How to (legally) travel faster than light

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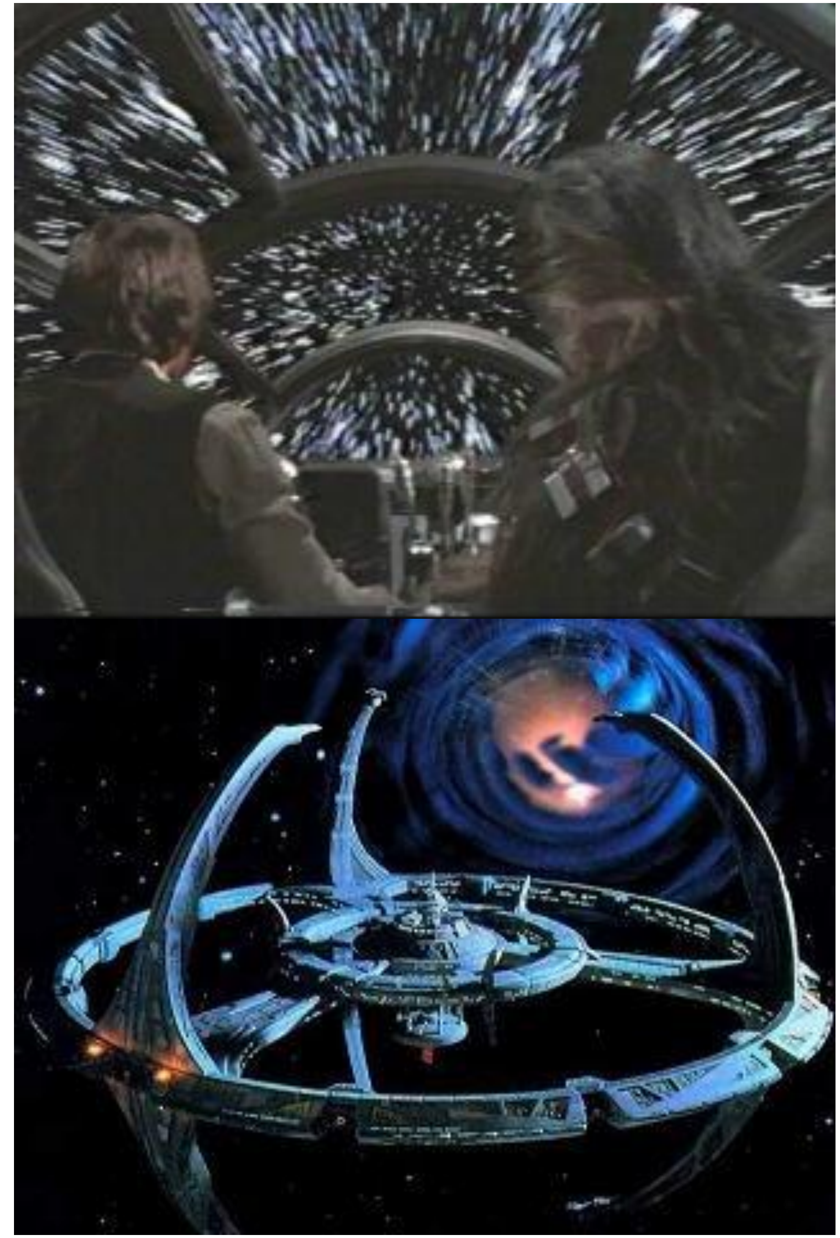
Faster than the Speed of Light

Traveling faster than the speed of light is a staple of science fiction (usually accompanied by groovy special effects).

But we know that within special relativity, the speed of light is the ultimate speed limit; you cannot go faster.

So, are we destined only to explore our cosmic backyard in a typical human life-time?



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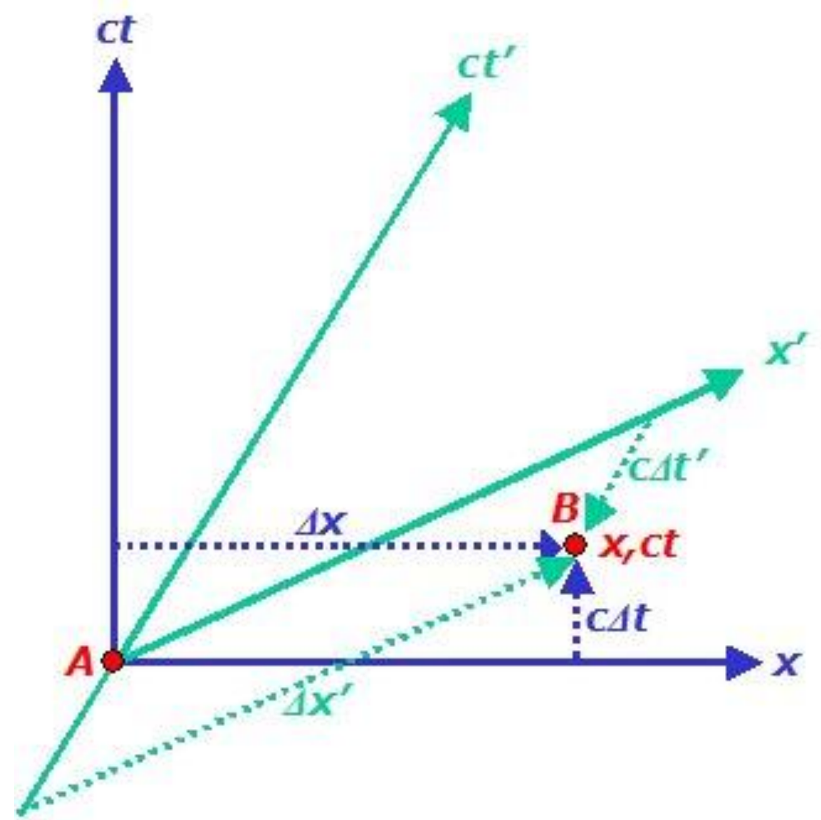
Special Relativity & The Speed of Light

In relativity, it's all about coordinates.

In classical physics, time and space are separate, whereas in relativity we can mix them together.

This picture shows the coordinates for two observers, on stationary *(ct,x)* and one moving with a relative velocity *(ct',x')*.



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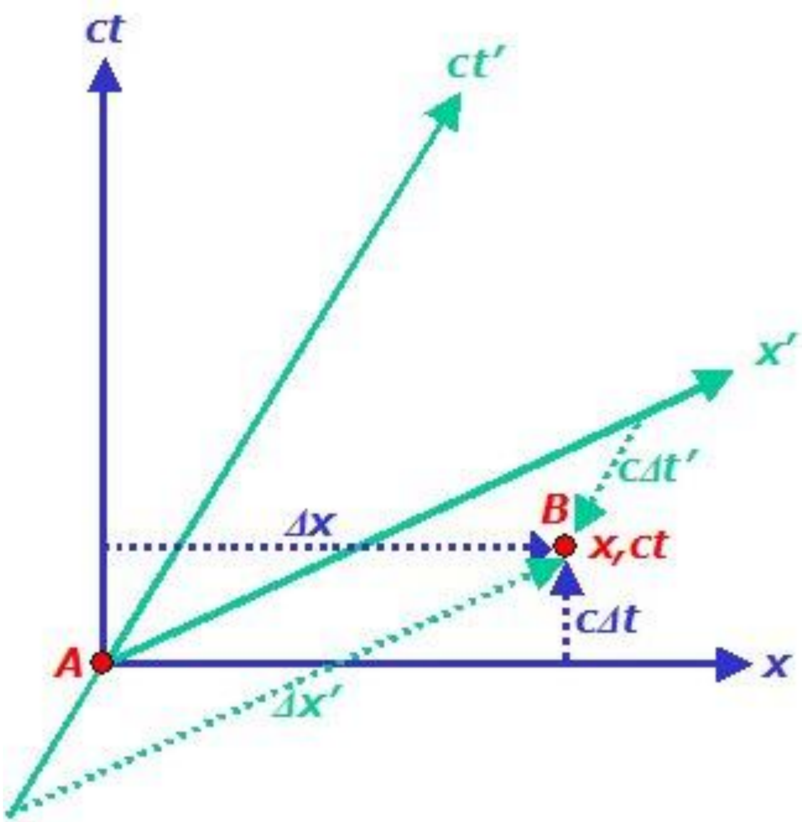
Special Relativity & The Speed of Light

If we have two distinct 'events', labeled A and B, then for our observer at rest, the separation of these events is *(cΔt,Δx)*.

However, our moving observer sees them with separations *(cΔt',Δx')*.



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Einstein showed that

$$\Delta s^2 = -(c\Delta t)^2 + (\Delta x)^2 = -(c\Delta t')^2 + (\Delta x')^2$$

This is the *Invariant Interval.*

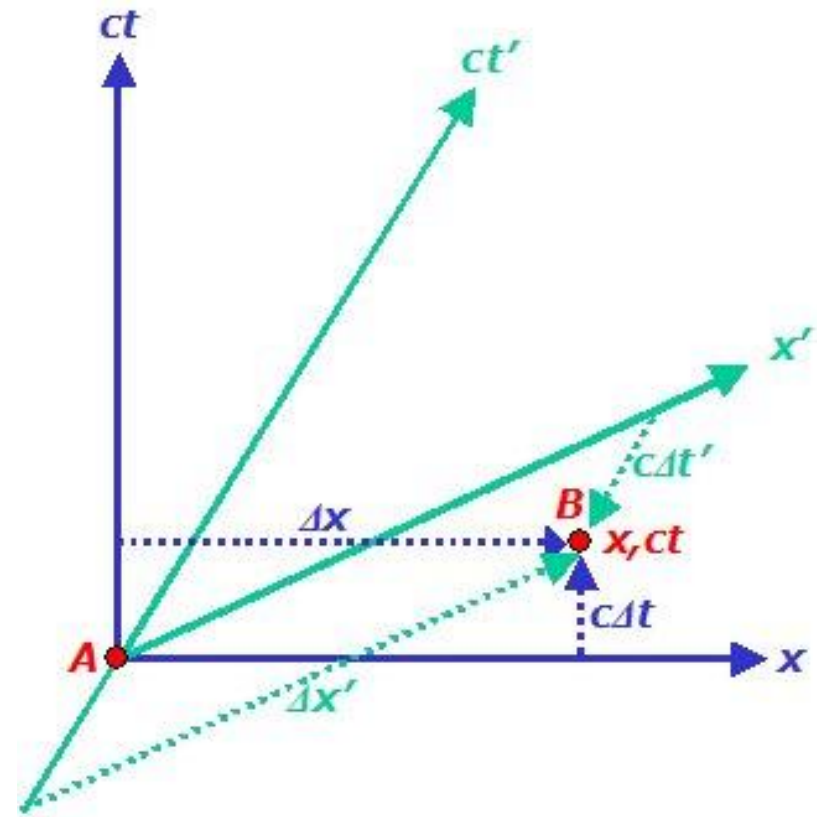
Special Relativity & The Speed of Light

It's easy to see that if $\Delta s^2=0$, then

$$\frac{\Delta x}{\Delta t} = \frac{\Delta x'}{\Delta t'} = c$$

Light rays travel along these *null paths,* and both observers see light moving at *c.*

This corresponds to a line at 45° on this *spacetime diagram.*



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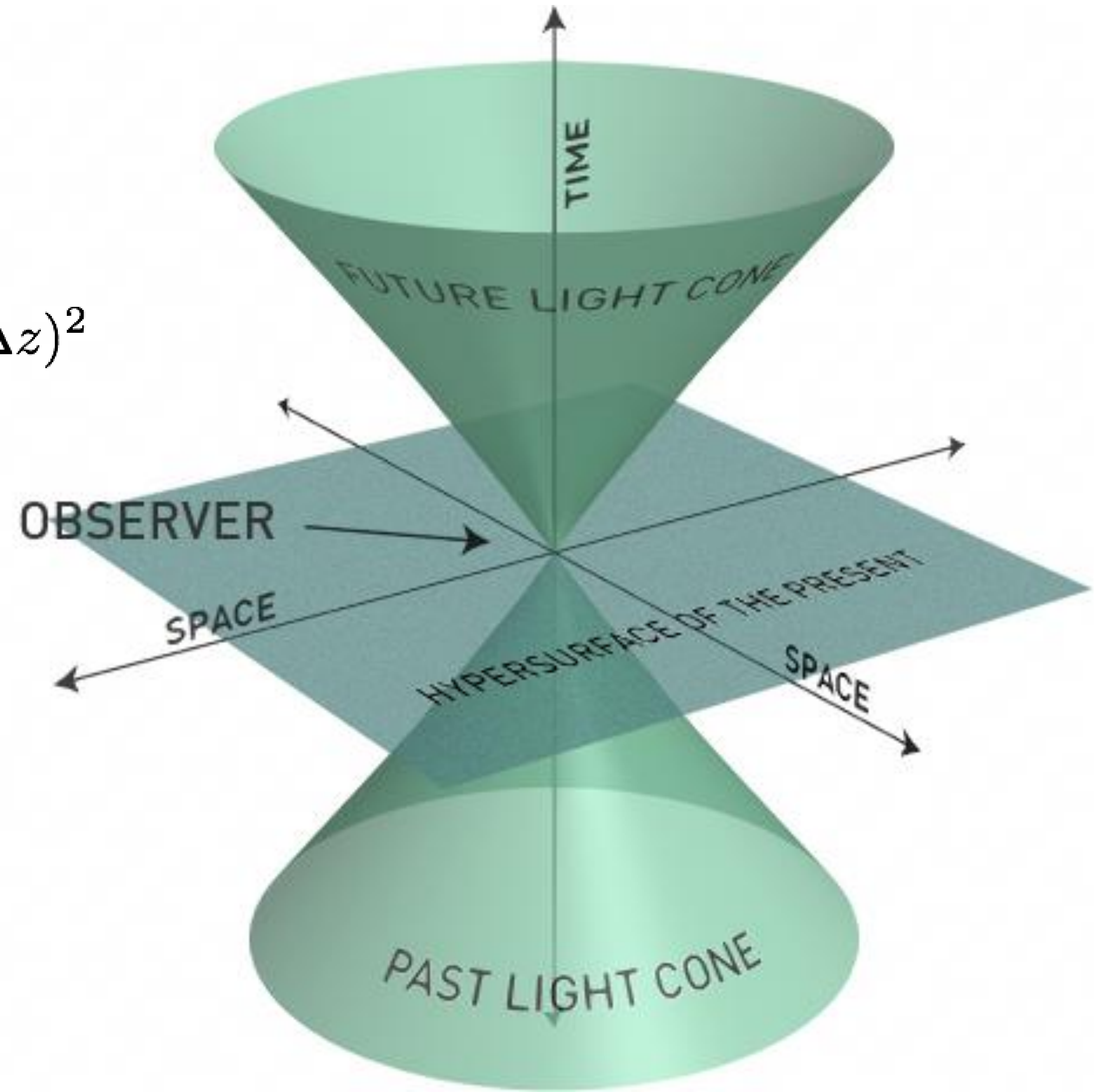
Special Relativity & The Speed of Light

Extending this into higher dimensions, the invariant interval is defined as

$$\Delta s^2 = -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

Light rays still travel along paths with $\Delta s^2=0$ and travel out *light cones.*

The paths of all massive objects have $\Delta s^2<0$ and their *world lines* always sit inside their light cones.



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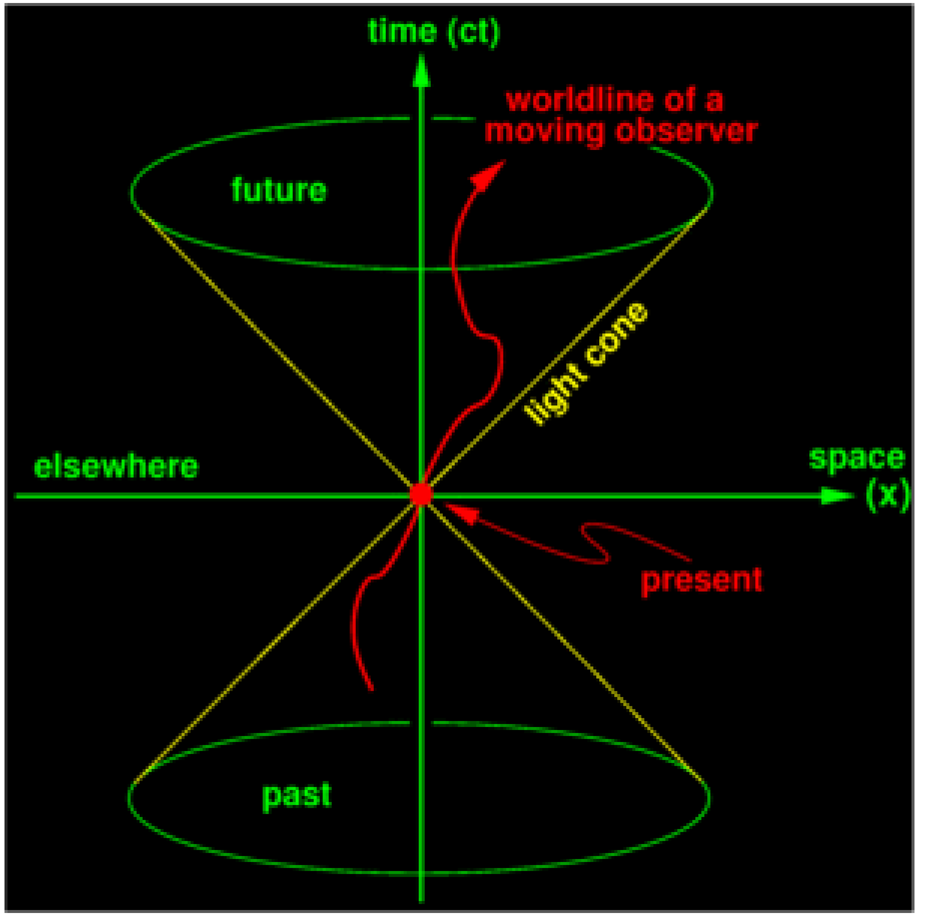
Special Relativity & The Speed of Light

So, how much of the universe we can explore is constrained to be within our future light cone, and we can only have been influenced by events in our past light cone.

We can, however, explore our light cone are speeds arbitrarily close to *c*, although we have inconvenient issues of *time dilation* to deal with.

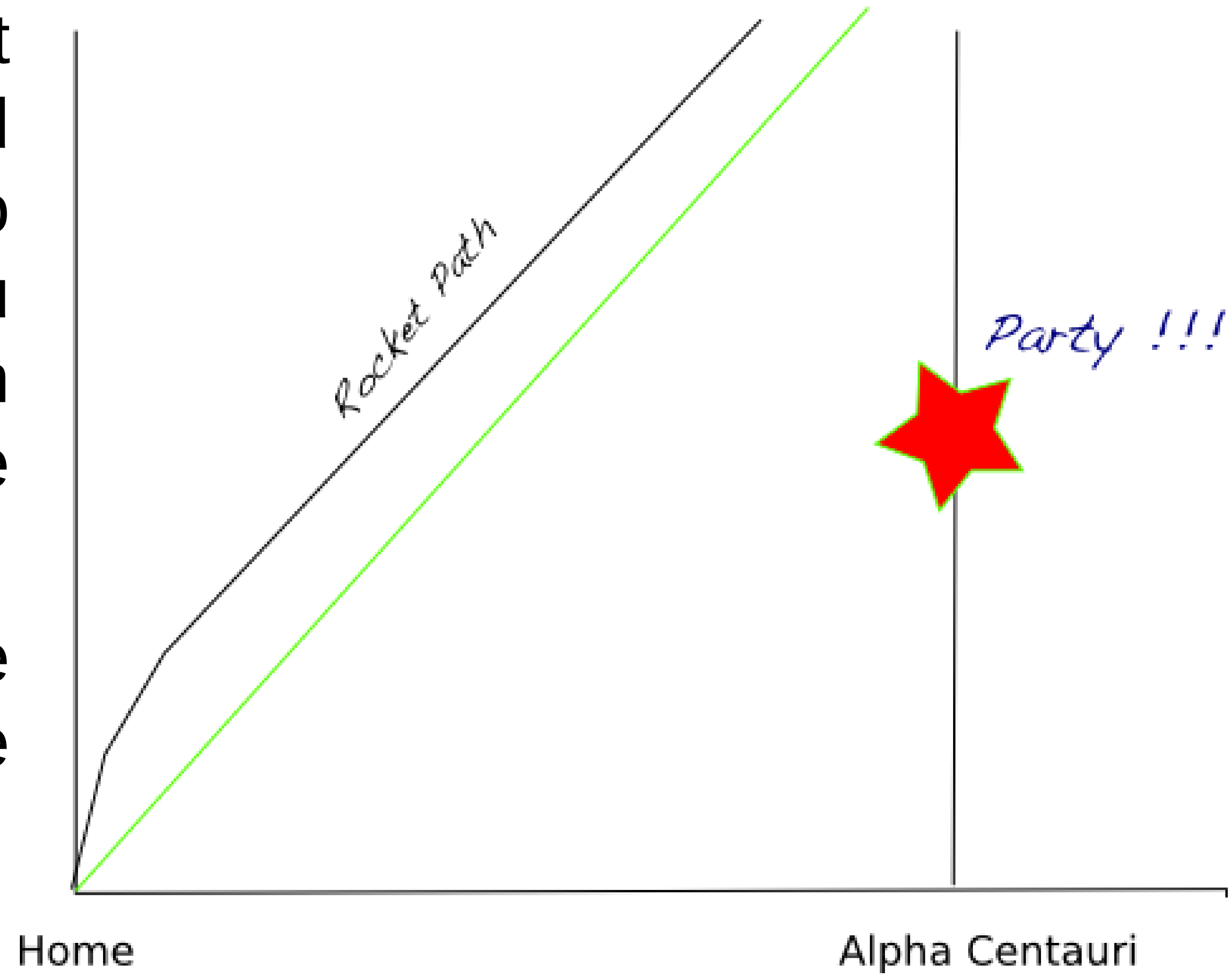


However, you never reach events outside your light cone.

Special Relativity & Causality

So, if there is a party at Alpha Centauri, and you leave too late, no matter how fast you travel, the best you can do is approach the slope of a light cone.

Hence, you are destined to miss the party - or are you?



The Alcubierre Warp Drive: Space-Time Structure

All we have looked at so far has been in the *flat space-time* of special relativity. What if we consider *general relativity* where we can bend and flex space-time?

Worried about missing parties in Alpha Centauri, in 1994 Alcubierre proposed a modification of flat space-time of the form

$$ds^2 = -dt^2 + [dx - V_s(t) f(r_s) dt]^2 + dy^2 + dz^2$$

(assuming *c=1).* Here, $V_s(t) = dx/dt$ is the (inverse) slope of a path across our space-time diagram.

The Alcubierre Warp Drive: Space-Time Structure

So, looking at the interval, there is one remaining term to consider

$$ds^2 = -dt^2 + [dx - V_s(t) f(r_s) dt]^2 + dy^2 + dz^2$$

The function *f* is simply defined such that

$$f(0) = 1 \quad and \quad f(r_s > R) = 0$$

where $r_s = \sqrt{(V_s(t)t - x)^2 + y^2 + z^2}$

and that *f* declines smoothly from unity to 0.

The Alcubierre Warp Drive: Space-Time Structure

With this, and looking at the interval,

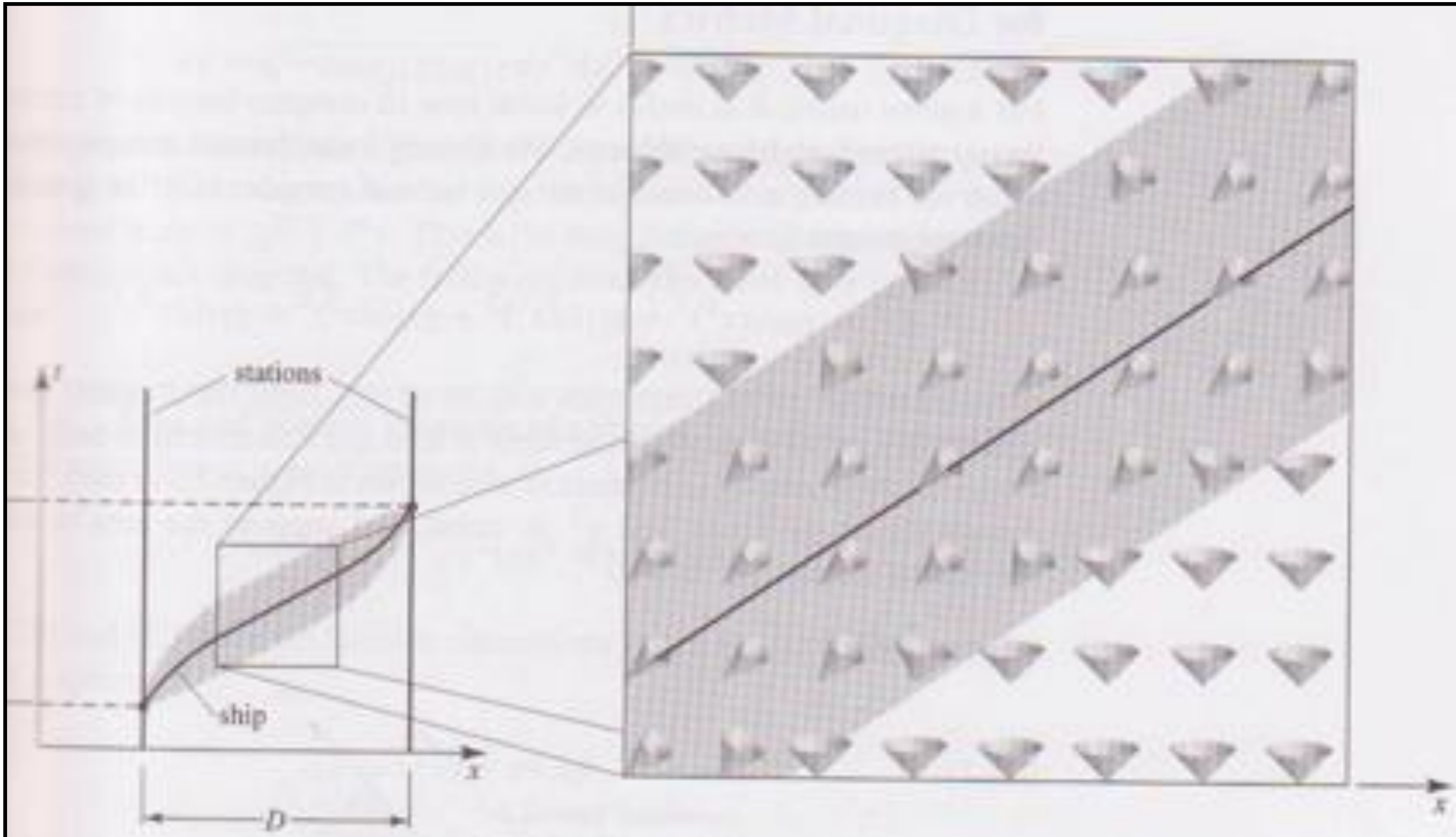
$$ds^2 = -dt^2 + [dx - V_s(t) f(r_s) dt]^2 + dy^2 + dz^2$$

when $r_s > R$ this is just the same as the flat, special relativity space-time. Inside R the space-time is curved; to understand this, it is easiest of consider what happens to light cones ($\Delta s^2 = 0$).

$$\frac{dx}{dt} = \pm 1 + V_s(t) f(r_s)$$

The bubble of curved space-time has tiled the light-cones away from what you would expect in special relativity.

The Alcubierre Warp Drive: Space-Time Structure



From Hartle's Gravity (Honours Textbook)

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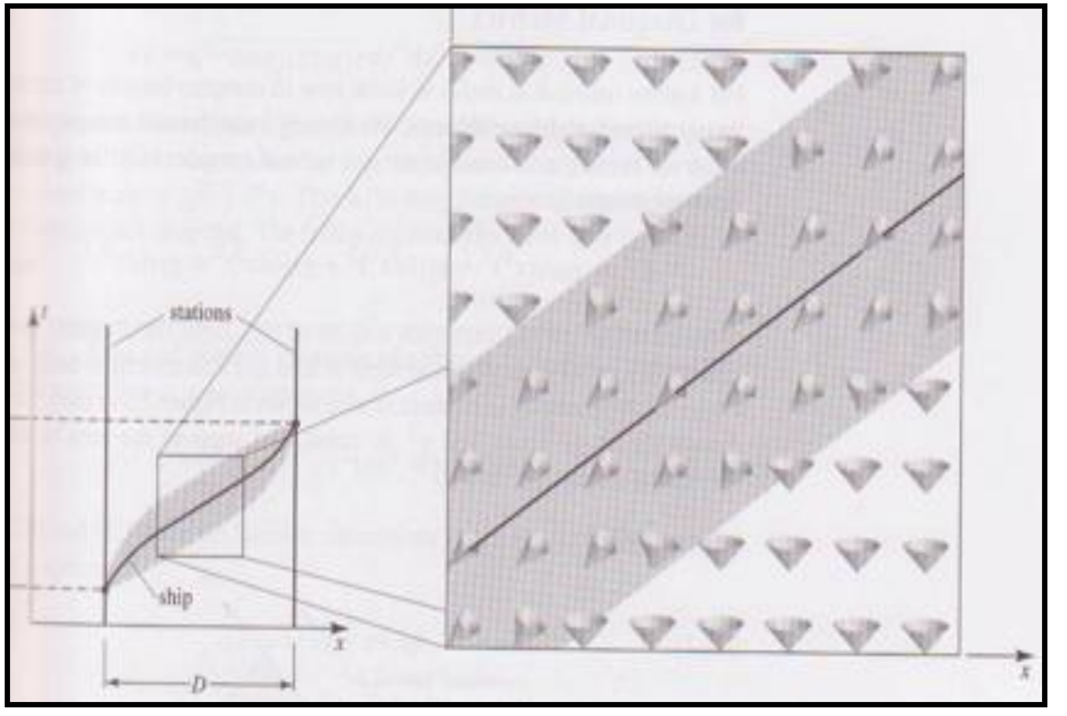
The Alcubierre Warp Drive: Space-Time Structure

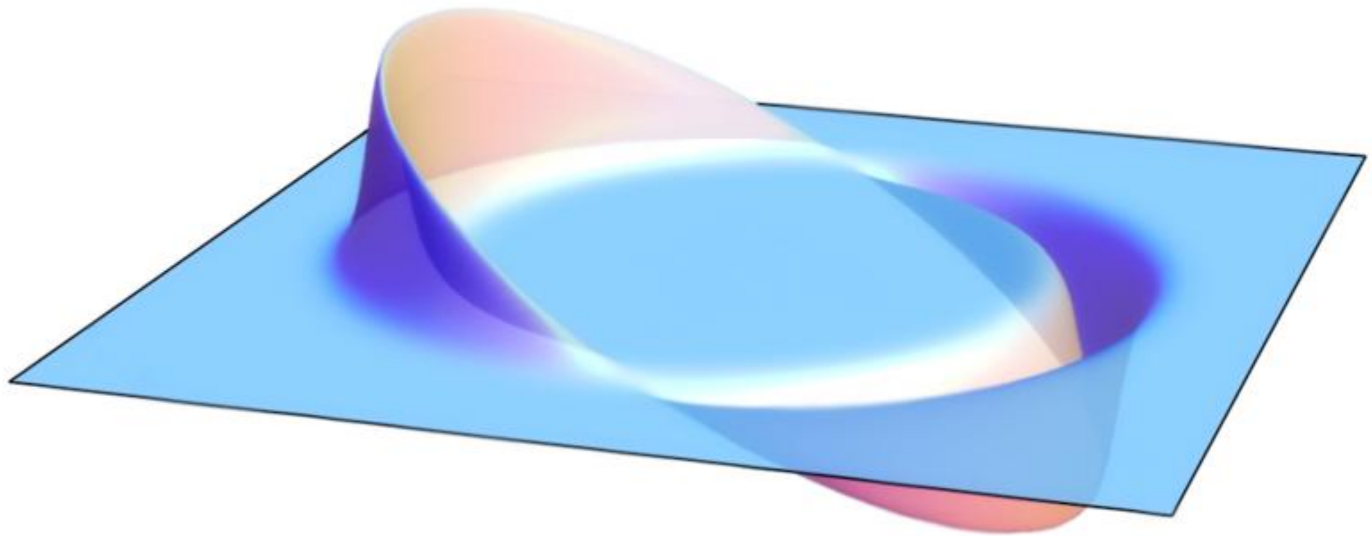
We can adjust $V_S(t)$ so that we can get an arbitrary slope (velocity) over the space-time.

If we look at the path of the massive particle, then we can happily follow this path and remain in the light cone.



So, with the bubble, our rocketeer can cross our space-time diagram at any velocity they want (although they remain within their local light cone and so never actually exceed the local speed of light).

The Alcubierre Warp Drive: Space-Time Structure



From Wikipedia

Basically, the warp drive works by (approx.) squeezing the space-time in front of it, while expanding the space-time behind it (although this is not how all warp drives work). So, apparently, you can travel faster than the speed of light, without ever going faster than the speed of light.

The Alcubierre Warp Drive: Advantage

The Alcubierre Warp drive has several major advantages:

✦ **non-inertial**: The warp drive does not accelerate. The rocketeer follows a free-fall path and so for the duration of the journey, they are weightless (just happily float around).

✦ **no time dilation**: unlike other relativistic travel, those in the Alcubierre bubble suffer no time dilation, so there are no problems about getting home and being the same age as your great-great-grandchildren.

So, on the face of it, this is fantastic! It means we can happily explore as much of the universe as we want. Surely, there is a flaw somewhere?

Metric-Mechanics: The wrong way round?

Remember, Alcubierre defined the structure of space-time, although this is really only one side of the story. The *Einstein Field Equations*

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = 8\pi T_{\alpha\beta}$$

relates geometry (lhs) to the distribution of matter and energy (rhs). We would like to specify the rhs and then calculate the lhs, but this is very difficult.

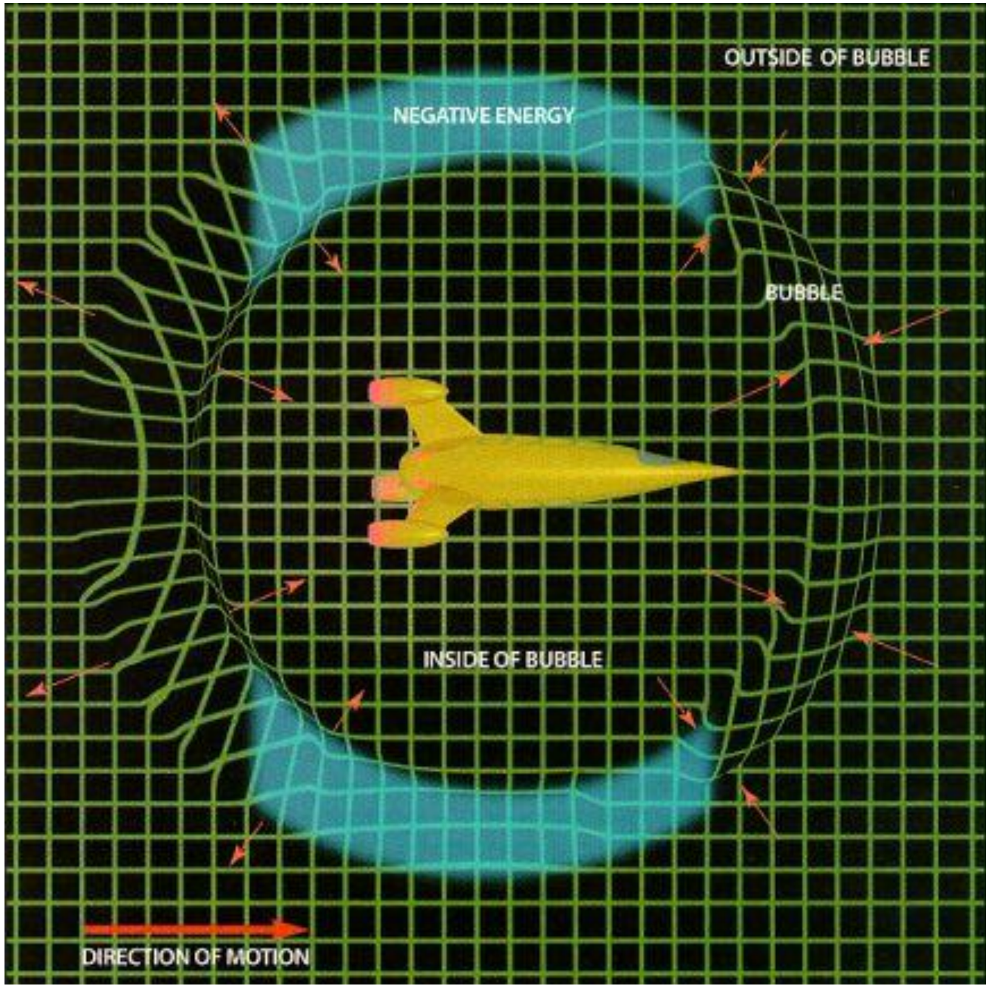
But, once we specify the geometry, we can calculate the mass/energy distribution. So, what do we get for the Alcubierre warp dive?

Metric-Mechanics: The need for negative energy

The result is that to make the Alcubierre drive work, we need material with *negative energy density.*

This sounds bizarre, but represents a tension, rather than a pressure.

While we don't know of such material in the lab, the dominant energy in the universe, *dark energy,* appears to have similar properties.

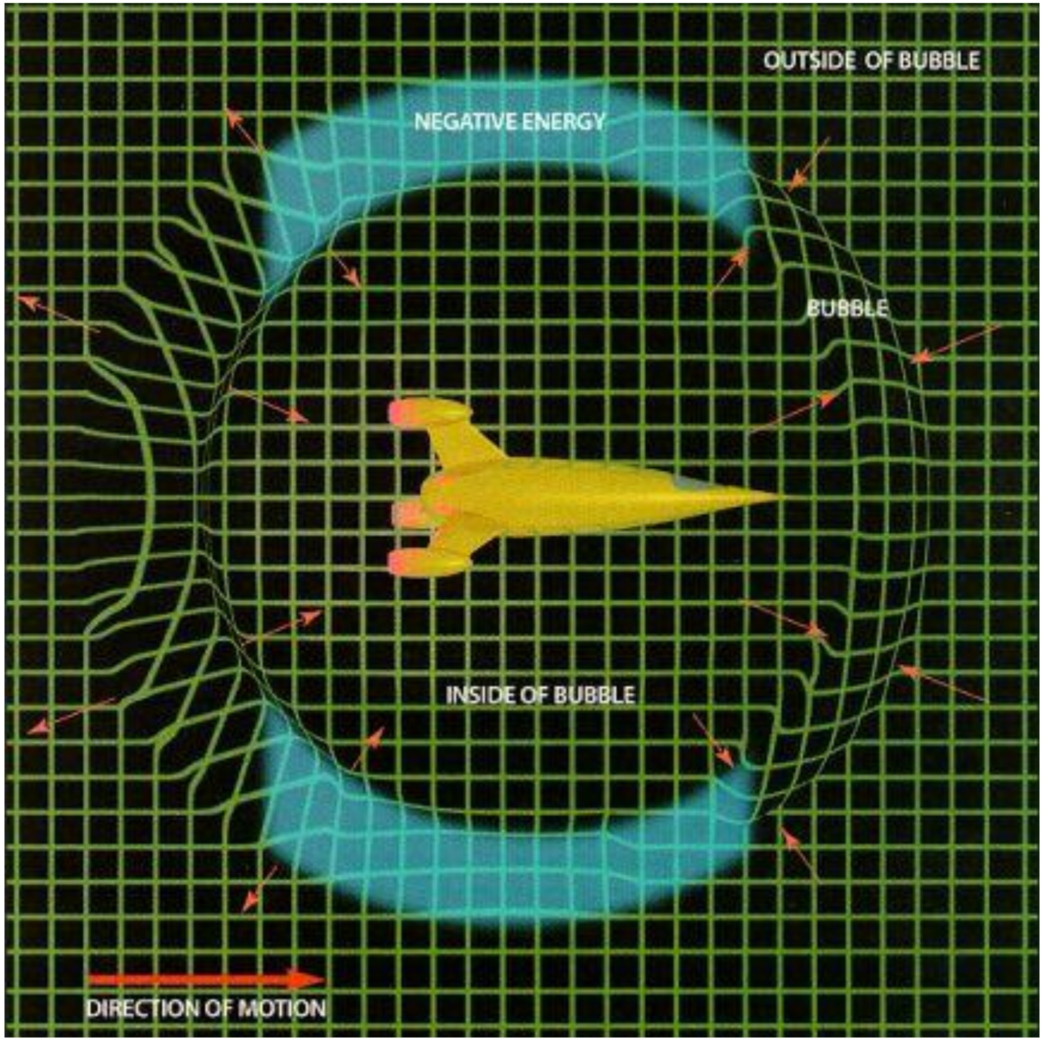


http://www.orbitalvector.com/

Metric-Mechanics: The need for negative energy

Other than negative energy, the Alcubierre warp drive has other rather worrying properties.

The curvature of space-time at the edge of the bubble means that the tidal gravitational forces are huge, and if the bubble runs into you, it will be bad (although the driver may not see it coming!)



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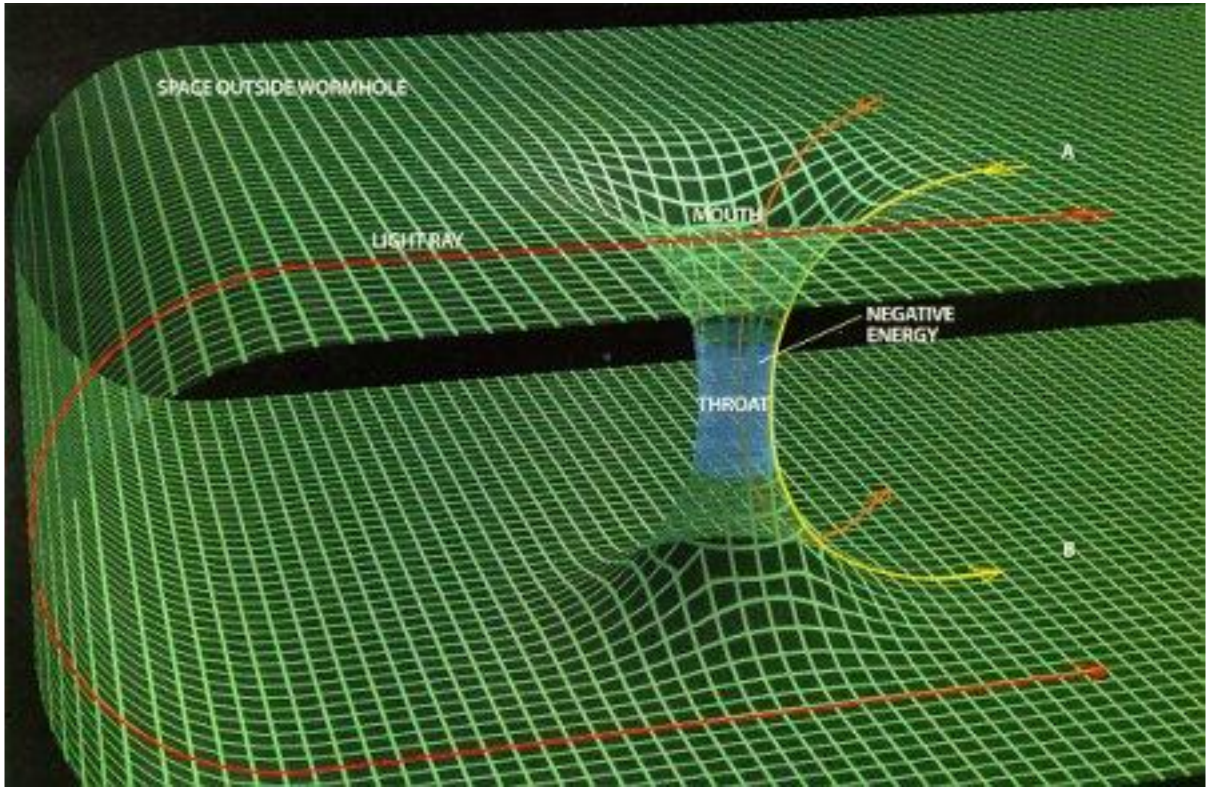
Metric-Mechanics: The awful truth



There are other geometries which allow weird (faster than light) through the twisting and bending of space-time (such as wormholes), although these rely on defining geometry and then calculating mass/energy distribution.

Final Message

General Relativity will soon be 100 years old. However, the mathematical complexity of solving the field equations for general mass distributions is still a major problem.

However, the inverse is (relatively) easy; we can specify a geometry of space-time and infer the properties of the mass distribution. This approach, however, usually produces unphysical results (this is not new - we get the same in classical physics).

Currently computers are making significant head-way in (brute-force) solutions, and we can expect further ‘weirdness’ to come.

The End