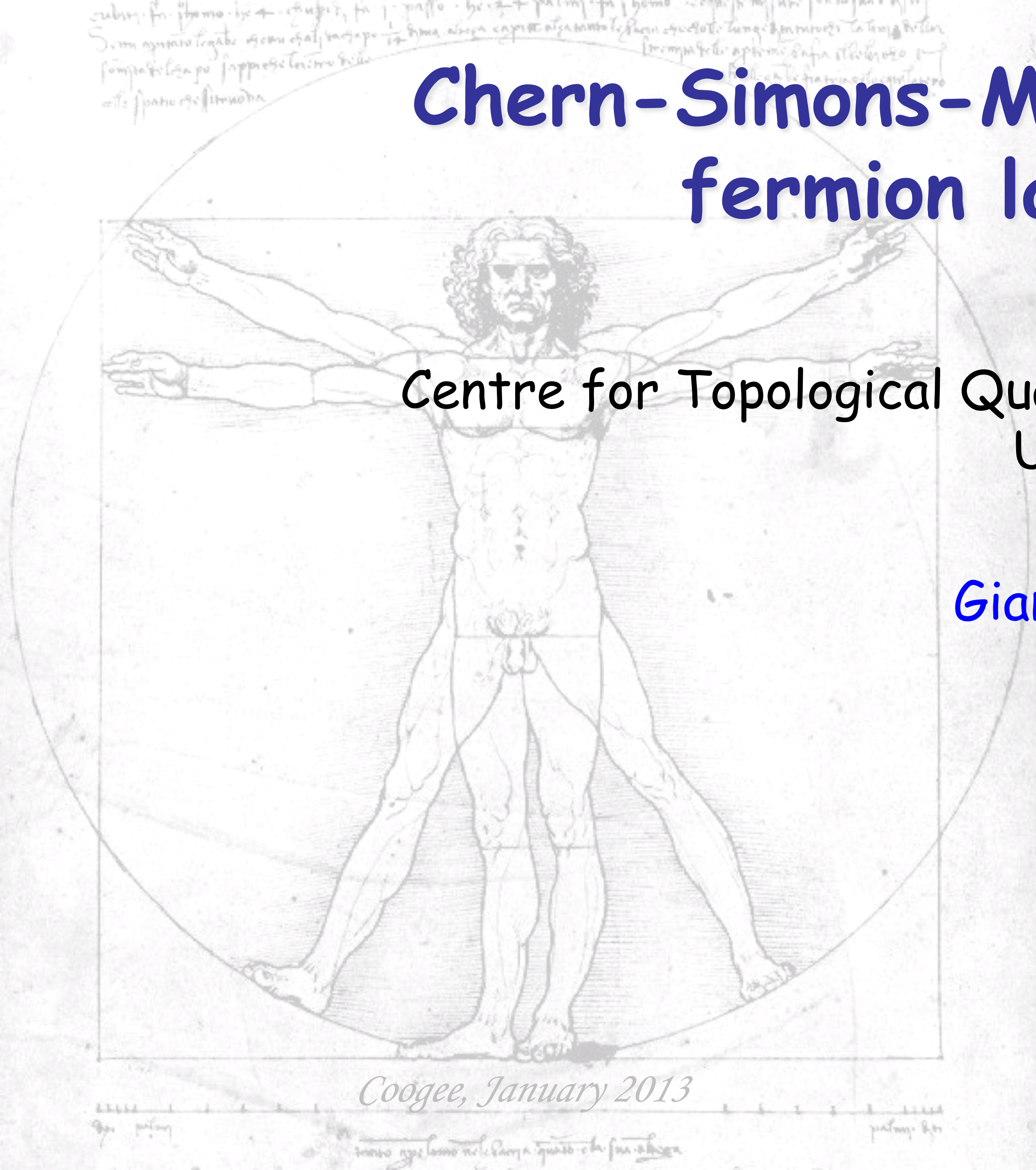
Chern-Simons-Maxwell from fermion lattice model



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Coogee, January 2013

EPSRC
Pioneering research and skills

UNIVERSITY OF LEEDS

Motivation

Topological systems fascinate:

- New *physics* (material science)
- New *reliable* technologies (quantum computation)

They can support **anyons**

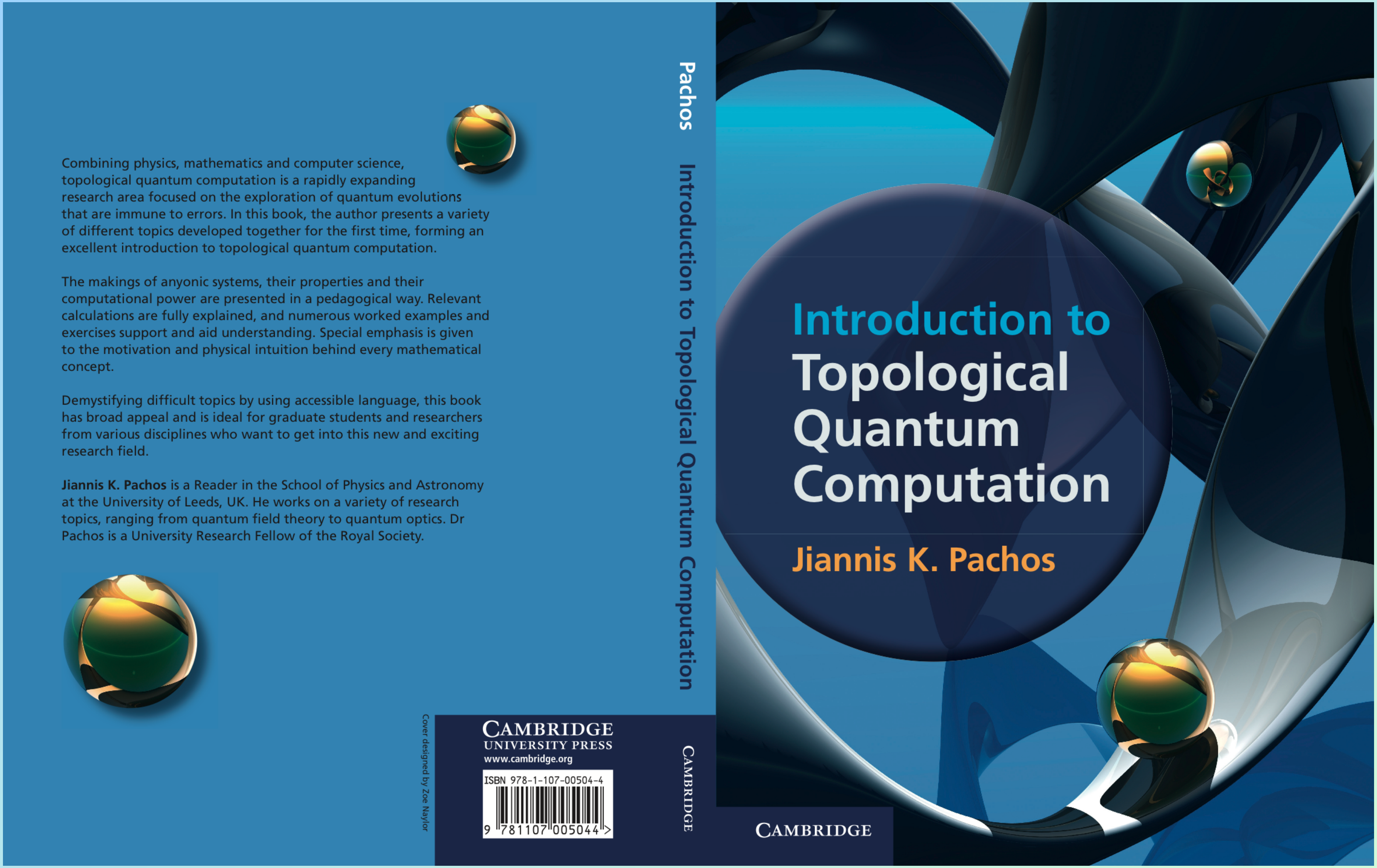
Anyonic particles are interesting on their own right:

- Experimental detection (Abelian/Non-Abelian anyons)
- Kinematics, interactions, transport physics

Main problems: **realisation**

detection

Commercial Break:



Introduction

Abelian Chern-Simons-Maxwell:

Gauge theory, *U(1)*

Chern-Simons: *topological*

Maxwell is non-topo., *confining in 2+1 dimensions*

Massive Thirring model:

Relativistic Dirac fermions with mass, *interacting*

Lattice models:

"Easy" to implement in the laboratory (atoms in optical lattices, engineered materials etc.)

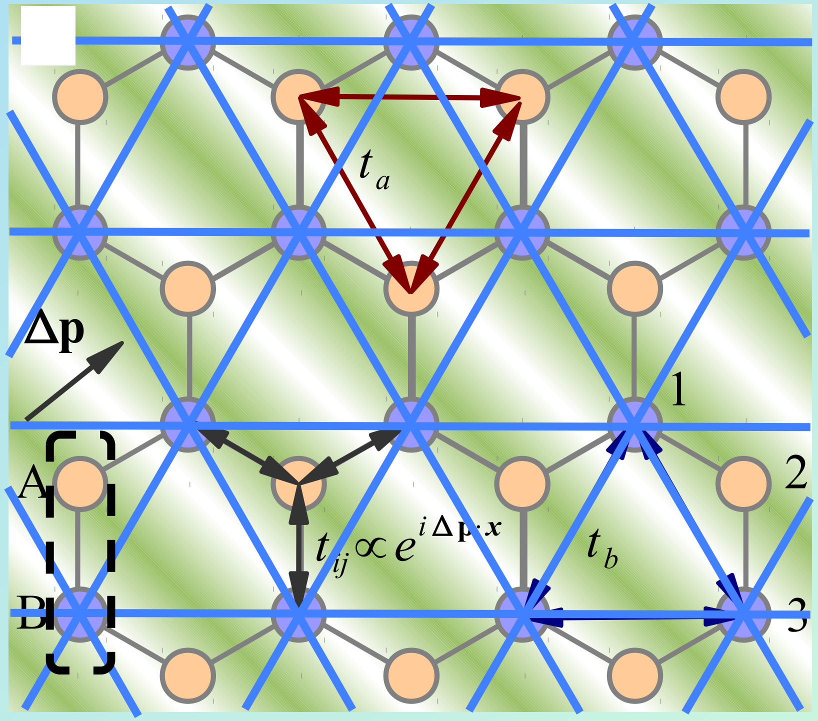
From Haldane (IQHE) + interactions (FQHE) **analytically**

Extended Haldane's model

Two triangular sublattices: A, B, each loaded with **fermionic atoms** in internal states *b, w*

Hamiltonian:

$$H = -t\sum_{\langle j,k\rangle}(b_j^+ w_k + w_k^+ b_j) - \sum_{\langle\langle j,k\rangle\rangle}(-t_w w_j^+ w_k + e^{i\phi_{jk}} t_b b_j^+ b_k) + U\sum_j w_j^+ w_j b_j^+ b_j$$



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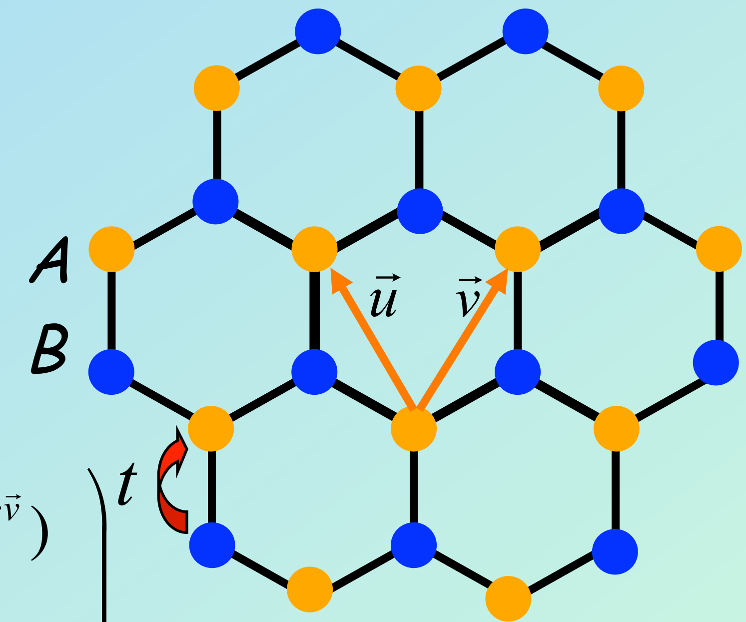
Two Fermi points
Two massless Dirac particles

Gives mass to
Dirac fermions and *maybe more...*

Interactions between
Dirac fermions

Graphene-like structure

$$H = -t \sum_{<i,j>} (w_i^+ b_j + b_j^+ w_i)$$

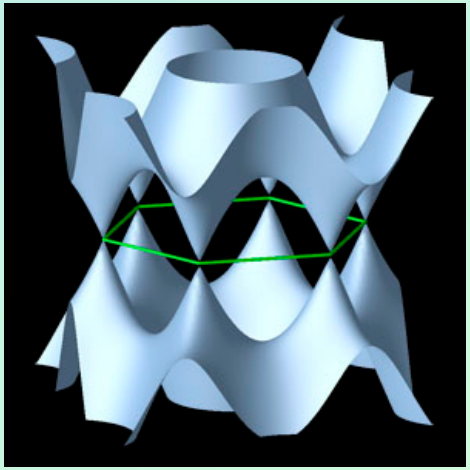


Fourier transformation:

$$H_{\vec{p}} = \begin{pmatrix} 0 & -t(1 + e^{-i\vec{p}\cdot\vec{u}} + e^{-i\vec{p}\cdot\vec{v}}) \\ -t(1 + e^{i\vec{p}\cdot\vec{u}} + e^{i\vec{p}\cdot\vec{v}}) & 0 \end{pmatrix}$$

$$E(\vec{p}) = \pm t\sqrt{3 + 2\cos\vec{p}\cdot\vec{u} + 2\cos\vec{p}\cdot\vec{v} + 2\cos\vec{p}\cdot(\vec{u}-\vec{v})}$$

$E(p_x,p_y)$:



Fermi points: *E(p)=0*

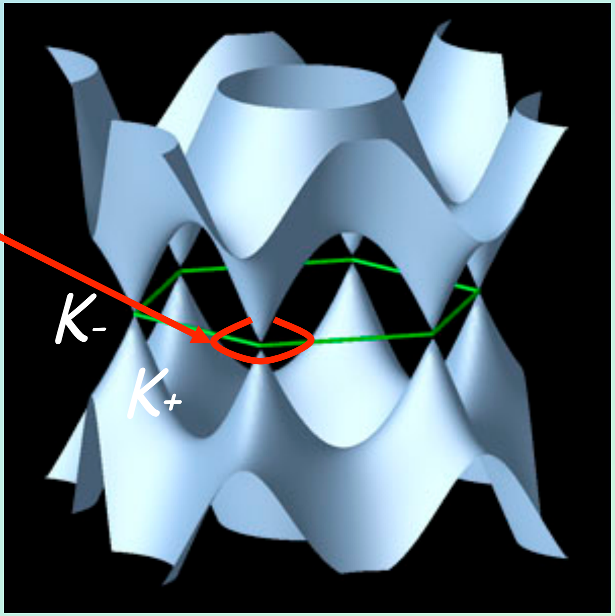
Graphene-like structure

$$E(\vec{p}) = \pm t\sqrt{3 + 2\cos\vec{p}\cdot\vec{u} + 2\cos\vec{p}\cdot\vec{v} + 2\cos\vec{p}\cdot(\vec{u}-\vec{v})}$$

Linearise energy $E(\vec{k})$ around a conical point:

$\vec{p} = \vec{K} + \vec{k}$

$$H_{\vec{k}} \approx \pm\frac{3t}{2}\begin{pmatrix} 0 & k_x + ik_y \\ k_x - ik_y & 0 \end{pmatrix} = \pm\frac{3t}{2}\vec{\sigma}\cdot\vec{k}$$



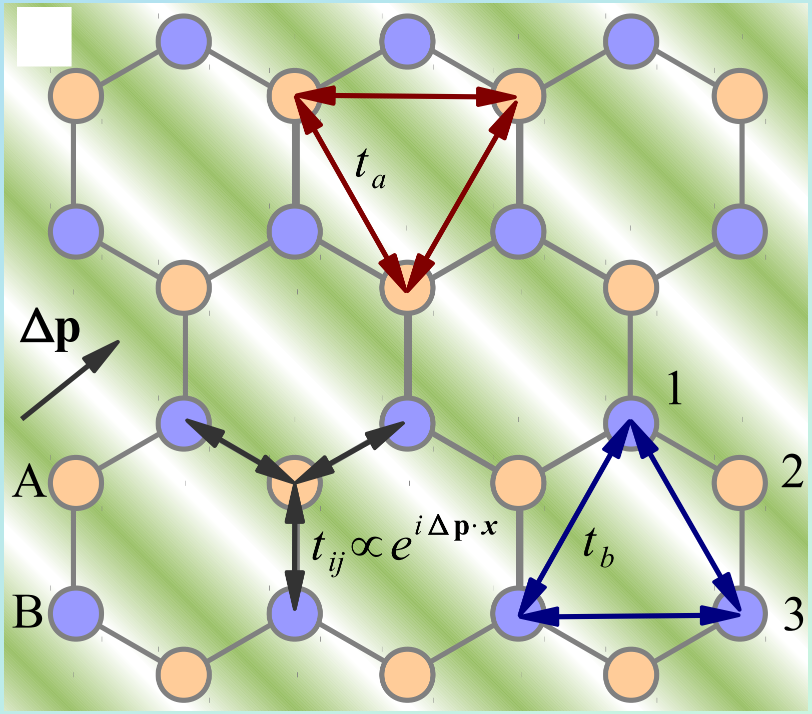
Relativistic Dirac equation!

-> Relativistic QFT.

Extended Haldane's model

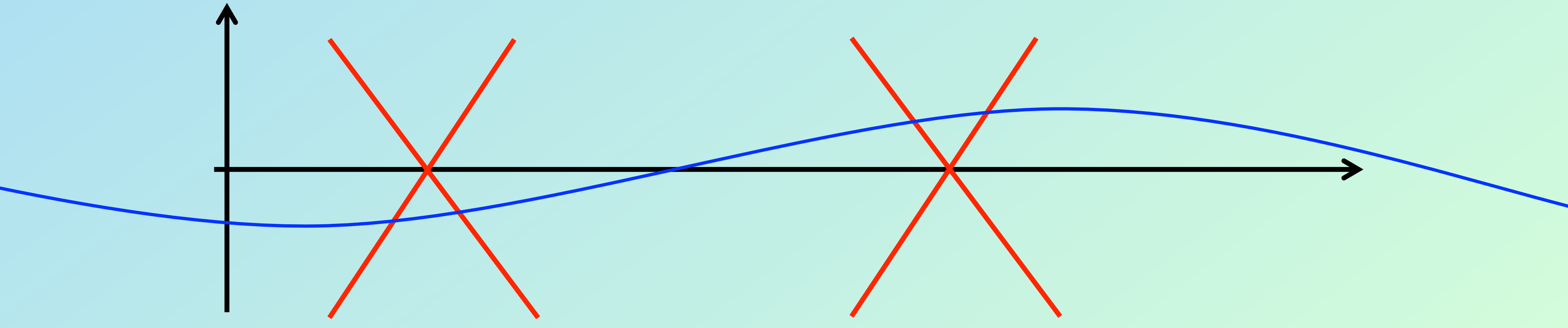
Linearisation of Hamiltonian around Fermi points

$$H = -E \sum_k (w_k^+ \ b_k^+) \, \mathbf{S}(k) \cdot \sigma \begin{pmatrix} w_k \\ b_k \end{pmatrix}$$



$$\mathbf{S}(k)\cdot\sigma \approx \underbrace{k_x\sigma_x + k_y\sigma_y}_{t\text{-term}} + \underbrace{m(k)\sigma_z}_{t_w,\,t_b\text{ -terms}}$$

"mass" m(k) can be +ve or -ve



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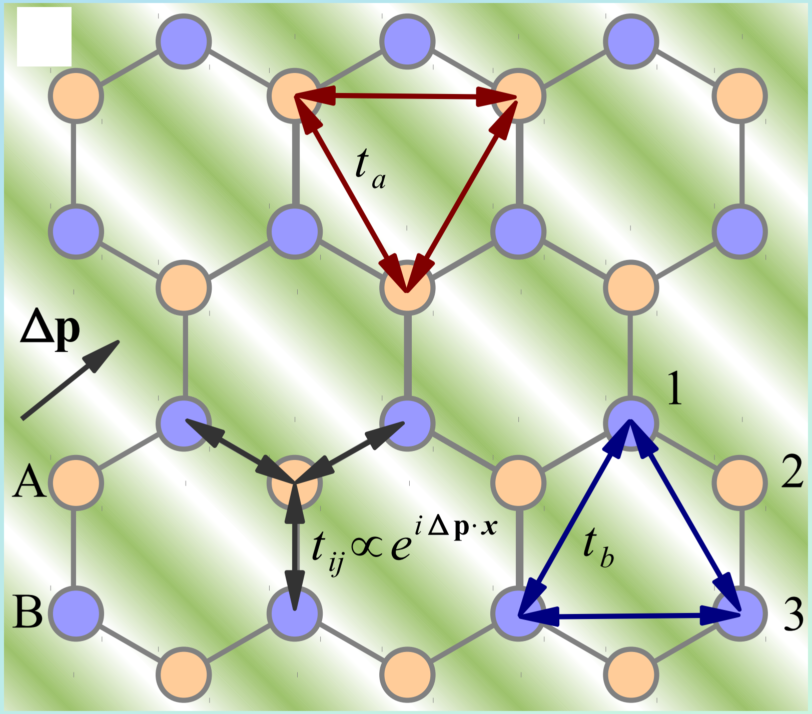

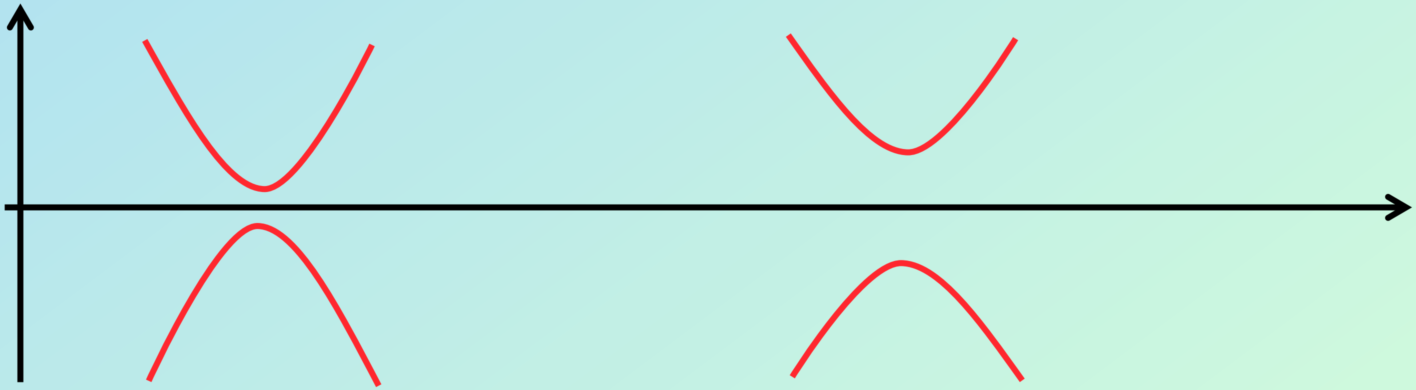

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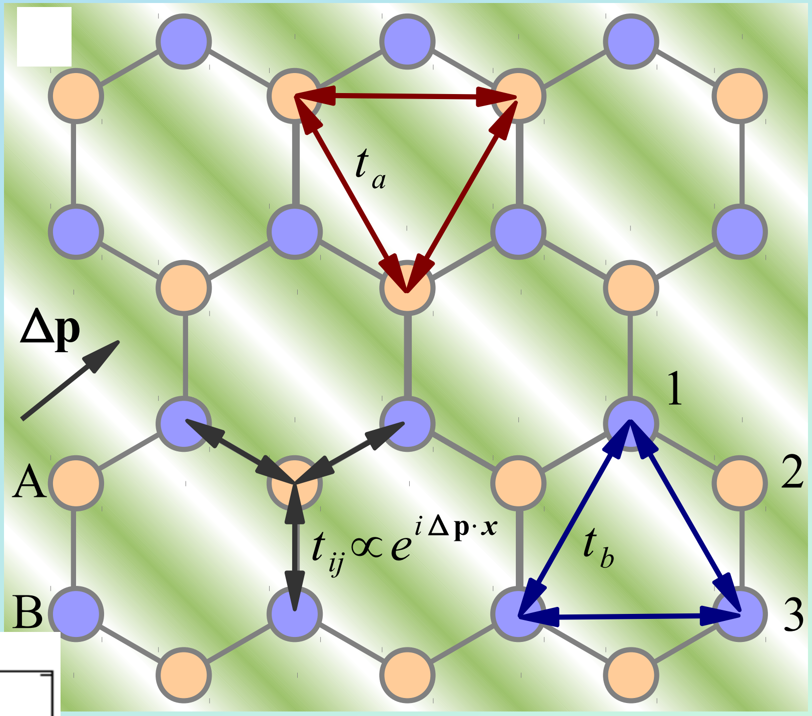
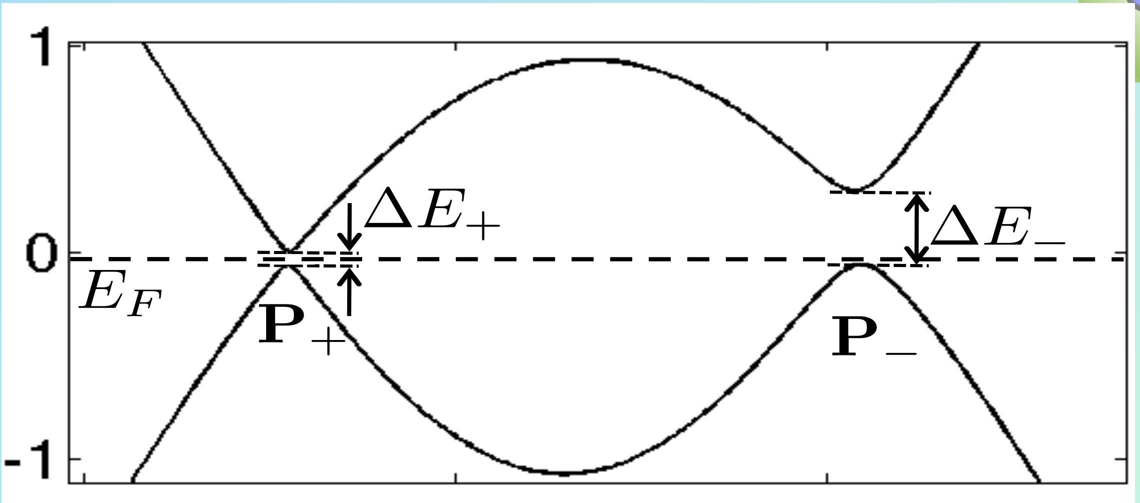
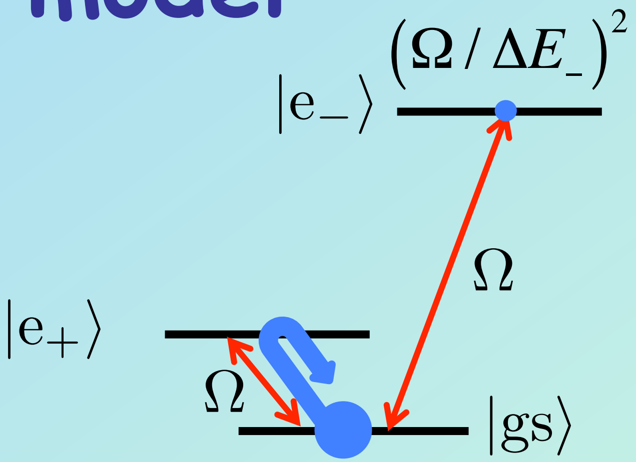




Adiabatic elimination of P_-

2+1 dim Thirring model

Eliminate one Fermi point so only one Dirac fermion out of the lattice model:



Introduce interactions:

$$U\sum_j w_j^+ w_j b_j^+ b_j \rightarrow U\int w^+(r)w(r)b^+(r)b(r)$$

Altogether:

$$H \approx \int d^2r\Big[\psi^\dagger(c\boldsymbol{\sigma}\cdot\mathbf{p}+\sigma_z Mc^2)\psi+\frac{g^2}{2}j^\mu j_\mu\Big]$$

$$g^2=\frac{U}{3},\ c=\frac{3}{2}t \text{ and } M=\frac{2}{3}\frac{t_b}{t^2}$$

Chern-Simons-Maxwell

2+1 dim Thirring -> Chern-Simons-Maxwell theory

$$Z_{\rm Th} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int d^3x \left[\bar{\psi}(c\,\partial\!\!\!/ - Mc^2)\psi - \frac{g^2}{2} j^\mu j_\mu\right]\right\}$$

$$\exp\left(\int d^3x\, \frac{g^2}{2}\, j^\mu j_\mu\right) = \int \mathcal{D}a_\mu \exp\left[-\int d^3x\, \left(\boxed{\frac{1}{2}a^\mu a_\mu} + g\, j^\mu a_\mu\right)\right]$$

$$\int \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\, \exp\left[-c\int d^3x\, \bar{\psi}\left(\partial\!\!\!/ + \frac{g}{c}\, a\!\!\!/ - Mc\right)\psi\right] =$$

$$= \exp\left\{c\log\left[\det\left(\partial\!\!\!/ + \frac{g}{c}\, a\!\!\!/ - Mc\right)\right]\right\} = \exp(-S_{\rm eff}[a])$$

[Fradkin & Schaposnik 1994]

Chern-Simons-Maxwell

2+1 dim Thirring -> Chern-Simons-Maxwell theory

$$S_{\text{eff}}[a] = \frac{ig^2}{8\pi c}\frac{Mc}{|Mc|}\int d^3x\, \epsilon^{\lambda\mu\nu} a_\lambda \partial_\mu a_\nu + \mathcal{O}\left(\frac{\partial}{Mc}\right)$$

$$S_{\text{I}}[a, A] = \int d^3x \Big(\frac{1}{2}\, a^\mu a_\mu - i\epsilon^{\lambda\mu\nu} a_\lambda \partial_\mu A_\nu + \frac{2\pi i c}{g^2}\epsilon^{\lambda\mu\nu} A_\lambda \partial_\mu A_\nu\Big)$$

$$Z_{\text{I}} = \int \mathcal{D}a_\mu \mathcal{D}A_\mu e^{-S_{\text{I}}[a,A]} =$$

$$\int \mathcal{D}a_\mu \exp\left[-\int d^3x\, \left(\frac{ig^2}{8\pi c}\epsilon^{\lambda\mu\nu} a_\lambda \partial_\mu a_\nu + \frac{1}{2} a^\mu a_\mu\right)\right] =$$

$$\int \mathcal{D}A_\mu \exp\left[-\int d^3x\, \left(\frac{2\pi i\, c}{g^2}\epsilon^{\lambda\mu\nu} A_\lambda \partial_\mu A_\nu + \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right)\right] = Z_{\text{CSM}}$$

[Deser & Jackiw 1984]

Measuring protocol

Observables of Chern-Simons theory: Wilson loop ops

$$W(L) = \exp\left(\frac{i}{g}\oint_L A_\mu dx^\mu\right)$$

$$\langle W(L)\rangle_{\mathrm{CSM}} = \exp(\pm i\Phi_L/8\pi)$$

Take L on the space plane, L_0, and a single loop: $\Phi_{L_0} = 0$

$$\langle \Psi_{\mathrm{CSM}}|\, W(L_0)\, |\Psi_{\mathrm{CSM}}\rangle = 1.$$

Ground state is stabilised by **all** Wilson loop operators
-> topologically ordered

- Topo entanglement entropy (Levin and Wen)
- Topo degeneracy (Freedman et al.)

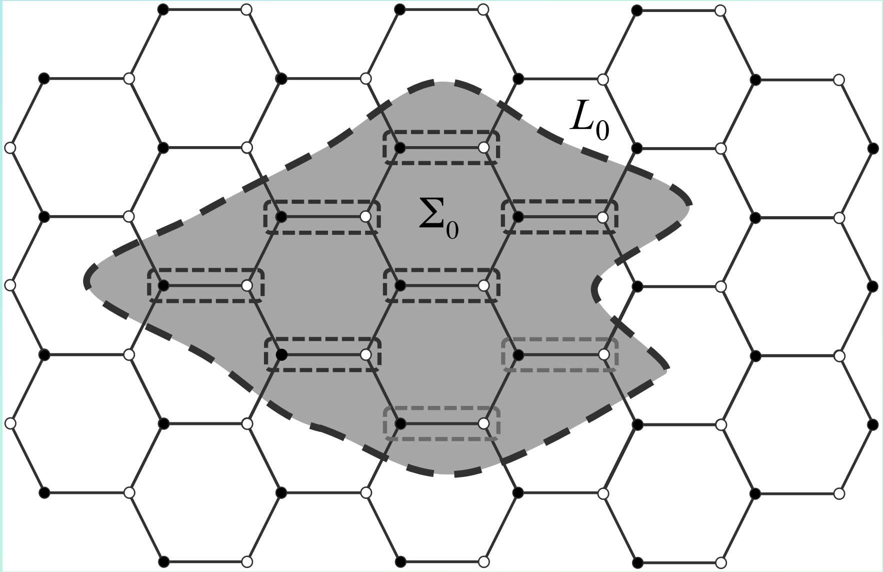
Measuring protocol

Bosonisation procedure: [Fradkin & Schaposnik 1994]

$$\langle W(L)\rangle_{\rm CSM} = \langle \exp\left(i\int_\Sigma dS_\mu \bar{\psi}\gamma^\mu\psi\right)\rangle_{\rm Th}$$

$$\int_{\Sigma_0} dS_\mu \bar{\psi}\gamma^\mu\psi = \int_{\Sigma_0} dS\,\left[b(\mathbf{r})^\dagger b(\mathbf{r}) + w(\mathbf{r})^\dagger w(\mathbf{r})\right]$$

$$\langle\Psi_{\rm TB}|\exp\left[i\sum_{i\in\Sigma_0}(b_i^\dagger b_i + w_i^\dagger w_i)\right]|\Psi_{\rm TB}\rangle$$



~ 1 Chern-Simons

$\sim \exp(-k|\Sigma_0|)$ Maxwell

Conclusions

- Method to **eliminate the "doubling"** of Fermi points in lattice models -> possible to realise chiral models.
- **Analytically tractable** model for FQHE.
- **No** need for **magnetic** field.
- Wilson loop behaviour by **local density** measurements.

- Future:
    - Non-Abelian anyons
    - SU(N) Yang-Mills theories 2+1 dims
    - Extension to 3+1 dims

G. Palumbo & JKP, arXiv:1301.2625