Quantum Computing on the Boundary

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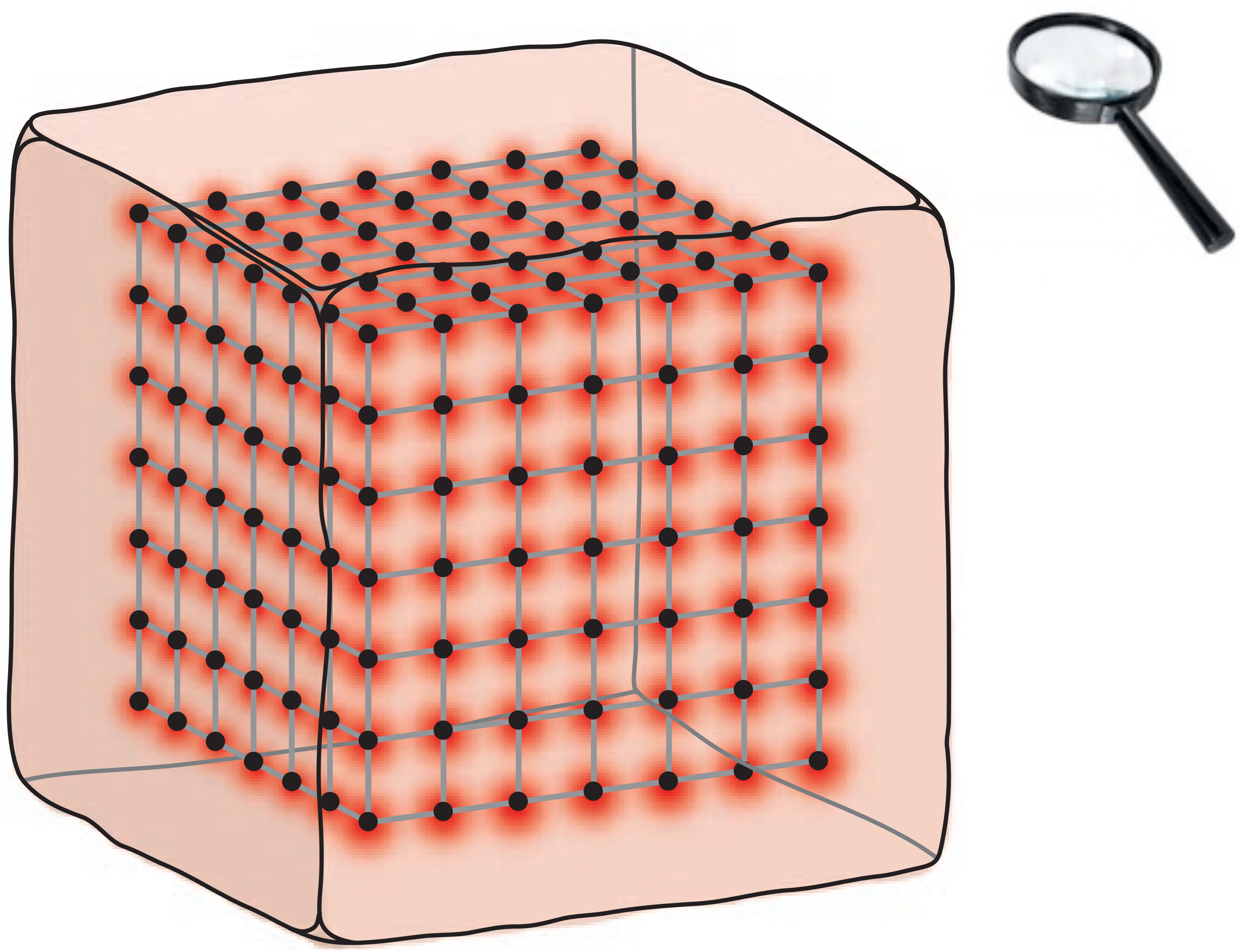
Goal

- Make quantum computers easier to scale up
- Can we do uniform blind operations in the bulk and all the info processing on the boundary?
- Can we accommodate very bad measurements?



Outline

- Quantum error correction made unitary
- A choice of code: Bacon-Shor QECC
- Error thresholds
  - Unitary gate threshold: $p_g = 3.76 \times 10^{-5}$
  - Measurement and preparation threshold: $p_p = p_m = 1/3$
- An architecture using boundary control
- Summary & Outlook

G. Paz-Silva, GKB, J. Twamley, New J. Phys. 13, 013011 (2011)
G. Paz-Silva, GKB, J. Twamley, Phys. Rev. Lett. 105, 100501 (2010)
G. Paz-Silva, GKB, J. Twamley, Phys. Rev. A 80, 052318 (2009)

Quantum error correction

- Stages of ideal circuit based QC

  i Preparation of states.

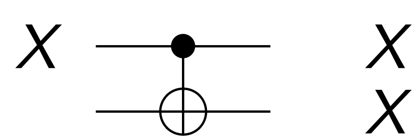
  ii Unitary gates.

  iii Measurement in some basis.

- Problem: Real operations have errors and take time

$$\begin{aligned} \text{Error rates} &\longrightarrow \{p_p, p_g, p_m\} \\ \text{Execution time} &\longrightarrow \{t_p, t_g, t_m\} \end{aligned}$$

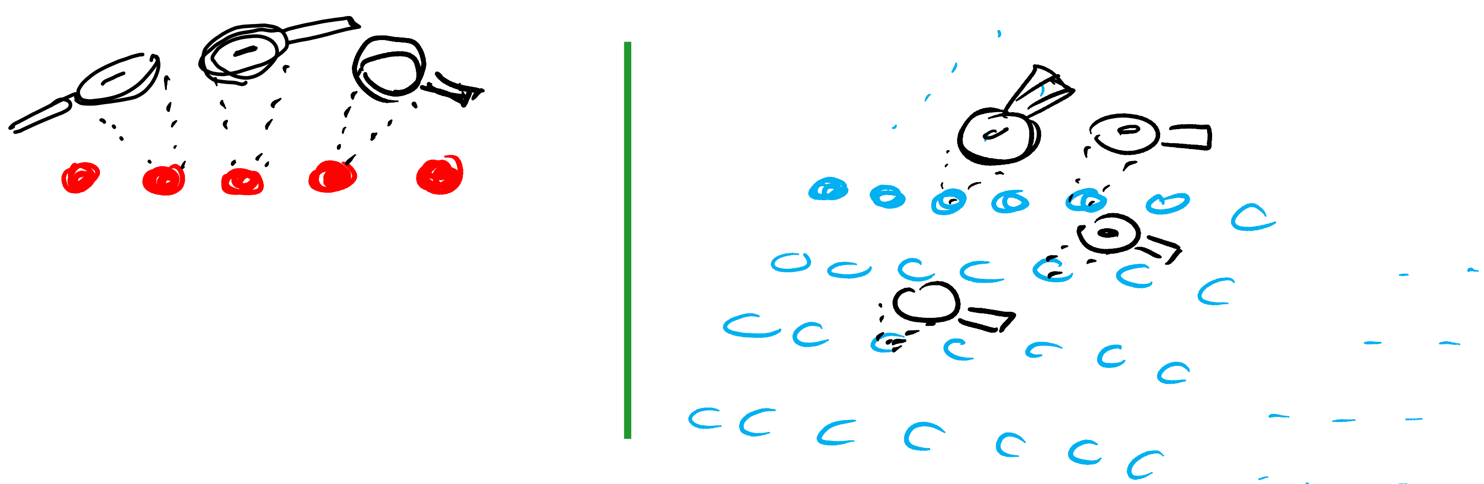
  - The bigger the computation more accuracy we need
  - Errors tend to propagate



Problems with measurement

In a 100 logical qubit quantum computer, e.g. 9-qubit Bacon-Shor code, $k = 4$ levels of concatenation.

- Physical qubits $\sim N \times (9^k + 2 \times 9^k) = 1968300 \sim 10^6$
- $N \times 3^k = 8100 \sim 10^3 - 10^4$ **simultaneous and distinguishable** measurements at every EC step.
- Key issues: lack of space, time, ...



$p_m \sim 2 \times 10^{-3}$ ion traps, 3×10^{-2} quantum dots

[3] Aliferis and Cross, PRL 98, 220502 (2007)
[4] Myerson et al, PRL 100, 200502; Benhelm et al. Nat. Phys. 4, 463 (2008)

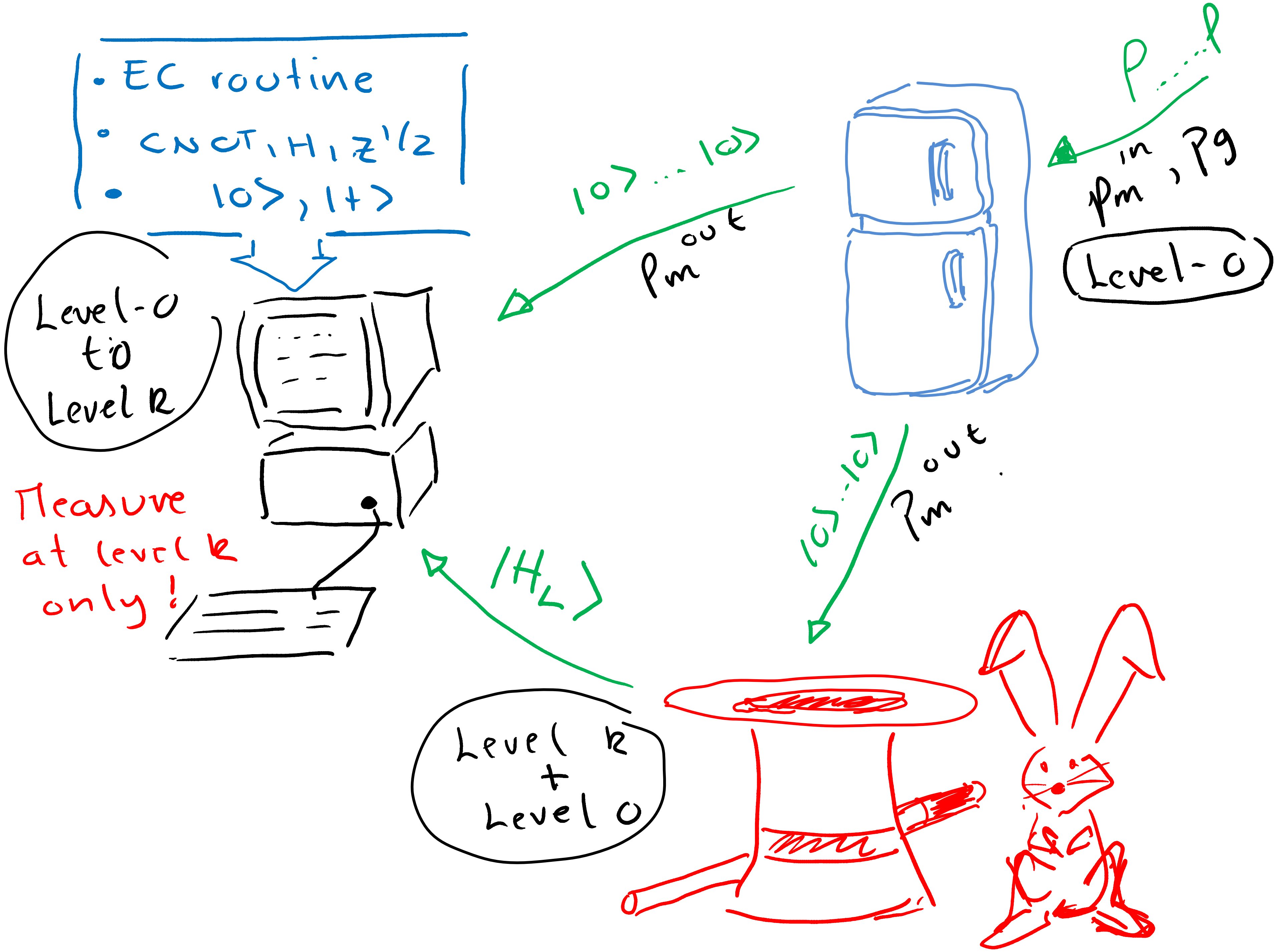
Problems with measurement

- Computations of threshold usually assume
  - all error rates are the same: $p_g = p_p = p_m$
  - all operation times are the same: $t_g = t_p = t_m$
  - but in many systems measurements are slow and very faulty
- Slow measurements can be allowed during error correction by compensating with rotated Pauli frame

  DiVencenzo and Aliferis, PRL 98, 020501 (2007)
- Faulty unitary gates can be improved using dynamical decoupling strategies

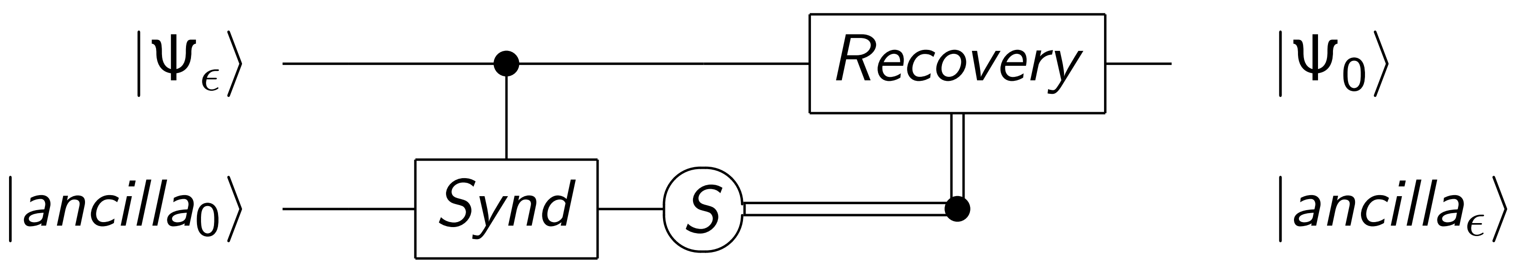
  K. Khodjasteh, D. A. Lidar and L. Viola, Phys. Rev. Lett. **104**, 090501 (2010); K. Khodjasteh and L. Viola, Phys. Rev. Lett. **102**, 080501 (2009); Phys. Rev. A **80**, 032314 (2009).]
  - doesn’t work for measurement step
- Goal to find a way to accommodate slow and noisy measurements/preparation with small impact to unitary gate threshold value

A QC with bad measurement

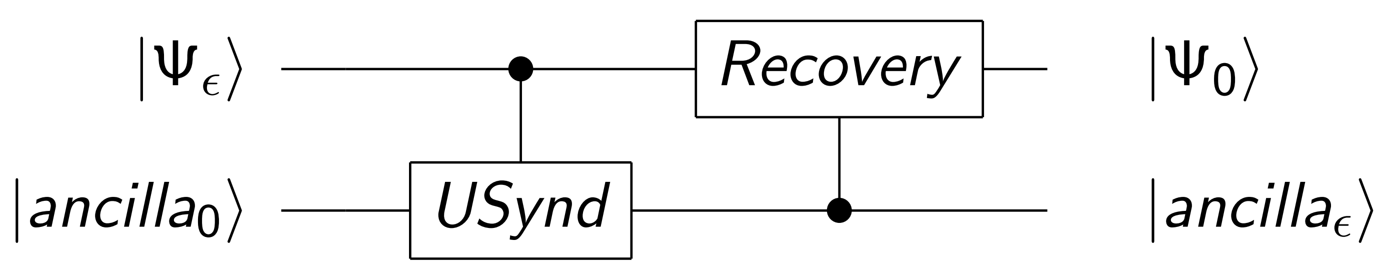


Removing measurement from QEC

- Measurement based syndrome extraction and correction



- Coherent version



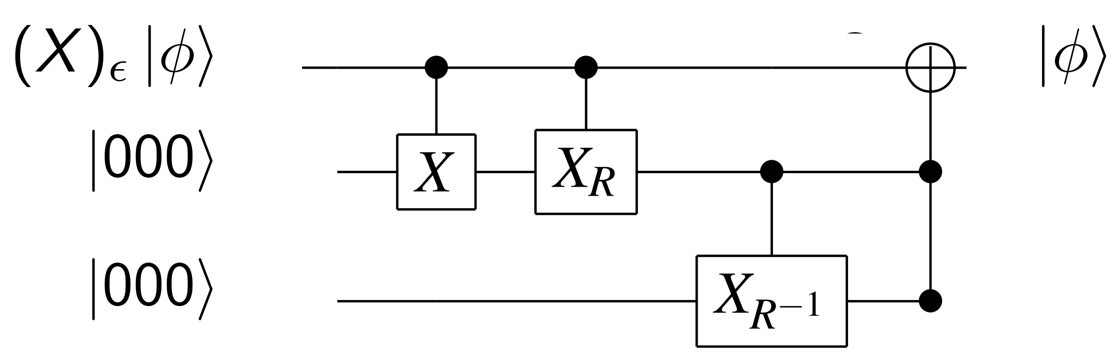
Unitary QEC

- Quantum repetition (QR) code

$$a\,|0\rangle + b\,|1\rangle \rightarrow a\,|000\rangle + b\,|111\rangle$$

- A majority voter: the $\mathcal{M}$ gate (version $\mathcal{M}^{(X)}$)
  - Corrects one bit flip error

$$|\phi\rangle = a\,|000\rangle + b\,|111\rangle$$

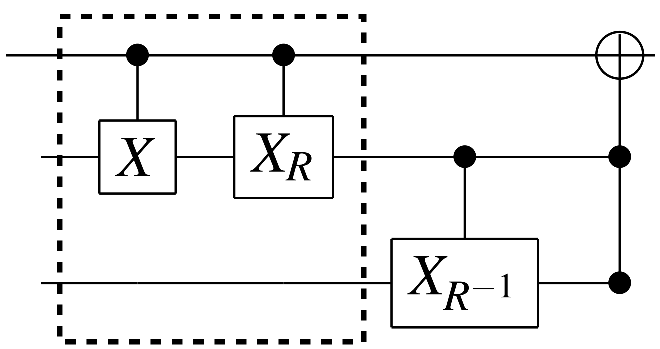


R: cyclic rotation of qubits.

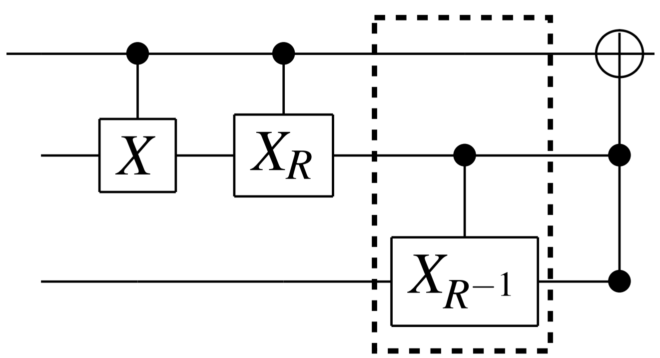
The $\mathcal{M}$ gate

- Correct a single bit flip error error

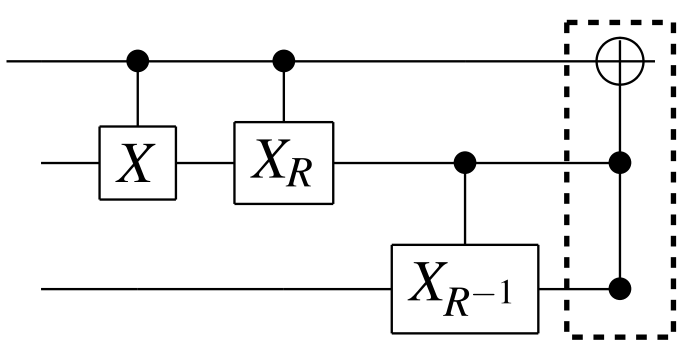
$$(a\,|100\rangle + b\,|011\rangle) \otimes \left|\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}\right\rangle$$



$$\rightarrow \quad a\left|\begin{matrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}\right\rangle + b\left|\begin{matrix} \mathbf{0} & \mathbf{1} & \mathbf{1} \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}\right\rangle$$



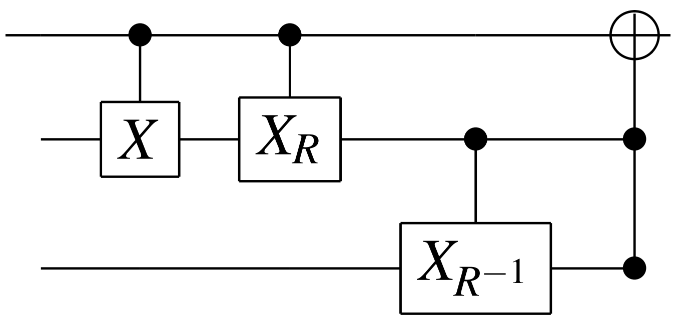
$$\rightarrow \quad a\left|\begin{matrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}\right\rangle + b\left|\begin{matrix} \mathbf{0} & \mathbf{1} & \mathbf{1} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}\right\rangle$$

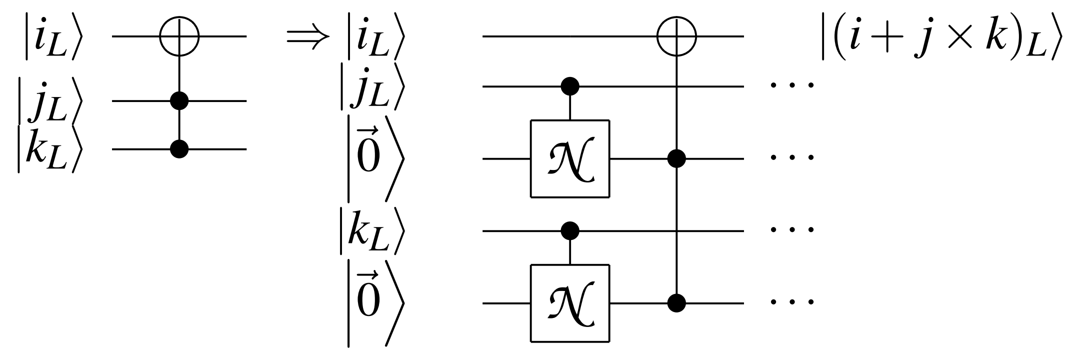


$$\rightarrow \quad (a\,|000\rangle + b\,|111\rangle) \otimes \left|\begin{matrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}\right\rangle$$

Correcting QR codes

- How to correct logical QR code $a\,|0_L 0_L 0_L\rangle + b\,|1_L 1_L 1_L\rangle$





$$\text{where}^{9}\ C\mathcal{N}_{(9,3)} : \begin{cases} |0_L\rangle\,|\vec{0}\rangle \rightarrow |0_L\rangle\,|\vec{0}\rangle \\ |0_L\rangle\,|\vec{1}\rangle \rightarrow |0_L\rangle\,|\vec{1}\rangle \\ |1_L\rangle\,|\vec{0}\rangle \rightarrow |1_L\rangle\,|\vec{1}\rangle \\ |1_L\rangle\,|\vec{1}\rangle \rightarrow |1_L\rangle\,|\vec{0}\rangle \end{cases} \sim CNOT$$

[9] Boykin et al., 2004 International Conf. on Dependable Systems and Networks, 157 (2004).

Bacon-Shor QECC

- A [[9,1,3]] code: encodes 1 logical qubit in 9 and corrects for 1 error

Defined by the stabilizers

$$\{S_i\} = \left\{ \begin{matrix} X & X & I \\ X & X & I \\ X & X & I \end{matrix}, \begin{matrix} I & X & X \\ I & X & X \\ I & X & X \end{matrix}, \begin{matrix} Z & Z & Z \\ Z & Z & Z \\ I & I & I \end{matrix}, \begin{matrix} I & I & I \\ Z & Z & Z \\ Z & Z & Z \end{matrix} \right\}.$$

Logical operators:

$$X_L = \begin{array}{|c|c|c|} \hline X & \cdot & \cdot \\ \hline X & \cdot & \cdot \\ \hline X & \cdot & \cdot \\ \hline \end{array} \; ; \; Z_L = \begin{array}{|c|c|c|} \hline \cdot & \cdot & \cdot \\ \hline Z & Z & Z \\ \hline \cdot & \cdot & \cdot \\ \hline \end{array}$$

Gauge operations:

$$X_G = \begin{array}{|c|c|c|} \hline X & X & \cdot \\ \hline \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot \\ \hline \end{array} \; ; \; Z_G = \begin{array}{|c|c|c|} \hline \cdot & Z & \cdot \\ \hline \cdot & \cdot & \cdot \\ \hline \cdot & Z & \cdot \\ \hline \end{array}$$

- Subsystem structure

$$\mathcal{H} = \oplus_{\nu^X, \nu^Z} \mathcal{H}_{\nu^X, \nu^Z}$$

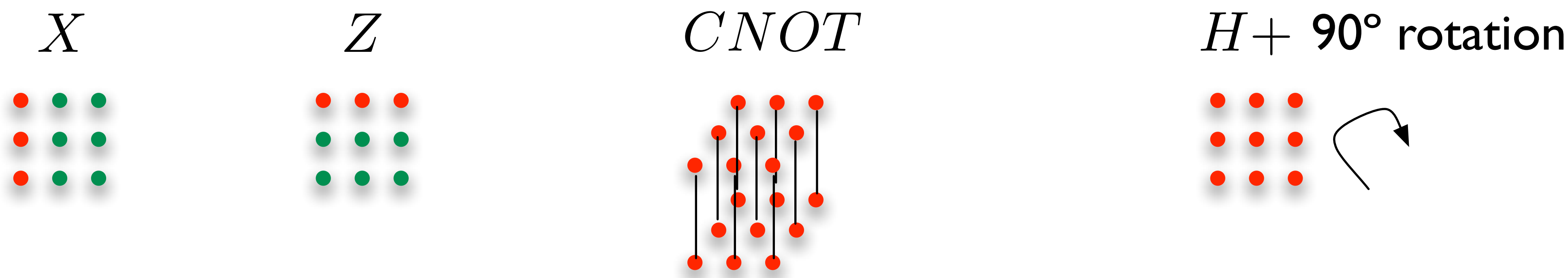
Eigenspaces of stabilizers

$$\mathcal{H}_{\nu^X, \nu^Z} = \mathcal{H}^{\mathcal{T}}_{\nu^X, \nu^Z} \otimes \mathcal{H}^{\mathcal{L}}_{\nu^X, \nu^Z}$$

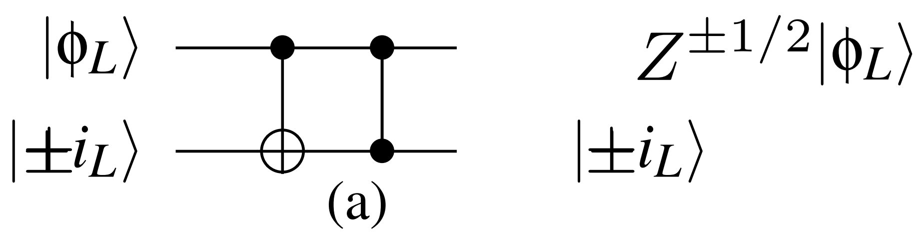
Gauge degrees of freedom | Logical qubit

Clifford operations in BS code

- Transversal gates



- $Z^{1/2}$ gate
  - The only complex gate & is not transversal for BS code: need ancilla state

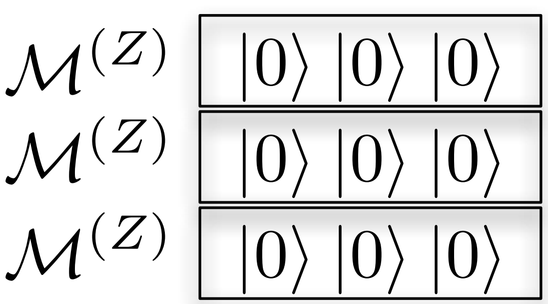


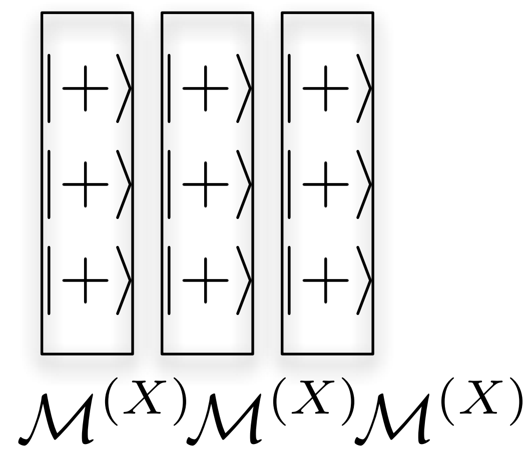
  - Fixed logical qubit at end of register prepared in $|0_L\rangle = \frac{1}{\sqrt{2}}(|+i_L\rangle + |-i_L\rangle)$ and always use this to perform gate
  - Entire computation spits into two paths $|\psi_{\text{final}}\rangle = \frac{|+i\rangle \otimes U|\psi_{\text{initial}}\rangle + |-i\rangle \otimes U^*|\psi_{\text{initial}}\rangle}{\sqrt{2}}$
  - Final computational observables A are Hermitian and can choose Real

$$\langle A\rangle = \langle\psi_{\text{initial}}|\frac{U^\dagger A U + (U^*)^\dagger A U^*}{2}|\psi_{\text{initial}}\rangle = \langle\psi_{\text{initial}}|U^\dagger A U|\psi_{\text{initial}}\rangle$$

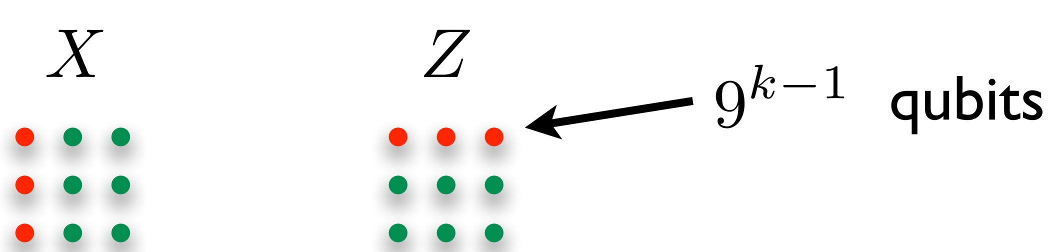
Clifford gates cont.

- Preparation of $|0_L\rangle, |+_L\rangle$





- Measurements in X, Z bases
  - Only required at highest level k of concatenation



Key observation:

$\mathcal{M}$ is an EC gadget for QR codes
Bacon-Shor code = composition of quantum repetition codes

But...

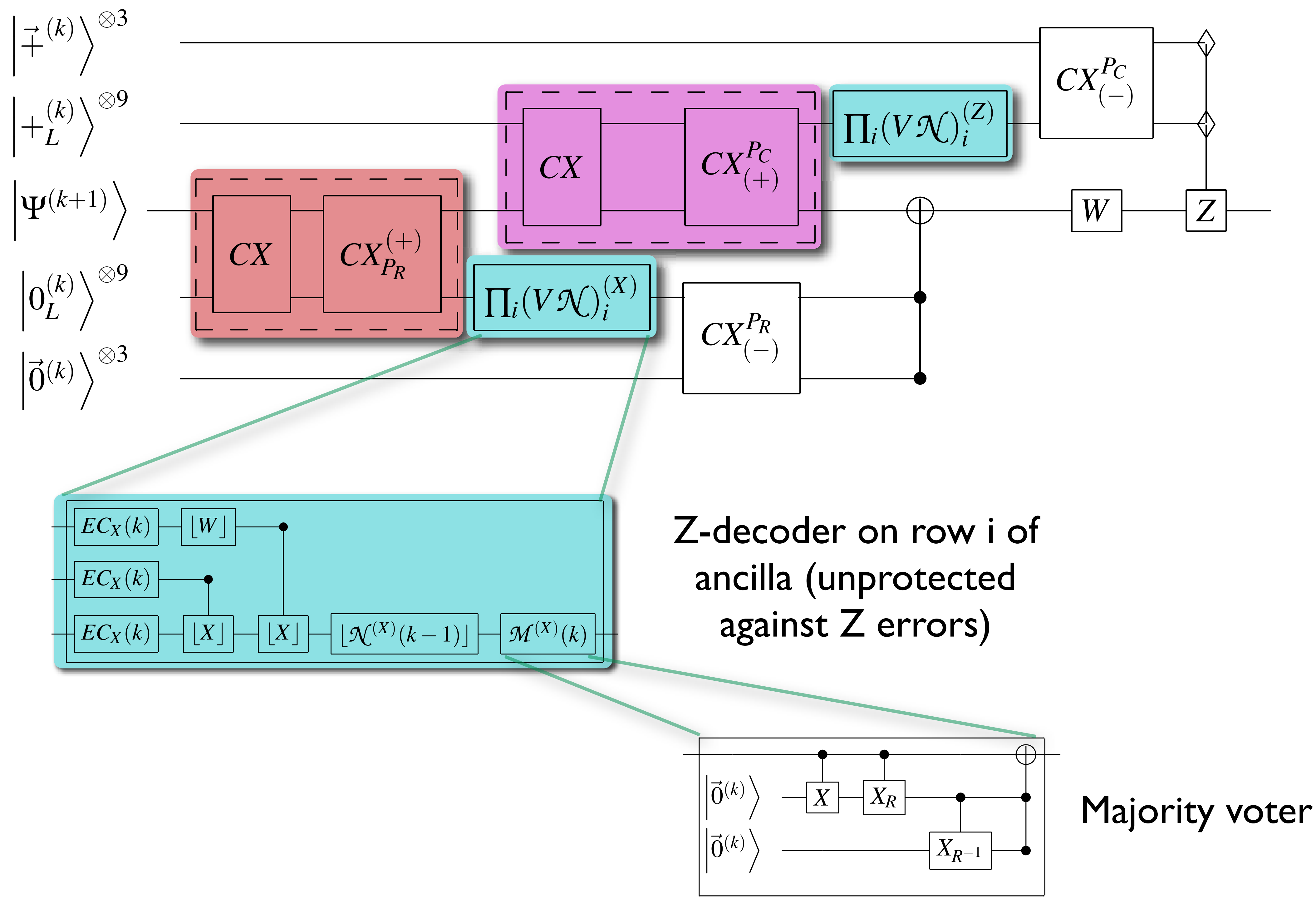
i Toffoli gate must be built at every level of concatenation!

ii What about propagation of errors ?

iii How to account for Bacon-Shor gauge freedom?

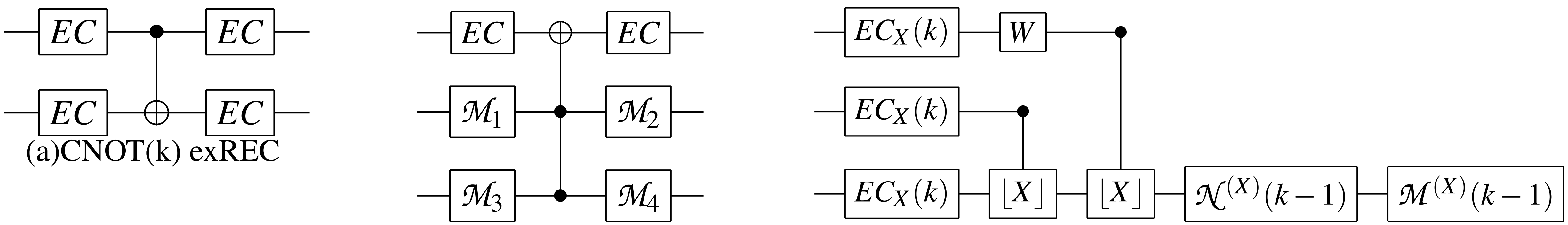
Error correction

- Unitary gadget for combined X (lower half) and Z (upper half) correction



Error threshold

- To get threshold we count disastrous error events (two data errors after EC +gate+EC). Most error prone modules:



(a)CNOT(k) exREC (b)$bTOFFOLI(k)$ exRec (c)$V\mathcal{N}_i(k)$ contracted exREC

- CNOT exREC has most possible error locations

$$
\begin{aligned}
p^{(1)} &\leq A_{(k=1)}(p^{(0)})^2 \\
p^{(k)} &\leq A_{(k>1)}(p^{(k-1)})^2 \quad , \text{ for } k > 1
\end{aligned}
$$

$$
p_{th} = \frac{1}{\sqrt{A_{(k=1)}A_{(k>1)}}} = 3.76 \times 10^{-5}
$$

- So need $p_p, p_g < p_{th}$

Measurement threshold

- Threshold for Clifford measurements (X and Z basis)
  - They are only needed at the highest level of encoding thus

$$p_{(m)}^{(k+1)} \ \leq \ 3(p_{(m)}^{(k)})^2 + O(p^{(k)})$$

  - for k large enough $p^{(k)} \ll 1$, negligible and

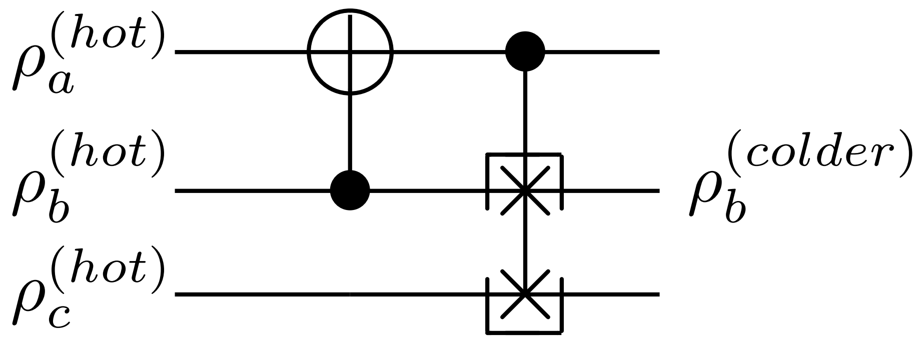
$$p_m^{(k+1)} < 3^{2^{k+1}-1}(p_m^{(0)})^{2^{k+1}}$$

  - Threshold value for Clifford basis measurement

$$p_m < \frac{1}{3}$$

Preparation threshold

- Variant of algorithmic cooling. Prepare 3 ancilla close to |0> with error $p_p = \epsilon^{(0)}$



$$\begin{pmatrix} 1-\epsilon^{(0)} & 0 \\ 0 & \epsilon^{(0)} \end{pmatrix}_b \rightarrow \begin{pmatrix} 1-\epsilon^{(1)} & 0 \\ 0 & \epsilon^{(1)} \end{pmatrix}_b$$

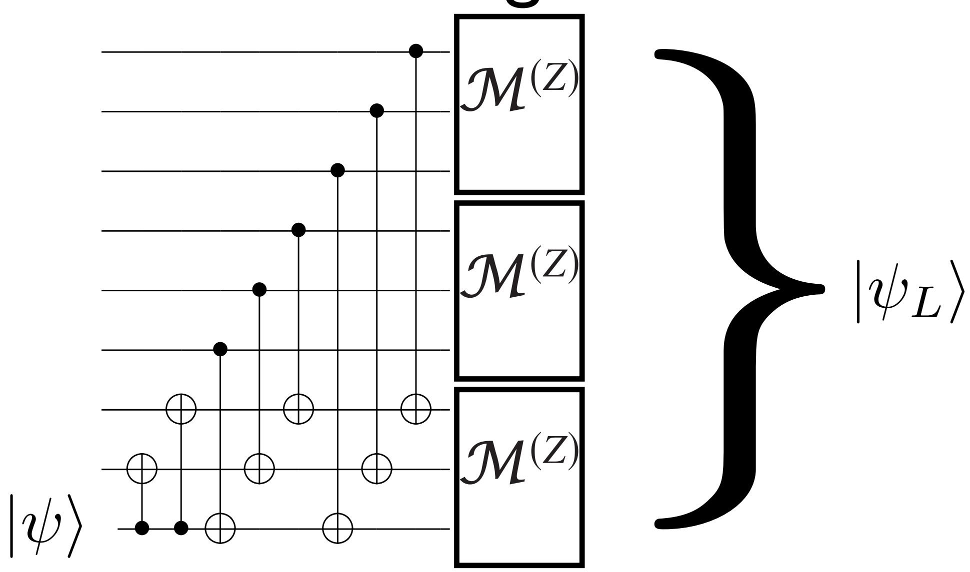
$$\epsilon^{(1)} = (\epsilon^{(0)})^2(3 - 2\epsilon^{(0)})$$

- Iterate for j rounds on 3^j ancilla
- Total preparation error of single output ancilla is $p_p^{(j)} = \epsilon^{(j)} + 5(j+1)3^j p_g^{(0)}$
- For small enough gate errors can always purify to $p_p^{(j)} < p_{th}$
- Since measurement is preparation, threshold is

$$p_p < \frac{1}{3}$$

Magic state distillation

- Protocol*: many badly prepared physical states to one good logical state $|H_L\rangle$
- Prepare ancillary qubit state close to $|H\rangle = \frac{1}{2}(|0\rangle + e^{i\pi/8}|1\rangle)$
  - Encode into a logical state



* S. Bravyi and A. Kitaev, Phys. Rev. A **71**, 022316 (2005); B. Reichardt, Quant. Inf. Comp. **9**, 1030 (2009)

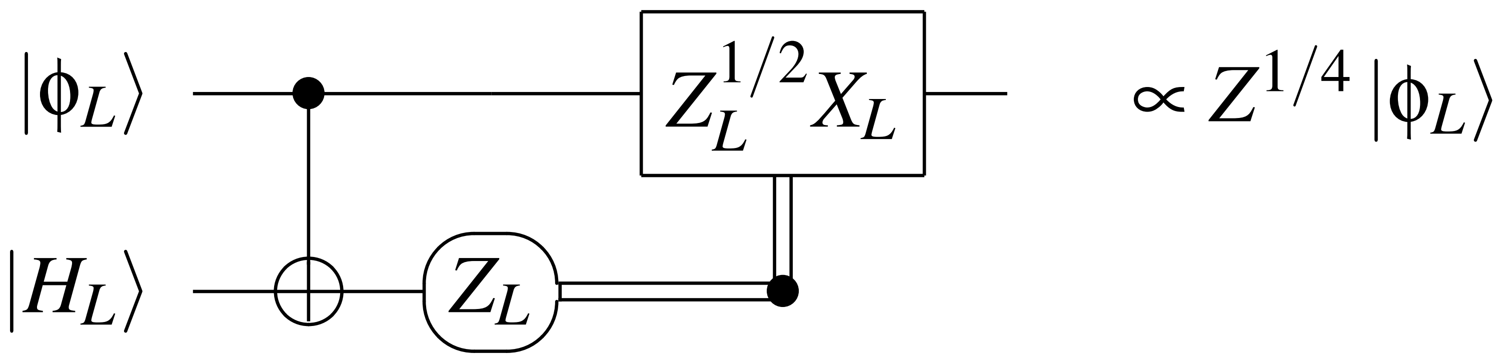
  - Apply recursively to prepare ancillary $|\psi_L\rangle$ at level L concatenation w/error

$$p_{\mathrm{anc}}^{(L)} \leq 10p^{(0)} + 108\sum_{j=0}^{L-1} p^{(j)}$$

  - Provided gate errors are below threshold then $p_{anc} = 1 - |\langle H_L|\psi_L\rangle|^2 < \sin^2(\frac{\pi}{8})$
  - Now can distill a better state. Prepare many copies (e.g. 15 copies for Reed-Muller Code). Use FT Clifford ops to distill one purer state closer to logical state.

Non-Clifford gate

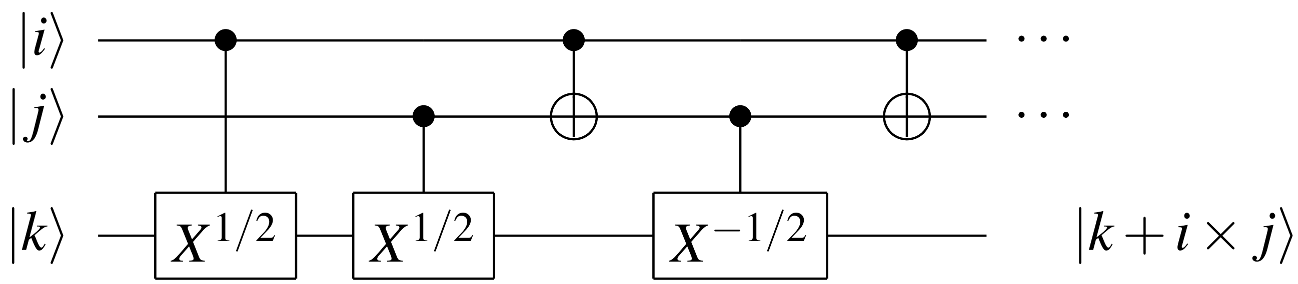
- Teleport non-Clifford gate using logical magic state



- Errors of the gate dominated by magic state preparation which can be made as small as encoded gate errors using the distillation protocol

Some issues

- Summary: $p_g = 3.76 \times 10^{-5}$ $p_m = 1/3$
- We assumed ability to perform 3 body gates (Toffoli)
  - If we only allow two body gates then replace Toffoli with

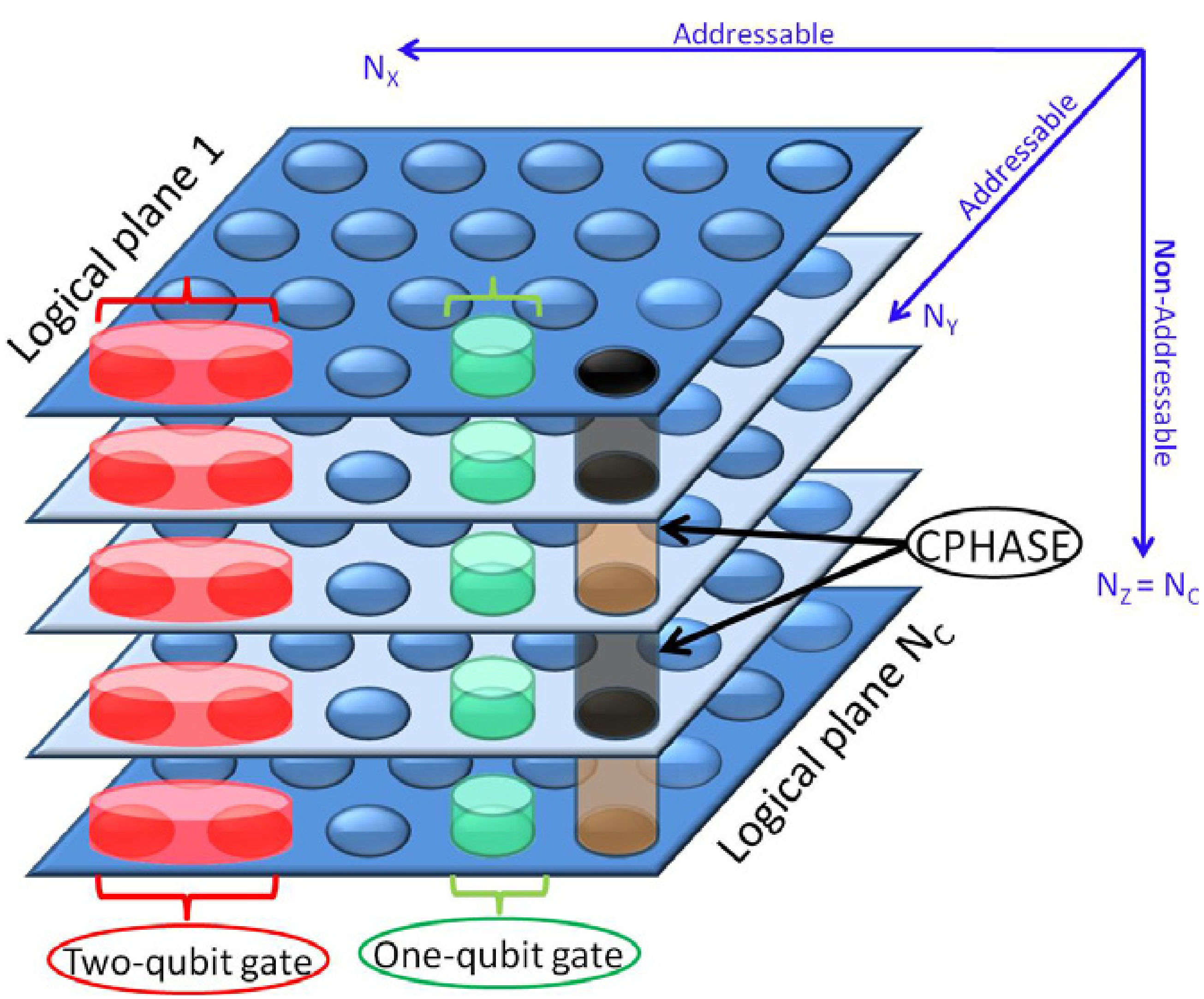


  - gate threshold reduces to $p_{(p,g)thresh} = 2.68 \times 10^{-5}$

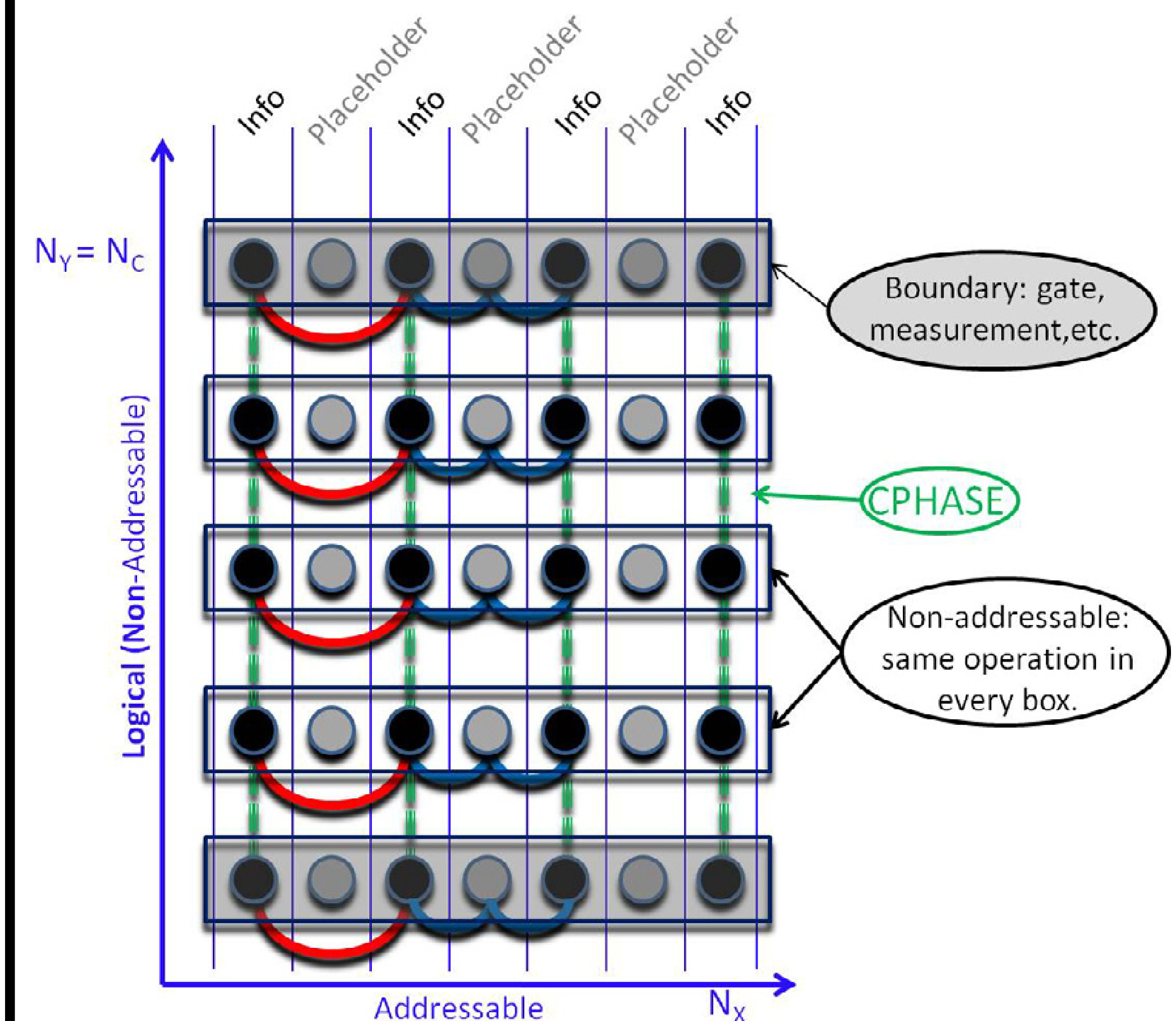
- We assumed arbitrary connectivity of qubits. With nearest neighbor connectivity only say in 2D would expect a gate threshold reduction by ~3.
- Thresholds could be improved by optimization

FT computing with boundary control

- Use global pulses to control the computer
  - Measurements done only at one boundary



3D

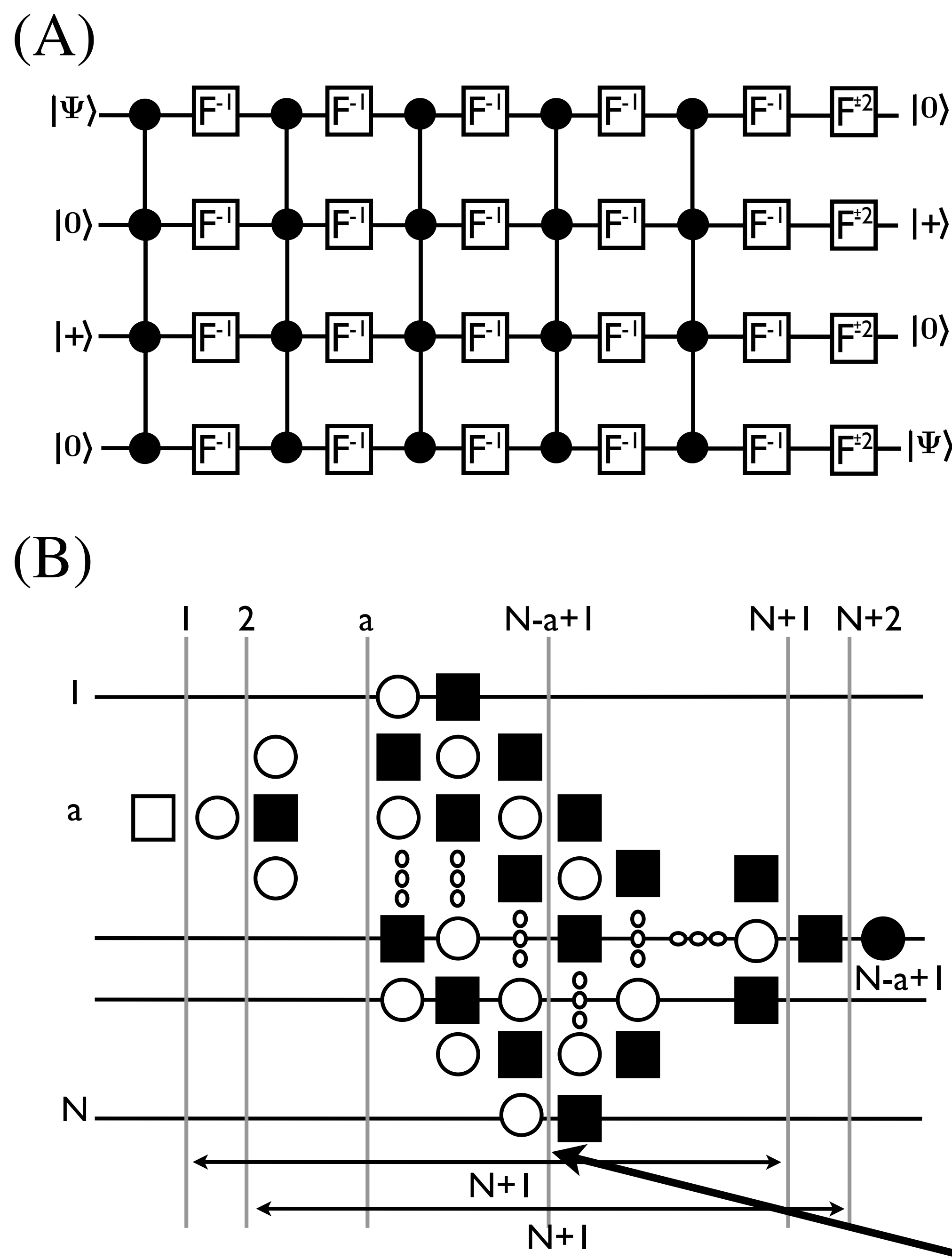


2D

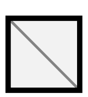
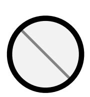
- Global pulses move info

F=Hadamard
(for qubits)

 =CPHASE



Global Clifford operations to swap a state on a chain

 =Z  =X

Heisenberg picture. Mirroring of a register independent of input.

Can address qubit a at the boundary at time step N-a+1

- Only O(N) overhead in circuit complexity

Performance

- Semi-global approach effectively reduces the number of control modes by a factor proportional to N, number of logical qubits
  - True even compared to circuits which allow for measurement and hence have higher threshold
- E.g. Shor's algorithm

Bit size of integer to factor	# logical qubits	Improvement over addressable circuit without measurement	Improvement over addressable circuit with measurement
768	1540	86	84
2048	4100	2048	225
4096	8196	4096	451

Summary & Outlook

- Quantum computers can be controlled/addressed on the boundary in a fault tolerant way
- Measurement free error correction with threshold only factor ~3 worse than similar model with measurement
- To do: Show how gate error rates could be improved with dynamical decoupling schemes, also include gate errors
- Can measurement free QECC be useful for adiabatic QC?
- A fault tolerant 1D QCA?
  - global control protocol can be modified to do all operations homogeneously without boundary control (need to schedule magic state injection for Pi/8 and $Z^{1/2}$ gates at each site)
  - has the right flavour except the QCA cell dimension grows as log(N)