{f_1}^{f_2} \Re[Z(f)]\, df \tag{1}$$

where $\overline{V^2}$ is the average of the voltages squared (known as **mean squared voltage**), k is Boltzmann's constant, T is the absolute temperature, $Z(f)$ is the impedance of the device (a resistor) at the frequency f, and f_1 and f_2 define the frequency range over which the measuring device is sensitive. If the device is a simple resistor, this becomes

$$\overline{V^2} = 4kTR(f_2 - f_1) \tag{2}$$

In this experiment, we will not measure $\overline{V^2}$ directly but will use an amplifier. The amplifier output $V_{\rm out}$ is related to the input $V_{\rm in}$ by

$$V_{\rm out} = A(f)V_{\rm in} \tag{3}$$

where $A(f)$ is the frequency dependent voltage gain of the amplifier. Hence

$$\overline{V^2} = 4kTR \int_{f_1}^{f_2} A(f)\, df \tag{4}$$

The integral can be easily calculated as $\overline{A(f)}\Delta f$, where $\overline{A(f)}$ is the average gain over the frequency range $\Delta f = f_2 - f_1$. We will choose a frequency range of f_1 to f_2 where $A(f)$ is almost constant, and calibrate $\overline{A(f)}$ in terms of the amplifier's input voltage V_{in} using a known sinusoidal signal. This way the gain of the amplifier (and the sound card in the PC) will be included in the calibration. Taking into account the noise produced by the amplifier itself when the resistance $R = 0$, the final equation can be written as

$$\overline{V^2} = \overline{V^2_{\mathrm{R=0}}} + 4kTR(f_2 - f_1) \tag{5}$$

2.2 Noise amplitude distribution

We can think of the (amplified) noise amplitude from a resistor as a continuous statistical function of voltage $f(V)$. We expect the noise amplitudes to have a normal distribution (also known as a Gaussian function), given by

$$f(V) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(V-\mu)^2}{2\sigma^2}\right) \tag{6}$$

where μ is the mean, and σ is the standard deviation. The **Central Limit Theorem** of statistics says that the sum of a very large number of arbitrary random distributions will result in a normal or Gaussian distribution. Each electron in the resistor, through its thermal motion, generates its own particular distribution at the terminals of the resistor. If it moves towards one electrode, then that electrode will become slightly more negative. Many such random motions will produce a normal distribution of positive and negative voltages across the resistor, about a mean μ of zero.

2.3 Electric power and $\mathrm{V_{RMS}}$

Consider the 240 volt AC power line coming into your home. The average voltage in this signal is zero. (So, why pay the electricity bill?) In fact, electricity is billed in *kilowatt hours (kWh)*, the time integral of power. The power is what is important for running electrical equipment. In a resistor, electric power varies as voltage squared, $P = V^2/R$. The square root of $\overline{V^2}$ is called **Root Mean Square voltage** and usually written as V_{RMS}. It is commonly used to measure alternating voltages when the waveform is complex and the average is zero. In the case of a sinusoidal signal, $v = V_0 \sin(\omega t)$, V_{RMS} can be easily expressed in terms of the **peak voltage** V_0 or the **peak-to-peak voltage** $V_{\mathrm{pp}} = 2V_0$:

$$V_{\mathrm{RMS}} = \frac{1}{\sqrt{2}} V_0 = \frac{1}{2\sqrt{2}} V_{\mathrm{pp}}. \tag{7}$$

The 240 V power line coming to your home refers to the RMS voltage; the peak voltage is approximately 340 V, and peak-to-peak voltage 680 V (quite a lot!). In terms of electric power the Nyquist equation 2 can now be expressed as

$$\overline{P} = 4kT(f_2 - f_1) \tag{8}$$

where $\overline{P}$ is the average power of the noise in the frequency range f_1 to f_2. You probably noticed that if $f_2 - f_1$ increases without limit then $\overline{P}$ becomes infinitely large! This is a consequence of our idealised model of a resistor. In reality every resistor has its own self capacitance that limits the highest frequency of the noise. In addition, a quantum-mechanical model introduces a fundamental limit on the highest frequency in the noise spectrum. At room temperature those limits are calculated to be 1.6 THz and 6.25 THz, respectively. There are no electronic devices that work at such high frequencies, so for any practical application the Nyquist equation is correct.

3 Experimental set-up

The set-up shown in Fig. 5-1 consists of a carefully shielded low-noise amplifier with its output fed to the computer. The sound adaptor in the computer samples the signal and the digitised data is analysed by the *Intune* program. The input of the amplifier can be connected to different signal sources. During the process of calibration a signal from the function generator is measured by the oscilloscope and fed to the amplifier input through the 1000:1 attenuator. To investigate the dependence of thermal noise on resistance, the input of the amplifier will be connected to a set of 10 resistors. Finally, a resistor probe will be used to measure noise as a function of temperature. All input devices (the attenuator, the set of resistors and the resistor probe) should be connected *directly* to the amplifier (without any cable) to avoid stray capacitance that can act as a low pass filter and reduce the bandwidth Δf, especially when working with higher source resistances.

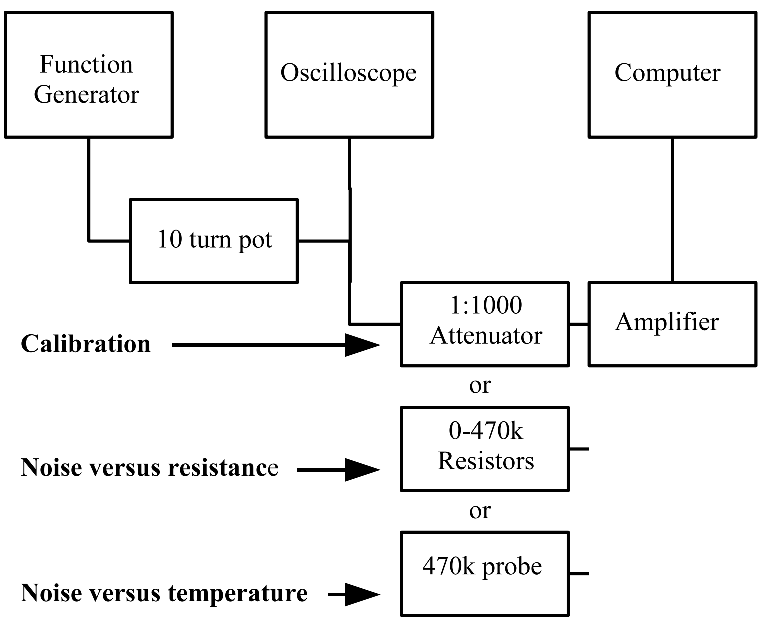

Fig. 5-1 : Block diagram of the noise experiment

3.1 Function generator

The function generator used in this experiment is the HAMEG HM8030. We will use it to generate a sinusoidal signal to calibrate our equipment. The maximum output signal from the generator is $20\,\mathrm{V_{pp}}$. To reduce this signal make sure that both $-20\,\mathrm{dB}$ switches, located close to the output socket, are pushed in. Now the maximum output signal has been reduced to $0.2\,\mathrm{V_{pp}}$. In our experiment the voltage at the amplifier input should be around $30\,\mu\mathrm{V_{pp}}$, to avoid signal clipping by the amplifier and the sound card. Such a small signal cannot be observed directly by the oscilloscope. The trick we will use is to adjust the signal to $30\,\mathrm{mV_{pp}}$, which is easy to measure, and then divide it precisely by 1000 before the amplifier input.

3.2 10-turn potentiometer (pot)

Although the output voltage from the function generator can be adjusted using the built-in level control, it is really difficult to achieve the desired value. To obtain a stable voltage from the function generator, set the built-in control to maximum and use the external 10-turn potentiometer — a voltage divider with fine adjustment — to set the signal level.

3.3 Fixed attenuator/voltage divider

As mentioned above, a precision 1000:1 attenuator is used to divide the $30\,\mathrm{mV_{pp}}$ signal (from the function generator and potentiometer) to $30\,\mu\mathrm{V_{pp}}$ at the input of the amplifier.

3.4 Set of resistors

The test resistors are contained in a cylindrical housing with a knob at the front that can be switched from positions 0 to 10. Each switch position selects a different resistor. The numbers are arbitrary and *do not* indicate the value of the resistance. You will need to use the digital multimeter to measure the resistances.

3.5 Resistor probe

To observe the dependence of noise power on temperature, the resistor needs to be heated up or cooled down. For convenience, and to shield the resistor from stray signals, we have placed the resistor at one end of a thin metal tube. The other end of the tube is crimped onto a BNC socket that is connected to the resistor.

3.6 Amplifier

The amplifier consists of two OPA129 operational amplifiers. The first stage is a non-inverting amplifier with a nominal gain of 101 and the second stage is an inverting amplifier with a gain of -100. Both stages have built-in high pass filters that cut off low frequency environmental noise. The high frequency gain is limited by the operational amplifier itself and drops by approximately $6\,\mathrm{dB}$ ($3\,\mathrm{dB}$ each stage) at $10\,\mathrm{kHz}$. The amplifier is powered by two $9\,\mathrm{V}$ batteries in the amplifier box. The circuit is shown in Fig. 5-2.

PLEASE . . . switch the amplifier off when not in use, to save the batteries

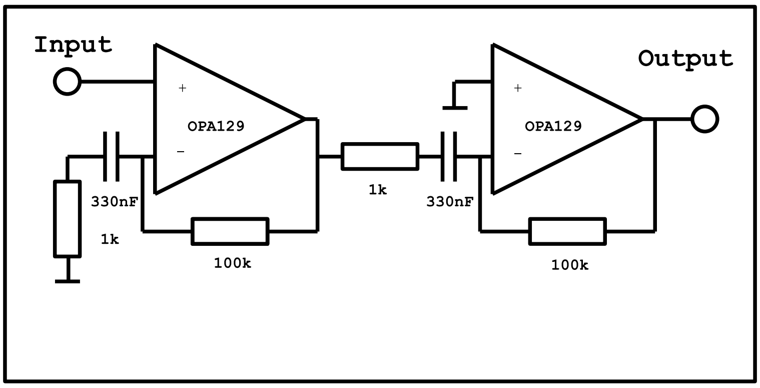

Fig. 5-2 : Amplifier circuit.

4 Experimental procedure

4.1 Calibration

The output of the amplifier is connected to the *Line In* input of the sound card in the computer. This signal is recorded and analysed by the *Intune* program. The amplification by the sound card is unknown and *Intune* uses arbitrary units to measure the waveform. The maximum value that can be recorded is set to +100% and minimum is -100%. As a result, we have to calibrate the amplifier and sound card by measuring how the output responds to a known signal at the amplifier input. We will use a sinusoidal input signal of constant voltage amplitude and vary the frequency to determine the gain:frequency response.

The calibration procedure is as follows:

- Connect the equipment as shown in Fig. 5-1.
- Using the 10-turn pot, adjust the signal amplitude of a sine wave from the function generator to 30 $\mathrm{mV_{pp}}$ (using the oscilloscope at 5 mV per division). Set the frequency to 800 Hz.
- Run the *Intune* program and record the signal for a few seconds. Save it using a meaningful filename for future reference. Observe the waveform of the recorded signal. It should look like the sine wave you saw on the oscilloscope, with added noise introduced by the amplifier. To measure the amplitude of the sinusoidal part of the signal alone we will observe its spectrum using the FFT (Fast Fourier Transform) feature of the program.
  - Open the FFT window using *View* → *FFT* menu or simply press the **F** key.

- Set the number of samples to 32768 and data windowing method to *Blackman Window* using buttons on the FFT window toolbar.[1] You should see a sharp peak around 800 Hz.
- Place the frequency indicator at the maximum of this peak and write down the values displayed after *Peak Freq[Hz]* and *Peak Ampl[%]*.

- Repeat the above measurements for frequencies up to 5000 Hz in 200 Hz steps. Make sure that the input signal amplitude remains constant.

- Run the *Origin* program and type your results into the worksheet. Make a scatter plot of peak amplitude versus frequency.
  Find the range of frequencies over which the amplitude is $\geq$0.9 times the maximum value and calculate the average value of the output amplitude over this frequency range. We will use the same frequency range for the noise measurements.
  Now we can assign a 30 μV_{pp} input voltage to this average output amplitude. For measurements of the noise voltage, we have to use V_{RMS} voltage rather than V_{pp}, so you will need to convert the input voltage from p-p to RMS. For convenience you can still use the *Intune* arbitrary units for future measurements and apply the above calibration after transferring data to the worksheet.

Questions:

1. Why should we multiply by a windowing function before applying the FFT? Try different windowing methods and observe what happens. *Hint*: use the vertical scroll bar to increase the display scale to see more detail around the base of the peak.

2. What is the maximum voltage that can be fed to the *Line In* sound card input without clipping the output signal?

C1 ▷

4.2 Variation of noise with resistance

Determine the noise voltage dependence on resistance using the following method:

- With a multimeter, measure the resistances in the resistor box for each position 0 to 10.
- Connect the resistor box to the input of the amplifier.
- Set the switch position to 0 and record a few seconds of the noise signal.
- Open the FFT window. Select the frequency range that you found during calibration. To do this, set the frequency indicator as close as possible to the lower frequency, f_1, and press the **L** key. Repeat this for the highest frequency, f_2, and press the **R** key. You should observe the highlighted frequency range f_1 to f_2. On the information bar you can see the displayed information *Range Ampl[%]* with the value calculated for the highlighted frequency range. *Intune*

[1] More information about FFT and windowing methods as well as how to use the *Intune* program can be found under *Help* → *Contents* menu.

calculates this value the same way as it does for a peak. The only difference is that frequencies f_1 and f_2 are selected automatically for a peak. Because of this, the *same* calibration factor will apply to both cases.

- Change the position of the time indicator in the main window by clicking the left mouse button over the displayed waveform. You should see the part of the waveform selected. This is the part of the signal that is analysed by the FFT. Make sure that the whole selected part is inside the waveform (do not place the time indicator close to the beginning or the end of the waveform). To achieve more accurate results take a few measurements of the *Range Ampl* at different time indicator positions, and find the mean and standard error.
- Repeat these measurements for the other resistors (positions 1 to 10).
- Enter the results into the worksheet. Add new columns to accommodate the calibrated values of $\overline{V}_{\mathrm{RMS}}$, $\overline{V^2}_{\mathrm{RMS}}$ and their errors. Don't forget to record the room temperature.
- Plot $\overline{V^2}$ versus R, including the errors in $\overline{V^2}$. The points should fall on a straight line.
- From a linear fit to your data obtain a value for Boltzmann's constant. Does your value agree with the accepted value for k within experimental error? If not, can you think of other (unaccounted for) sources of error?

Question:

What is the significance of the value of $\overline{V^2}$ for $R = 0$?

C2 ▷

4.3 Noise amplitude distribution

The aim of this section is to determine the distribution function, $f(V)$, that describes the resistor's noise amplitudes (refer to section 2.2). For this part of the experiment (and the following part) we will use the 470 kΩ resistor probe.

- Record a few seconds of noise with the resistor probe connected to the amplifier's input and determine V_{RMS}, as before.
- In the main *Intune* window select around a one second portion of the waveform using the same method as used above or simply press the *Ctrl* key and the left mouse button together and drag the cursor over the desired range. Use the *WAV Range* → *Export As Text* menu to save the waveform data as an ASCII file.
- Open a new worksheet and import this file into *Origin*.
- Using the *Statistics* → *Descriptive Statistics* → *Frequency Count* menu and carry out a statistical analysis on the data. In the dialog box choose a bin size to obtain approximately 20 bins.
- Plot the number of counts in each bin versus the centre value of the bin.

- Fit a normal (Gaussian) distribution to the plot, excluding the two end bins near $\pm 100\%$ where *Intune* saturates. Hence find the mean (μ) and the standard deviation (σ) of the noise signal.
- Compare σ with the V_{RMS} value taken at the beginning of this section.

Questions:

1. Is your fit consistent with a normal distribution? Comment.

2. Do you expect the mean of the noise distribution to be zero? If it isn't, can you suggest a reason?

3. One of the methods used to test the quality of audio amplifiers is to apply a pure sinusoidal signal to the input and obtain the amplitude distribution of the output. What do you expect this distribution function to look like if the amplifier is noisy? Check this directly using one of the datafiles you recorded for the gain calibration.

C3 ▷

4.4 Noise dependence on temperature

As can be seen from Eq. 1 the noise voltage will increase if the temperature is raised and decrease if the temperature is lowered. We will now examine this temperature dependence using the following method:

- Connect the resistor probe to the input of the amplifier and place the probe in the oven. Ensure that the thermocouple is positioned close to the resistor to monitor its temperature.
- Using the same frequency range as before, measure the RMS noise and the resistance of the resistor at a few (≥ 4) well spaced temperatures in the range from room temperature up to approximately 150 °C. Make sure that the temperature remains constant during a measurement. We recommend setting the regulator knob to the maximum value and observing the temperature rise. When the temperature reaches the desired value, turn the regulator knob to the marked position. Wait a minute or two to ensure that the temperature remains constant and record the noise signal. Then disconnect the resistor probe from the input and measure its resistance using the multimeter.
- Make a scatter plot of $\overline{V^2}$ versus RT, remembering to convert temperatures to Kelvin.
- Fit a straight line to your data points, measure the slope (and its error) and compare with what you expect from equation 2.

Question:

What would you expect to happen as the temperature of the resistor approaches 0 K?

C4 ▷