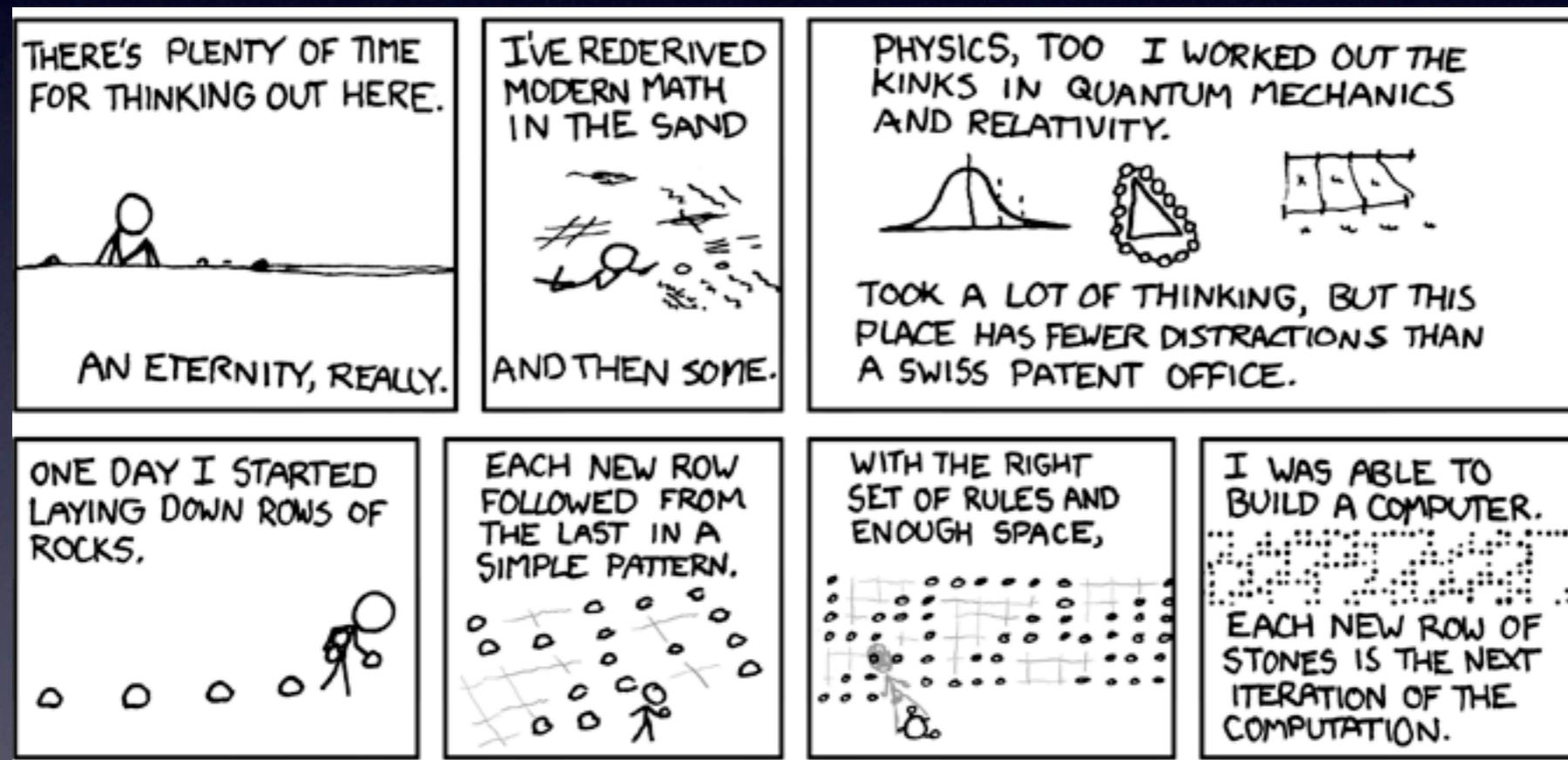


# Turning Time Into Space for Codes and Profit\*

Dave Bacon  
University of Washington



\*no warrantee, void where prohibited, not valid in WA, Australia, or anywhere else in this universe

# Spacetime Joke



spacetime joke

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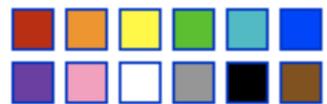
Clip art

Line drawing

Any color

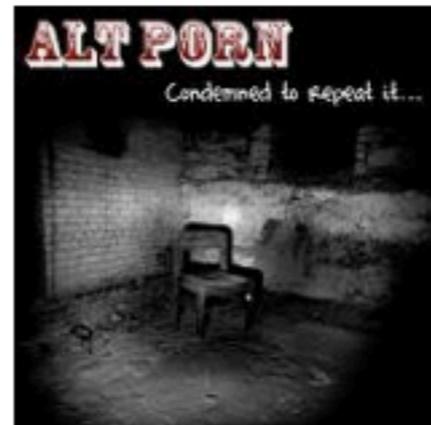
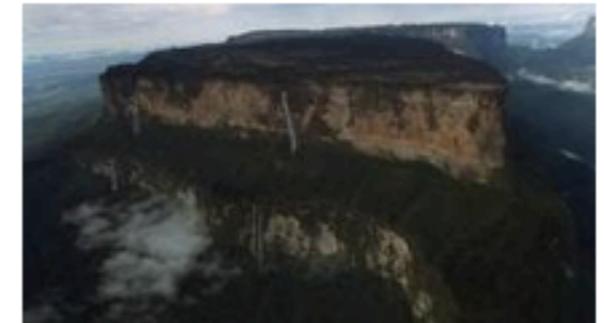
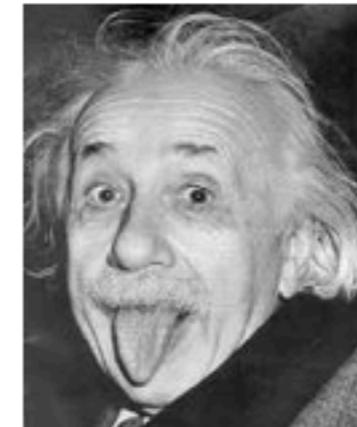
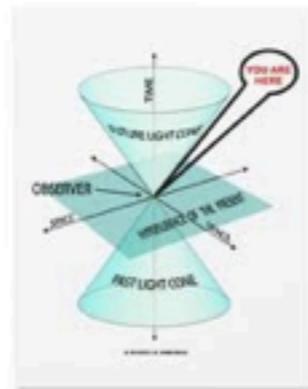
Full color

Black and white



Standard view

Show sizes



# TQC 2011

6th Conference on Theory of Quantum Computation, Communication and Cryptography  
24-26 May, Madrid, Spain

## Menu

- [General information](#)
- [Scope](#)
- [Program](#)
  - [Invited speakers](#)
- [Submissions](#)
- [Registration](#)
- [Location and travel info](#)
- [Submission policy](#)
- [Committees](#)

## Organizer



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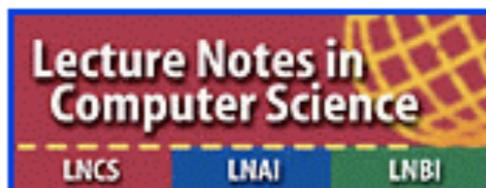


## General information

The sixth Conference on the Theory of Quantum Computation, Communication and Cryptography will be held at Madrid, Spain, from 24th - 26th May 2011. [Download the poster!](#)

Quantum computation, quantum communication, and quantum cryptography are topics of a new and interdisciplinary field in the intersection of computer science, information theory, and quantum mechanics. The aim of the TQC'11 conference is to allow deep coverage of new and original research on these topics and to raise important problems that can benefit from theoretical investigation and analysis.

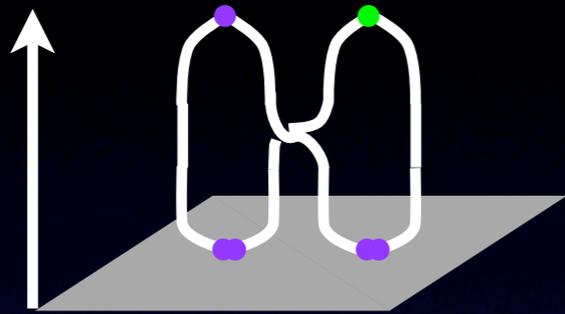
As for previous TQC conferences, a post-conference proceedings volume will be published in [Springer's Lecture Notes in Computer Science](#) to which selected speakers will be invited to contribute.



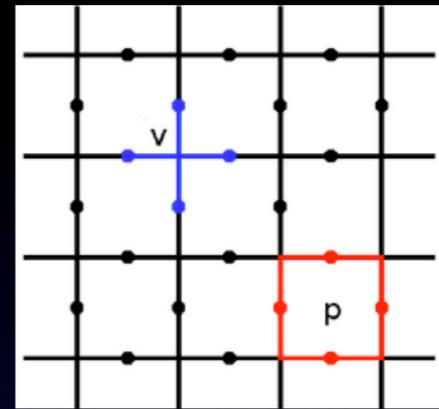
## Important dates

- Submission deadline: **January 24, 2011 (23:59 CET local time)**
- Notification of acceptance/rejection: March 14, 2011
- Final version of extended abstracts: March 31, 2011
- Registration deadline: May 10, 2011
- Conference: May 24-26, 2011
- Post-proceedings submission deadline: End of June 2011
- Publication date: October 2011

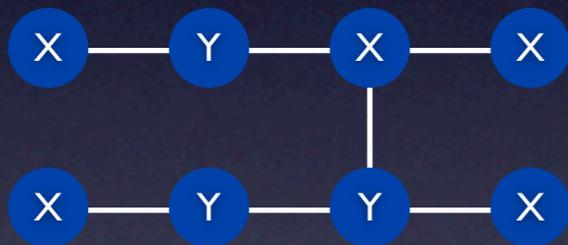
# What Are We Doing?



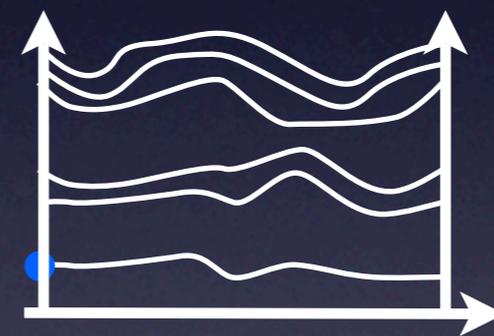
Topological QC



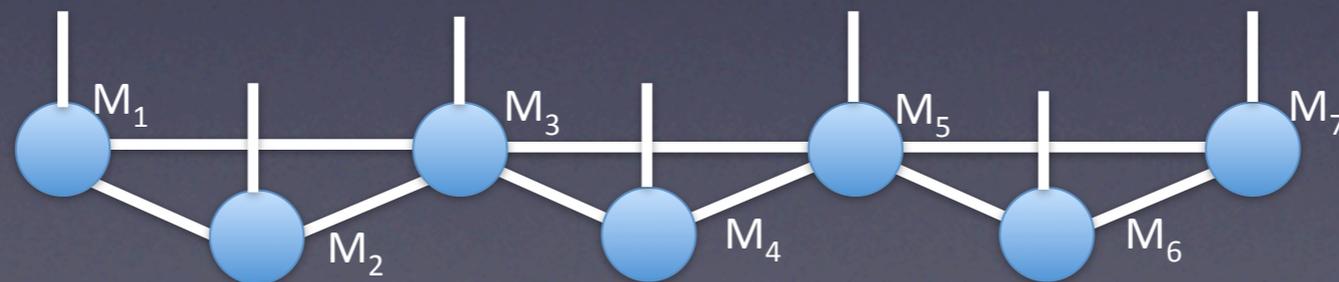
Local Fault-tolerant QC



Measurement-based QC



Adiabatic QC



Matrix Product States, PEPS, MERA, tensor networks

# Quantum Computational Matter?

A focus on systems, states, models, phases where space-time locality is an important limiting constraint

Systems, states, models, phases where quantum information or quantum computational are front and center

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A focus on systems, states, models, phases where space-time locality is an important limiting constraint

WE ARE PHYSICISTS, but for materials yet to be engineered?

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A focus on systems, states, models, phases where space-time locality is an important limiting constraint

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Systems, states, models, phases where quantum information or quantum computational are front and center

WE AREN'T SCARED BY INFORMATION?

# Self-Correcting System



Physics of the device enacts quantum error correction and allows fault-tolerant quantum computing?

# One Code, Two Models

Redundancy code:  $0 \rightarrow 000 \dots 0$   
 $1 \rightarrow 111 \dots 1$

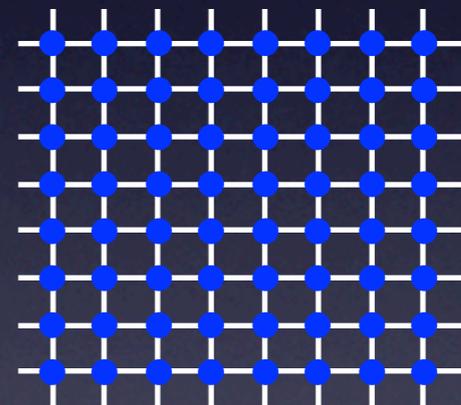
Ising model:  $E = -J \sum_{\langle i,j \rangle} s_i s_j$   $s_i \in \{+1, -1\}$

1D



$\dots 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \dots$   
- - + - + - -

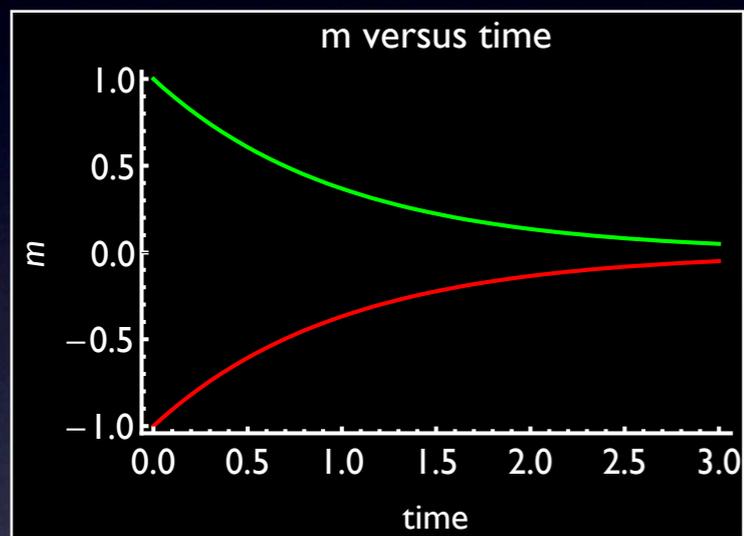
2D



0 0 0 0  
0 1 0 0  
0 1 1 0  
0 0 0 0

# Memory

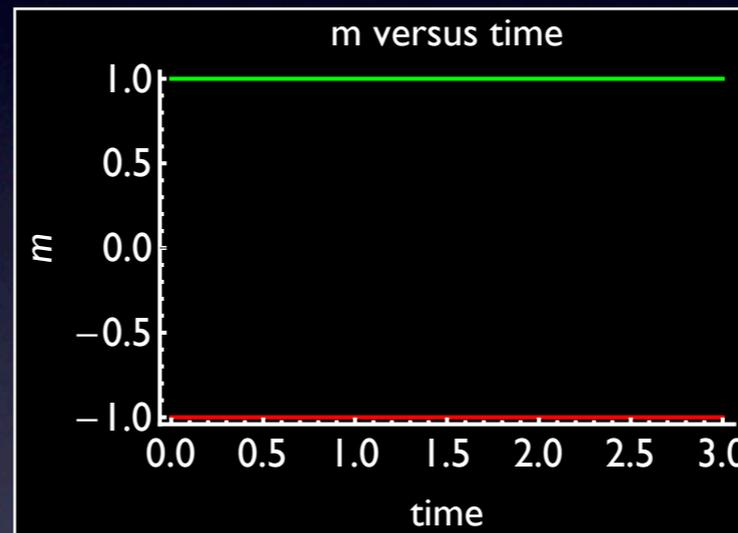
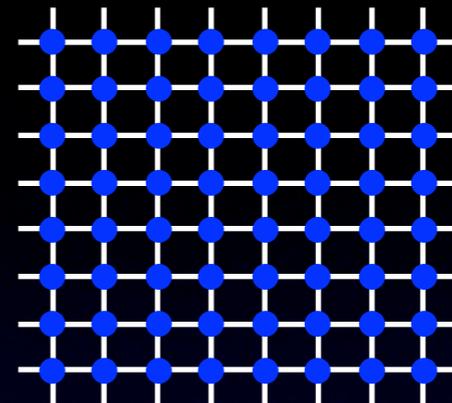
1D



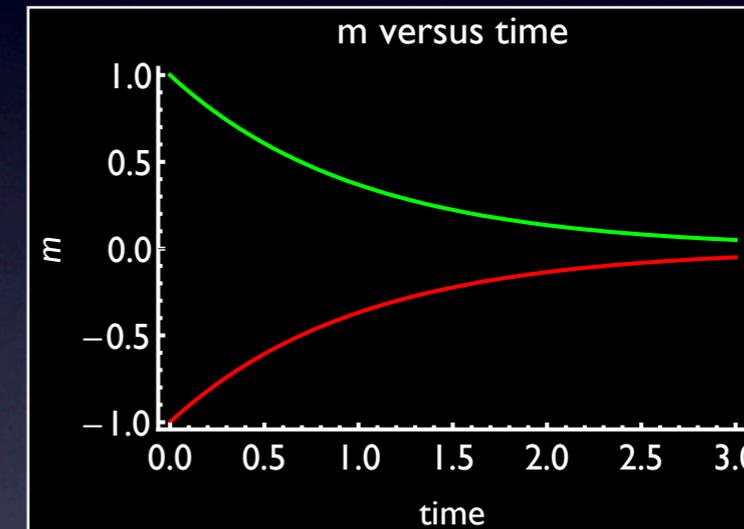
rate  $\sim \exp(-c\beta J)$

$\beta = T^{-1}$

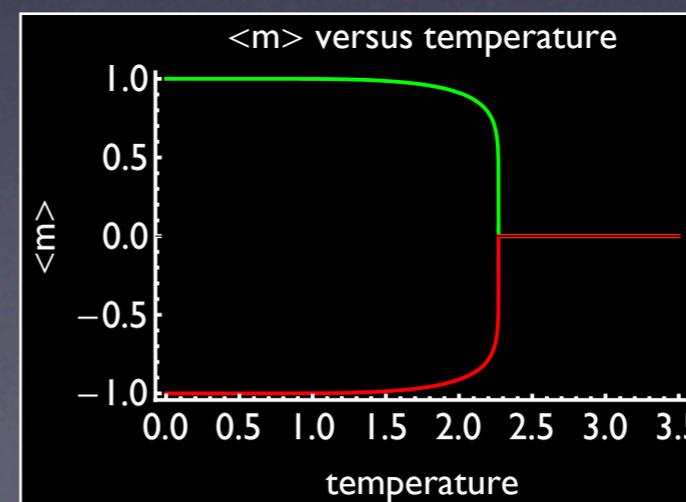
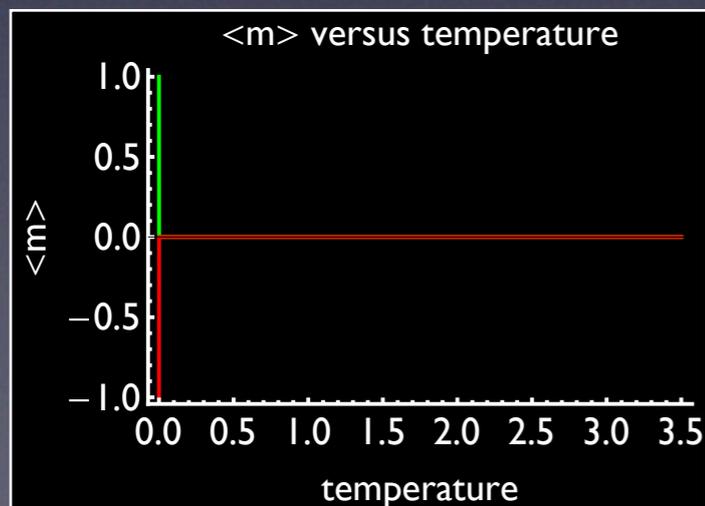
2D



$T < T_C$

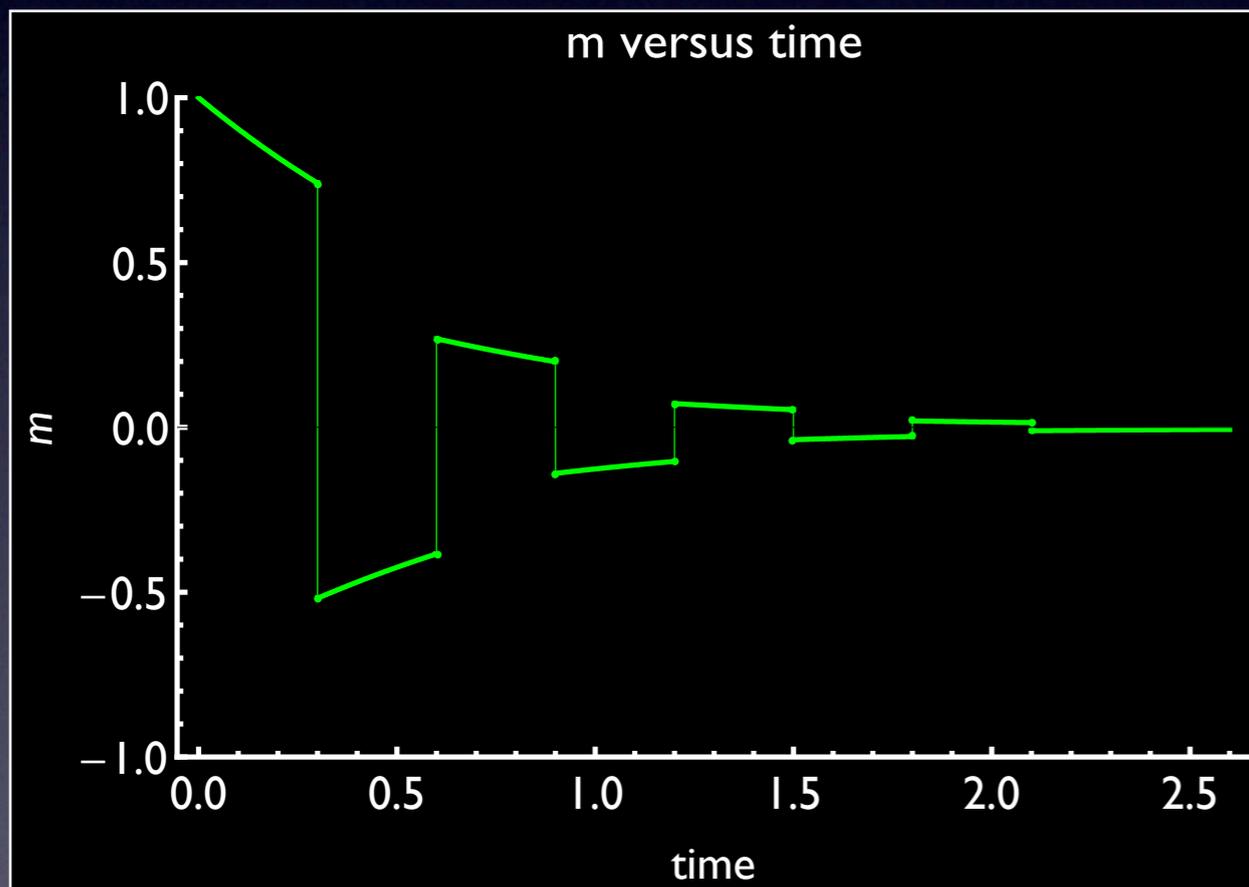
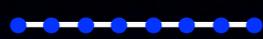


$T > T_C$

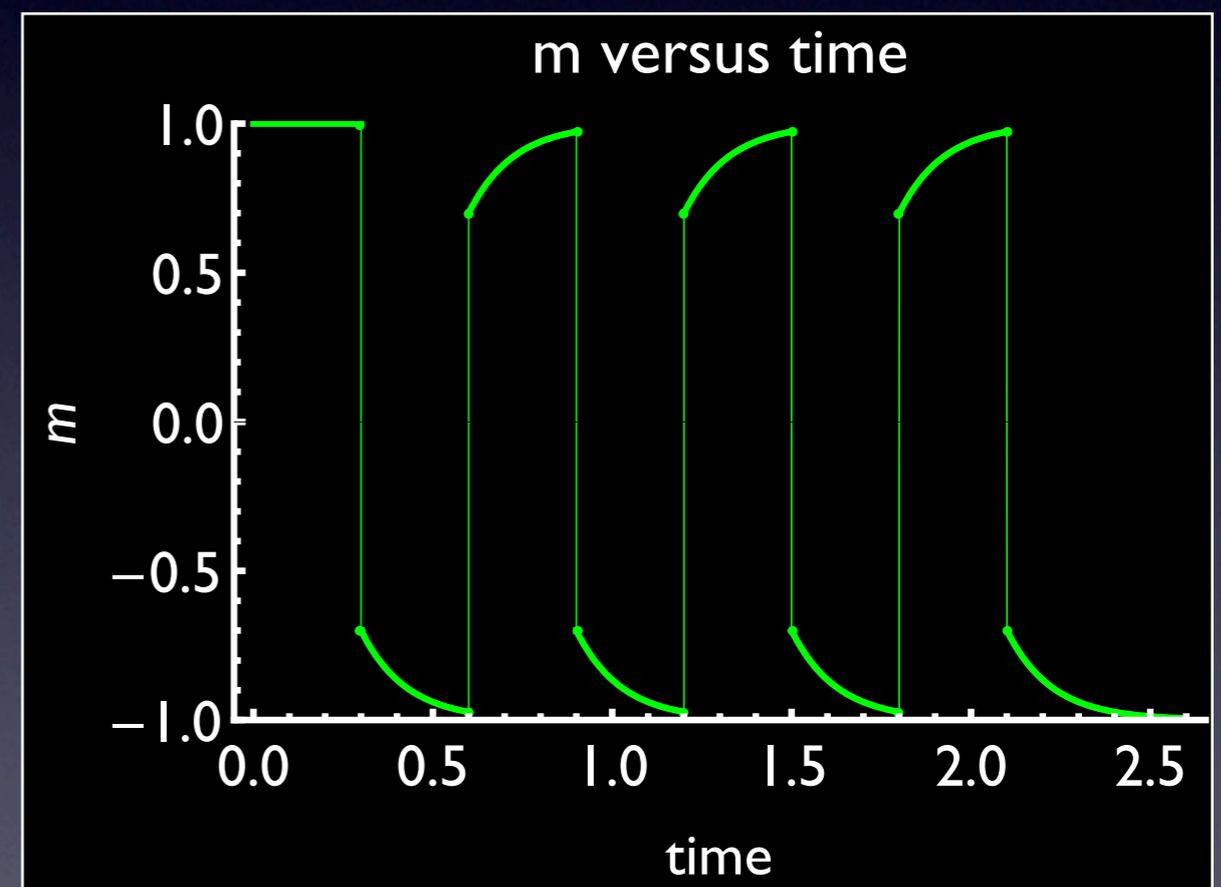
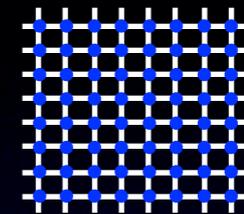


# Fault-Tolerance

1D



2D



Resilience to “gate” of flipping spins: self-correcting

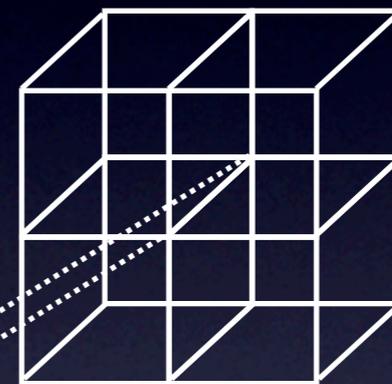
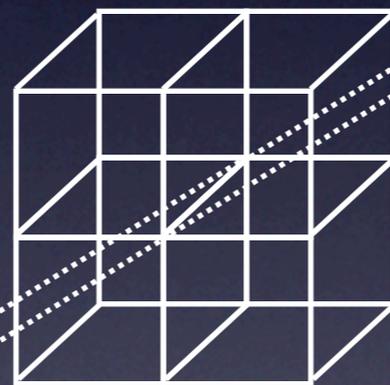
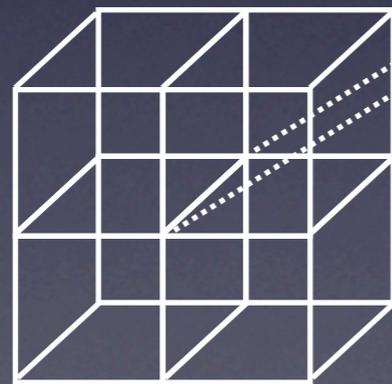
# Self-Correcting Quantum Systems?

cube operators

$$S_c = Z^{\otimes 6}$$

link operators

$$S_l = X^{\otimes 6}$$



qubits on  
plaquettes

$$H = -J \left( \sum_c S_c + \sum_l S_l \right)$$

excitations = strings

4D toric code is self-correcting

# Bravyi-Terhal Bounds

[Bravyi-Terhal arXiv:0810.1983]

For  $k$ -local  $D$  dimensional stabilizer codes (subspace and subsystem) on an  $\{1, \dots, L\}^D$  lattice, the distance of the code is bound by

$$d \leq cL^{D-1}$$



$$D = 3$$

D	bound	best known
1	$c$	$c$ , many
2	$cL$	toric, color codes, $d=cL$
3	$cL^2$	3D toric, color codes, $d=cL$
4	$cL^3$	4D toric, color code, $d=cL^2$

# Without Spatial Locality

Classical

distance =  $n$

redundancy code  
(works with bits)

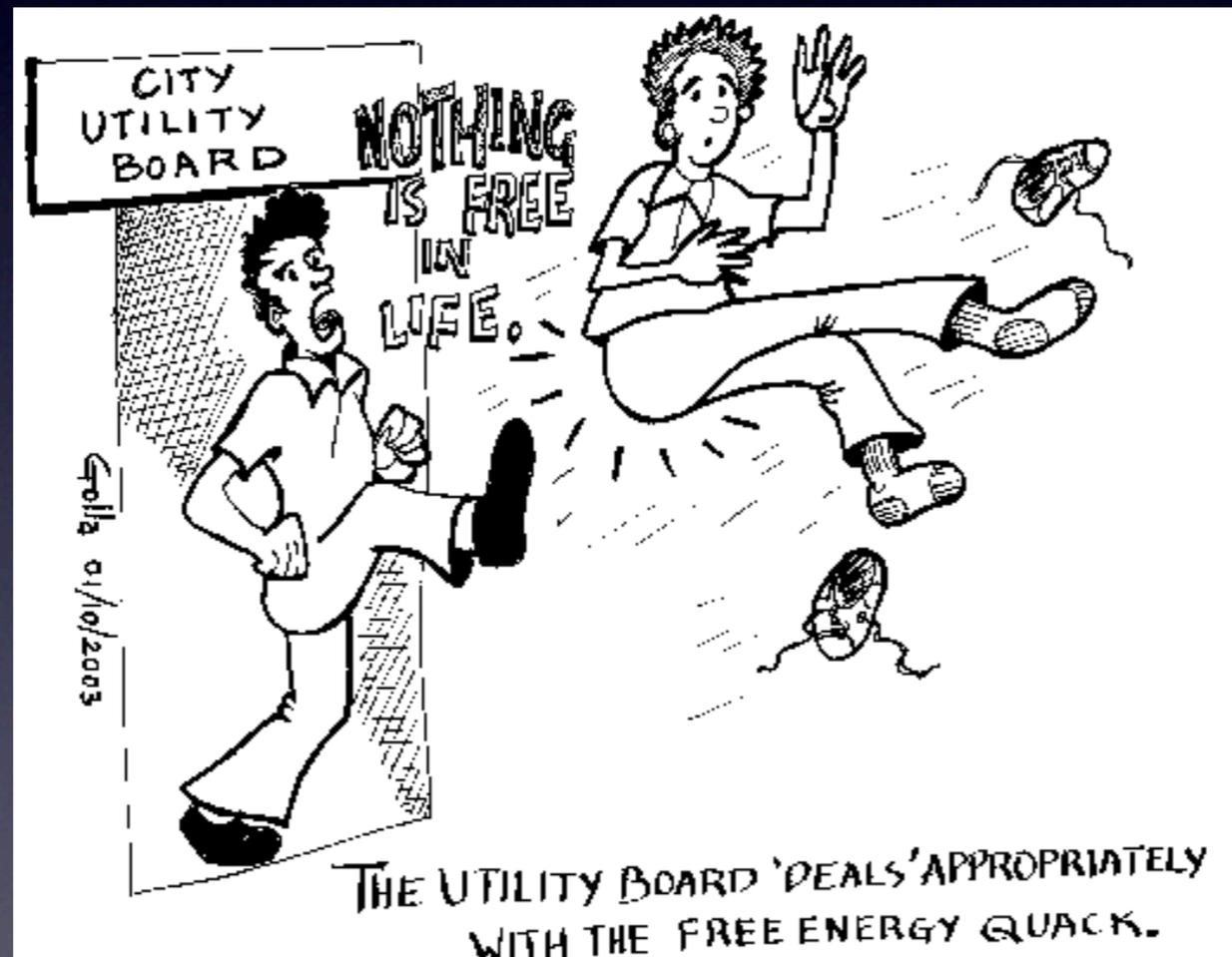
Quantum

distance =  $n/2$

polynomial code  
(uses qudits)

# Looking For?

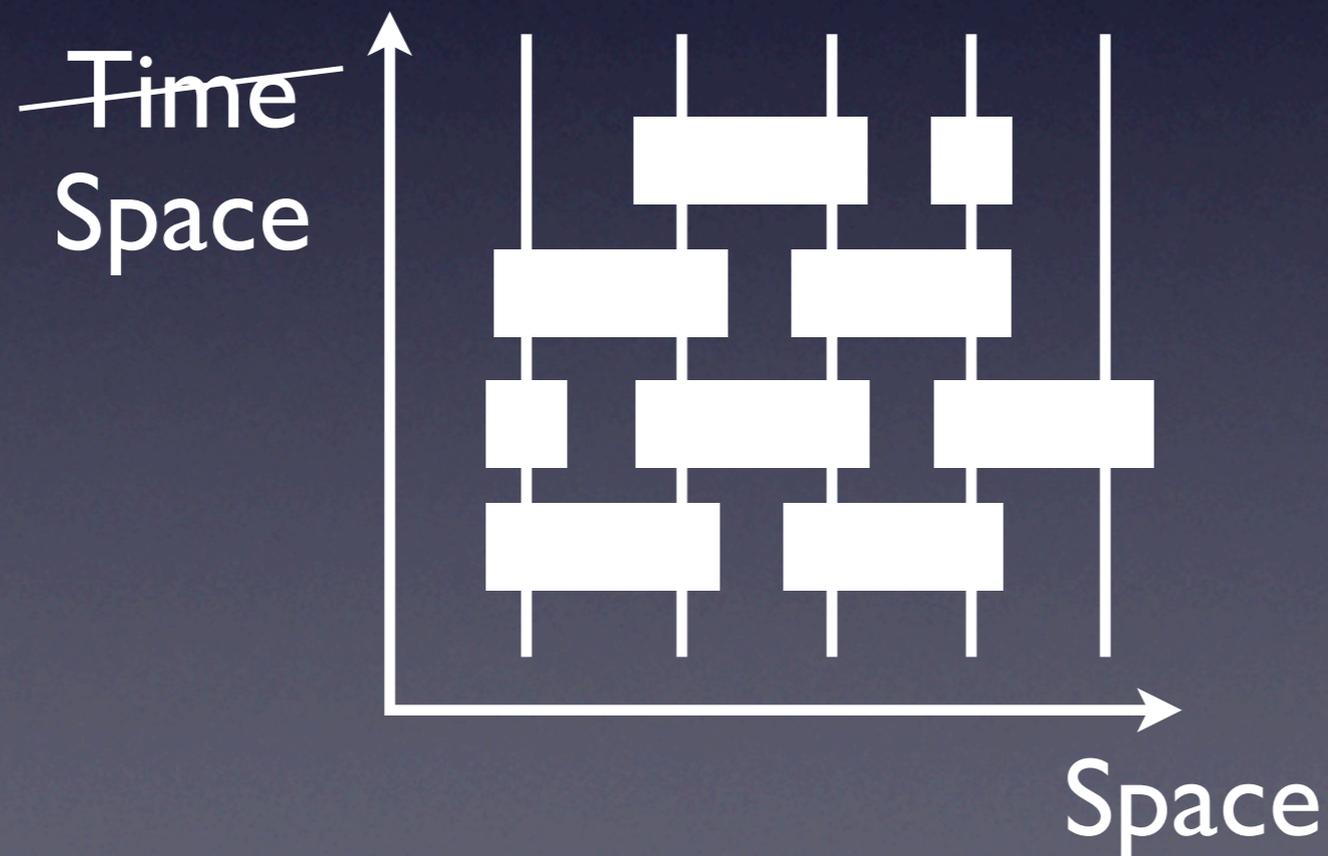
Competition between most likely errors and cost of these errors (energy versus entropy)



Codes with local check operators.....

# Turn Time to Space?

We know how to perform quantum error correction, locally in space-time. Can we leverage this to design codes that are local in space?



# Stabilizer Subsystem Codes

[Poulin, arXiv:quant-ph/0508131]

n physical qubits



k encoded qubits

$[n, k, d]$  subspace code

---

# Stabilizer Subsystem Codes

[Poulin, arXiv:quant-ph/0508131]

n physical qubits



$[n, k, d]$  subspace code



k encoded qubits

n physical qubits



$[n, r, k, d]$  subsystem code



k encoded  
qubits

k gauge  
qubits

# Baloney Sandwich Example

## Gauge

$$\tilde{X}_1 = \begin{pmatrix} X & X & I \\ I & I & I \\ I & I & I \end{pmatrix} \quad \tilde{X}_2 = \begin{pmatrix} I & X & X \\ I & I & I \\ I & I & I \end{pmatrix} \quad \tilde{X}_3 = \begin{pmatrix} I & I & I \\ I & I & I \\ X & X & I \end{pmatrix} \quad \tilde{X}_4 = \begin{pmatrix} I & I & I \\ I & I & I \\ I & X & X \end{pmatrix}$$

$$\tilde{Z}_1 = \begin{pmatrix} Z & I & I \\ Z & I & I \\ I & I & I \end{pmatrix} \quad \tilde{Z}_2 = \begin{pmatrix} I & I & Z \\ I & I & Z \\ I & I & I \end{pmatrix} \quad \tilde{Z}_3 = \begin{pmatrix} I & I & I \\ Z & I & I \\ Z & I & I \end{pmatrix} \quad \tilde{Z}_4 = \begin{pmatrix} I & I & I \\ I & I & Z \\ I & I & Z \end{pmatrix}$$

## Stabilizer

$$S_1 = \begin{pmatrix} X & X & I \\ X & X & I \\ X & X & I \end{pmatrix} \quad S_2 = \begin{pmatrix} I & X & X \\ I & X & X \\ I & X & X \end{pmatrix} \quad S_3 = \begin{pmatrix} Z & Z & Z \\ Z & Z & Z \\ I & I & I \end{pmatrix} \quad S_4 = \begin{pmatrix} I & I & I \\ Z & Z & Z \\ Z & Z & Z \end{pmatrix}$$

# Baloney Sandwich Example

## Gauge

$$\begin{array}{cccc} \tilde{X}_1 = & X & X & I \\ & I & I & I \\ & I & I & I \end{array} \quad \tilde{X}_2 = \begin{array}{ccc} I & X & X \\ I & I & I \\ I & I & I \end{array} \quad \tilde{X}_3 = \begin{array}{ccc} I & I & I \\ I & I & I \\ X & X & I \end{array} \quad \tilde{X}_4 = \begin{array}{ccc} I & I & I \\ I & I & I \\ I & X & X \end{array}$$

$$\begin{array}{cccc} \tilde{Z}_1 = & Z & I & I \\ & Z & I & I \\ & I & I & I \end{array} \quad \tilde{Z}_2 = \begin{array}{ccc} I & I & Z \\ I & I & Z \\ I & I & I \end{array} \quad \tilde{Z}_3 = \begin{array}{ccc} I & I & I \\ Z & I & I \\ Z & I & I \end{array} \quad \tilde{Z}_4 = \begin{array}{ccc} I & I & I \\ I & I & Z \\ I & I & Z \end{array}$$

## Stabilizer

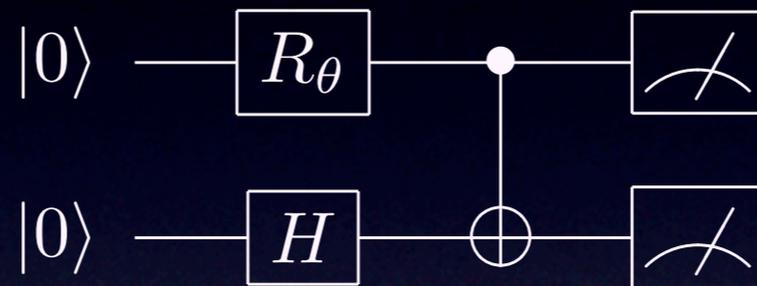
$$\begin{array}{cccc} S_1 = & X & X & I \\ & X & X & I \\ & X & X & I \end{array} \quad S_2 = \begin{array}{ccc} I & X & X \\ I & X & X \\ I & X & X \end{array} \quad S_3 = \begin{array}{ccc} Z & Z & Z \\ Z & Z & Z \\ I & I & I \end{array} \quad S_4 = \begin{array}{ccc} I & I & I \\ Z & Z & Z \\ Z & Z & Z \end{array}$$

Measure large stabilizer using only 2-qubit measurements

$$\tilde{X}_1, \tilde{X}_3, \tilde{X}_1\tilde{X}_3S_1 \quad \tilde{X}_2, \tilde{X}_4, \tilde{X}_2\tilde{X}_4S_2 \quad \tilde{Z}_1, \tilde{Z}_2, \tilde{Z}_1\tilde{Z}_2S_3 \quad \tilde{Z}_3, \tilde{Z}_4, \tilde{Z}_3\tilde{Z}_4S_4$$

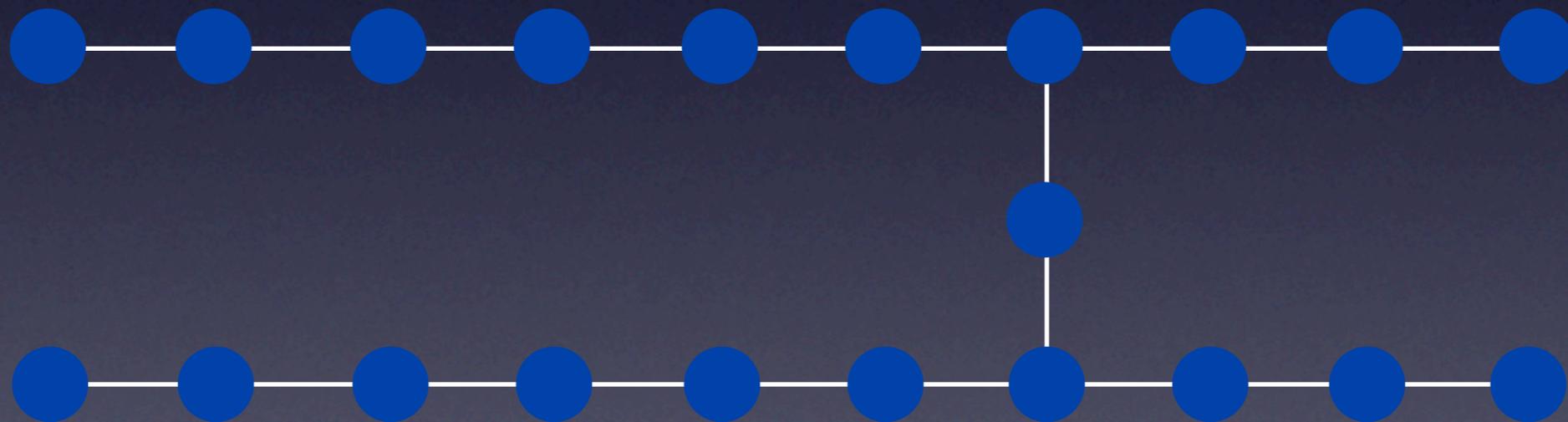
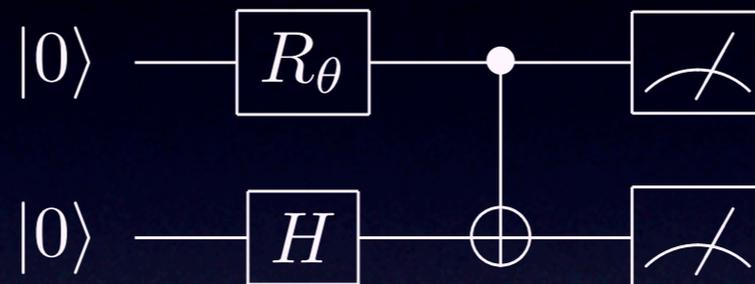
# One Way QC

[Raussendorf and Briegel, Phys. Rev. Lett. 86, 5188 (2001)]



# One Way QC

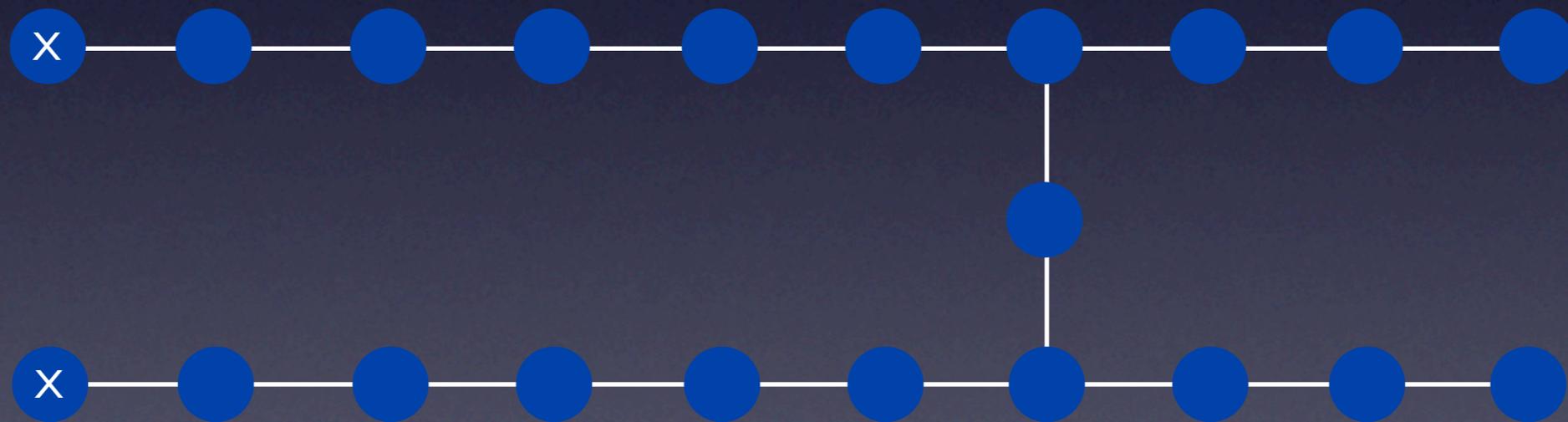
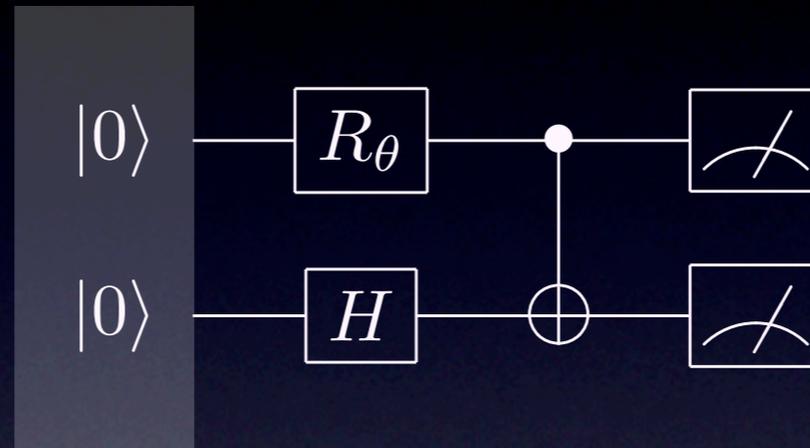
[Raussendorf and Briegel, Phys. Rev. Lett. 86, 5188 (2001)]



Create Entangled State

# One Way QC

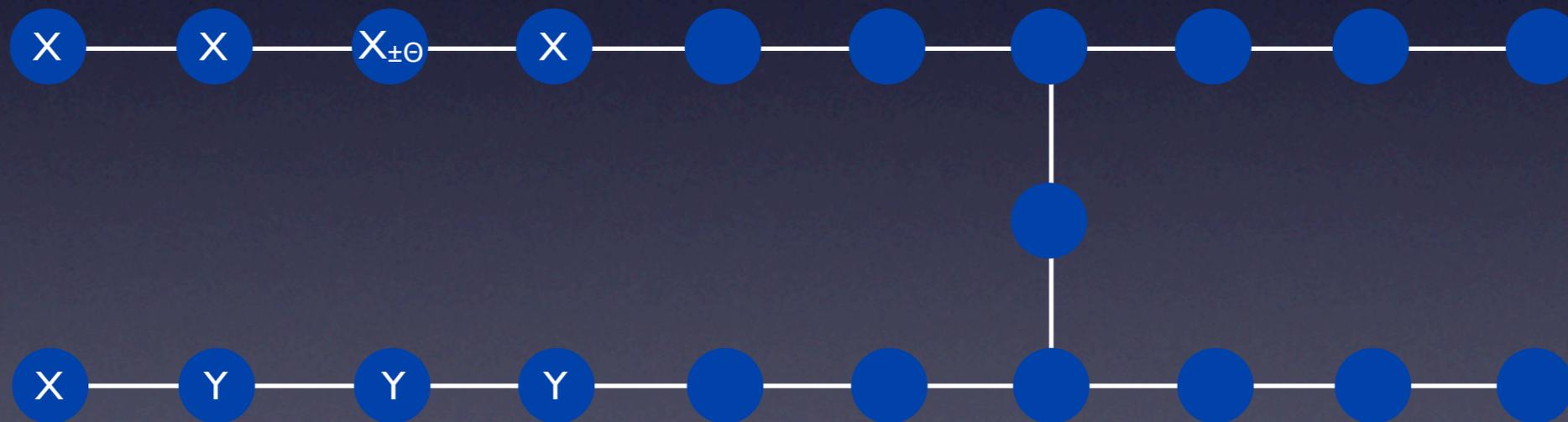
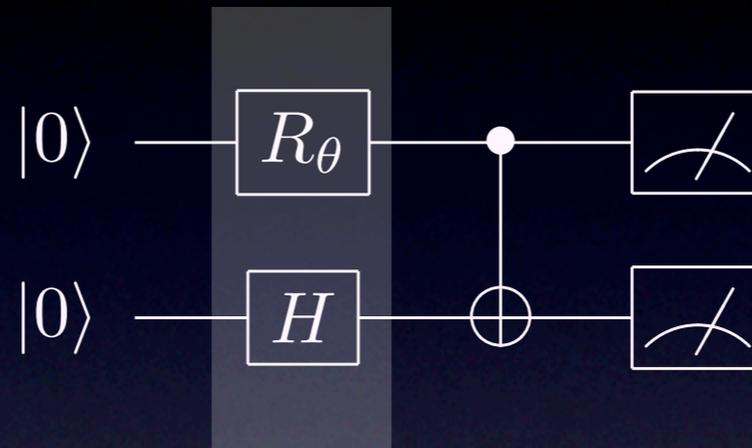
[Raussendorf and Briegel, Phys. Rev. Lett. 86, 5188 (2001)]



Adaptively measure to enact circuit

# One Way QC

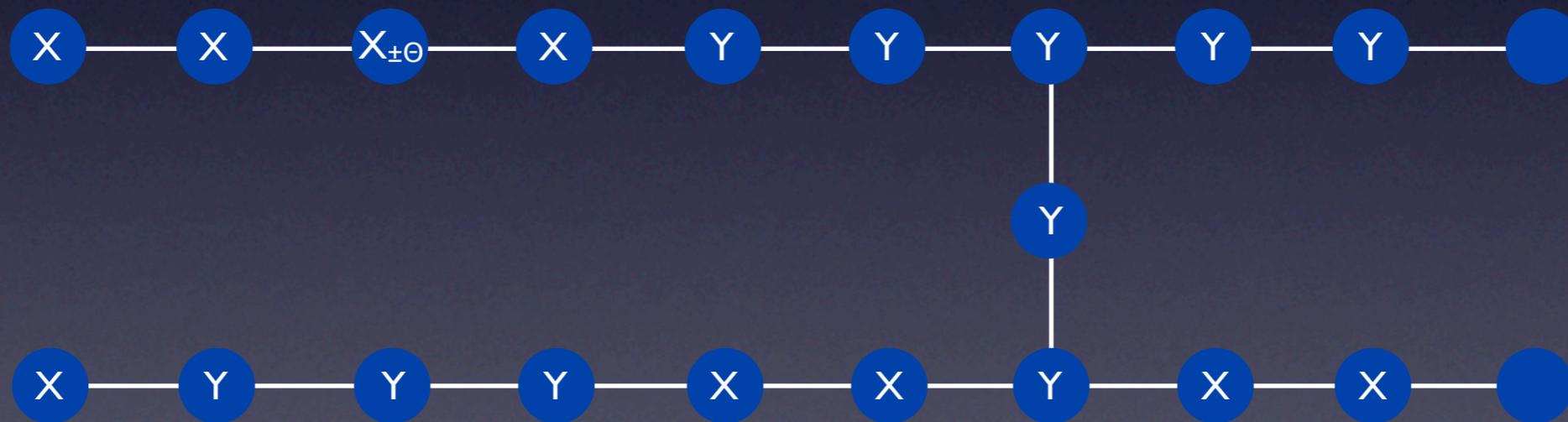
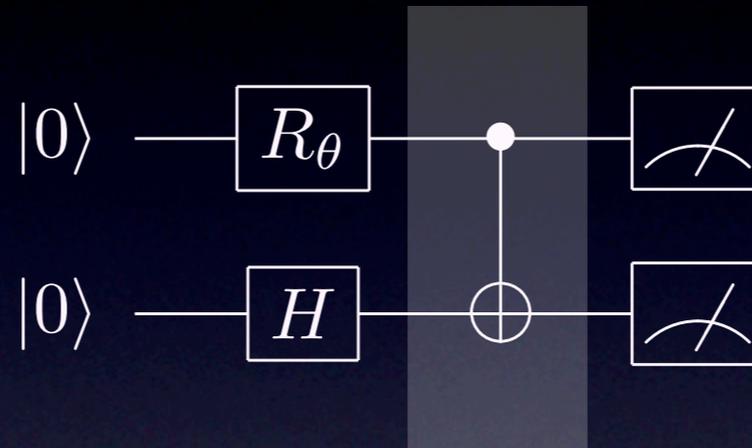
[Raussendorf and Briegel, Phys. Rev. Lett. 86, 5188 (2001)]



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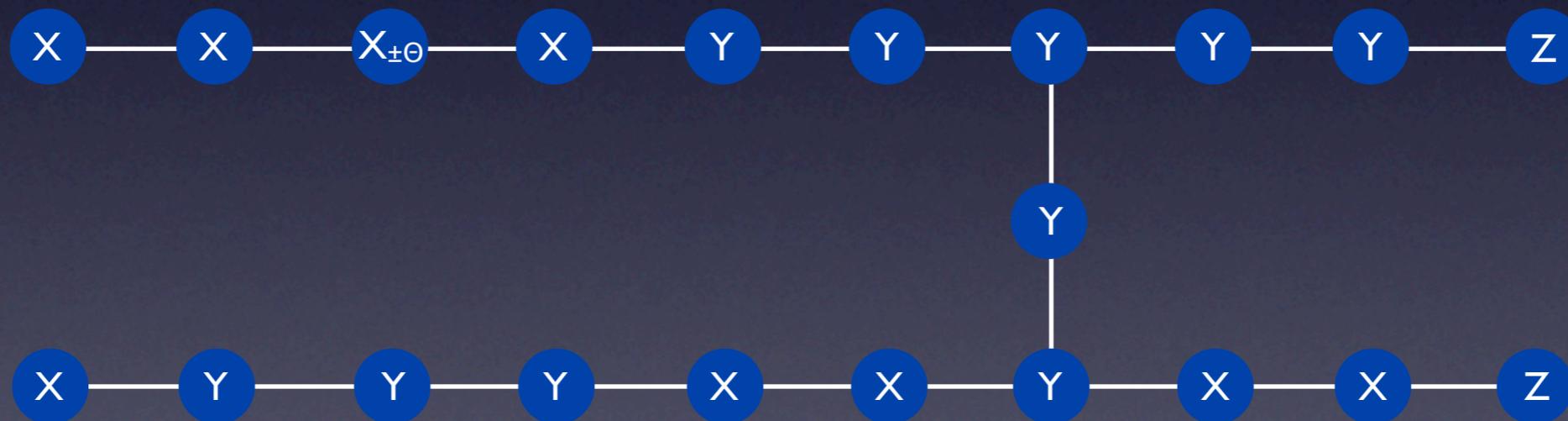
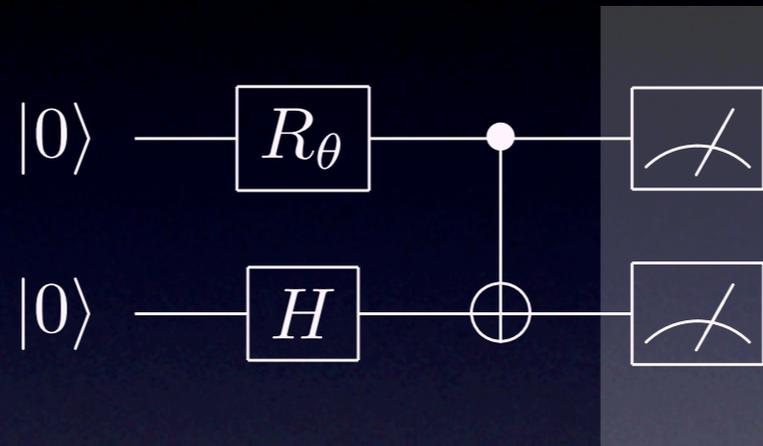
[Raussendorf and Briegel, Phys. Rev. Lett. 86, 5188 (2001)]



Adaptively measure to enact circuit

# One Way QC

[Raussendorf and Briegel, Phys. Rev. Lett. 86, 5188 (2001)]



Adaptively measure to enact circuit

Initial state described locally + measurements are all local

# There's a Code in My Wire



Vertex operators at  $v$ :  $X_v \prod_{(w,v) \in E} Z_w$

Consider stabilizer code with stabilizer generators all vertices in a line except first one:

$$S_i = [Z]_i [X]_{i+1} [Z]_{i+2} \quad 2 \leq i \leq n-2$$

$$S_{n-1} = [Z]_{n-1} [X]_n$$

$n-1$  independent stabilizers = one encoded qubit

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Z info

localized here



X info

localized here

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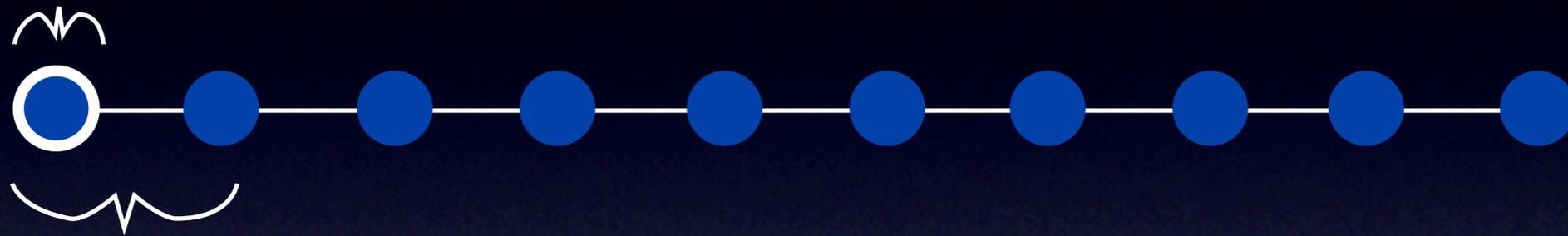
$$S_{n-1} = [Z]_{n-1} [X]_n$$

logical qubit:  $\bar{X} = [X]_1 [Z]_2, \bar{Z} = [Z]_1$

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# There's a Code in My Wire



Add a gauge qubit:

$$S_i = [Z]_i [X]_{i+1} [Z]_{i+2} \quad 3 \leq i \leq n-2$$

$$S_{n-1} = [Z]_{n-1} [X]_n$$

$$\tilde{X}_1 = [X]_1, \tilde{Z} = [Z]_1 [X]_2 [Z]_3$$

logical qubits?

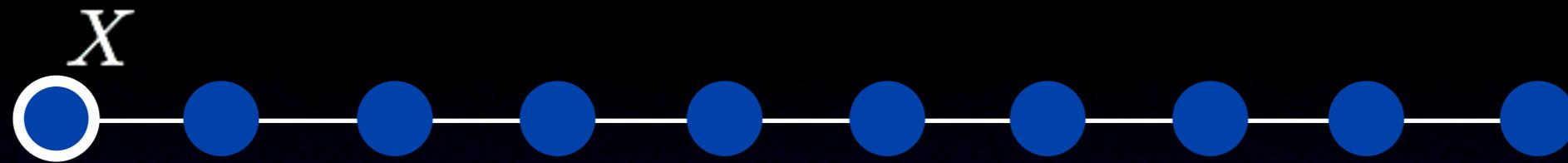
$$\bar{X} = [X]_1 [Z]_2, \bar{Z} = [Z]_1$$

does not commute with  
new gauge qubits

$$\bar{X} = [X]_1 [Z]_2, \bar{Z} = [X]_2 [Z]_3$$

commutes with new  
gauge qubits

# There's a Code in My Wire



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$$S_i = [Z]_i [X]_{i+1} [Z]_{i+2} \quad 3 \leq i \leq n-2$$

$$S_{n-1} = [Z]_{n-1} [X]_n$$

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logical qubits?

$$\bar{X} = [X]_1 [Z]_2, \bar{Z} = [Z]_1$$

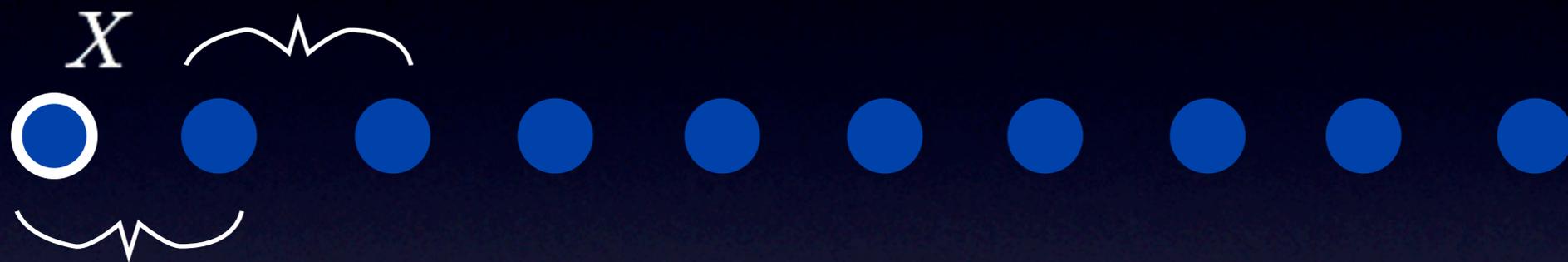
does not commute with  
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$$\bar{X} = [X]_1 [Z]_2, \bar{Z} = [X]_2 [Z]_3$$

commutes with new  
gauge qubits

# There's a Code in My Wire

Z info localized here  $\bar{X} = [X]_1[Z]_2, \bar{Z} = [X]_2[Z]_3$



X info localized here

modulo the gauge qubits:  $\tilde{X}_1 = [X]_1, \tilde{Z} = [Z]_1[X]_2[Z]_3$

Z info localized here

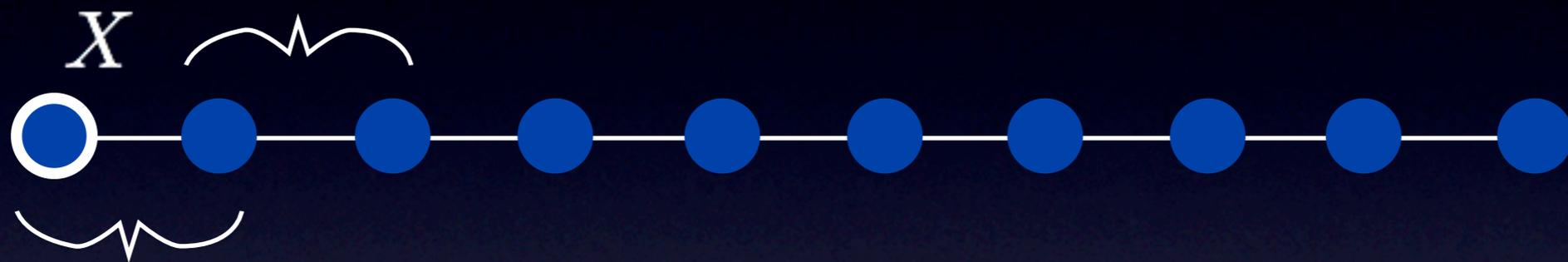


X info localized here  $\bar{X}\tilde{X}_1 = [Z]_2$

q-info has propagated down the wire with Hadamard

# There's a Code in My Wire

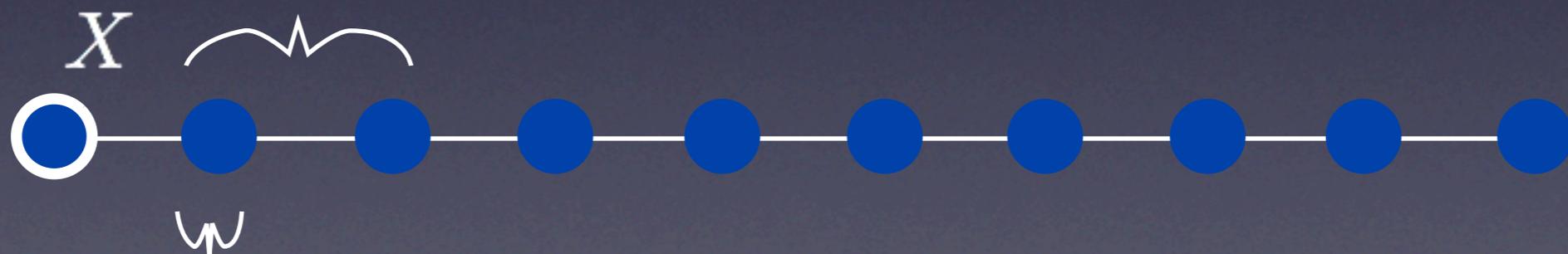
Z info localized here  $\bar{X} = [X]_1[Z]_2, \bar{Z} = [X]_2[Z]_3$



X info localized here

modulo the gauge qubits:  $\tilde{X}_1 = [X]_1, \tilde{Z} = [Z]_1[X]_2[Z]_3$

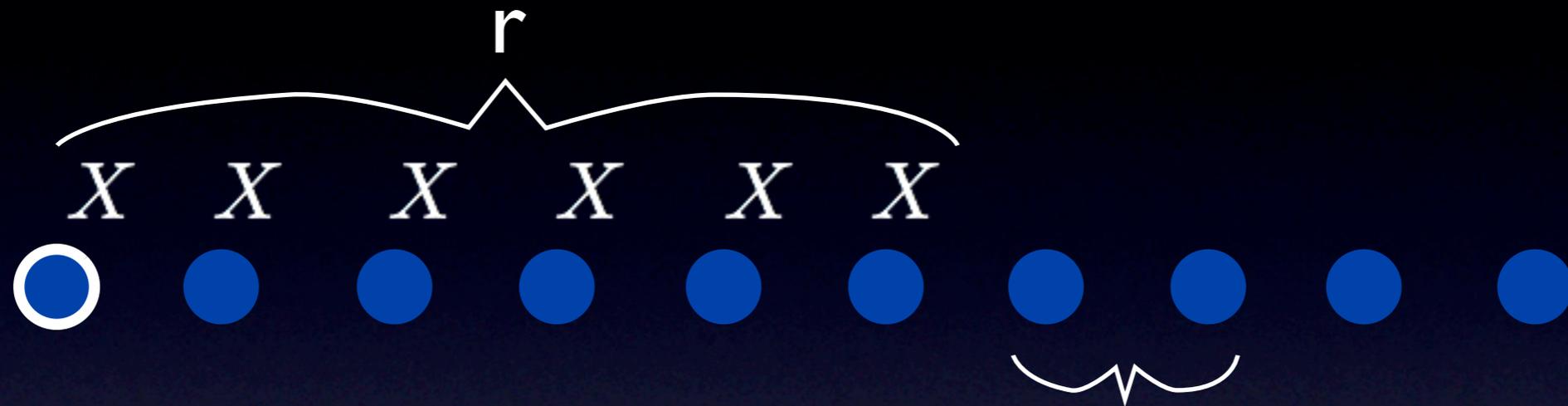
Z info localized here



X info localized here  $\bar{X}\tilde{X}_1 = [Z]_2$

q-info has propagated down the wire with Hadamard

# There's a Code in My Wire



$[n, l, r, l]$  subsystem code

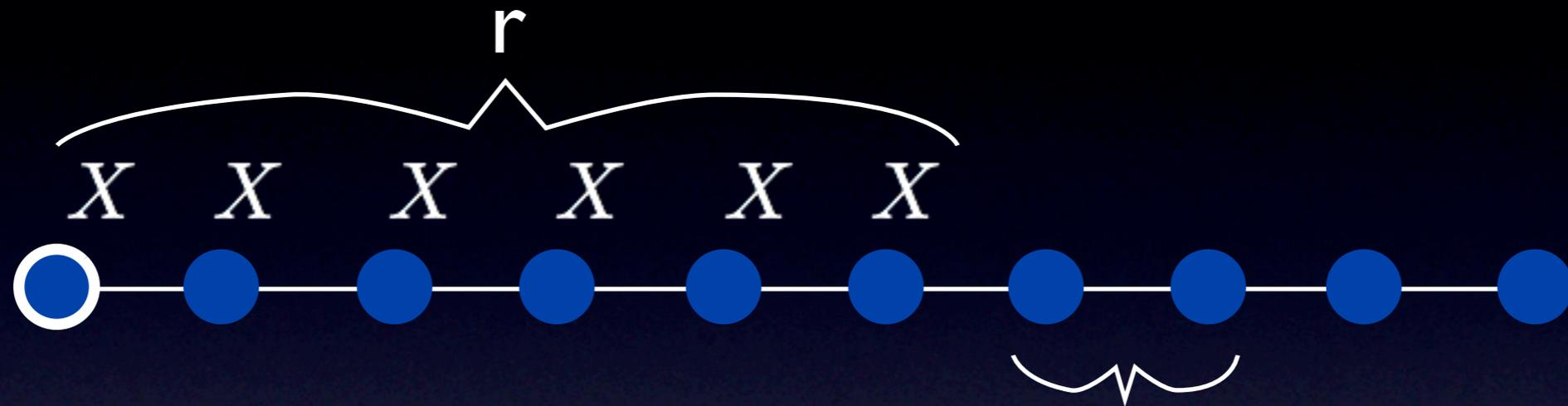
Modulo gauge,  
information can be  
localized here

$$\bar{X} = [X]_1 [X]_3 [X]_5 [X]_7 [Z]_8$$

$$\bar{Z} = [X]_2 [X]_4 [X]_6 [Z]_7$$

MBQC from a coding perspective

# There's a Code in My Wire



$[n, l, r, l]$  subsystem code

Modulo gauge,  
information can be  
localized here

$$\bar{X} = [X]_1 [X]_3 [X]_5 [X]_7 [Z]_8$$

$$\bar{Z} = [X]_2 [X]_4 [X]_6 [Z]_7$$

MBQC from a coding perspective

# Dave WTH Are You Doing?

*Take:* set of n-qubit Pauli operators

*These:* generate a group

*This:* group = r qubit Pauli group + abelian group

*Elements:* gauge operators and stabilizer operators

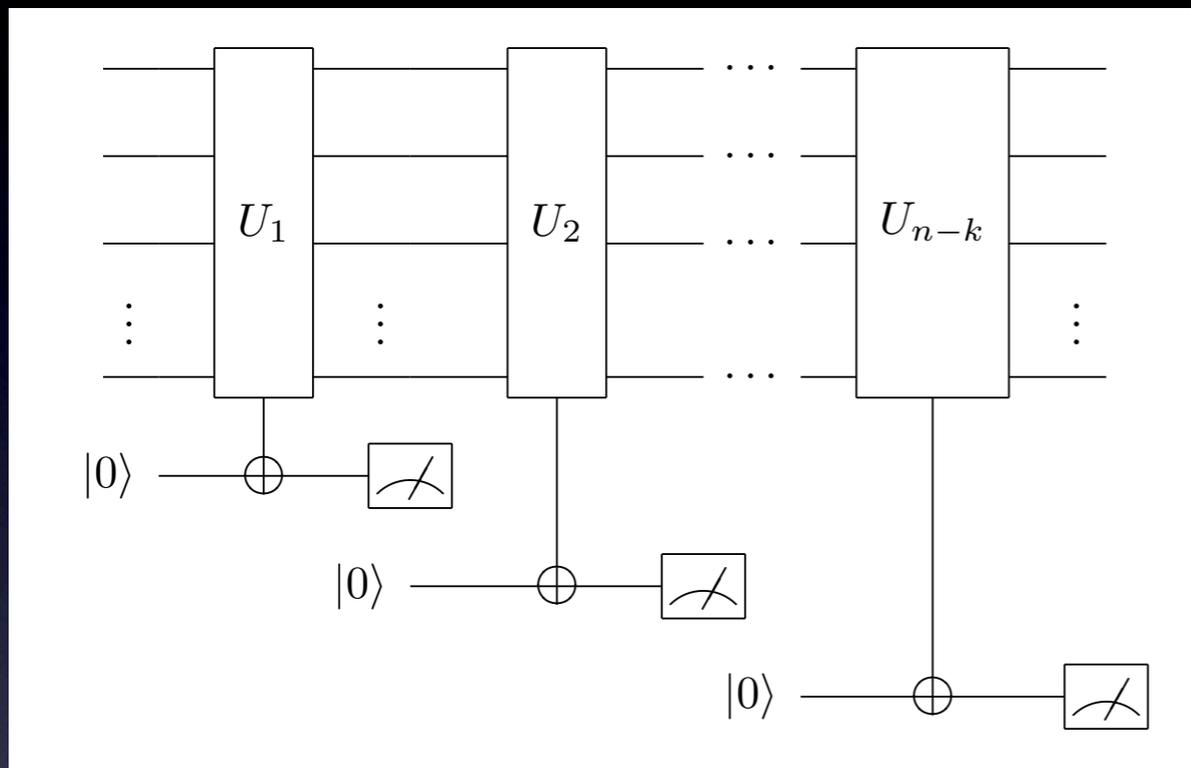
$$\mathcal{T} = \langle T_1, \dots, T_m \rangle \quad \mathcal{T} = \mathcal{G} \otimes \mathcal{S}$$

$\nearrow$   
 $\mathcal{P}_r$

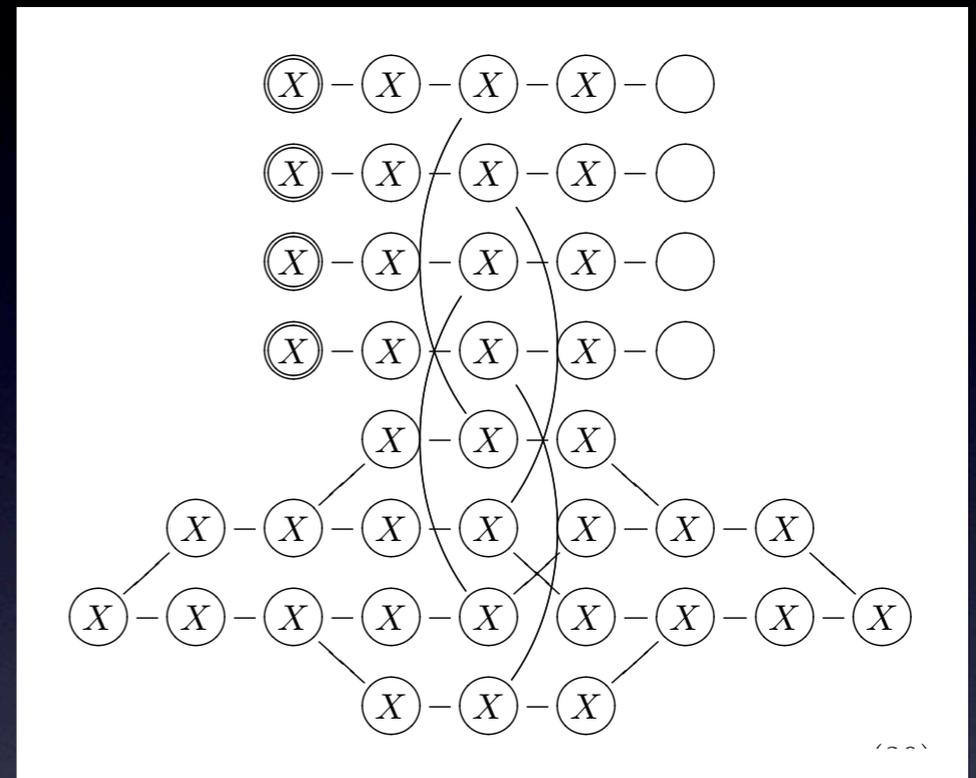
*Left over:* Pauli operators that commute with the group, but not in the group generate logical

$$\mathcal{L} = \mathcal{P}_k$$

# Main Idea (WAKE UP)



Circuit for measuring syndrome of a  $[n,k,d]$  stabilizer code



MBQC scheme for circuit, convert to subsystem code on  $N$  qubits

Claim: RHS code is  $[N,k,d]$  code\*

\* = subject to circuit having a particular FT like criteria

# Dictionary:

circuit to labeled graph

labeled graph describes group

$$\mathcal{T} = \langle T_1, \dots, T_m \rangle$$

include: vertex operators for non-double framed vertices

$$X_v \prod_{(w,v) \in E} Z_w$$

include: single qubit for each  $X$  labeled vertex

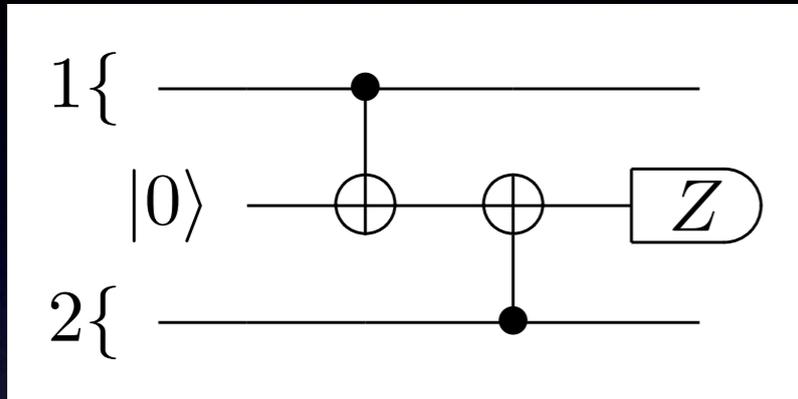
$$\cos \theta X_v + \sin \theta Y_v$$

$\theta = 0$  for no  $\theta$  label

Circuit Element	Graph Gadget
$ 0\rangle$ —	$(X) \text{---} (X) \text{---}$
$\underbrace{\quad}_{\text{input}}$	$(X) \text{---}$
$\underbrace{\quad}_{\text{internal}}$	$\text{---} (X) \text{---} (X) \text{---}$
$\underbrace{\quad}_{\text{output}}$	$\text{---} (X) \text{---} \bigcirc$
$\text{---} [H] \text{---} [R(\theta)] \text{---}$	$\text{---} (X)^\theta \text{---}$
$\begin{array}{c} \text{---} [H] \text{---} [R(\theta)] \text{---} \bullet \\ \text{---} [H] \text{---} [R(\phi)] \text{---} \bullet \\   \\   \end{array}$	$\begin{array}{c} \text{---} (X)^\theta \text{---} \\   \\ \text{---} (X)^\phi \text{---} \end{array}$
$\begin{array}{c} \text{---} [H] \text{---} [R(\theta_1)] \text{---} \bullet \\ \text{---} [H] \text{---} [R(\theta_2)] \text{---} \bullet \\ \vdots \\ \text{---} [H] \text{---} [R(\theta_{m-1})] \text{---} \bullet \\ \text{---} [H] \text{---} [R(\theta_m)] \text{---} \bullet \\   \\   \\   \end{array}$	$\begin{array}{c} \text{---} (X)^{\theta_1} \text{---} \\   \\ \text{---} (X)^{\theta_2} \text{---} \\ \vdots \\ \text{---} (X)^{\theta_{m-1}} \text{---} \\   \\ \text{---} (X)^{\theta_m} \text{---} \end{array}$
$\text{---} [H] \text{---} (X)$	$\text{---} (X)$
$\text{---} (X)$	$\text{---} (X) \text{---} (X)$

# Circuit, Circuit, Graph

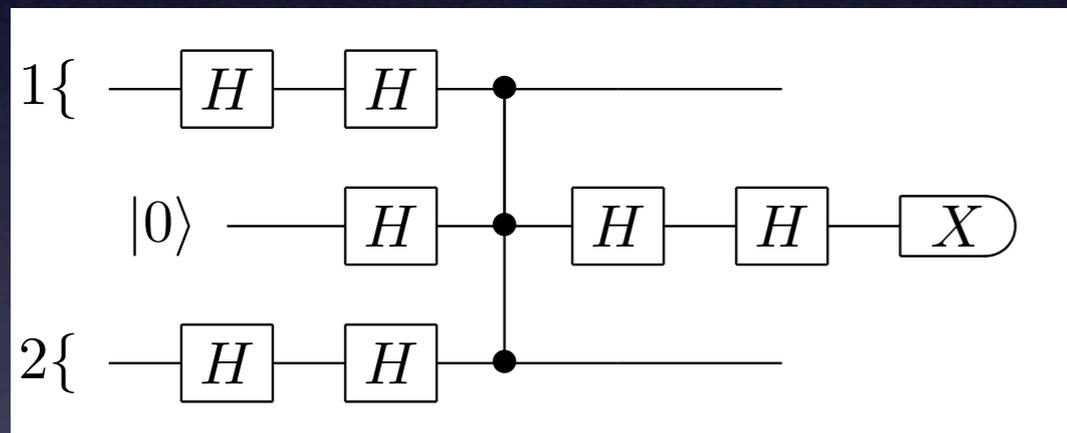
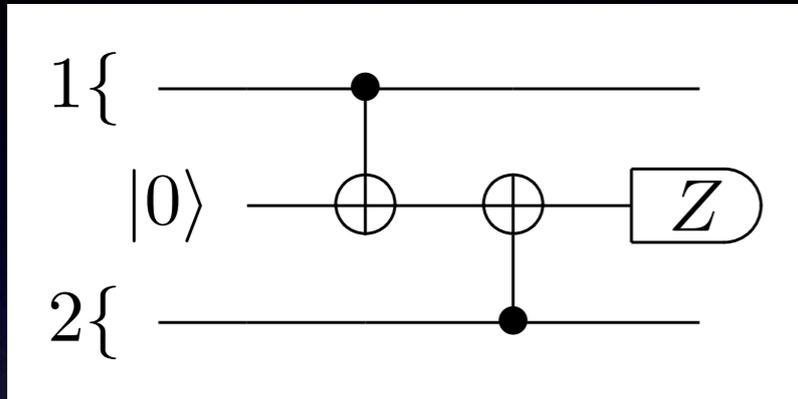
Simplest stabilizer code:  $S = Z \otimes Z$



Circuit Element	Graph Gadget
$ 0\rangle$ —	$(X) \text{---} (X) \text{---}$
$\underbrace{\quad}_{\text{input}}$	$(X) \text{---}$
$\underbrace{\quad}_{\text{internal}}$	$\text{---} (X) \text{---} (X) \text{---}$
$\underbrace{\quad}_{\text{output}}$	$\text{---} (X) \text{---} \bigcirc$
$\text{---} [H] \text{---} [R(\theta)] \text{---}$	$\text{---} (X)^\theta \text{---}$
$\text{---} [H] \text{---} [R(\theta)] \text{---} \bullet$ $\text{---} [H] \text{---} [R(\phi)] \text{---} \bullet$	$\text{---} (X)^\theta \text{---}$ $ $ $\text{---} (X)^\phi \text{---}$
$\text{---} [H] \text{---} [R(\theta_1)] \text{---} \bullet$ $\text{---} [H] \text{---} [R(\theta_2)] \text{---} \bullet$ $\vdots$ $\text{---} [H] \text{---} [R(\theta_{m-1})] \text{---} \bullet$ $\text{---} [H] \text{---} [R(\theta_m)] \text{---} \bullet$	$\text{---} (X)^{\theta_1} \text{---}$ $ $ $\text{---} (X)^{\theta_2} \text{---}$ $\vdots$ $ $ $\text{---} (X)^{\theta_{m-1}} \text{---}$ $ $ $\text{---} (X)^{\theta_m} \text{---}$
$\text{---} [H] \text{---} [X]$	$\text{---} (X)$
$\text{---} [X]$	$\text{---} (X) \text{---} (X)$

# Circuit, Circuit, Graph

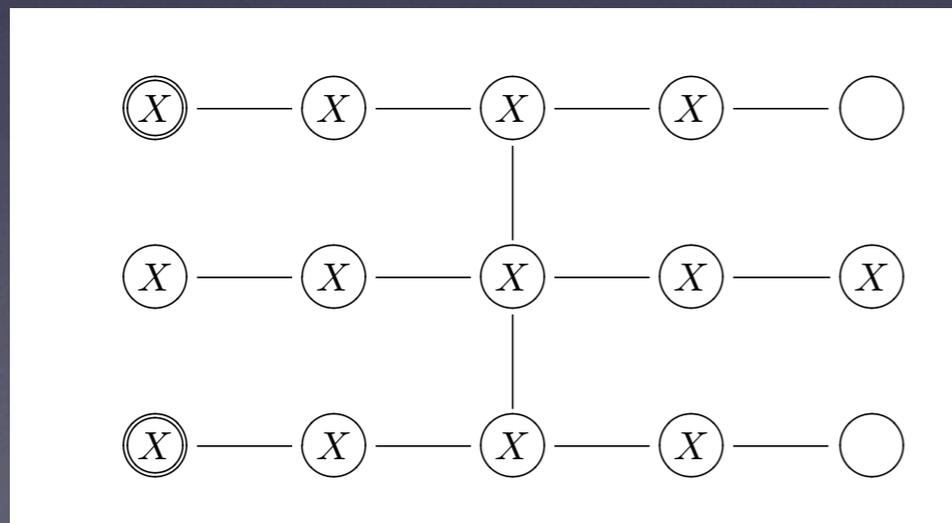
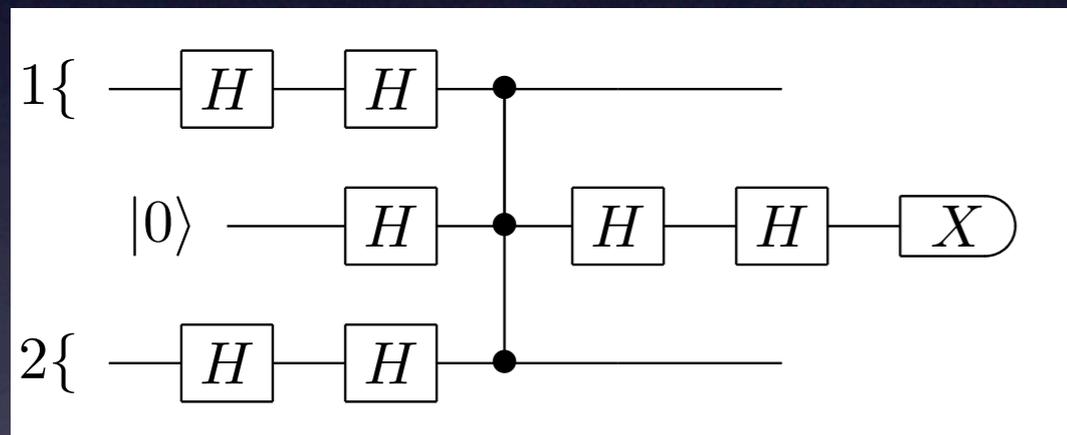
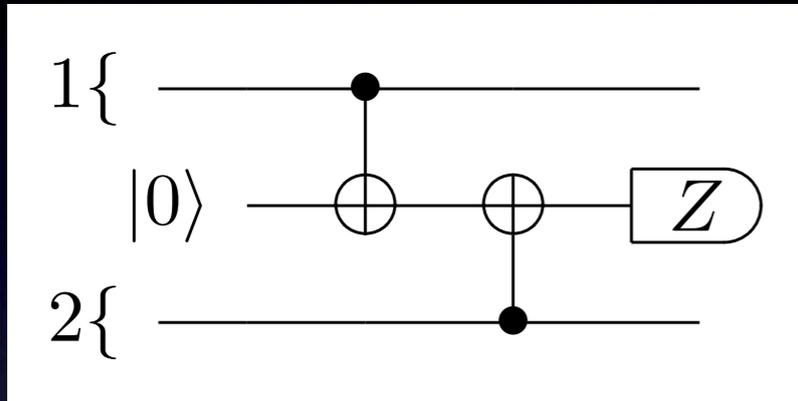
Simplest stabilizer code:  $S = Z \otimes Z$



Circuit Element	Graph Gadget
$ 0\rangle$ —	$(X) \text{---} (X) \text{---}$
$\underbrace{\quad}_{\text{input}}$	$\textcircled{X} \text{---}$
$\underbrace{\quad}_{\text{internal}}$	$\text{---} (X) \text{---} (X) \text{---}$
$\underbrace{\quad}_{\text{output}}$	$\text{---} (X) \text{---} \bigcirc$
$\text{---} [H] \text{---} [R(\theta)] \text{---}$	$\text{---} (X)^\theta \text{---}$
$\text{---} [H] \text{---} [R(\theta)] \text{---} \bullet$ $\text{---} [H] \text{---} [R(\phi)] \text{---} \bullet$	$\text{---} (X)^\theta \text{---}$ $ $ $\text{---} (X)^\phi \text{---}$
$\text{---} [H] \text{---} [R(\theta_1)] \text{---} \bullet$ $\text{---} [H] \text{---} [R(\theta_2)] \text{---} \bullet$ $\vdots$ $\text{---} [H] \text{---} [R(\theta_{m-1})] \text{---} \bullet$ $\text{---} [H] \text{---} [R(\theta_m)] \text{---} \bullet$	$\text{---} (X)^{\theta_1} \text{---}$ $ $ $\text{---} (X)^{\theta_2} \text{---}$ $\vdots$ $ $ $\text{---} (X)^{\theta_{m-1}} \text{---}$ $ $ $\text{---} (X)^{\theta_m} \text{---}$
$\text{---} [H] \text{---} [X]$	$\text{---} (X)$
$\text{---} [X]$	$\text{---} (X) \text{---} (X)$

# Circuit, Circuit, Graph

Simplest stabilizer code:  $S = Z \otimes Z$



Circuit Element	Graph Gadget
$ 0\rangle$ —	$(X) \text{---} (X) \text{---}$
$\underbrace{\hspace{2cm}}_{\text{input}}$	$(X) \text{---}$
$\underbrace{\hspace{2cm}}_{\text{internal}}$	$\text{---} (X) \text{---} (X) \text{---}$
$\underbrace{\hspace{2cm}}_{\text{output}}$	$\text{---} (X) \text{---} \bigcirc$
$\text{---} [H] \text{---} [R(\theta)] \text{---}$	$\text{---} (X)^\theta \text{---}$
$\text{---} [H] \text{---} [R(\theta)] \text{---}$ $\text{---} [H] \text{---} [R(\phi)] \text{---}$	$\text{---} (X)^\theta \text{---}$ $\text{---} (X)^\phi \text{---}$
$\text{---} [H] \text{---} [R(\theta_1)] \text{---}$ $\text{---} [H] \text{---} [R(\theta_2)] \text{---}$ $\vdots$ $\text{---} [H] \text{---} [R(\theta_{m-1})] \text{---}$ $\text{---} [H] \text{---} [R(\theta_m)] \text{---}$	$\text{---} (X)^{\theta_1} \text{---}$ $\text{---} (X)^{\theta_2} \text{---}$ $\vdots$ $\text{---} (X)^{\theta_{m-1}} \text{---}$ $\text{---} (X)^{\theta_m} \text{---}$
$\text{---} [H] \text{---} [X]$	$\text{---} (X)$
$\text{---} [X]$	$\text{---} (X) \text{---} (X)$

# 12 Gauge Qubits

$$\bar{X}_1 = X_{(1,1)}, \bar{Z}_1 = S_{(1,2)}$$

$$\bar{X}_2 = X_{(2,1)}, \bar{Z}_2 = S_{(2,2)}$$

$$\bar{X}_3 = X_{(a,1)}, \bar{Z}_3 = S_{(a,2)}$$

$$\bar{X}_4 = X_{(1,2)}, \bar{Z}_4 = S_{(1,3)}$$

$$\bar{X}_5 = X_{(2,2)}, \bar{Z}_5 = S_{(2,3)}$$

$$\bar{X}_6 = X_{(a,2)}, \bar{Z}_6 = S_{(a,1)}$$

$$\bar{X}_7 = X_{(1,1)}X_{(1,3)}, \bar{Z}_7 = S_{(1,4)}$$

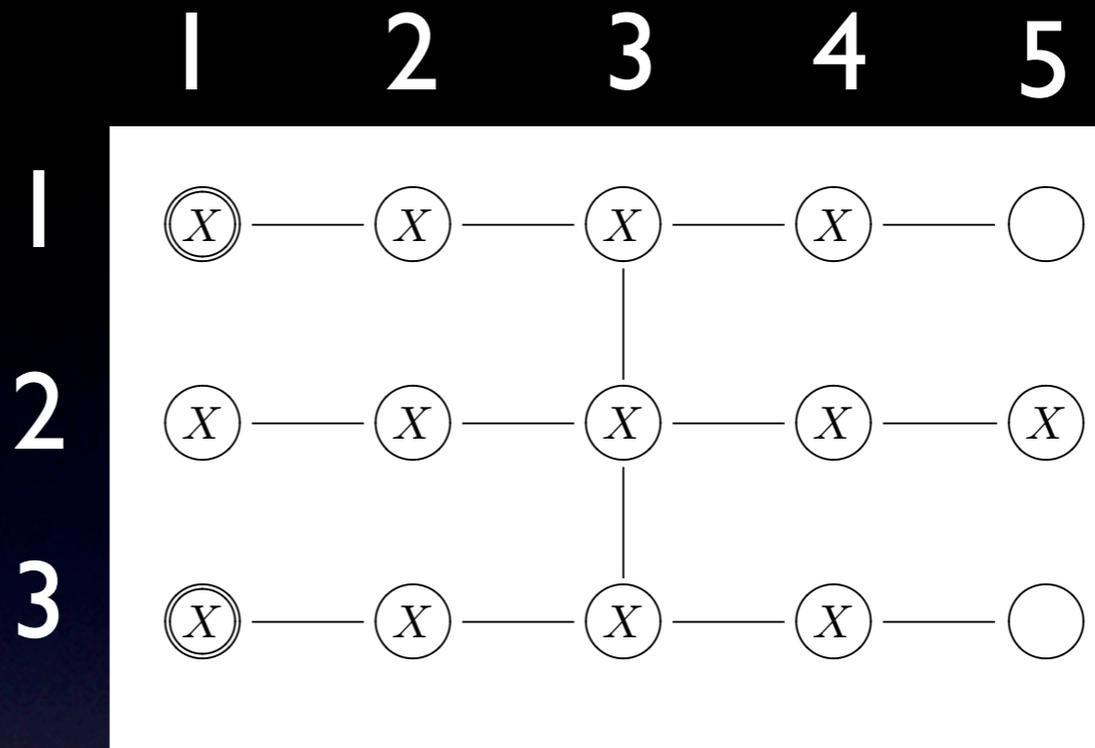
$$\bar{X}_8 = X_{(2,1)}X_{(2,3)}, \bar{Z}_8 = S_{(2,4)}$$

$$\bar{X}_9 = X_{(1,2)}X_{(1,4)}, \bar{Z}_9 = S_{(1,5)}$$

$$\bar{X}_{10} = X_{(2,2)}X_{(2,4)}, \bar{Z}_{10} = S_{(2,5)}$$

$$\bar{X}_{11} = X_{(a,4)}, \bar{Z}_{11} = S_{(a,5)}$$

$$\bar{X}_{12} = X_{(a,5)}, \bar{Z}_{12} = S_{(a,4)}$$



2 Stabilizers

$$S_1 = S_{(1,4)}S_{(2,4)}S_{(a,1)}S_{(a,3)}S_{(a,5)}$$

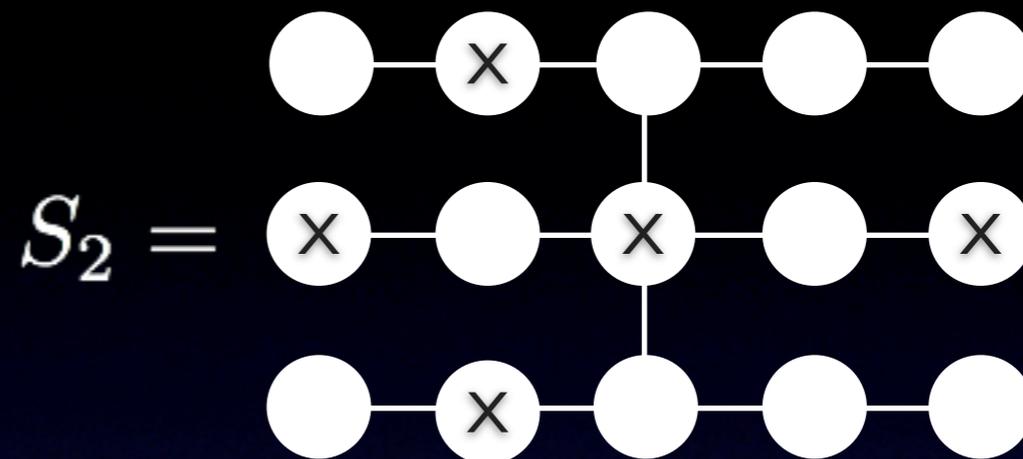
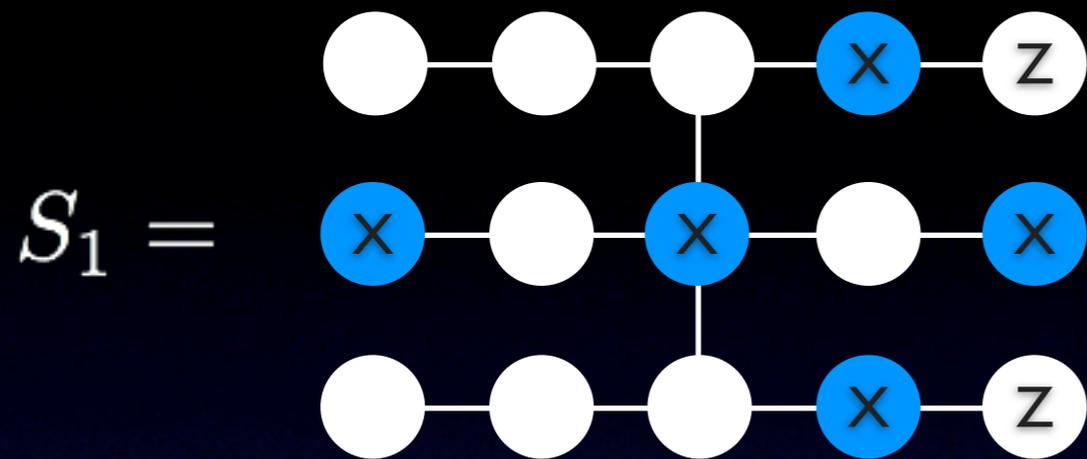
$$S_2 = X_{(1,2)}X_{(2,2)}X_{(a,1)}X_{(a,3)}X_{(a,5)}.$$

1 Logical Qubit

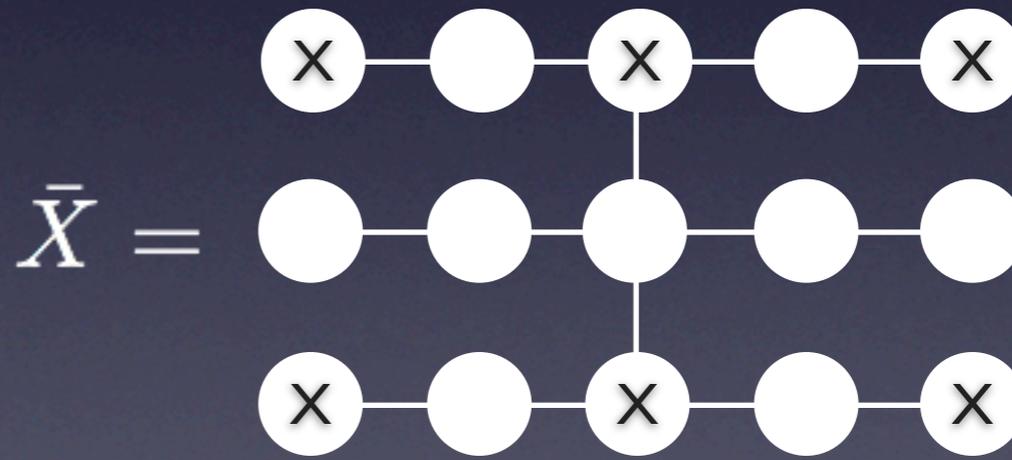
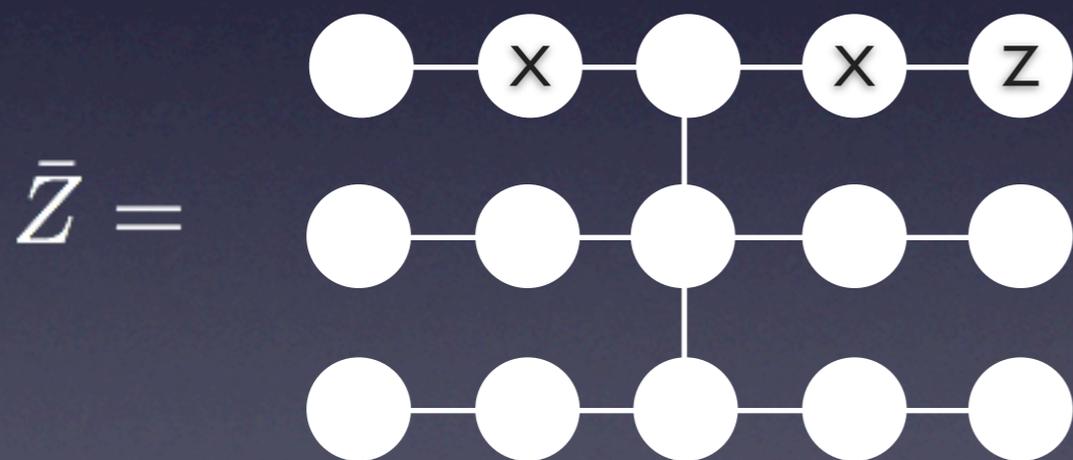
$$\bar{X}_{L,1} = X_{(1,1)}X_{(1,3)}X_{(1,5)}X_{(2,1)}X_{(2,3)}X_{(2,5)}$$

$$\bar{Z}_{L,1} = X_{(1,2)}X_{(1,4)}Z_{(1,5)}.$$

$$S_{(i,j)} = [X]_{(i,j)} \prod_{(v,(i,j)) \in E} [Z]_v$$

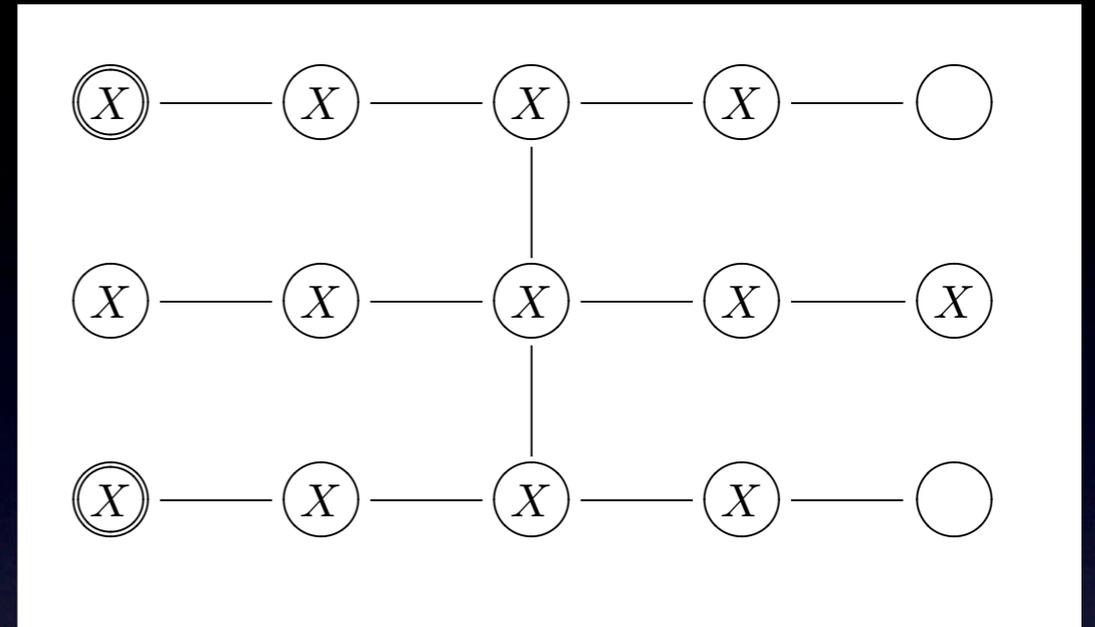
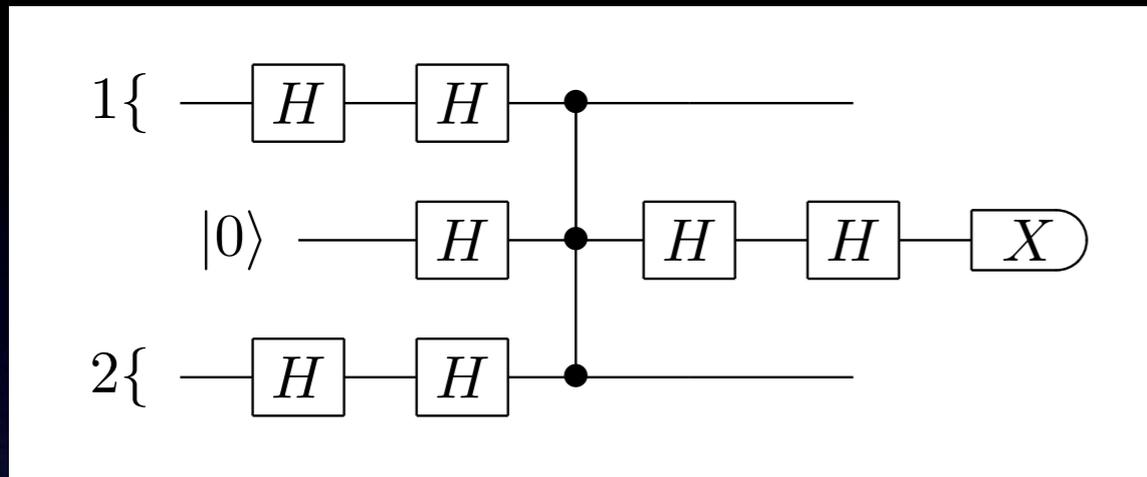


Stabilizers can be measured by low weight measurements



Modulo gauge and stabilizer group logical  $Z$  can be made weight 1, but logical  $X$  can only be made weight 2

# General Method

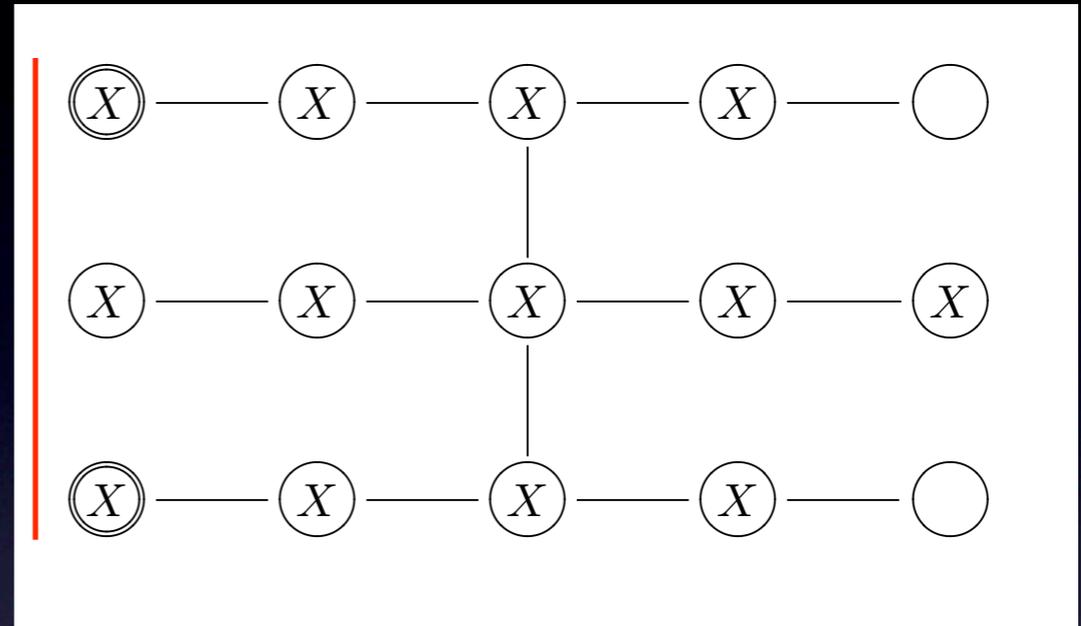
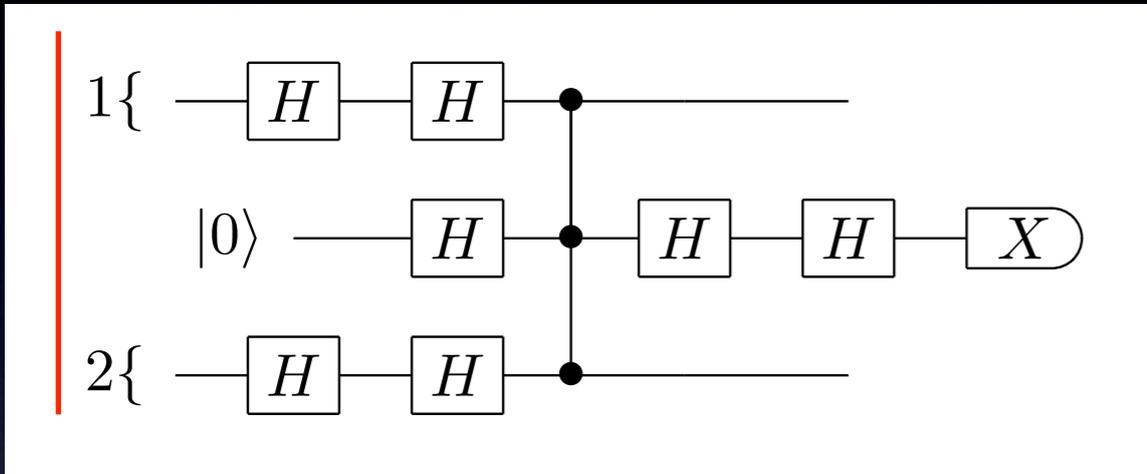


Generalized Heisenberg pict:

- Track  $X, Y, Z$  for input qubits
- Track single Pauli for state preps
- Update according to gates
- Update according to measurements

Subsystem code:

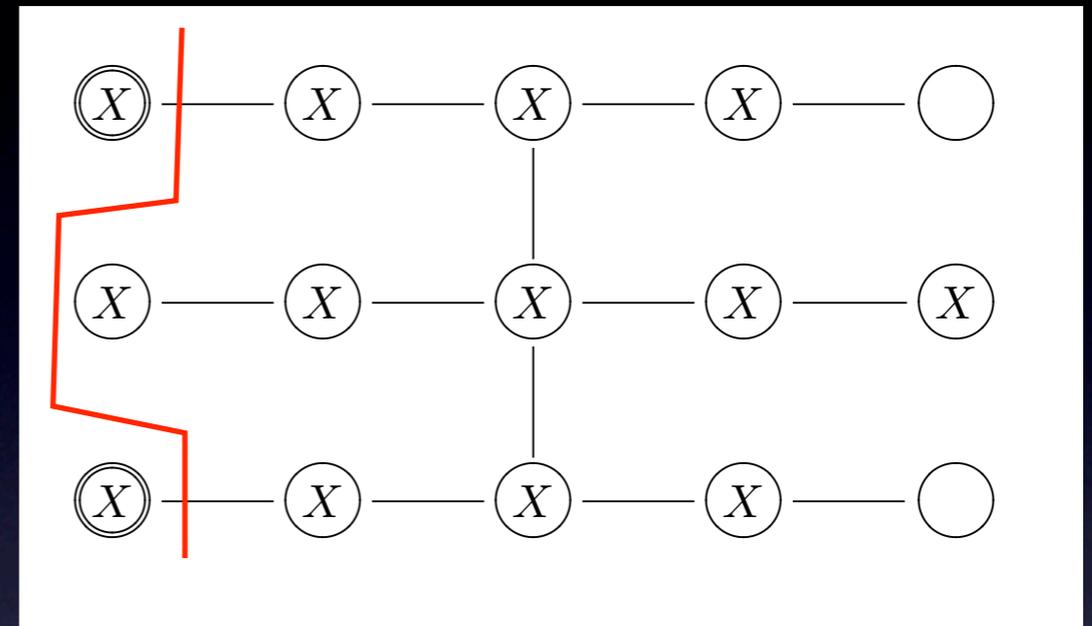
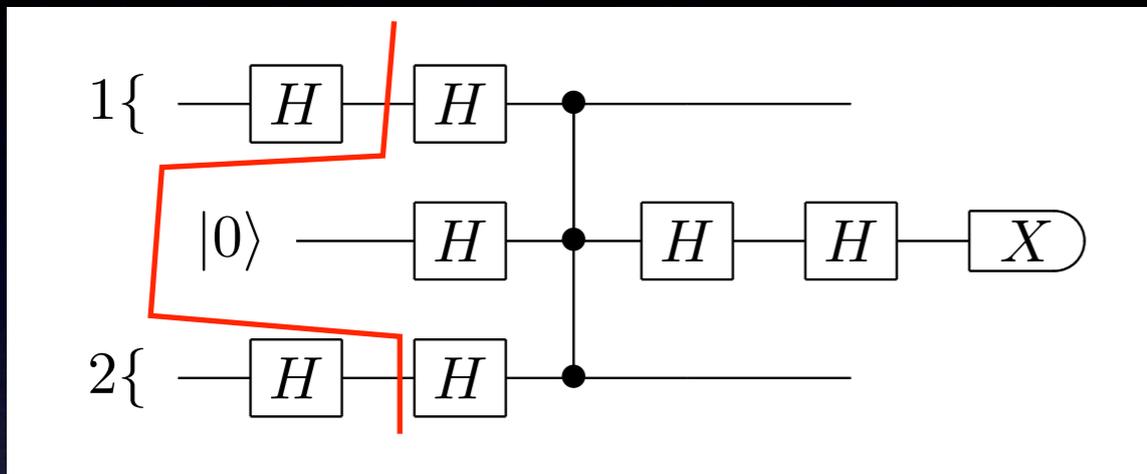
- Track group generated by vertex operators and  $X$  operators up to given level
- Group generates gauge and stabilizer group
- Keep track of logical qubits



$$\mathcal{S}_0 = \{S_{(1,2)}, S_{(1,3)}, S_{(1,4)}, S_{(1,5)}, S_{(2,2)}, S_{(2,3)}, S_{(2,4)}, S_{(2,5)}, S_{(a,1)}, S_{(a,2)}, S_{(a,3)}, S_{(a,4)}, S_{(a,5)}\},$$

$$\mathcal{G}_0 = \{\}$$

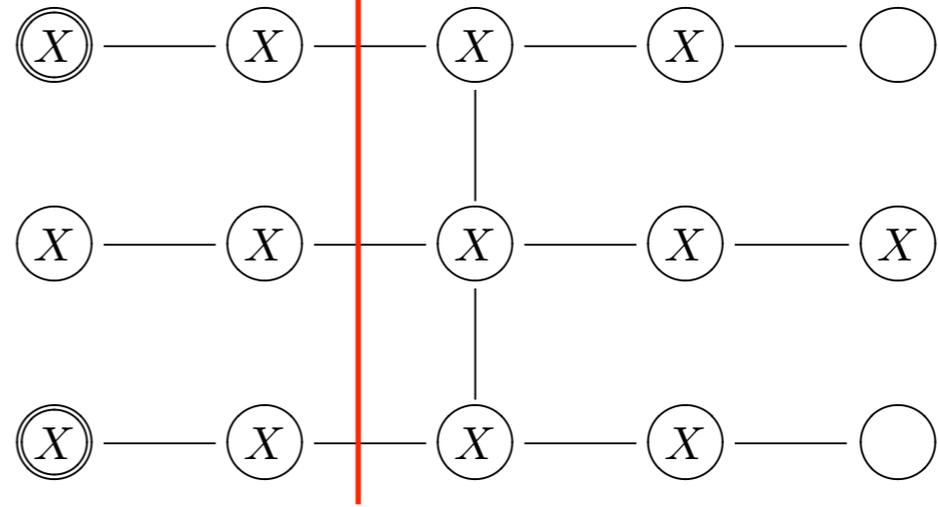
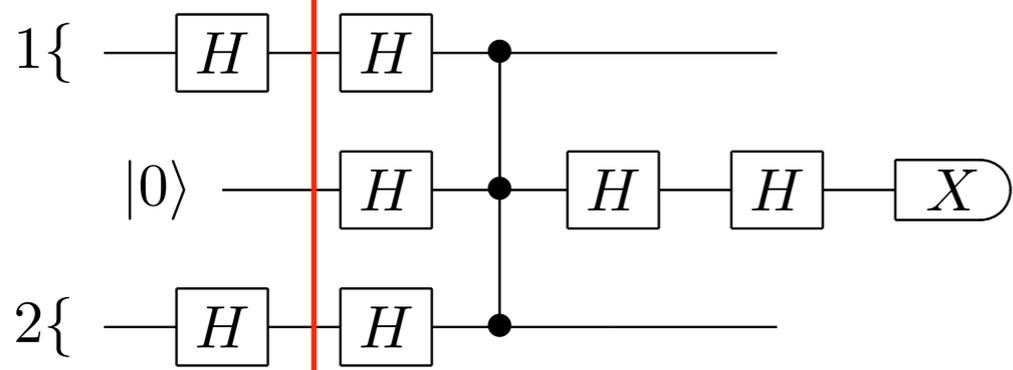
$$\mathcal{L}_0 = \{(X_{(1,1)}Z_{(1,2)}, Z_{(1,1)}), (X_{(2,1)}Z_{(2,2)}, Z_{(2,1)})\}$$



$$\mathcal{S}_1 = \{S_{(1,3)}, S_{(1,4)}, S_{(1,5)}, S_{(2,3)}, S_{(2,4)}, S_{(2,5)}, \\ S_{(a,1)}, S_{(a,2)}, S_{(a,3)}, S_{(a,4)}, S_{(a,5)}\},$$

$$\mathcal{G}_1 = \{(X_{(1,1)}, S_{(1,2)}), (X_{(2,1)}, S_{(2,2)})\}$$

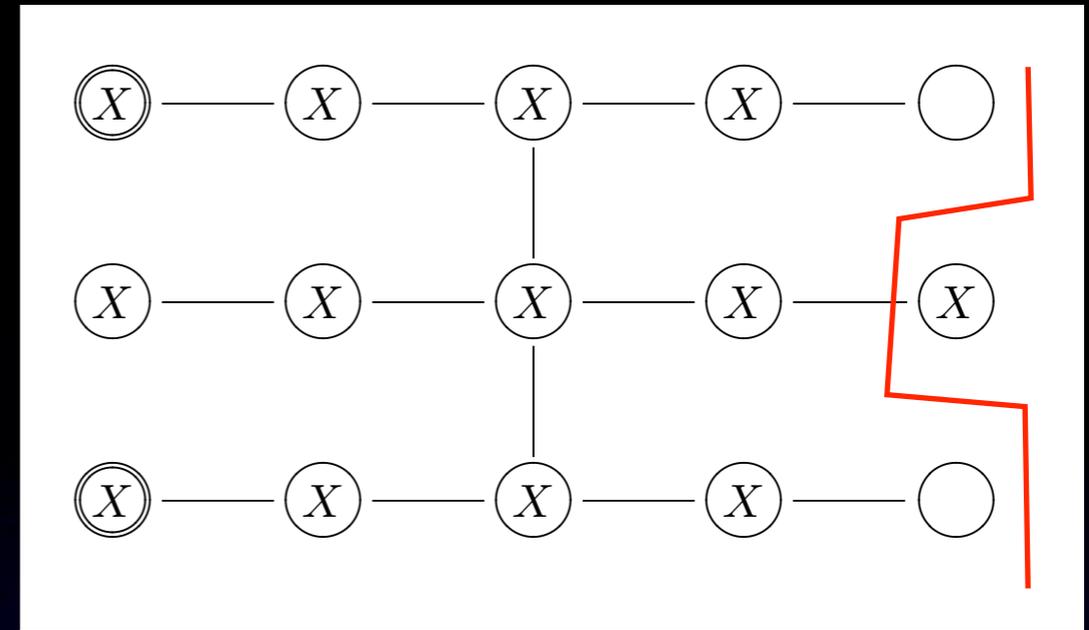
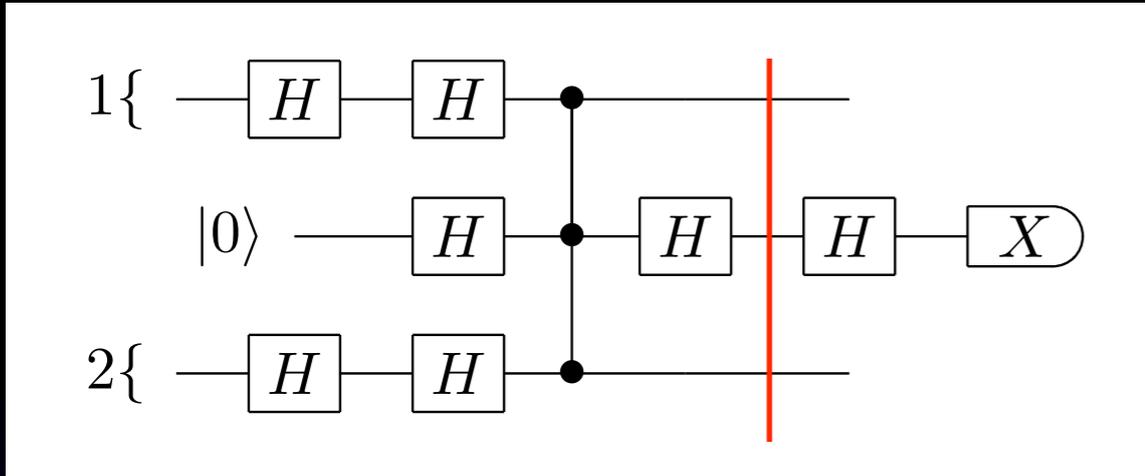
$$\mathcal{L}_1 = \{(X_{(1,1)}Z_{(1,2)}, S_{(1,2)}Z_{(1,1)}), \\ (X_{(2,1)}Z_{(2,2)}, S_{(2,2)}Z_{(2,1)})\}$$



$$\mathcal{S}_2 = \{S_{(1,4)}, S_{(1,5)}, S_{(2,4)}, S_{(2,5)}, \underline{S_{(a,3)} S_{(a,1)}}, S_{(a,4)}, S_{(a,5)}\},$$

$$\mathcal{G}_2 = \{(X_{(1,1)}, S_{(1,2)}), (X_{(2,1)}, S_{(2,2)}), (X_{(1,2)}, S_{(1,3)}), (X_{(2,2)}, S_{(2,3)}), (X_{(a,1)}, S_{(a,2)}), (X_{(a,2)}, S_{(a,1)})\}$$

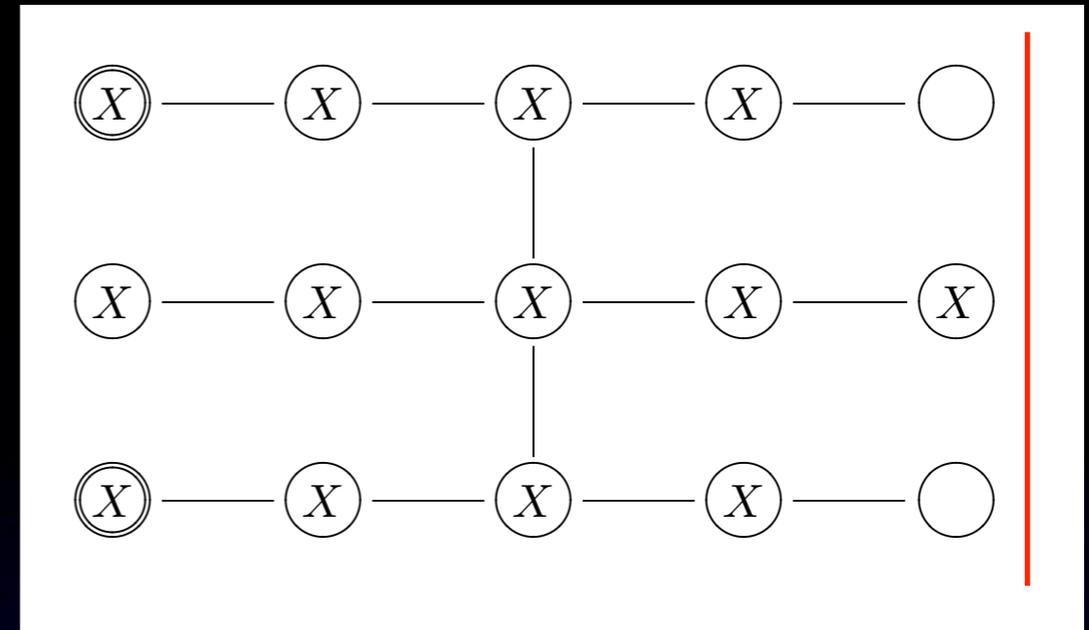
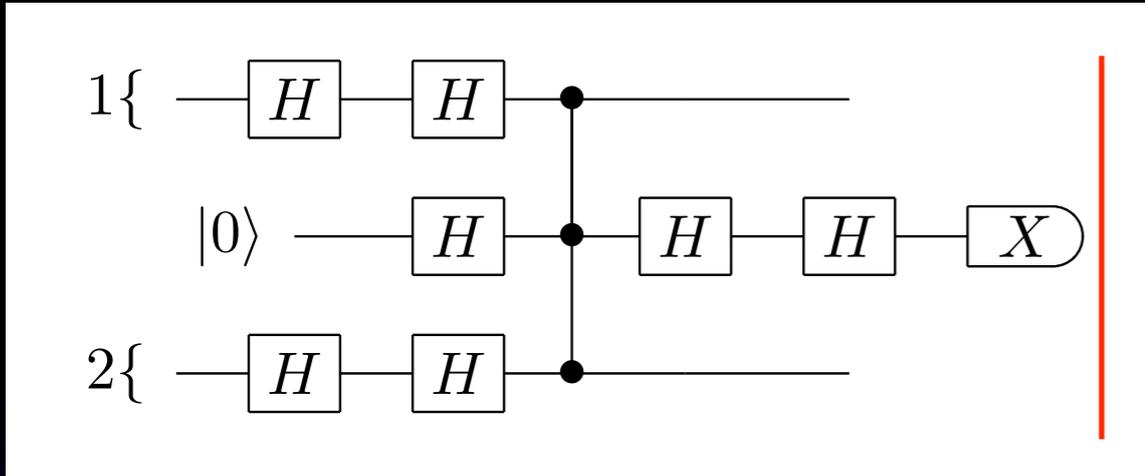
$$\mathcal{L}_2 = \{(S_{(1,3)} X_{(1,1)} Z_{(1,2)}, S_{(1,2)} Z_{(1,1)}), (S_{(2,3)} X_{(2,1)} Z_{(2,2)}, S_{(2,2)} Z_{(2,1)})\}$$



$$\mathcal{S}_4 = \{S_{(a,5)} S_{(a,3)} S_{(a,1)} S_{(1,4)} S_{(2,4)}\}$$

$$\mathcal{G}_4 = \{(X_{(1,1)}, S_{(1,2)}), (X_{(2,1)}, S_{(2,2)}), (X_{(1,2)}, S_{(1,3)}), \\ (X_{(2,2)}, S_{(2,3)}), (X_{(a,1)}, S_{(a,2)}), (X_{(a,2)}, S_{(a,1)}), \\ (X_{(1,1)} X_{(1,3)}, S_{(1,4)}), (X_{(2,1)} X_{(2,3)}, S_{(2,4)}), \\ (X_{(a,1)} X_{(a,3)} X_{(1,2)} X_{(2,2)}, S_{(a,4)}), (X_{(a,4)}, S_{(a,5)}), \\ (X_{(1,2)} X_{(1,4)}, S_{(1,5)}), (X_{(2,2)} X_{(2,4)}, S_{(2,5)})\}$$

$$\mathcal{L}_4 = \{(S_{(1,5)} S_{(1,3)} S_{(a,4)} X_{(1,1)} Z_{(1,2)}, S_{(1,4)} S_{(1,2)} Z_{(1,1)}), \\ (S_{(2,5)} S_{(2,3)} S_{(a,4)} X_{(2,1)} Z_{(2,2)}, S_{(2,4)} S_{(2,2)} Z_{(2,1)})\}$$



$$\mathcal{S}_5 = \{S_{(a,5)}S_{(a,3)}S_{(a,1)}S_{(1,4)}S_{(2,4)}, \\ X_{(a,5)}X_{(a,3)}X_{(a,1)}X_{(1,2)}X_{(2,2)}\}$$

$$\mathcal{G}_5 = \{(X_{(1,1)}, S_{(1,2)}), (X_{(2,1)}, S_{(2,2)}), (X_{(1,2)}, S_{(1,3)}), \\ (X_{(2,2)}, S_{(2,3)}), (X_{(a,1)}, S_{(a,2)}), (X_{(a,2)}, S_{(a,1)}), \\ (X_{(1,1)}X_{(1,3)}, S_{(1,4)}), (X_{(2,1)}X_{(2,3)}, S_{(2,4)}), \\ (X_{(a,1)}X_{(a,3)}X_{(1,2)}X_{(2,2)}, S_{(a,4)}), (X_{(a,4)}, S_{(a,5)}), \\ (X_{(1,2)}X_{(1,4)}, S_{(1,5)}), (X_{(2,2)}X_{(2,4)}, S_{(2,5)})\}$$

$$\mathcal{L}_5 = \{(S_{(1,5)}S_{(1,3)}S_{(2,5)}S_{(2,3)}X_{(1,1)}Z_{(1,2)}X_{(2,1)}Z_{(2,2)}, \\ S_{(1,4)}S_{(1,2)}Z_{(1,1)})\}$$

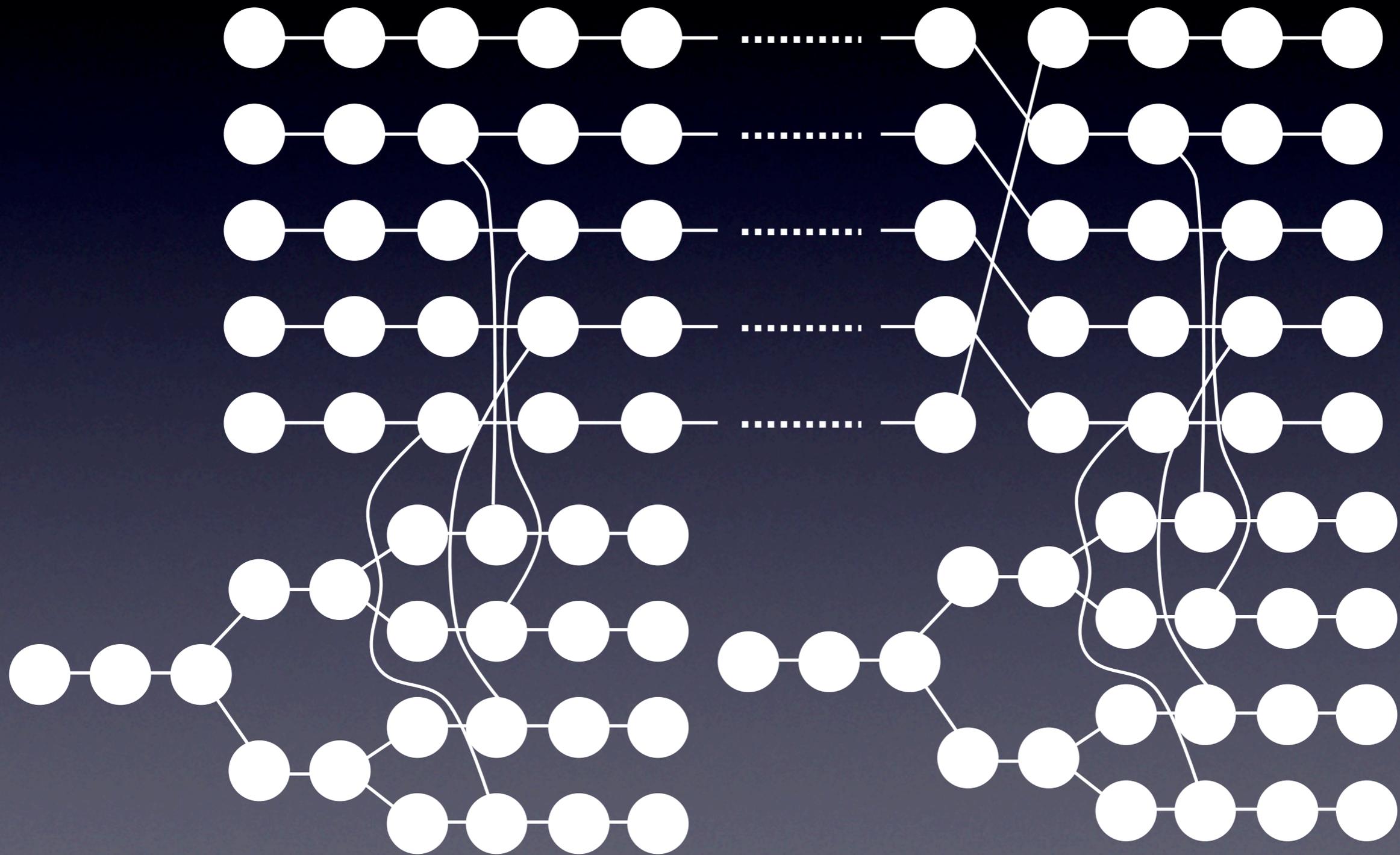
# Whew

Using this method (track gauge, stabilizer, logical) one can figure out what these operators are for any given circuit.

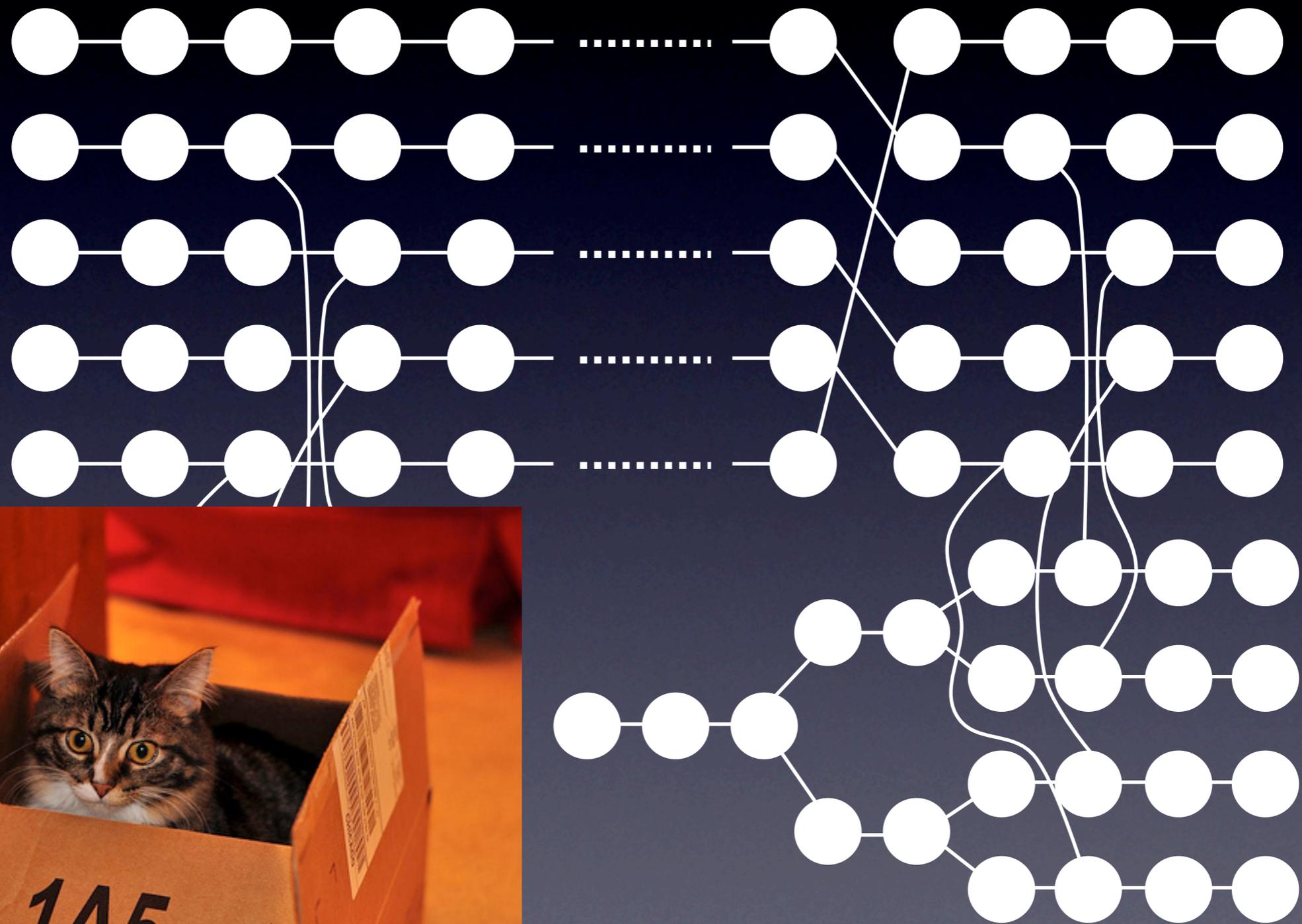
In particular we can work out what the code is when the circuit is a measurement of a syndrome of a  $[n,k,d]$  code



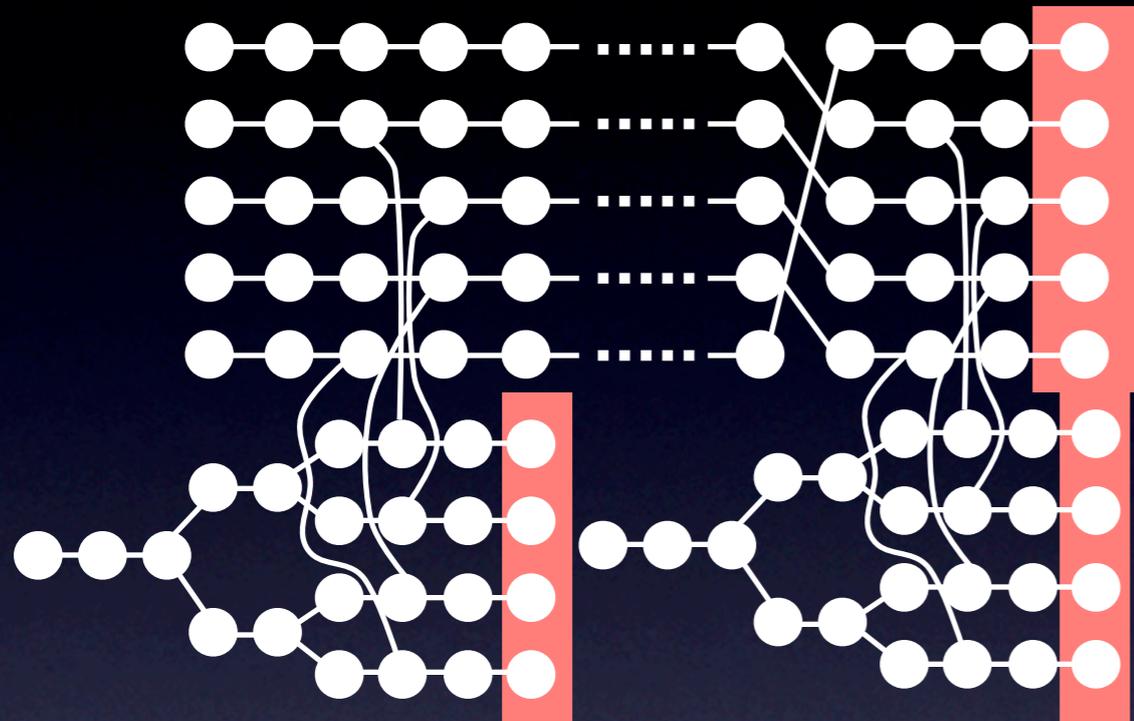
# Syndrome Circuit



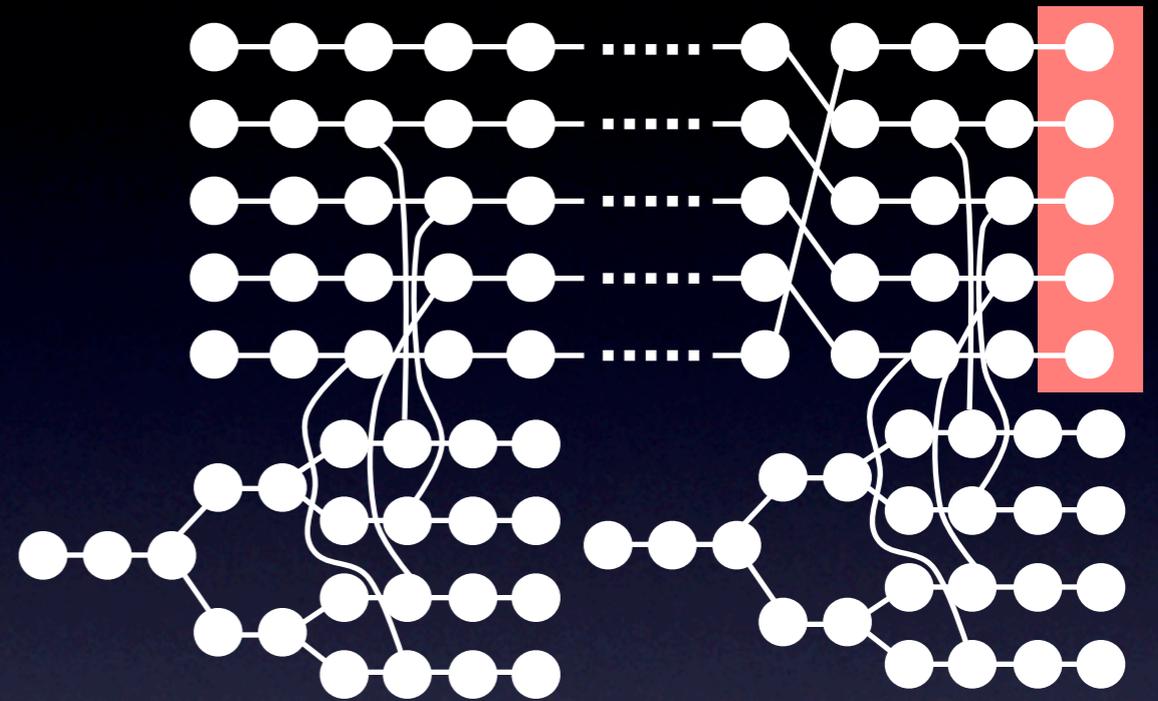
# Syndrome Circuit



# Syndrome Circuit

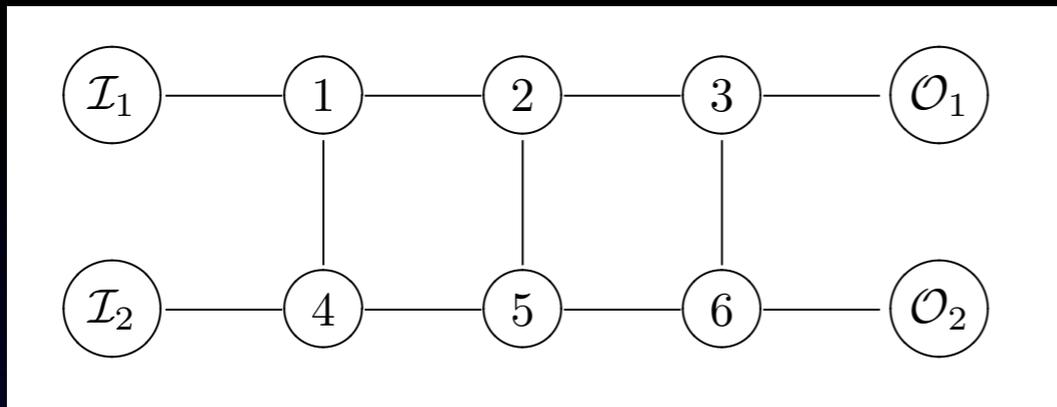


Modulo gauge operators stabilizers are (a) stabilizer of code with ancillas used to measure then (b) ancillas measurement operators

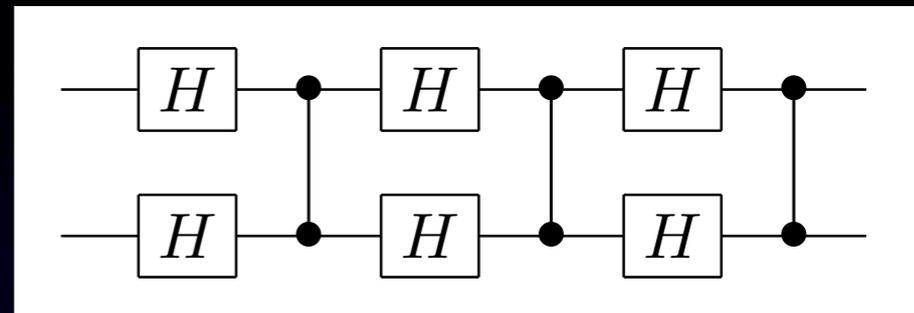


Modulo gauge/stabilizer operators, logical operators are logical operators for code on output qubits

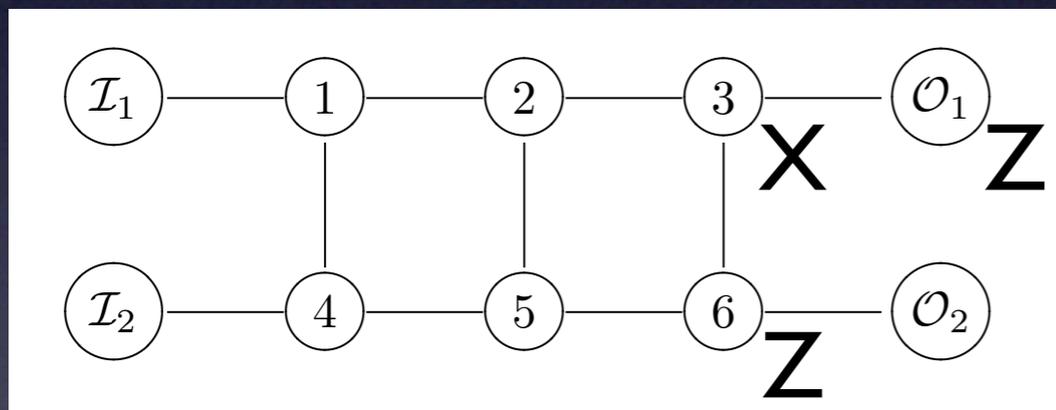
# An Obstacle



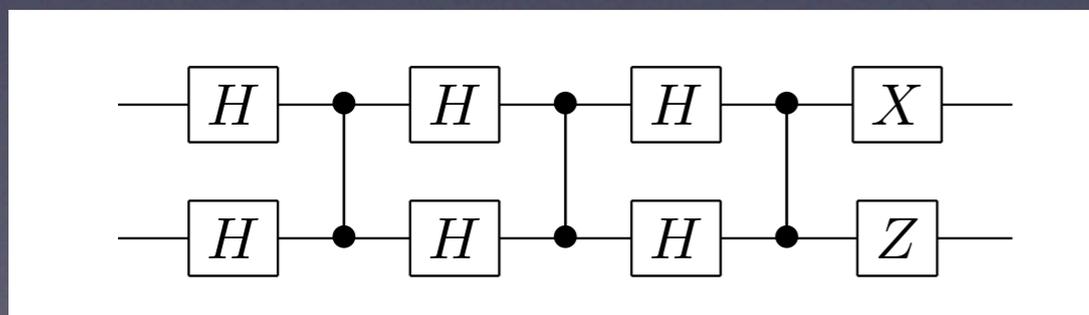
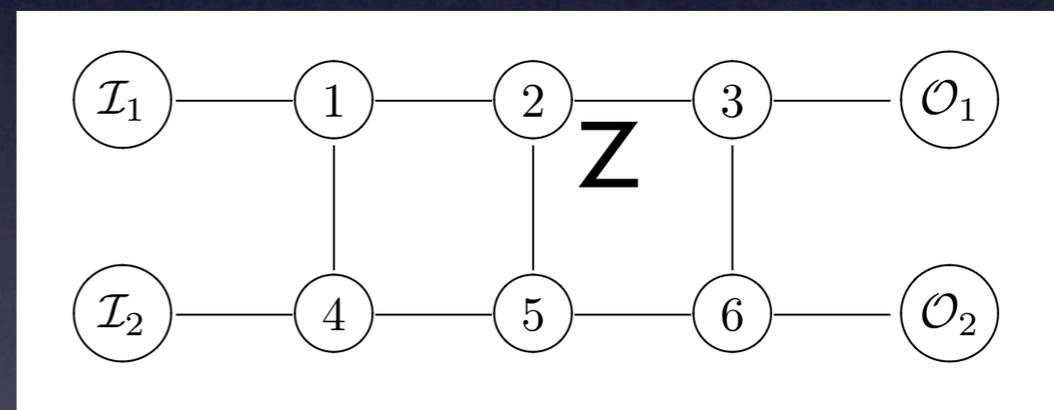
=



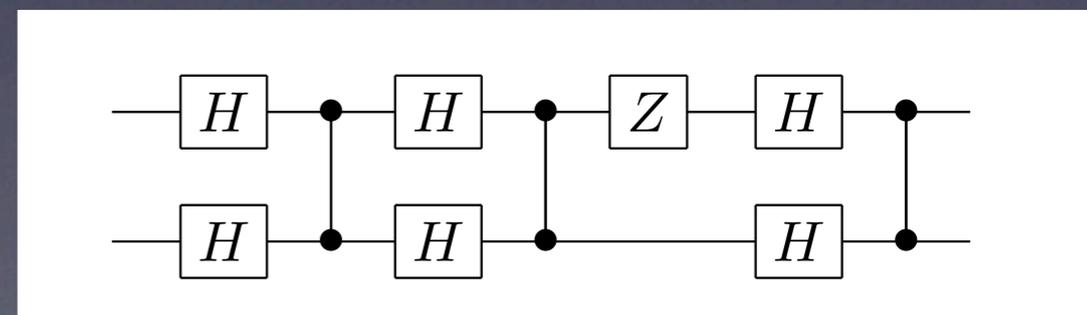
swap



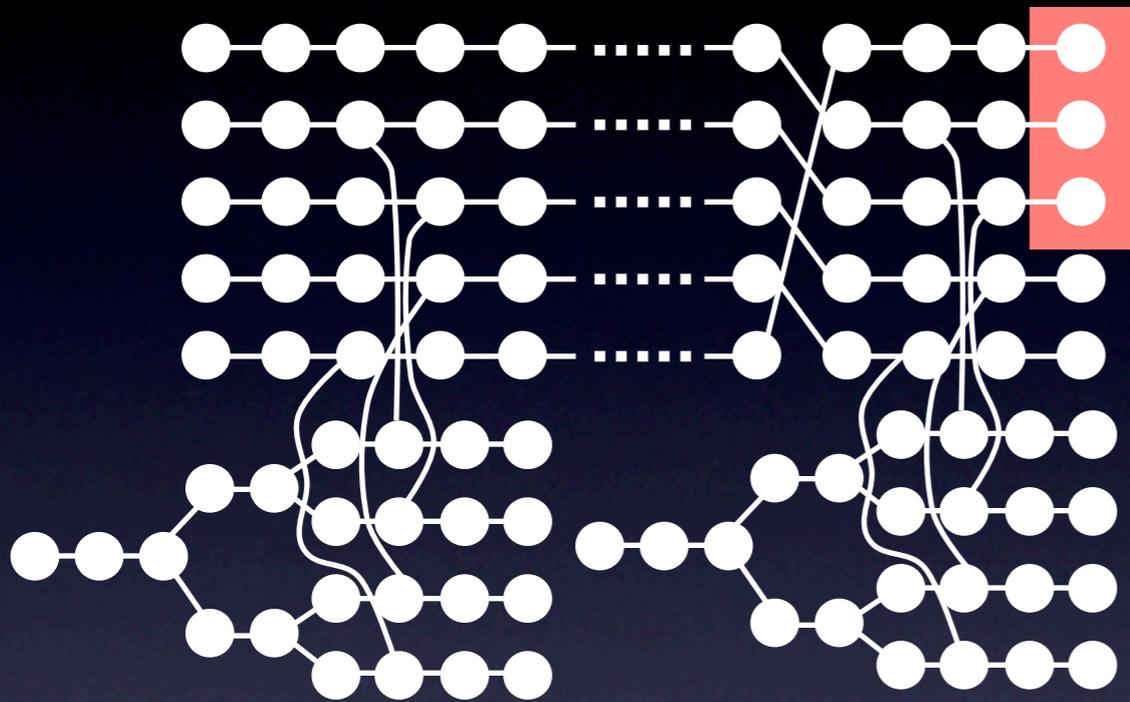
equiv



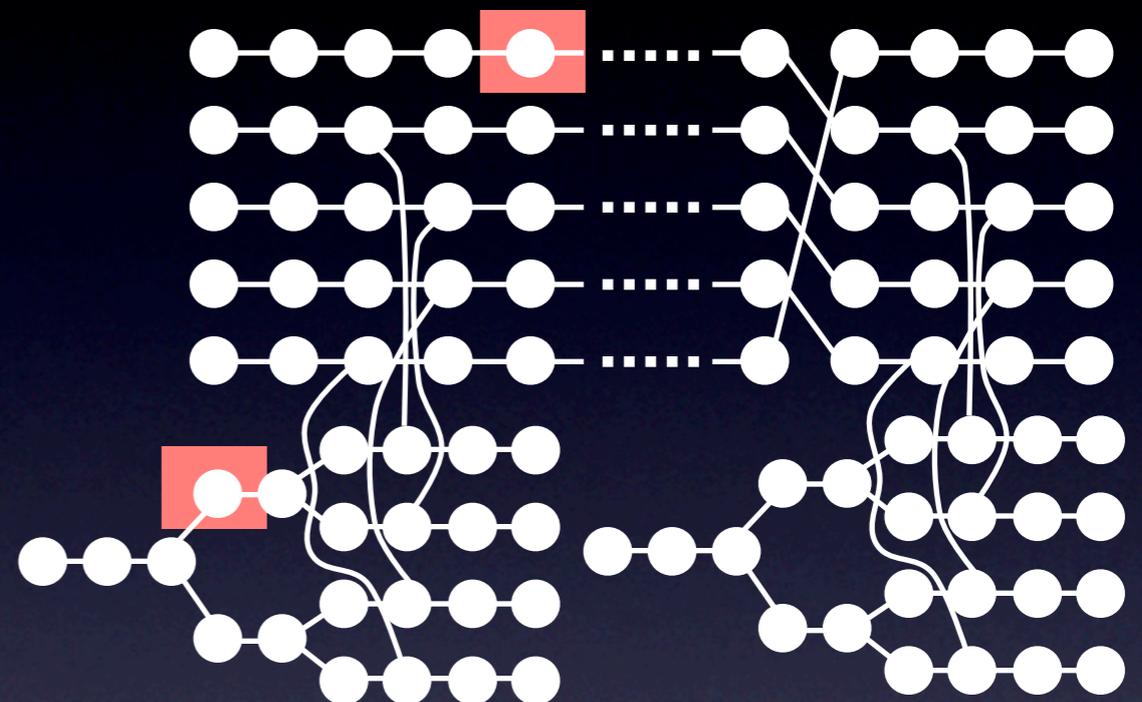
equiv



# An Obstacle



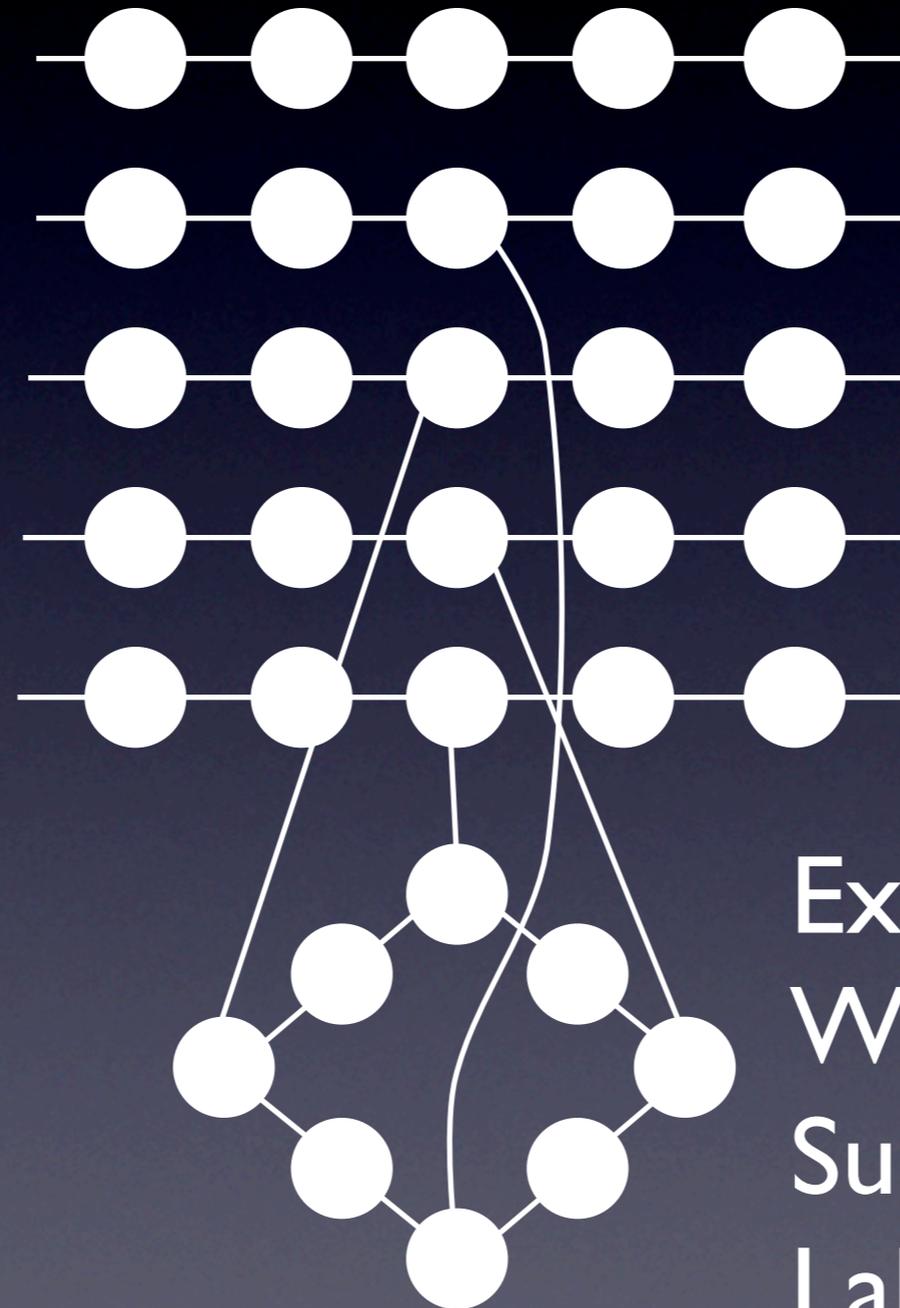
weight  $d$  logical



weight  $< d$  logical

Need to design circuits that do not lower weight of logicals as they are propagated back through code using gauge operators (FT criteria)

# One solution



Expander graph

Wire vertices to block

Subdivide

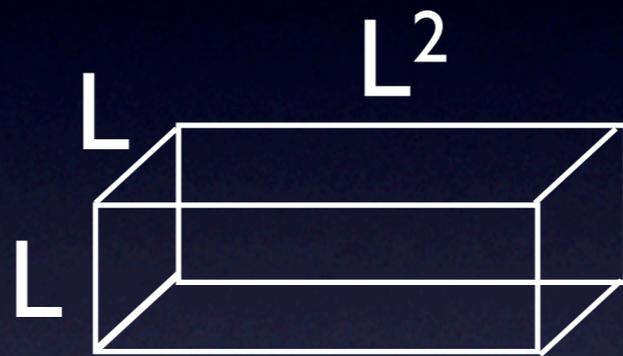
Label all by X

# Main Result (AGAIN)

With careful use of syndrome measuring circuits, one can turn any  $[n,k,d]$  stabilizer code into a spatially local code with  $[N,r,k,d]$  code, where  $N = O(\text{size of syndrome measuring circuit})$

# Bravyi-Terhal Bounds

Concatenated code:  $n^r$  qubits, distance  $d^r = n^r \log_n d$



$$\text{distance } d = L^2 \log_n d$$

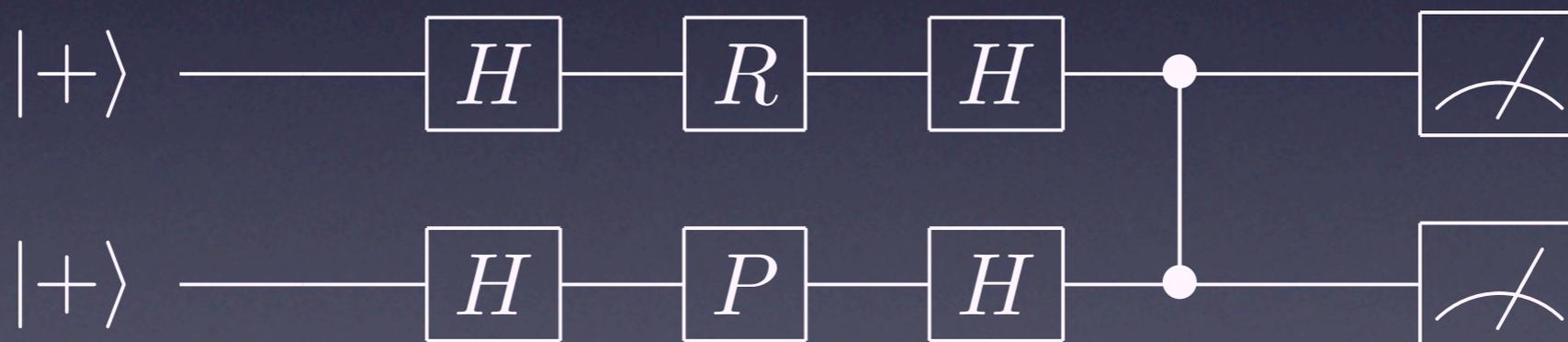
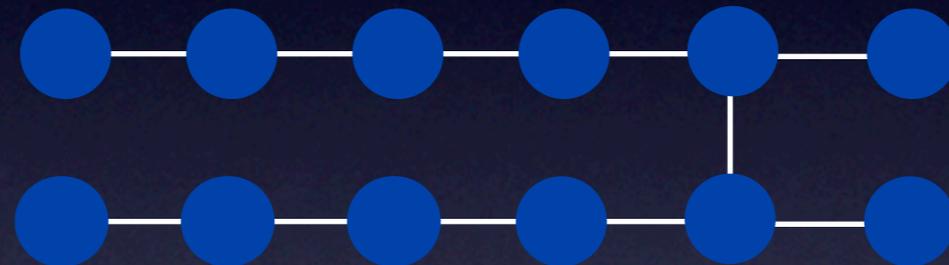
$$\text{distance } d = L^{1.365}$$

Further, this distance is **ONLY** in non-“time” direction

If one uses polynomial codes (qudits), one can saturate  $d = L^2$

# Adiabatic Cluster State QC

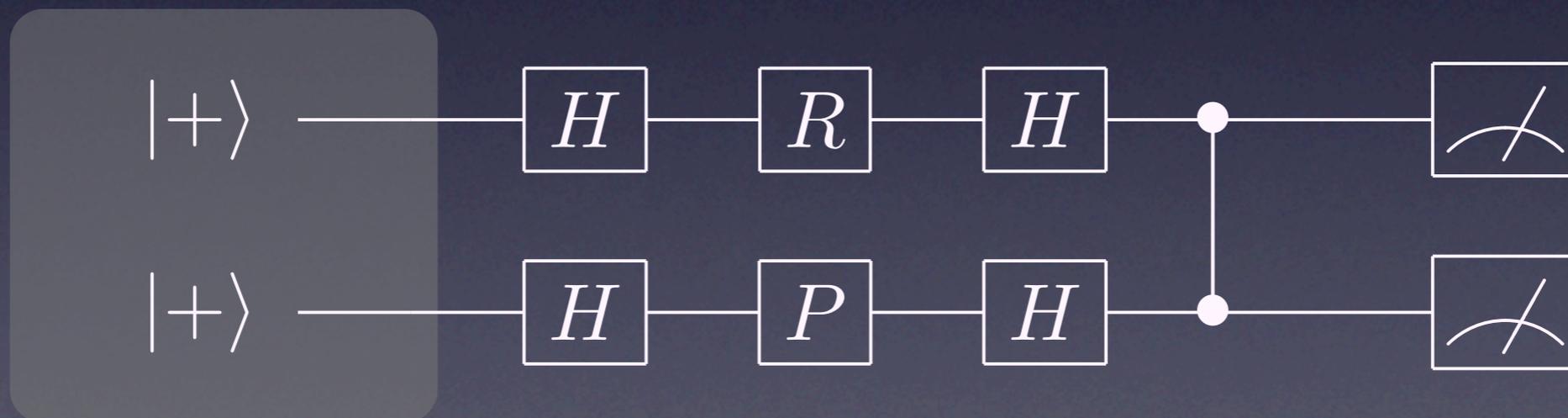
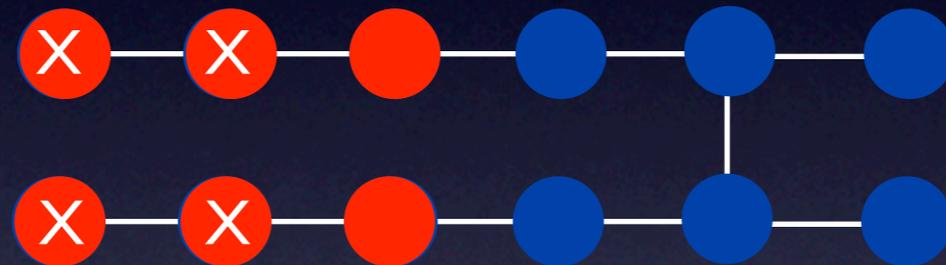
By turning on single qubit Hamiltonians while turning off parts of the cluster state Hamiltonian, we can enact a quantum circuit:



[Bacon and Flammia, Phys. Rev. A 82, 030303R (2010)]

# Adiabatic Cluster State QC

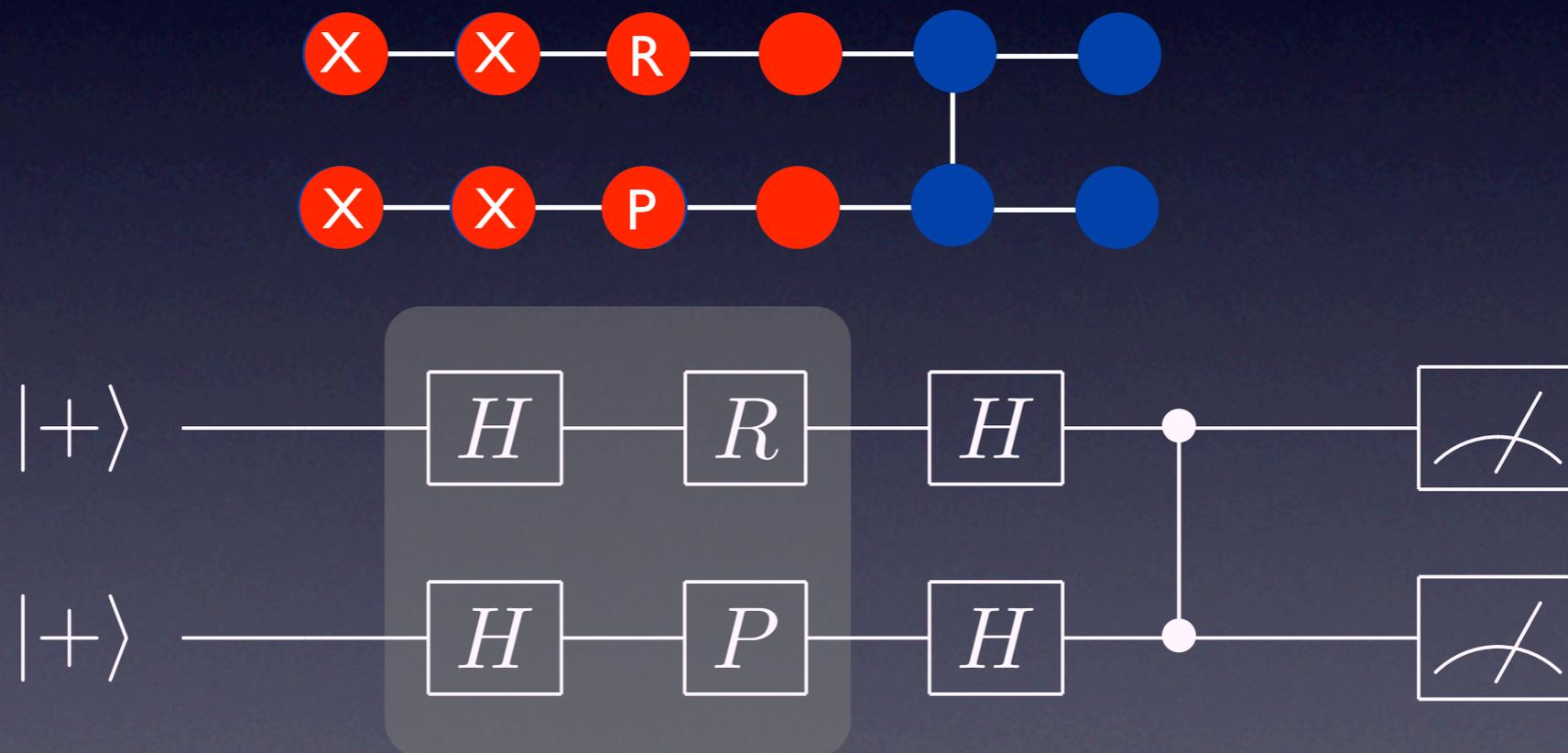
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# Adiabatic Cluster State QC

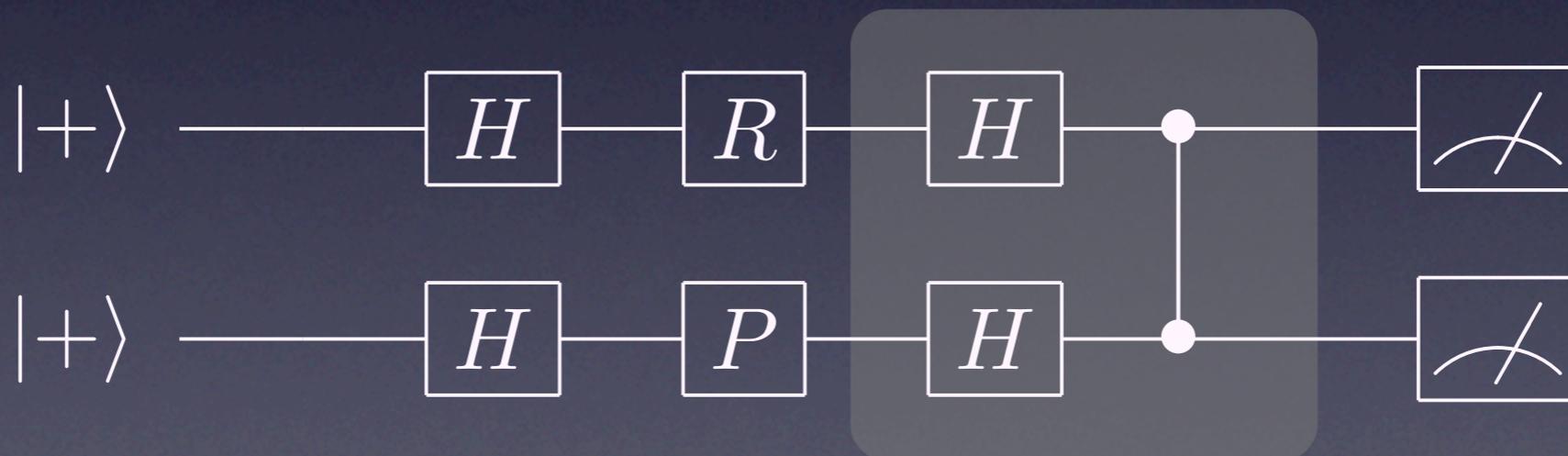
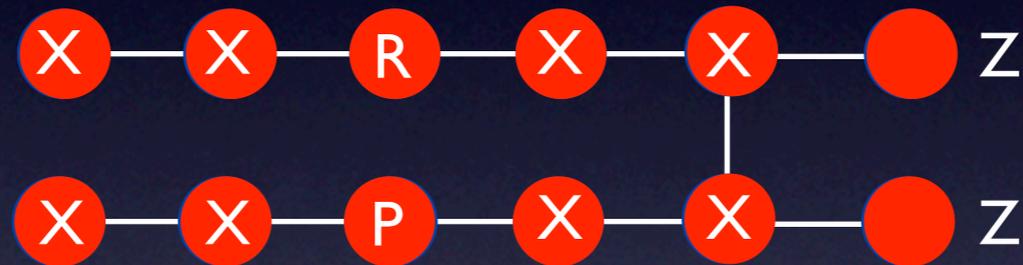
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# Adiabatic Cluster State QC

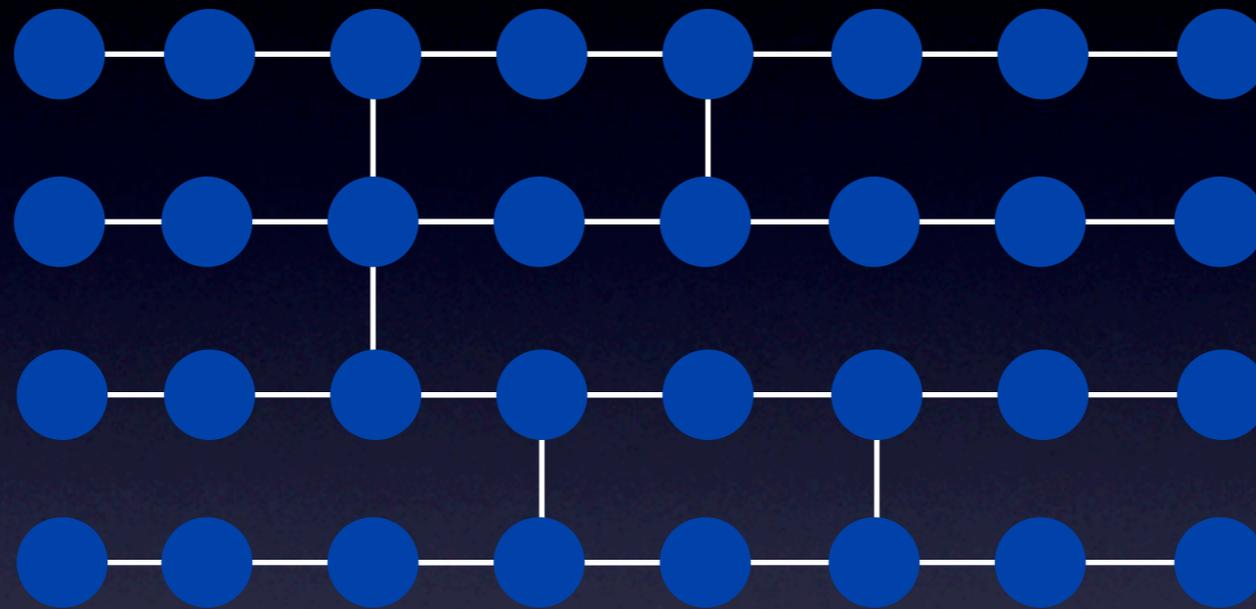
By turning on single qubit Hamiltonians while turning off parts of the cluster state Hamiltonian, we can enact a quantum circuit:



[Bacon and Flammia, Phys. Rev. A 82, 030303R (2010)]

# Quantum Transistor

[Bacon, Crosswhite, Flammia “Adiabatic Quantum Transistors” (2010)  
ask me for a copy]

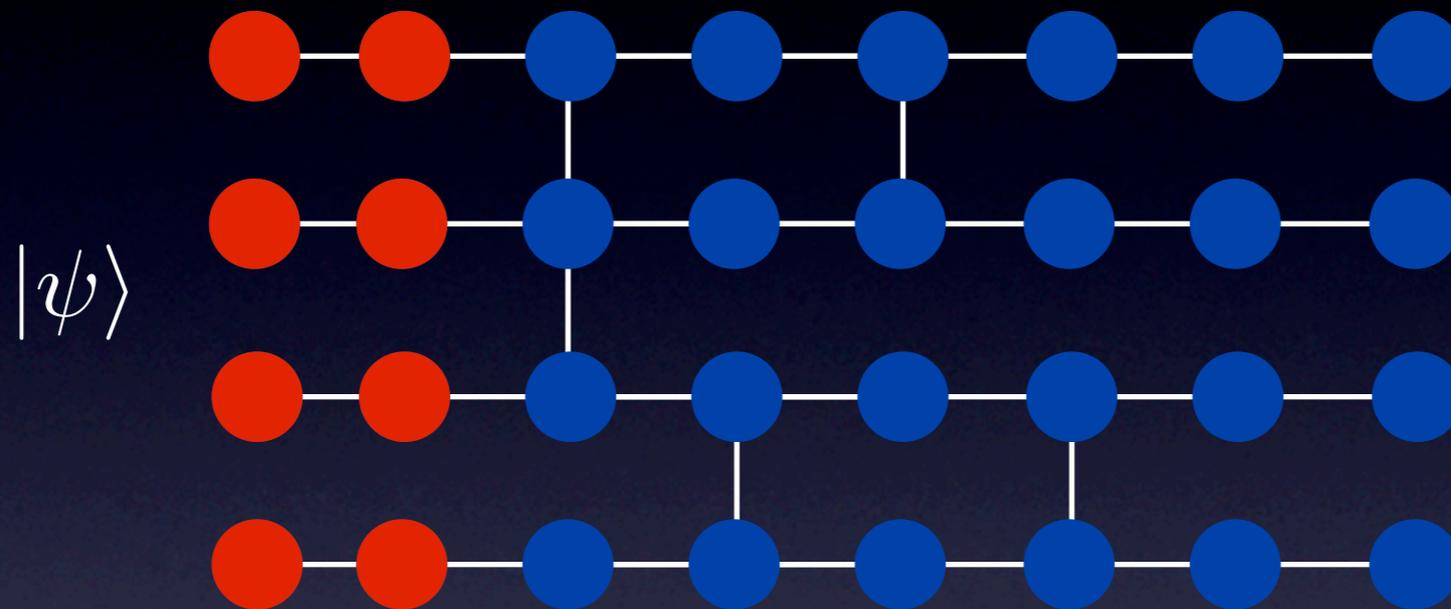


I. Many-body system in its ground state

# Quantum Transistor

[Bacon, Crosswhite, Flammia “Adiabatic Quantum Transistors” (2010)

ask me for a copy]

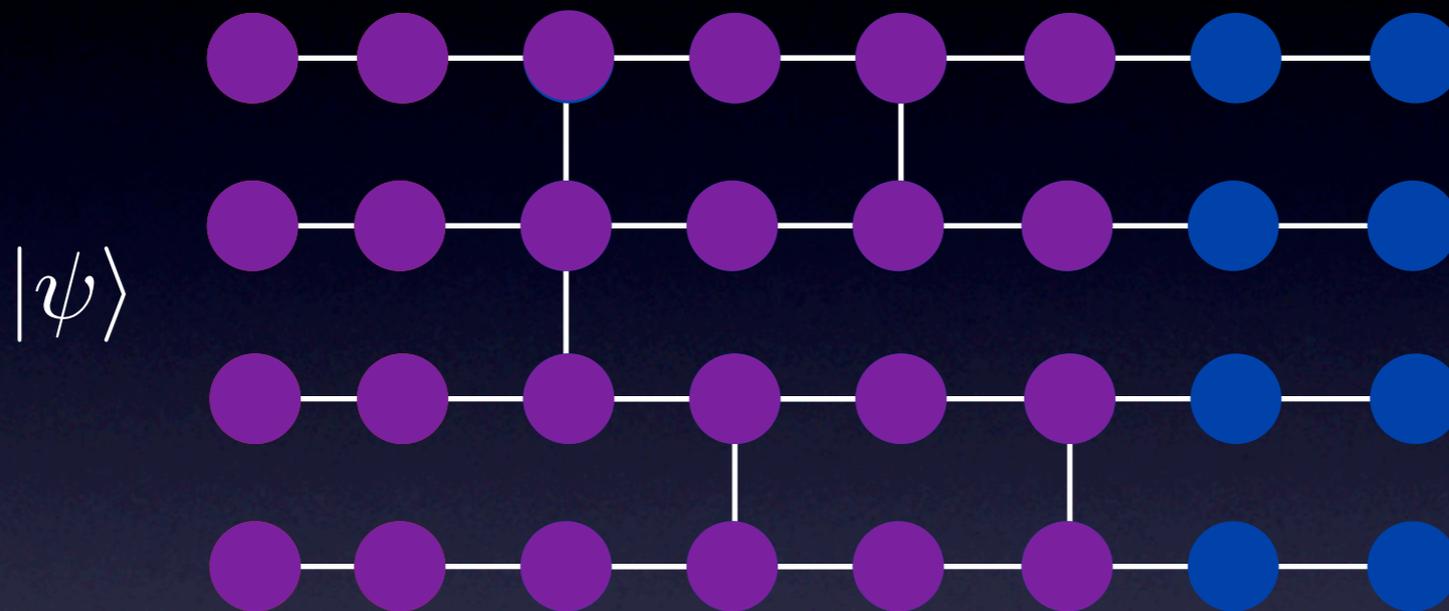


1. Many-body system in its ground state
2. Qubits localized on one side of the device

# Quantum Transistor

[Bacon, Crosswhite, Flammia “Adiabatic Quantum Transistors” (2010)

ask me for a copy]

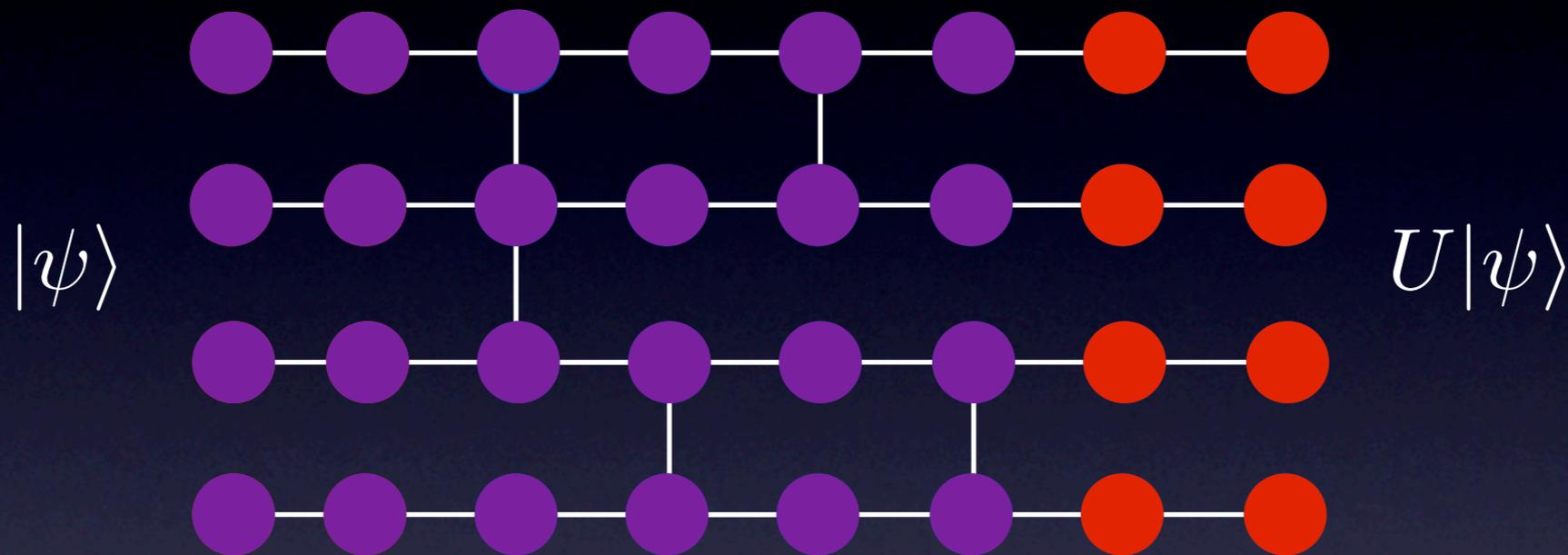


1. Many-body system in its ground state
2. Qubits localized on one side of the device
3. Apply a strong 1-qubit external field to device

# Quantum Transistor

[Bacon, Crosswhite, Flammia “Adiabatic Quantum Transistors” (2010)

ask me for a copy]



1. Many-body system in its ground state
2. Qubits localized on one side of the device
3. Apply a strong 1-qubit external field to device
4. Qubits now localized on other side of device with a quantum circuit applied to the qubits

# Questions

- Threshold?
- Hamiltonian which is sum of generators, is it self-correcting? (Related to questions about “adiabatic quantum transistor.”)
- Other methods for not lowering weight of error operators? Do traditional FT methods work?
- Can we reinterpret topological codes using these codes?
- Can we turn any code into 2-local subsystem code?

# “Q-Dub” Group

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