Classical and Quantum Simulation of Gauge Theories

Enrique Rico Ortega
15 January 2014

The team

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M. Müller

- IQOQI - Innsbruck University


M. Dalmonte


D. Marcos


P. Zoller

- Technical Univ. Vienna


P. Rabl

- Ulm University


T. Pichler


S. Montangero

The references

1. **Atomic quantum simulation of dynamical gauge fields coupled to fermionic matter: from string breaking to evolution after a quench,**
   D. Banerjee, M. Dalmonte, M. Muller, E. Rico, P. Stebler, U.J. Wiese, P. Zoller.
   Phys. Rev. Lett. 109, 175302 (2012)

2. **Atomic quantum simulation of U(N) and SU(N) non-abelian lattice gauge theories,**
   D. Banerjee, M. Bogli, M. Dalmonte, E. Rico, P. Stebler, U.J. Wiese, P. Zoller.
   Phys. Rev. Lett. 110, 125303 (2013)

3. **Superconducting circuits for quantum simulation of dynamical gauge fields,**
   D. Marcos, P. Rabl, E. Rico, P. Zoller.
   Phys. Rev. Lett. 111, 110504 (2013)

4. **Tensor networks for Lattice Gauge Theories and Atomic Quantum Simulation,**
   E. Rico, T. Pichler, M. Dalmonte, P. Zoller, S. Montangero.
   arXiv:1312.3127 (2013)

Why simulate Gauge theories?

The study of <u>Gauge</u> theories is the study of <u>Nature</u>.

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Gauge symmetry as a fundamental principle

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Gauge symmetry as an emergent phenomenon

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Gauge symmetry as a fundamental principle

Gauge symmetry as an emergent phenomenon

Gauge symmetry as a resource

Gauge symmetry as a fundamental principle

<u>Standard model</u>: for every force there is a gauge boson,

- The photon is the "carrier" of the electromagnetic force.

- The W^+, W^- and Z^0 are the "carriers" of the weak force.

- The gluons are the "carriers" of the strong force.

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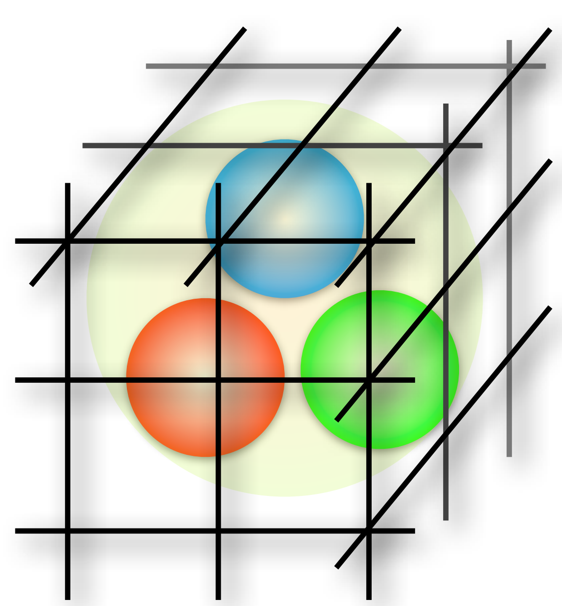
K. Wilson, Phys. Rev. D (1974)



Gauge theories on a discrete lattice structure.

Non-perturbative approach to fundamental theories of matter, e.g. Q.C.D.

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}\left[\psi, U\right] e^{-S[\psi,U]} O\left[\psi, U\right]$$

$$\sim \frac{1}{N} \sum_{n=1}^{N} e^{-S[\psi_n,U_n]} O\left[\psi_n, U_n\right]$$

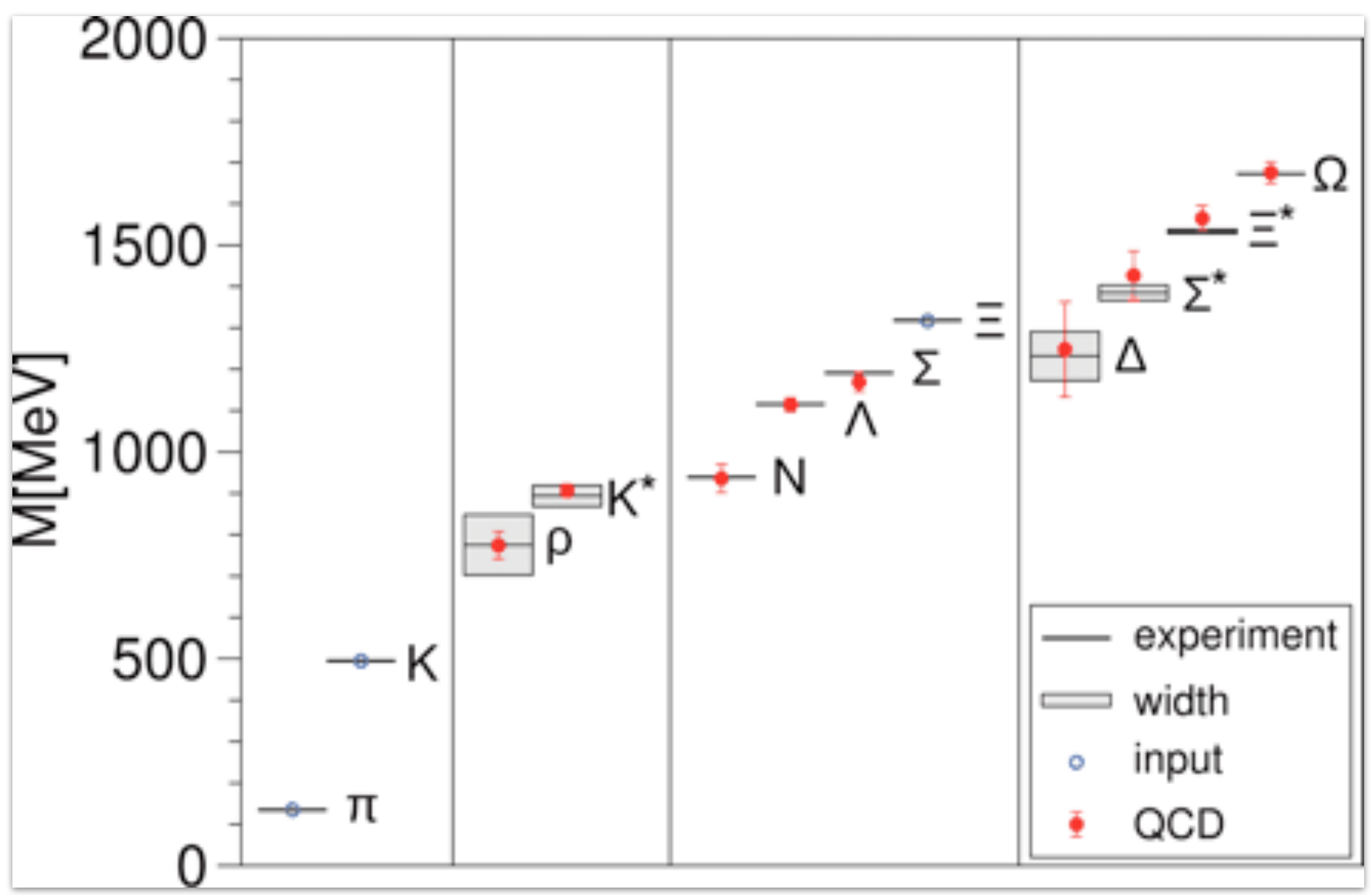
$$\sim \frac{1}{N} \sum_{P[U_n] \propto e^{-S[\psi_n,U_n]}} O\left[\psi_n, U_n\right]$$

Monte Carlo simulation = Classical Statistical Mechanics

Gauge symmetry as a fundamental principle

Achievements by classical Monte-Carlo simulations:

- first evidence of quark-gluon plasma
- ab-initio estimate of the entire hadronic spectrum



S. Dürr, et al., Science (2008)

Difficult problems on classical machines

Real time evolution:

Heavy ion experiments (collisions)

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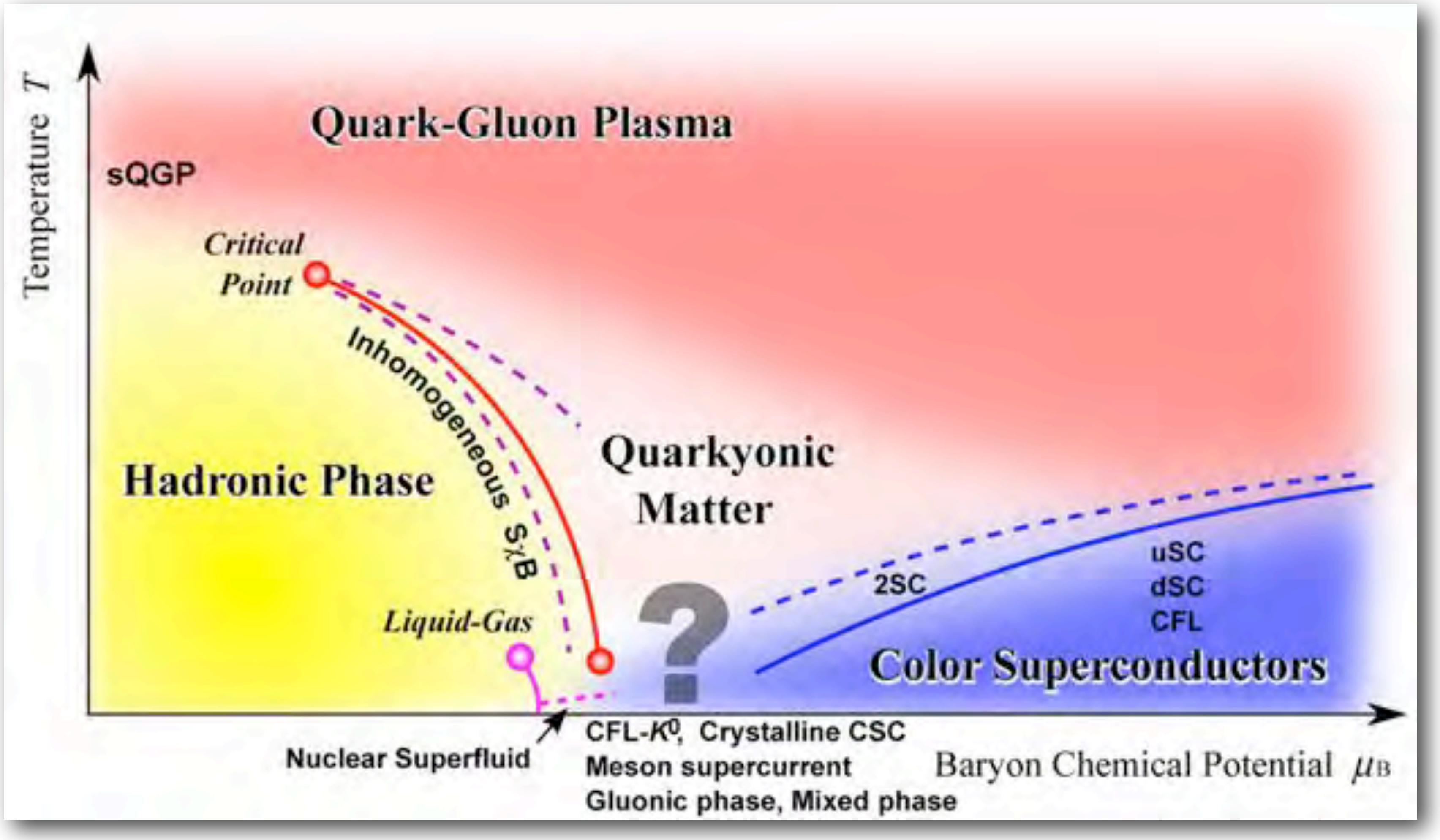
Heavy ion experiments
(collisions)

QCD with finite density of fermions:

Dense nuclear matter, color superconductivity
(phase diagram of QCD)

S. Hands, Contemp. Phys. (2001)
M.G. Alford, A. Schmitt, K. Rajagopal, T. Schäfer, Rev. Mod. Phys. (2008)
K. Fukushima, T. Hatsuda, Rep. Prog. Phys. (2011)



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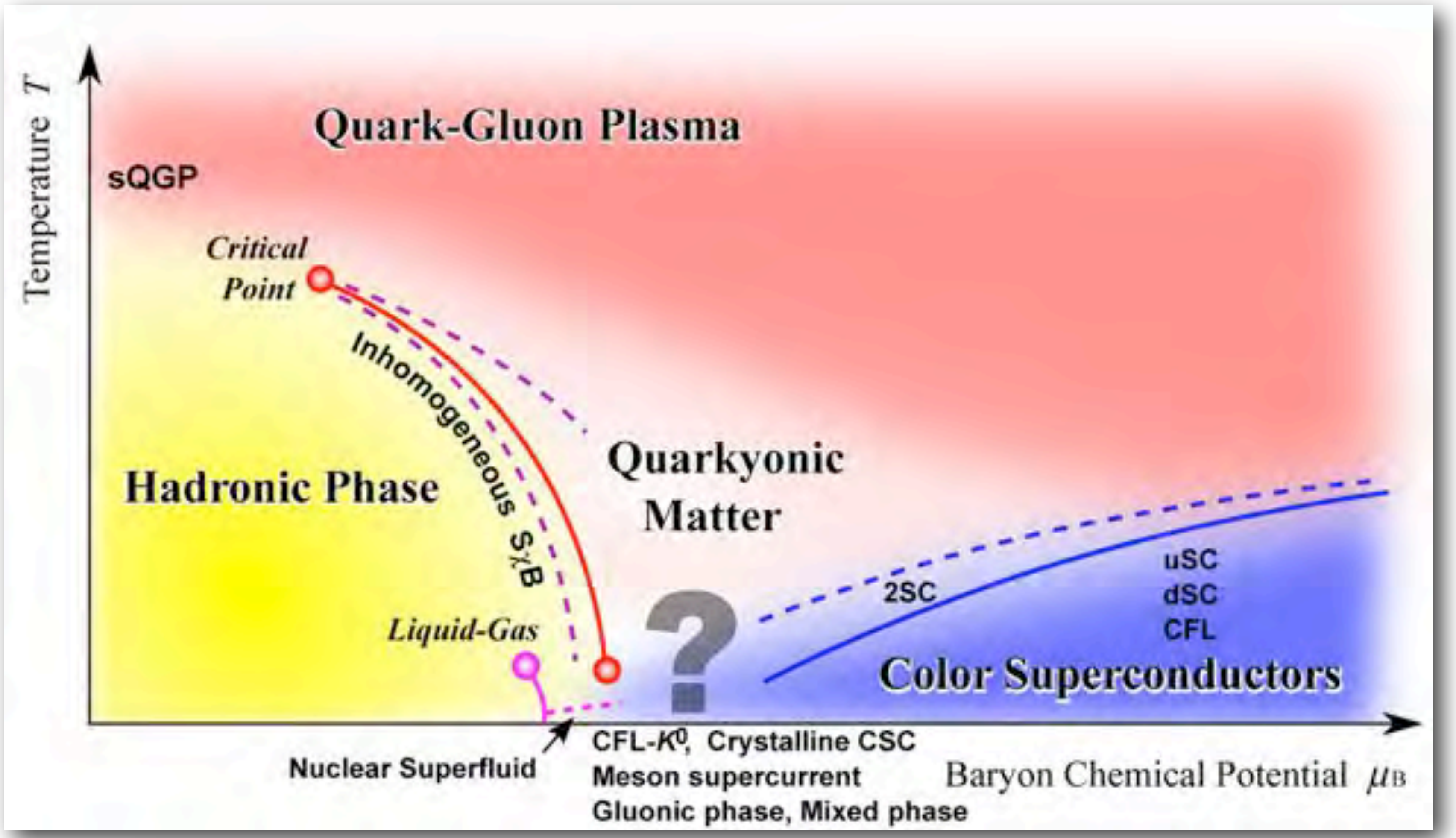
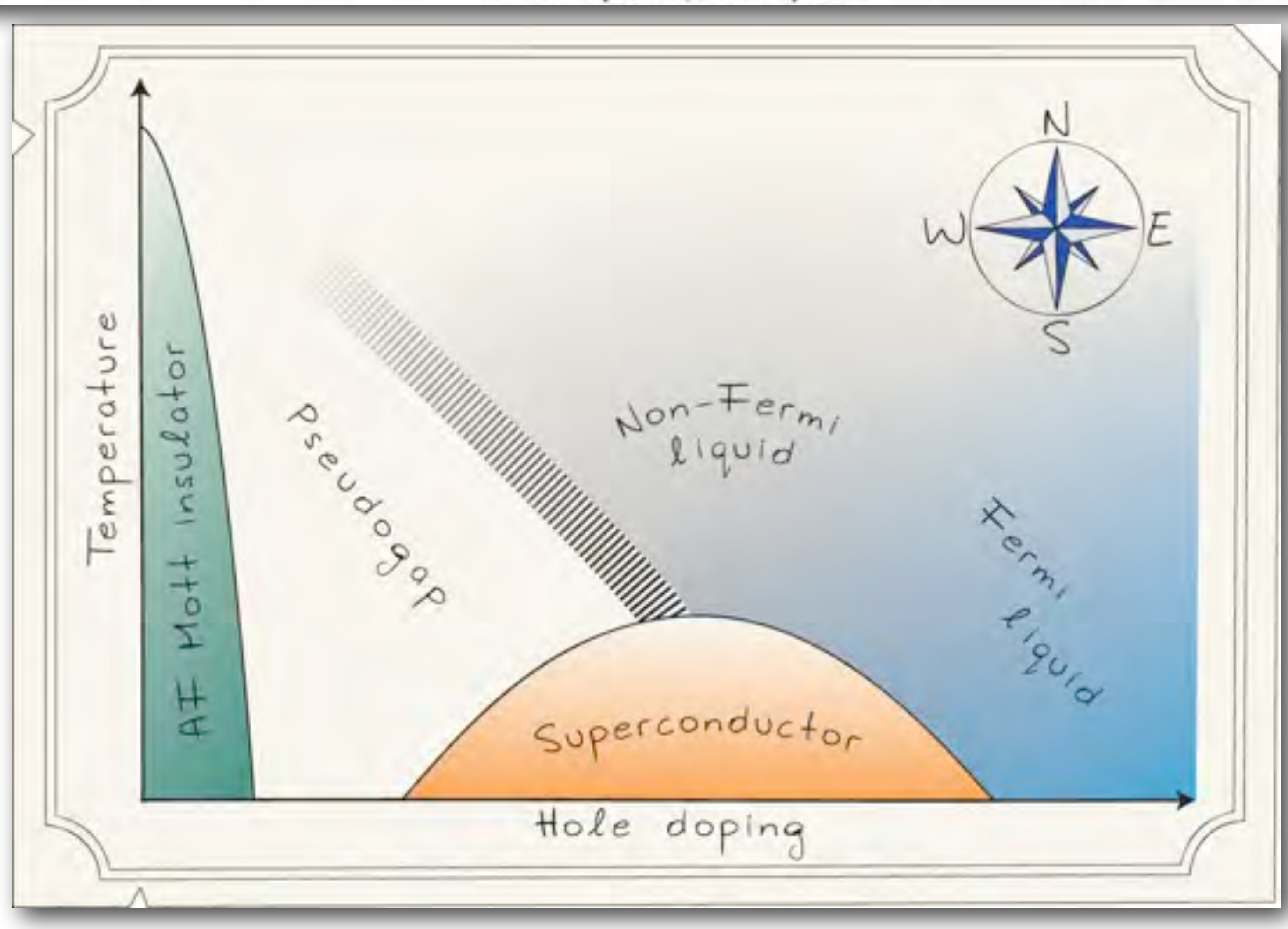
Frustrated spin models:

Spin liquid physics, RVB states
(High Tc superconductivity?)

E. Dagotto, Science (2005)
M.R. Norman, D. Pines, C. Kallinl, Adv. Phys. (2005)
P. Wahl, Nat. Phys. (2012)





What is this talk about?

Feynman's universal quantum simulator

Controlled quantum device which efficiently reproduces the dynamics of any other many-particle quantum system.

How?... cold atoms, ions, photons, superconducting circuit, etc.

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I. Bloch, J. Dalibard, S. Nascimbène
R. Blatt, C.F. Roos,
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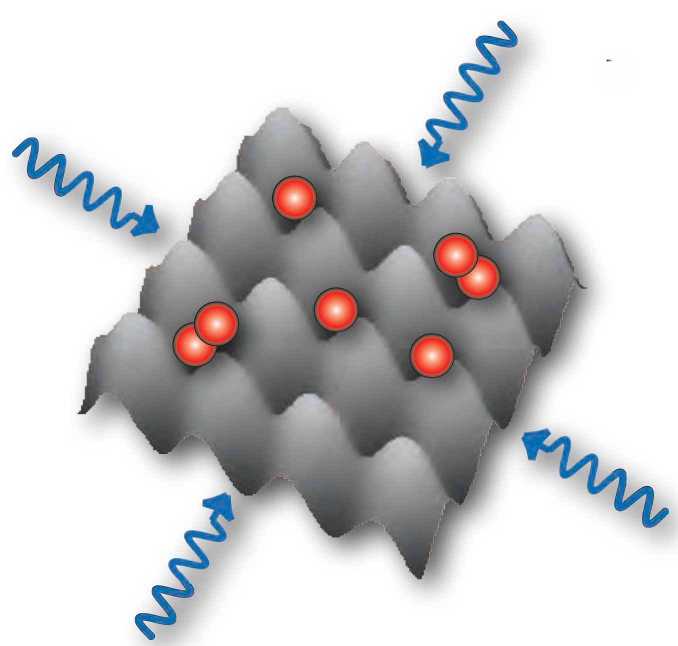

Related works at ICFO, Barcelona (M. Lewenstein's group) and MPQ, Munich (I. Cirac's group)

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See talks by G. Vidal,
G. Brennen and T. Osborne

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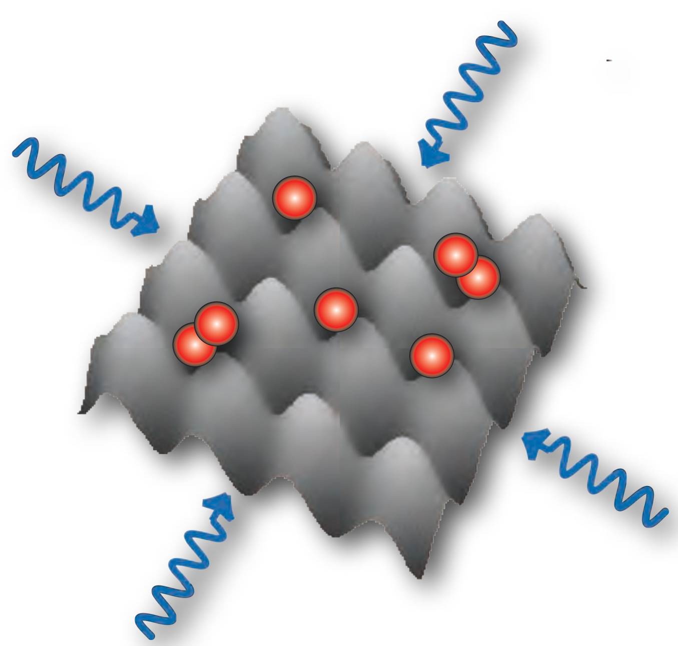
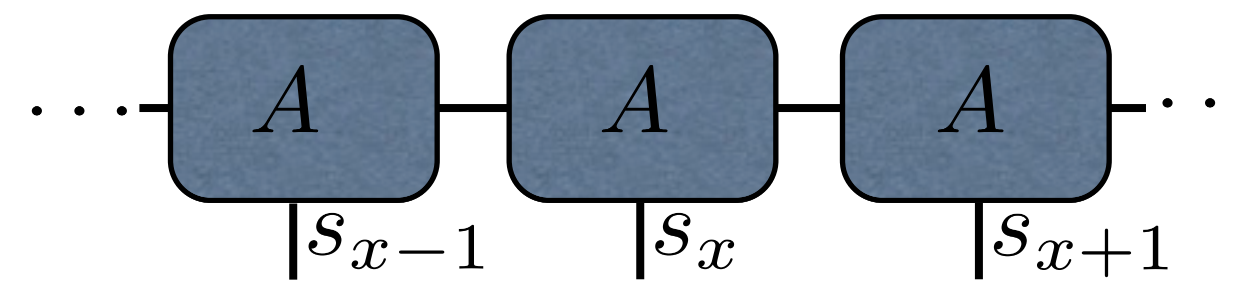


Tensor network simulation



Variational (non-perturbative) for Hamiltonian systems

Extremely useful in 1D systems (MPS)
Proposals and extensions in higher dimensions (TNS)

Ground states
Low-energy excitations
Thermal states
Time evolution

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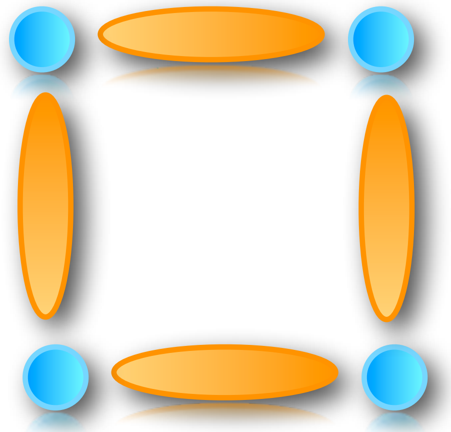
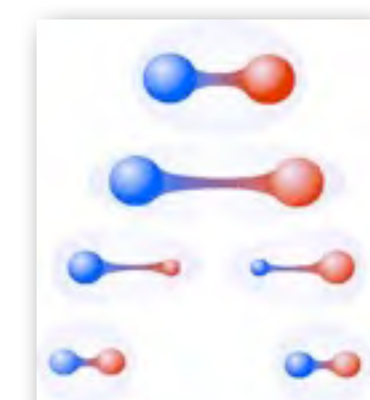


Lattice gauge theory

Simulation of Lattice Gauge Theories

ψ_x : fermion

$U_{x,x+1}$: gauge boson



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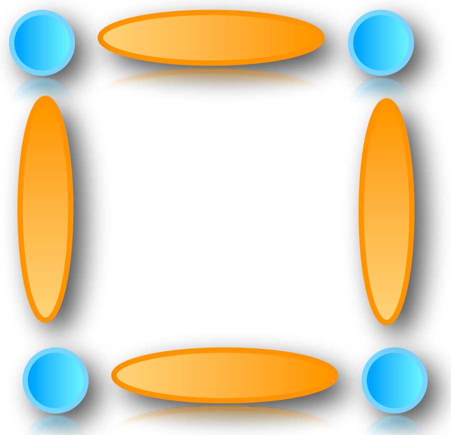

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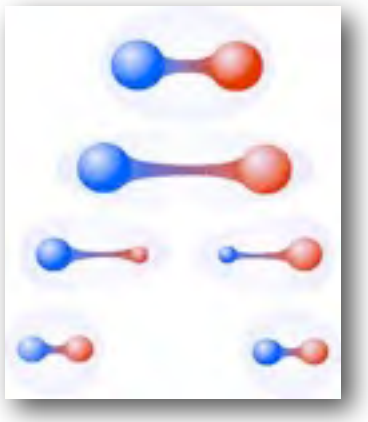

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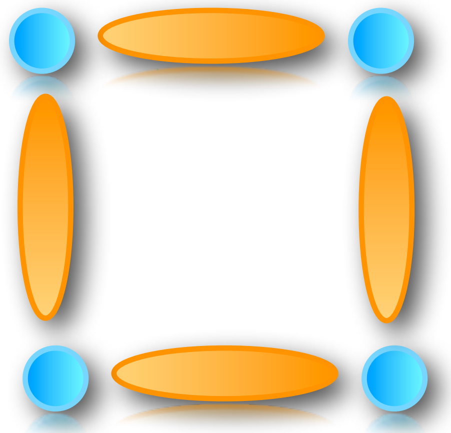

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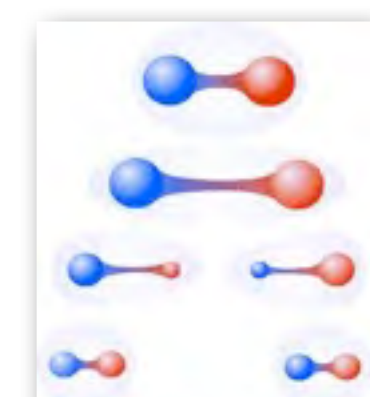
- **Hamiltonian formulation of lattice gauge theories.**
  (degrees of freedom, symmetry generators, dynamics)



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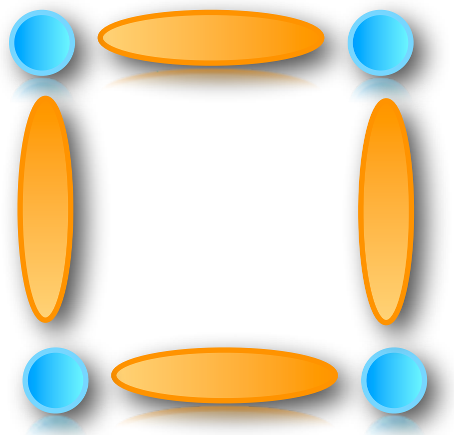

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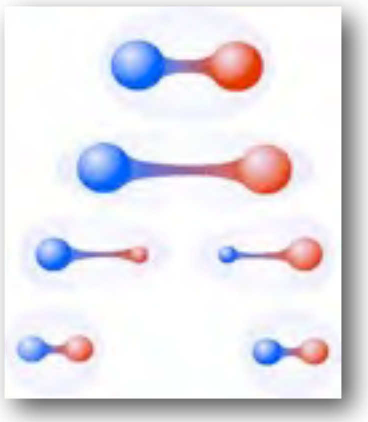

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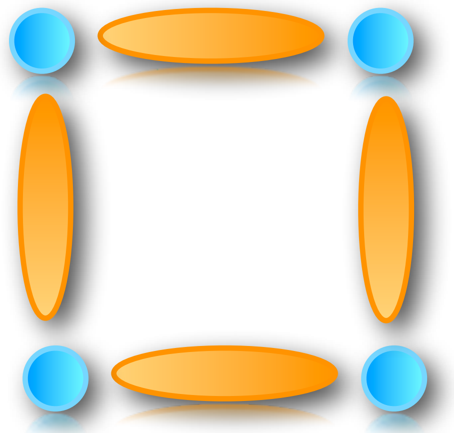

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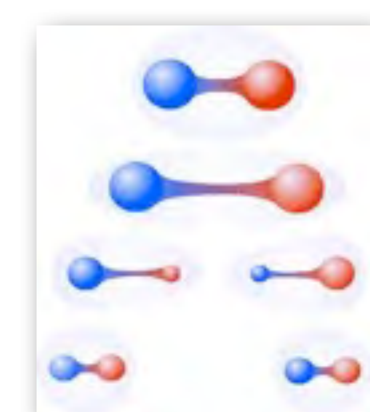

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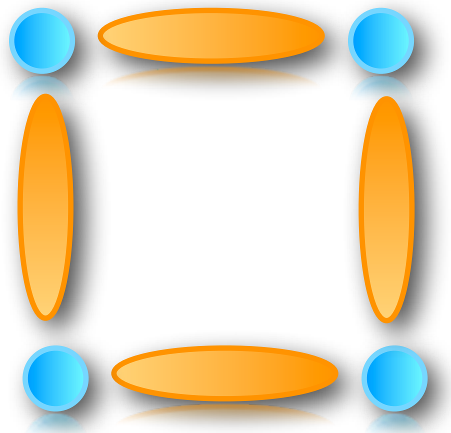

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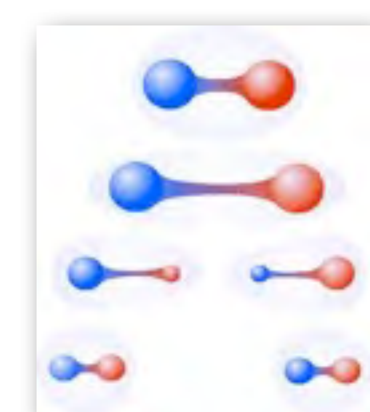
- **Implementation of quantum link models**



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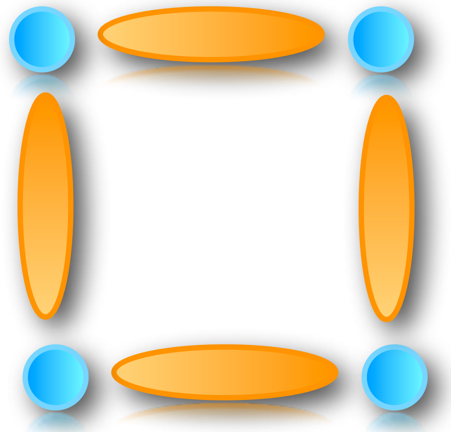
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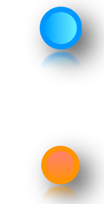
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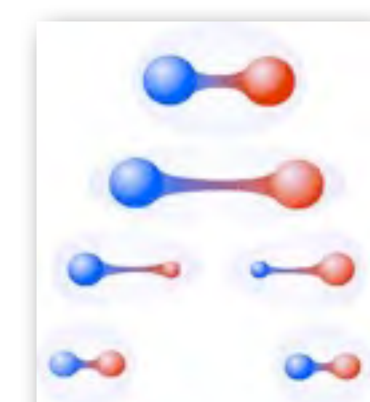
- Phase diagram of a U(1) Quantum Link Model in (1+1)-d
- Implementation of quantum link models
- Outlook & Conclusions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J.B. Kogut, L. Susskind, PRD (1975)

Global symmetries:

$$H = -t \sum_x \left(\psi_x^\dagger \psi_{x+1} + \mathrm{h.c.}\right)$$

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$$\psi_x \to \tilde{\psi}_x = V^\dagger \psi_x V = e^{-i\theta} \psi_x$$

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Force to introduce a link operator

$$H = -t \sum_x \left(\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \mathrm{h.c.}\right)$$

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Local conserved quantities

$$V = e^{-i\sum_y \theta_y G_y} \; : \quad [H, G_y] = 0, \, \forall y$$

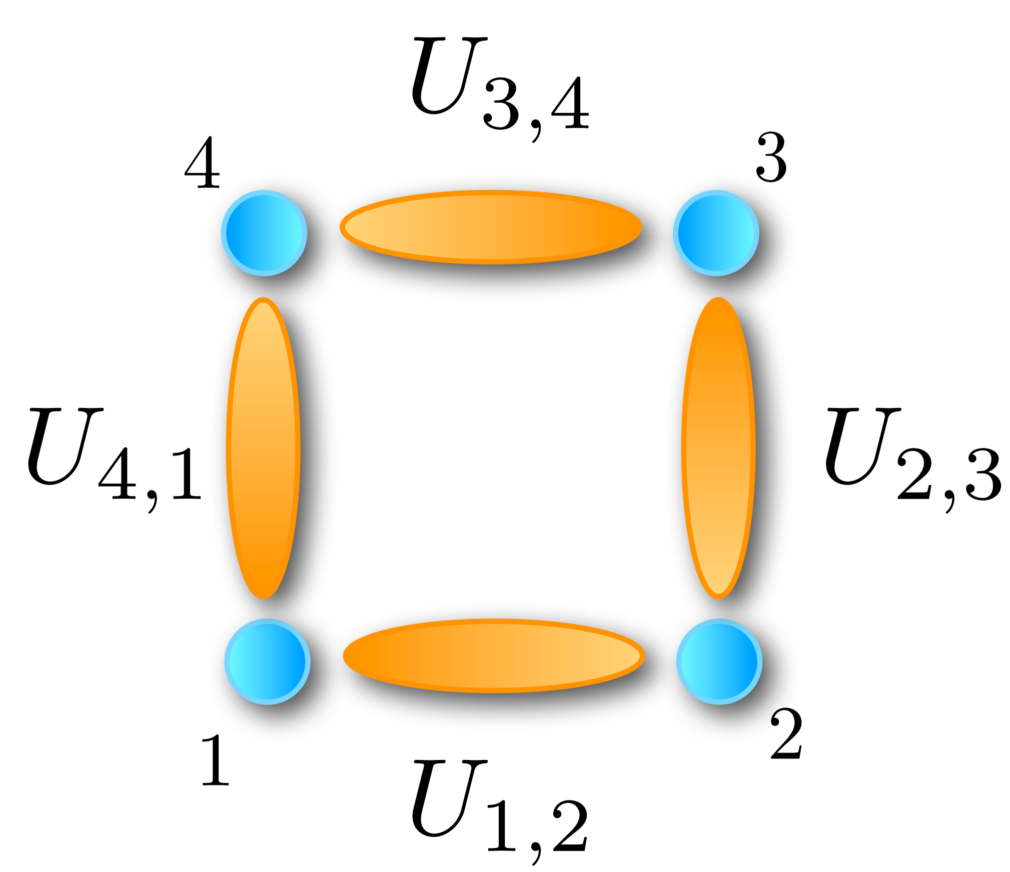
Hamiltonian formulation of lattice gauge theories

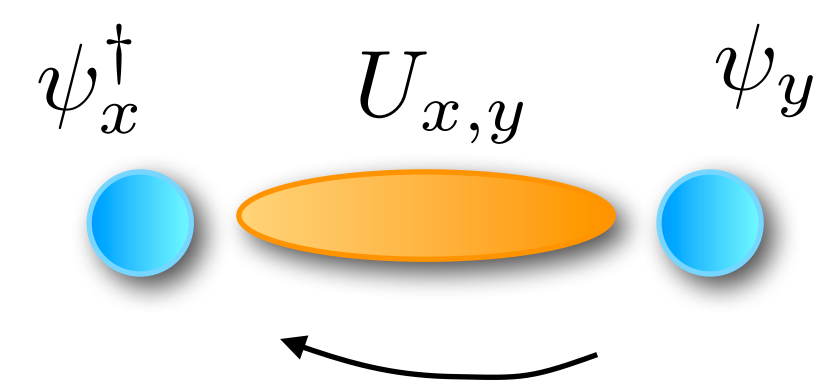
A gauge invariant model is defined by:

Hamiltonian formulation of lattice gauge theories

A gauge invariant model is defined by:

Set of local operators acting on the vertices (matter fields) and on the links (gauge fields)





(operators acting on a Hilbert space)

J.B. Kogut, L. Susskind, PRD (1975)
J.B. Kogut, Rev. Mod. Phys. (1979)
ref. Creutz and Montvay/Muenster books

Hamiltonian formulation of lattice gauge theories

Gauge invariant quantum Hamiltonian:

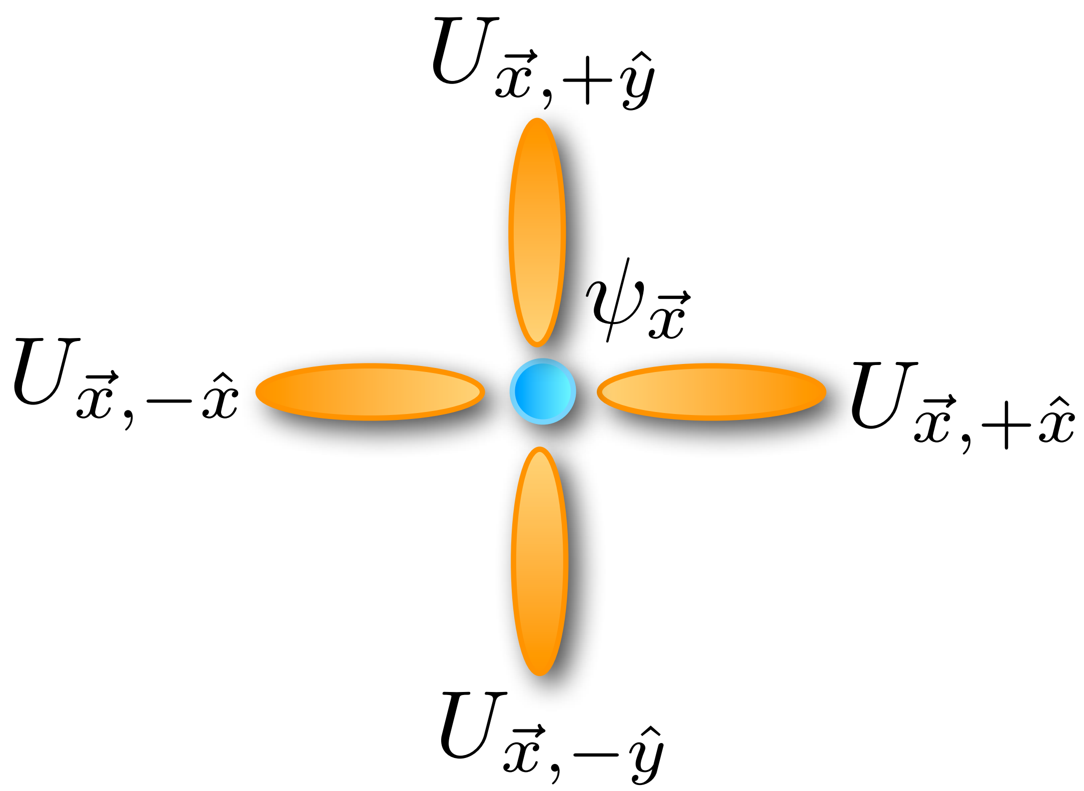
$$\underbrace{[H}_{\text{Hamiltonian}}, \underbrace{G_x}_{\text{Gauge (local) generators}}] = 0 \quad \forall x$$

Local conserved quantitites
Gauge (local) symmetries

Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations

Generators of the local U(1) symmetry:



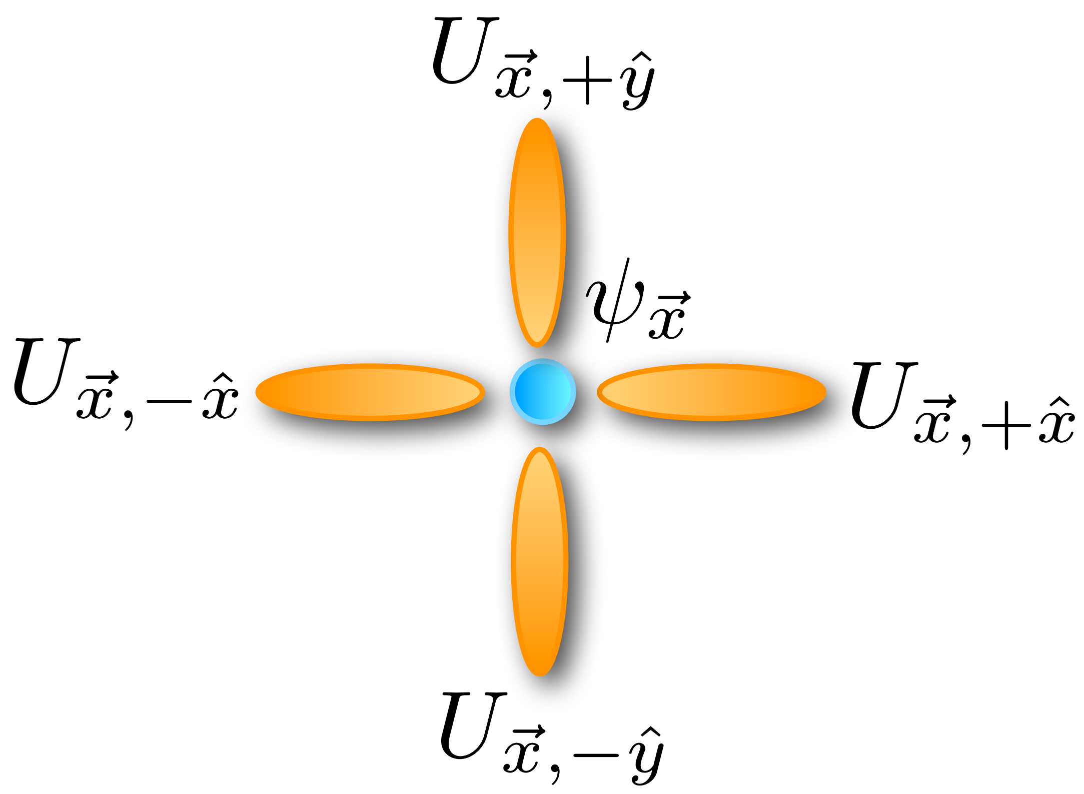
Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations

Generators of the local U(1) symmetry:

Local phase transformation

$$
\begin{aligned}
\tilde{\psi}_x &= e^{i\theta_x}\psi_x \\
&= e^{i\sum_z \theta_z G_z}\psi_x e^{-i\sum_z \theta_z G_z} \\
\tilde{U}_{x,y} &= e^{-i\theta_x}U_{x,y}e^{i\theta_y} \\
&= e^{i\sum_z \theta_z G_z}U_{x,y}e^{-i\sum_z \theta_z G_z}
\end{aligned}
$$



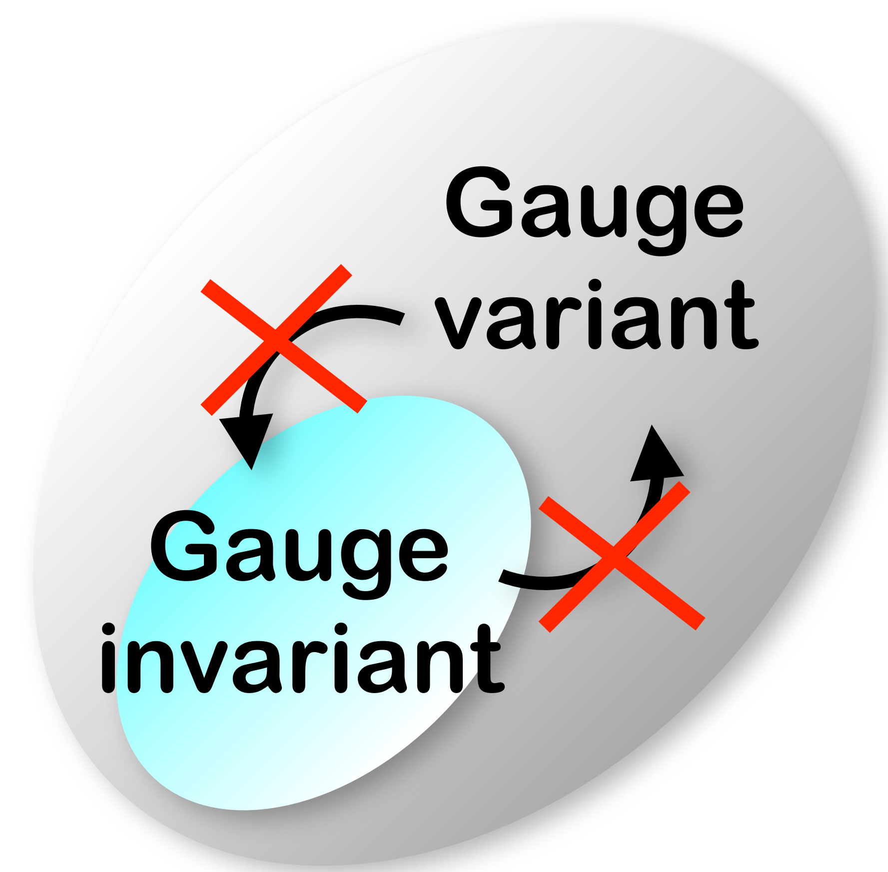
Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations

Define the "physical" Hilbert space:

$$[H, G_x] = 0 \quad \forall x$$

$$G_x |\text{physical}\rangle = 0$$



$$\mathcal{H} = \mathcal{H}_{\text{inv}} \oplus \mathcal{H}_{\text{var}}$$

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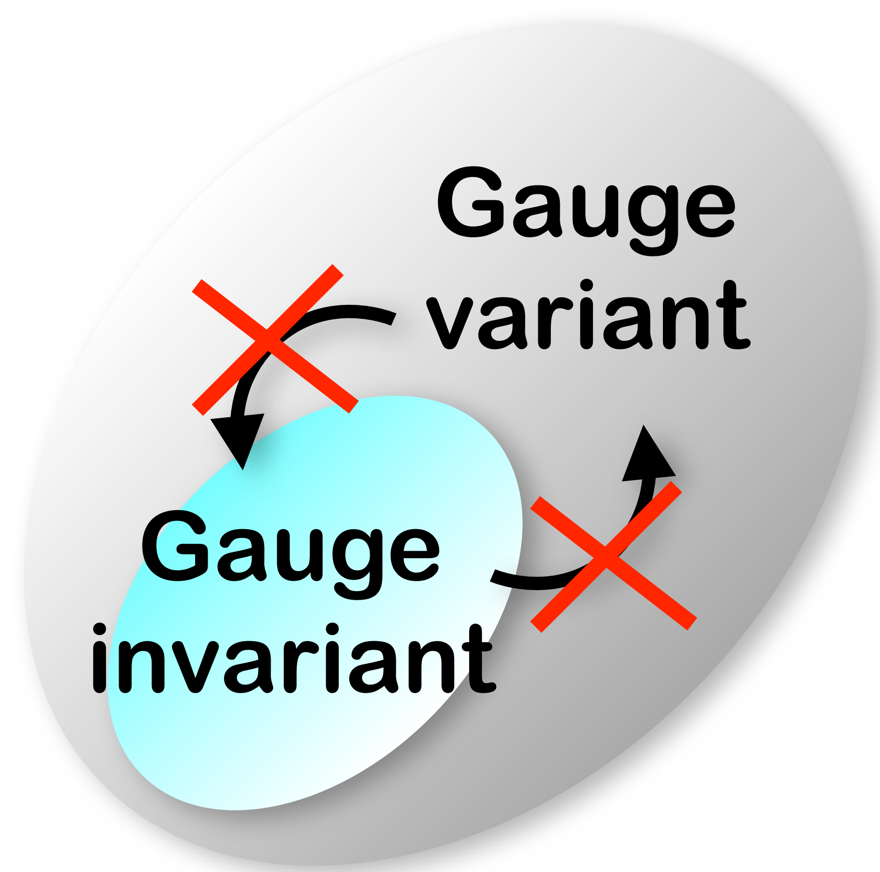
ex.- U(1) group

$$G_x = \psi_x^\dagger \psi_x - \sum_{\hat{\mu}} E_{x,x+\hat{\mu}} - E_{x-\hat{\mu},x}$$

matter **electric field**

$$\left[\rho - \vec{\nabla}\cdot\vec{E}\right]_{\mathrm{phys}} = 0$$

: Gauss' law



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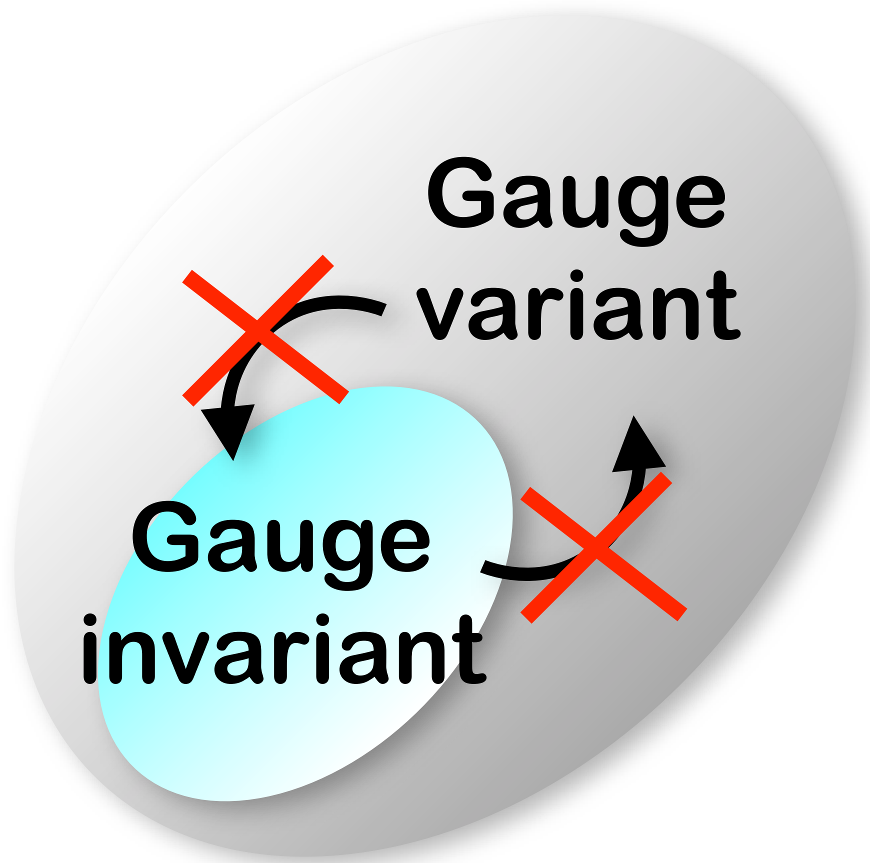


$$\left[\rho - \vec{\nabla} \cdot \vec{E}\right]_{\mathrm{phys}} = 0$$

: Gauss' law

Wilson's formulation:

$$U_{x,y} \rightarrow e^{i\phi_{x,y}} \qquad E_{x,y} \rightarrow -i\frac{\partial}{\partial \phi_{x,y}} \qquad \mathcal{H} = \mathcal{H}_{\mathrm{inv}} \oplus \mathcal{H}_{\mathrm{var}}$$

Quantum link formulation

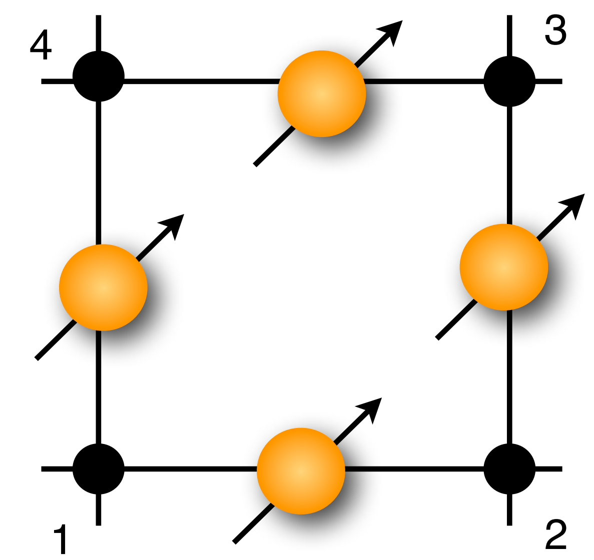
Gauge fields span a finite-dimensional local Hilbert space

D. Horn, Phys. Lett. B (1981)
P. Orland, D. Röhrlich, Nucl. Phys. B (1990)
S. Chandrasekharan, U.-J. Wiese, Nucl. Phys. B (1997)

$$U_{x,x+1} \to S^{+}_{x,x+1}$$

$$E_{x,x+1} \to S^{z}_{x,x+1}$$

Spin S = ½, 1, ...



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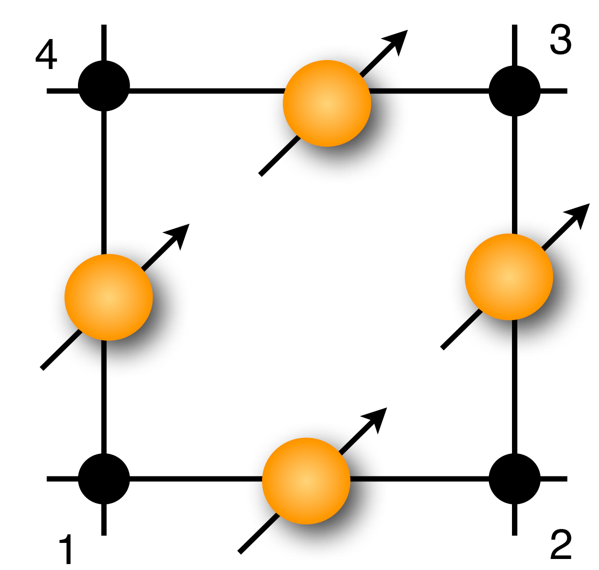


Spin S = ½, 1, ...

Q.C.D. can be formulated as a non-abelian quantum link model

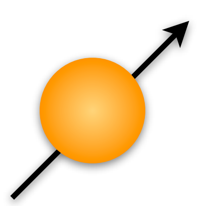
R. Brower, S. Chandrasekharan, U.-J. Wiese, Phys. Rev. D (1999)
R. Brower, S. Chandrasekharan, S. Riederer, U.-J. Wiese, Nucl. Phys. B (2004)

Quantum Link models

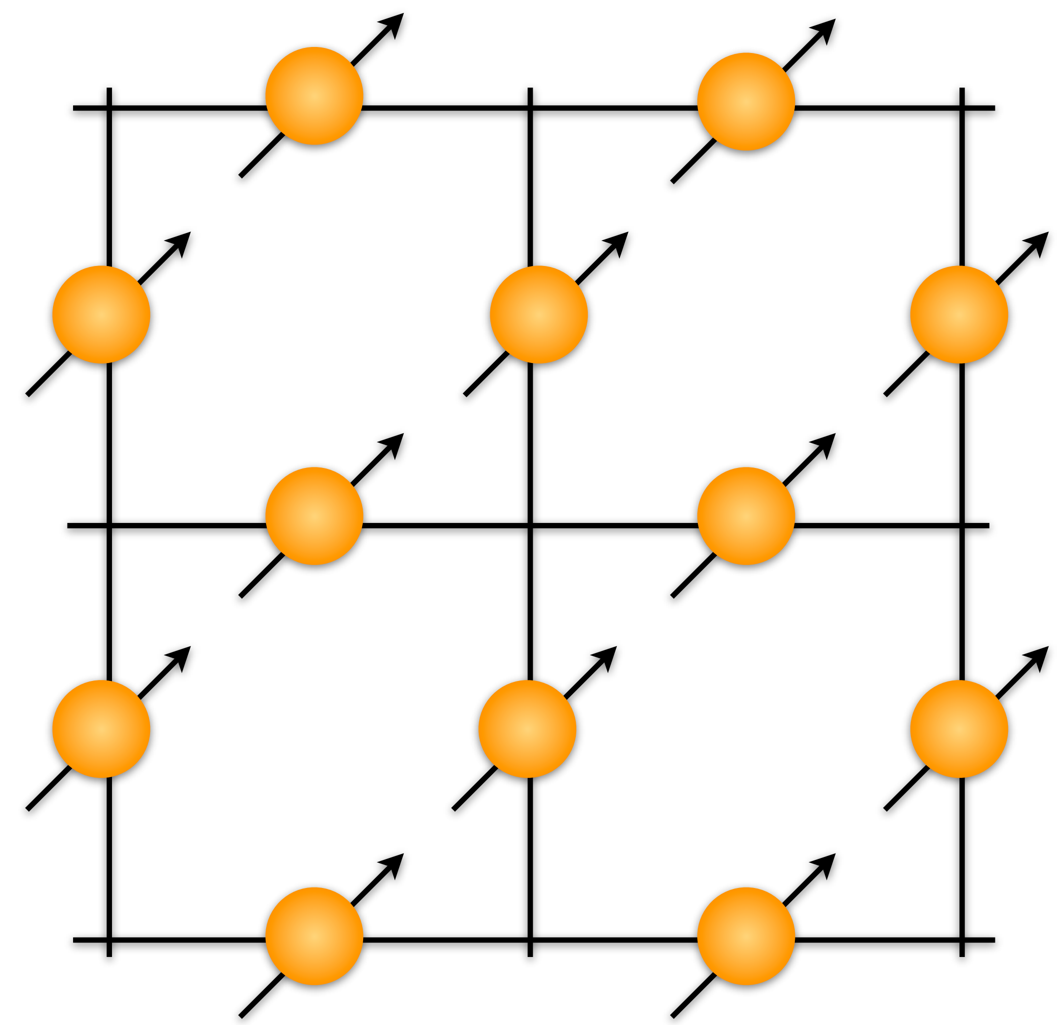
Connections with Condensed Matter (U(1) gauge theory-Quantum Spin Ice model)

Local degrees of freedom.-

Quantum two level system living on the link



$$\{\sigma^{(3)}_{x,y}, \sigma^{(+)}_{x,y}, \sigma^{(-)}_{x,y}\}$$



L. Balents, Nature (2010)
C. L. Henley, Ann. Rev. Cond. Matt. Phys. (2010)
C. Castelnovo, R. Moessner, and S.L. Sondhi, Ann. Rev. Cond. Matt. Phys. (2012)

Quantum Link models

Connections with Condensed Matter (U(1) gauge theory-Quantum Spin Ice model)

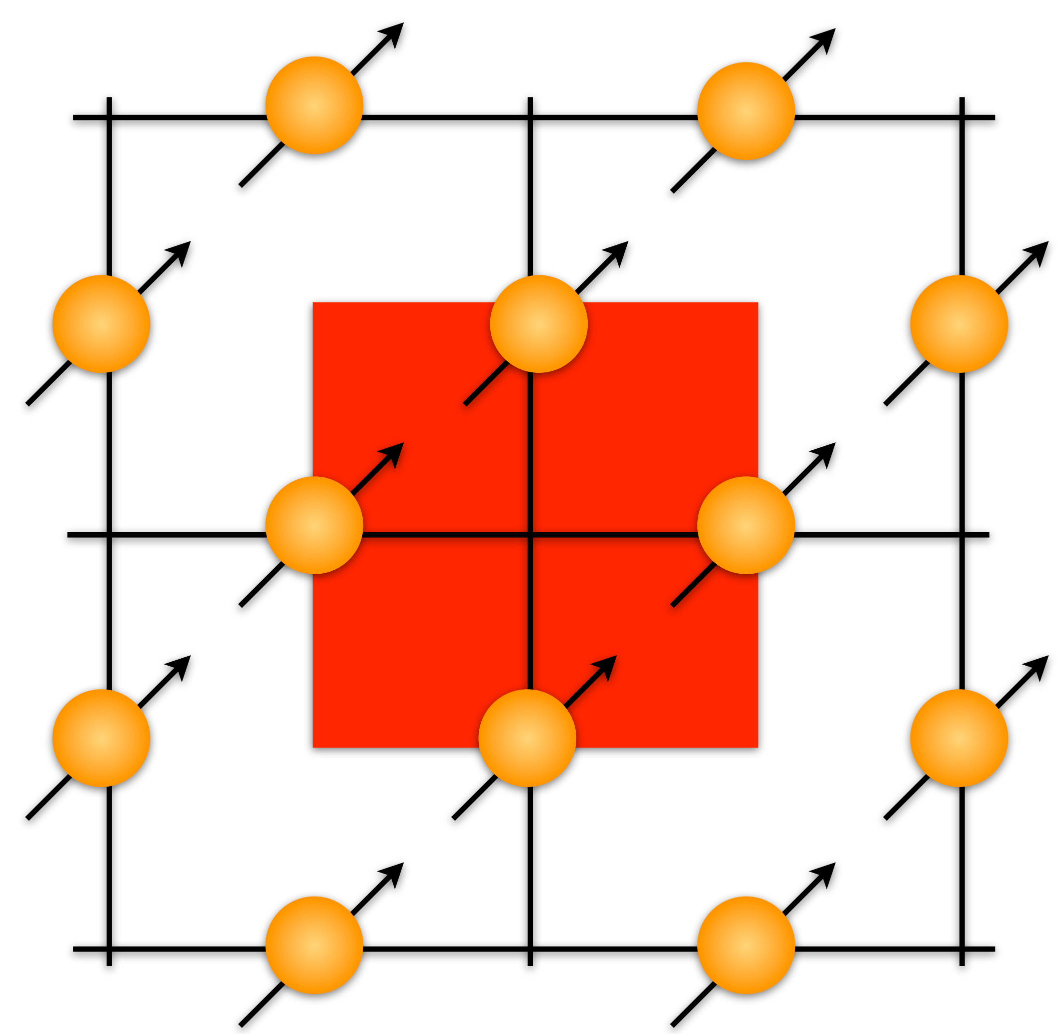
Local generator of gauge transformations.-

Local generator around every vertex

$$\exp\left[i\frac{\theta_{\text{vert}}}{2}G_{\text{vert}}\right]\sigma^{(+)}_{1,2}\exp\left[-i\frac{\theta_{\text{vert}}}{2}G_{\text{vert}}\right]=e^{i\theta_{\text{vert}}}\sigma^{(+)}_{1,2}$$

$$G_{\text{vert}}=\sigma^{(3)}_{1,2}+\sigma^{(3)}_{2,3}+\sigma^{(3)}_{3,4}+\sigma^{(3)}_{4,1}$$

U(1) gauge transformation



Quantum Link models

Connections with Condensed Matter
(U(1) gauge theory-Quantum Spin Ice model)

Local generator of gauge transformations.-

"Physical" Hilbert space (Gauss' law)

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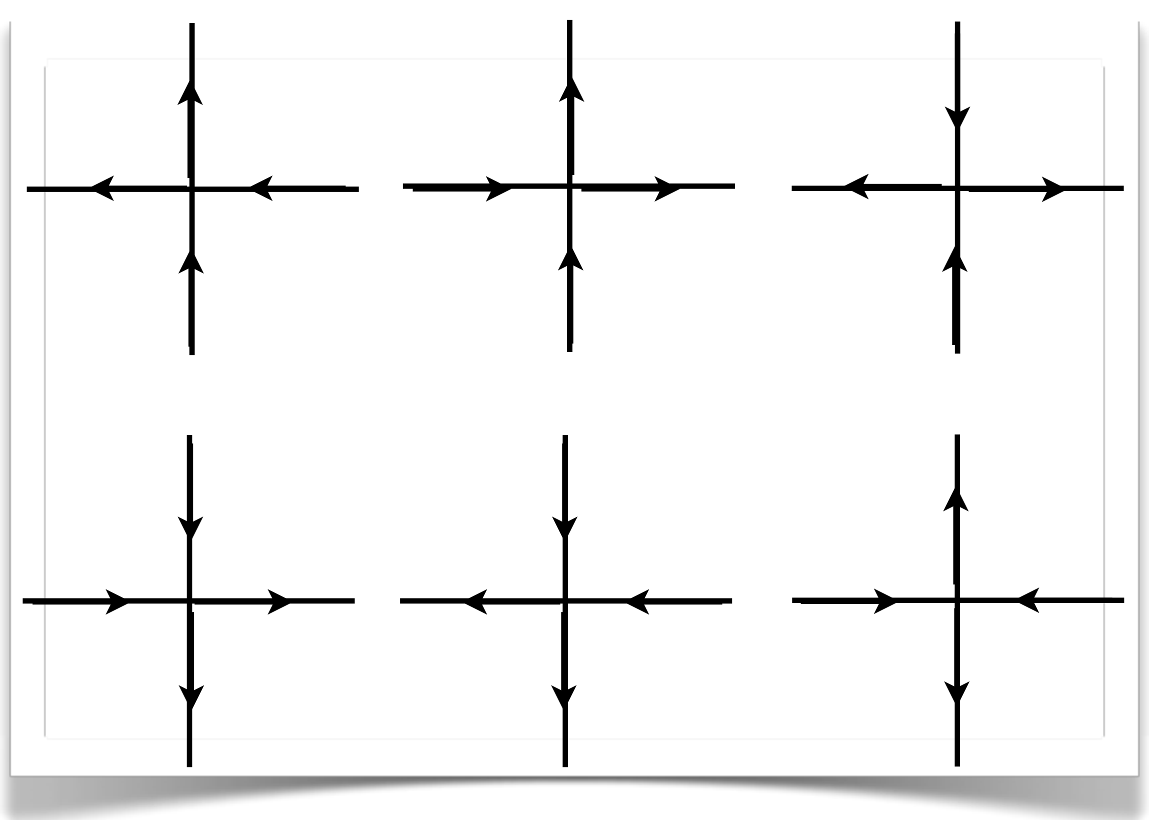
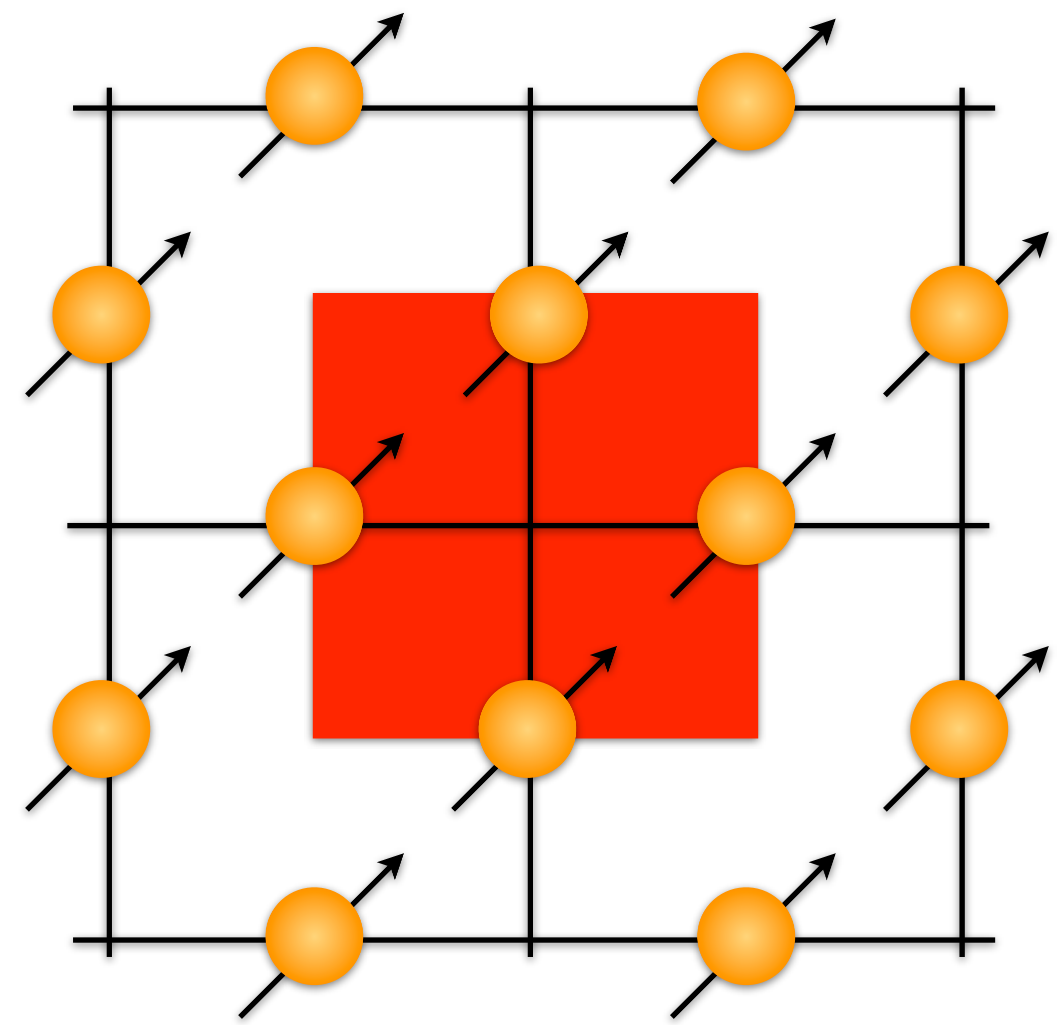


6-vertex model:
zero magnetization subspace



Quantum Link models

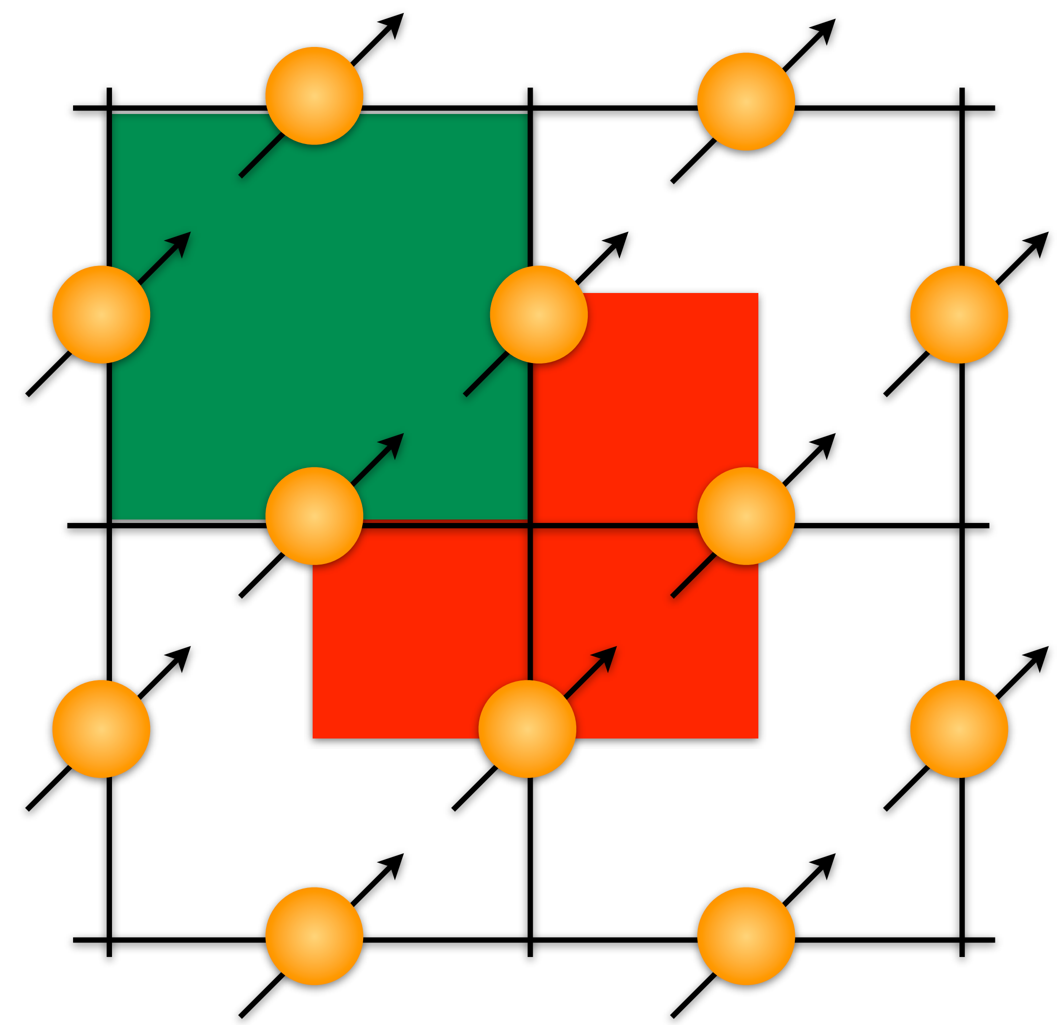
Connections with Condensed Matter (U(1) gauge theory-Quantum Spin Ice model)

Gauge invariant Hamiltonian.-

$$H = -\sum_{\text{plaq}} \left[\sigma^+_{1,2}\sigma^-_{2,3}\sigma^+_{3,4}\sigma^-_{4,1} + \sigma^-_{1,2}\sigma^+_{2,3}\sigma^-_{3,4}\sigma^+_{4,1}\right]$$

magnetic term

$$[H, G_{\text{vert}}] = 0, \quad \forall \text{ vertex}$$



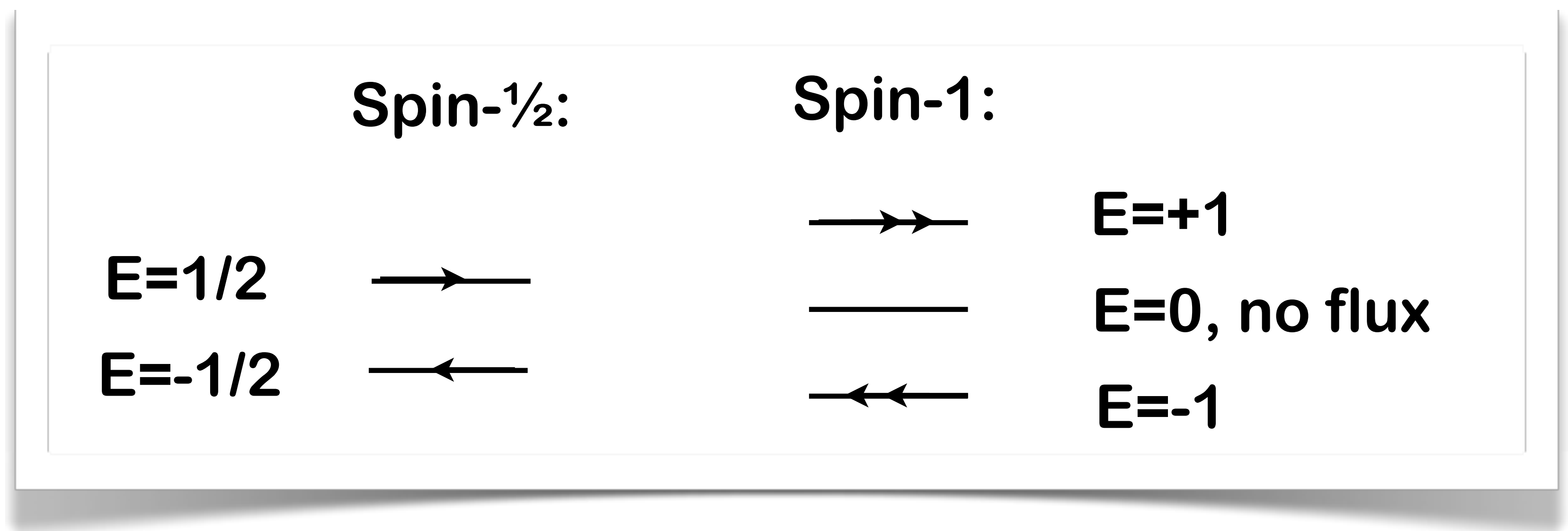
Quantum Link models

Local degrees of freedom.-

Quantum link carrying an electric flux

$$U_{x,y} \equiv S^{+}_{x,y}$$

$$E_{x,y} \equiv S^{(3)}_{x,y}$$



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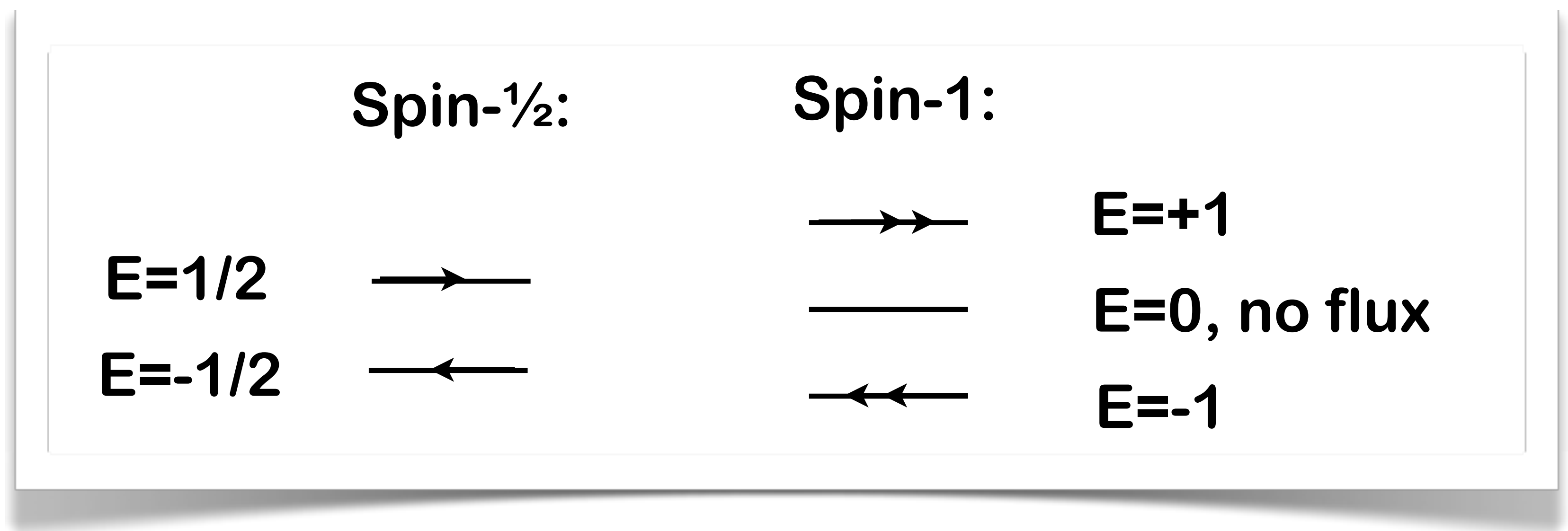


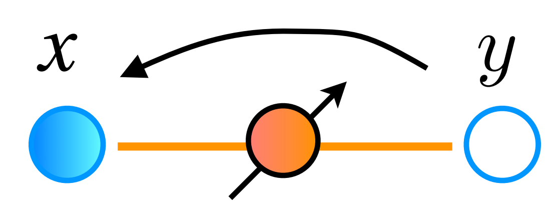
Gauge invariant Hamiltonian.-

$$H = \frac{g^2}{2} \sum_{\langle x,y \rangle} [\underset{\uparrow\, S^{(3)}_{x,y}}{E_{x,y}}]^2 - \frac{1}{4g^2} \sum_{\text{plaq}} \left[U^{\dagger}_{1,2} U_{2,3} U^{\dagger}_{3,4} U_{4,1} + U_{1,2} U^{\dagger}_{2,3} U_{3,4} U^{\dagger}_{4,1} \right]$$

with $U^{\dagger}_{x,y} \to S^{-}_{x,y}$ and $U_{x,y} \to S^{+}_{x,y}$

Electric term **Magnetic term**

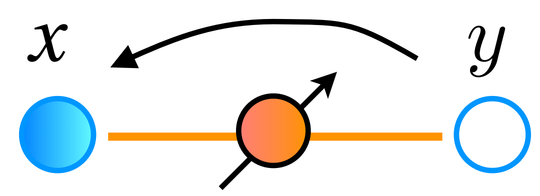
U(1) Quantum Link model with matter



$$H = -t \sum_{\langle x,y \rangle} \psi_x^\dagger S_{xy}^+ \psi_y + \mathrm{h.c.} + \dots$$

Matter - gauge interaction
= hopping of fermions mediated by a quantum link
= hopping of particle and flips spin

U(1) Quantum Link model with matter



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Matter - gauge interaction
= hopping of fermions mediated by a quantum link
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Gauge invariance

$[G_x, H] = 0 \quad \forall x$ **(physical Hilbert space)**

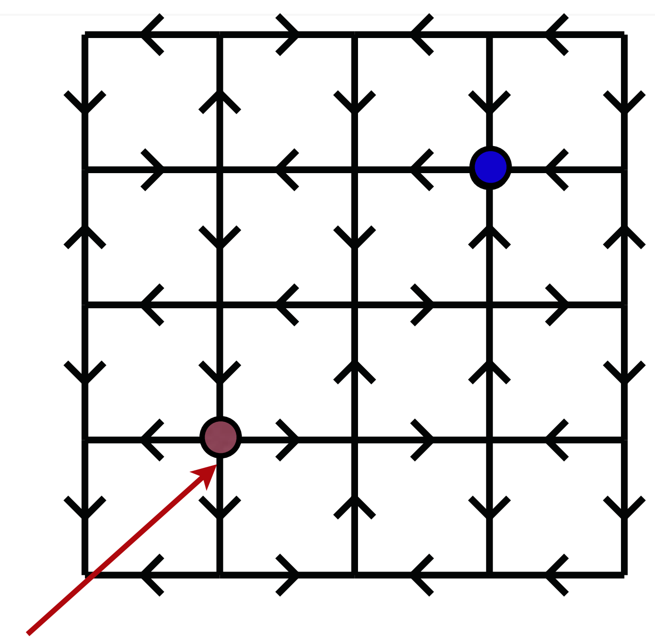
Gauss law

$G_x|\Psi\rangle = 0 \quad \forall x$

$$G_x = \psi_x^\dagger \psi_x - \sum_i \left(E_{x,x+\hat{i}} - E_{x-\hat{i},x} \right)$$

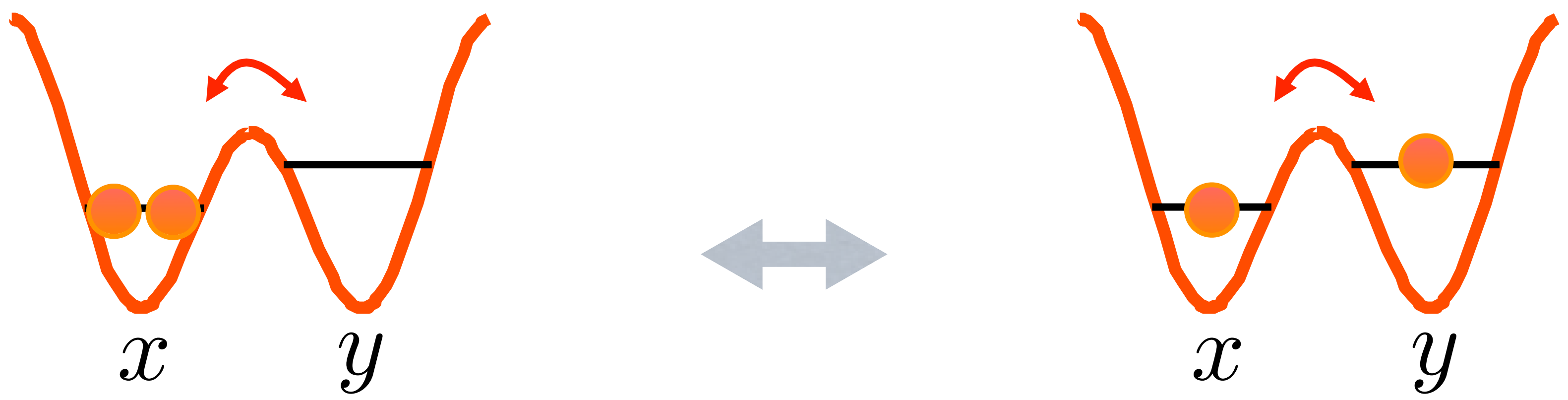
$$\updownarrow$$

$$\rho - \nabla \cdot E = 0$$



dynamical charge

Schwinger representation



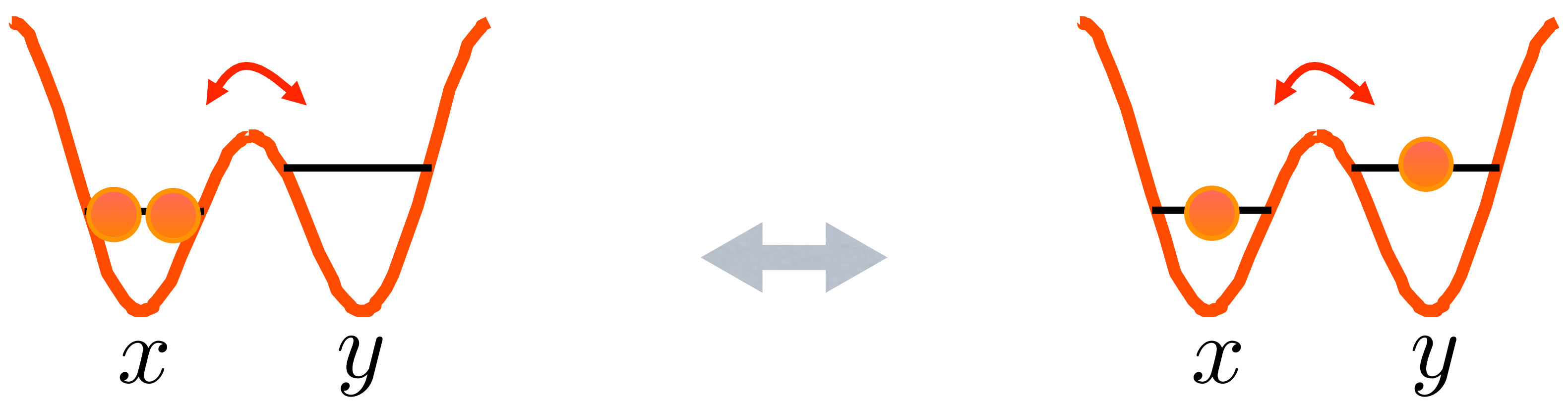
Link operator $$U_{x,y} \equiv S^{+}_{x,y} = c^{\dagger}_{y} c_{x}$$

Electric field [U(1) generator] $$E_{x,y} \equiv S^{(3)}_{x,y} = \frac{1}{2}\left[c^{\dagger}_{y} c_{y} - c^{\dagger}_{x} c_{x}\right]$$

$\{c_x, c^{\dagger}_{y}\} = \delta_{x,y}$ **Schwinger fermions (rishons)**

$[c_x, c^{\dagger}_{y}] = \delta_{x,y}$ **Schwinger bosons**

Schwinger representation



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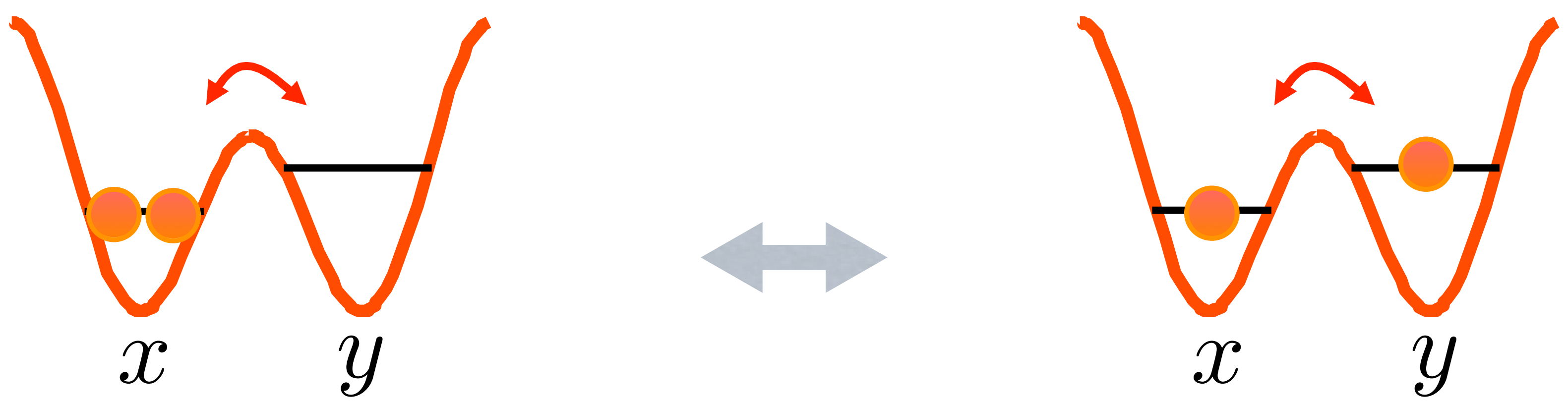
Spin representation:

$N_{x,y} = c^{\dagger}_{y} c_{y} + c^{\dagger}_{x} c_{x}$ $\quad \left[\vec{S}_{x,y}\right]^2 \equiv \frac{N_{x,y}}{2}\left[\frac{N_{x,y}}{2} + 1\right]$

Schwinger representation



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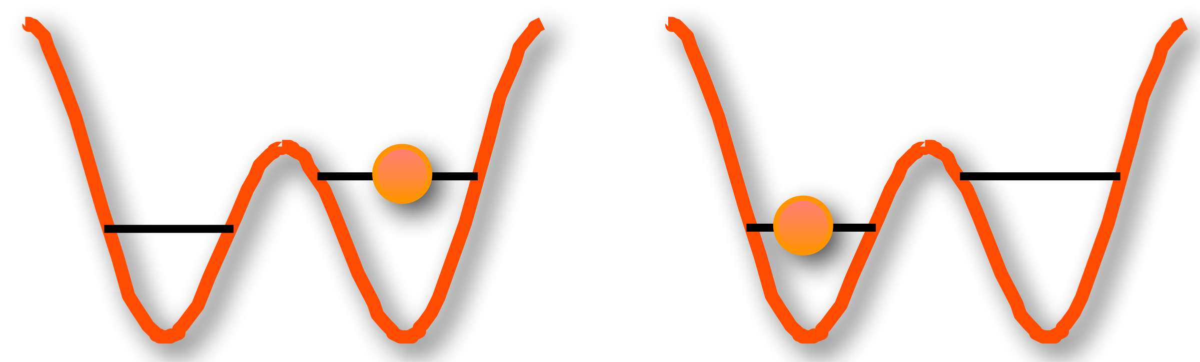
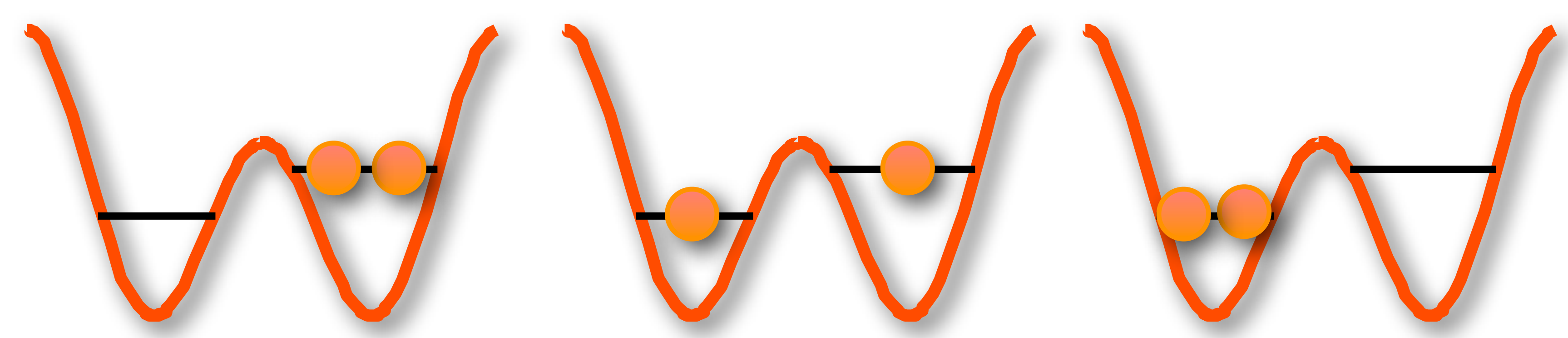
Spin-½:

E=1/2 $\rightarrow$ **E=-1/2** $\leftarrow$



Spin-1:

E=+1 $\twoheadrightarrow$ **E=0** — **E=-1** $\twoheadleftarrow$



Tensor networks and gauge symmetry

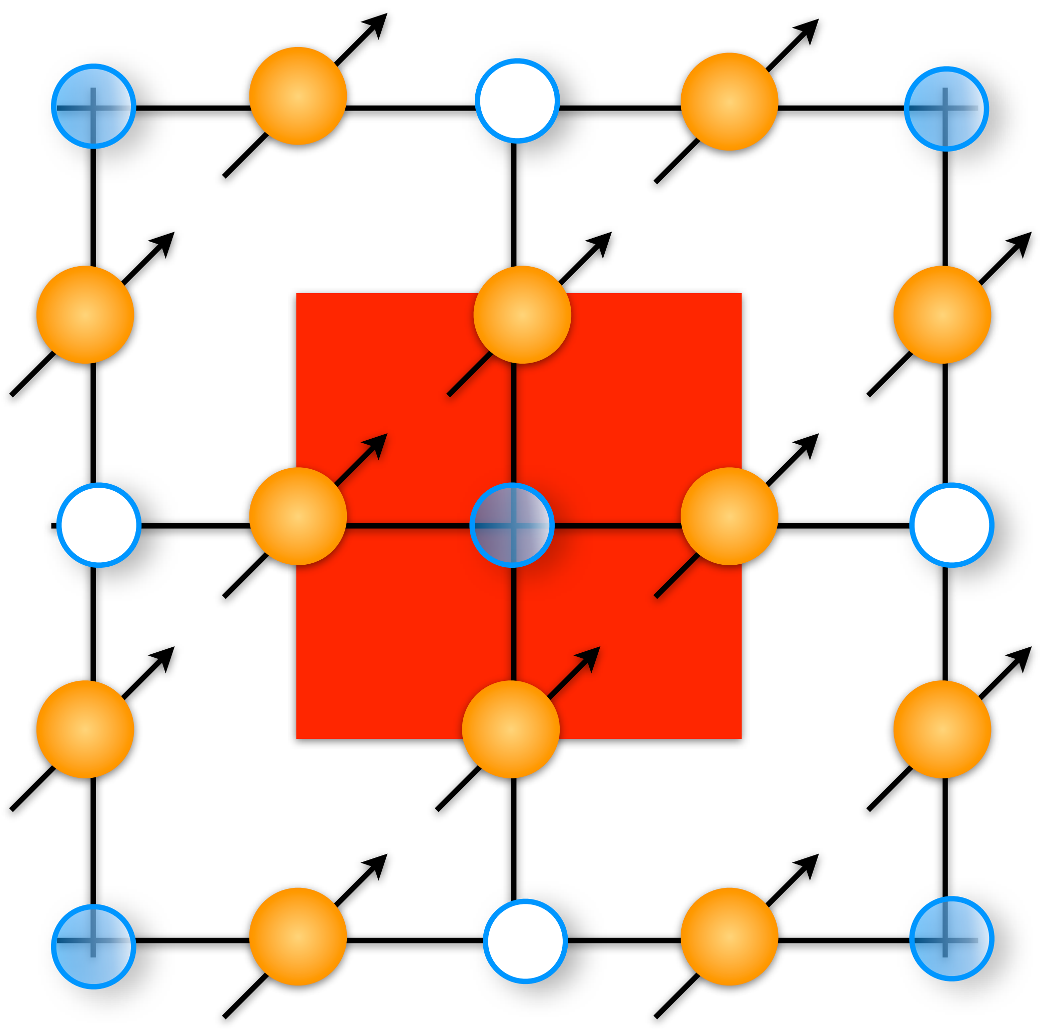
Two independent local constraints

"Physical" Hilbert space (Gauss' law)

Gauge generator

$$G_{\text{vert}}|\text{phys}\rangle = 0$$

$$G_x = \psi_x^\dagger \psi_x - \sum_i \left(E_{x,x+\hat{\imath}} - E_{x-\hat{\imath},x} \right)$$



Tensor networks and gauge symmetry

Two independent local constraints

"Physical" Hilbert space (Gauss' law)

All the gauge constraints are independent

Gauge generator

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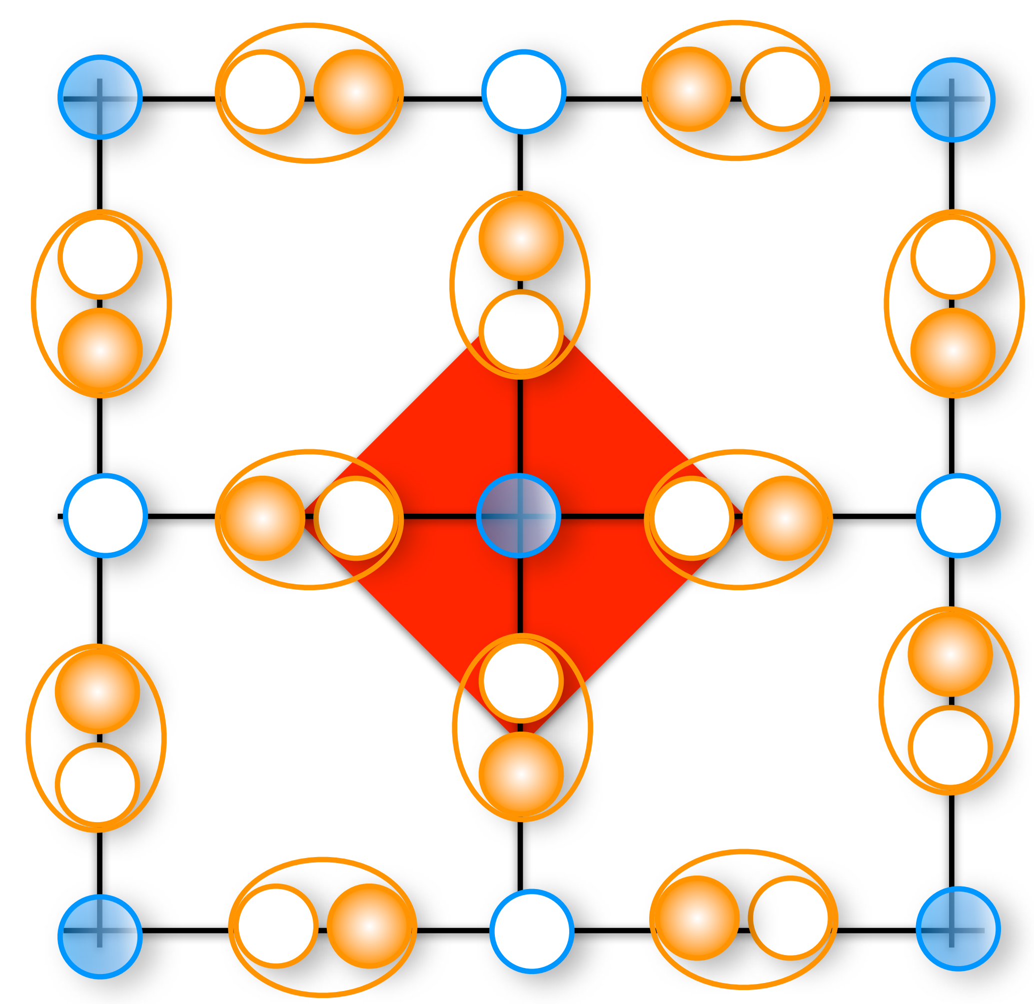
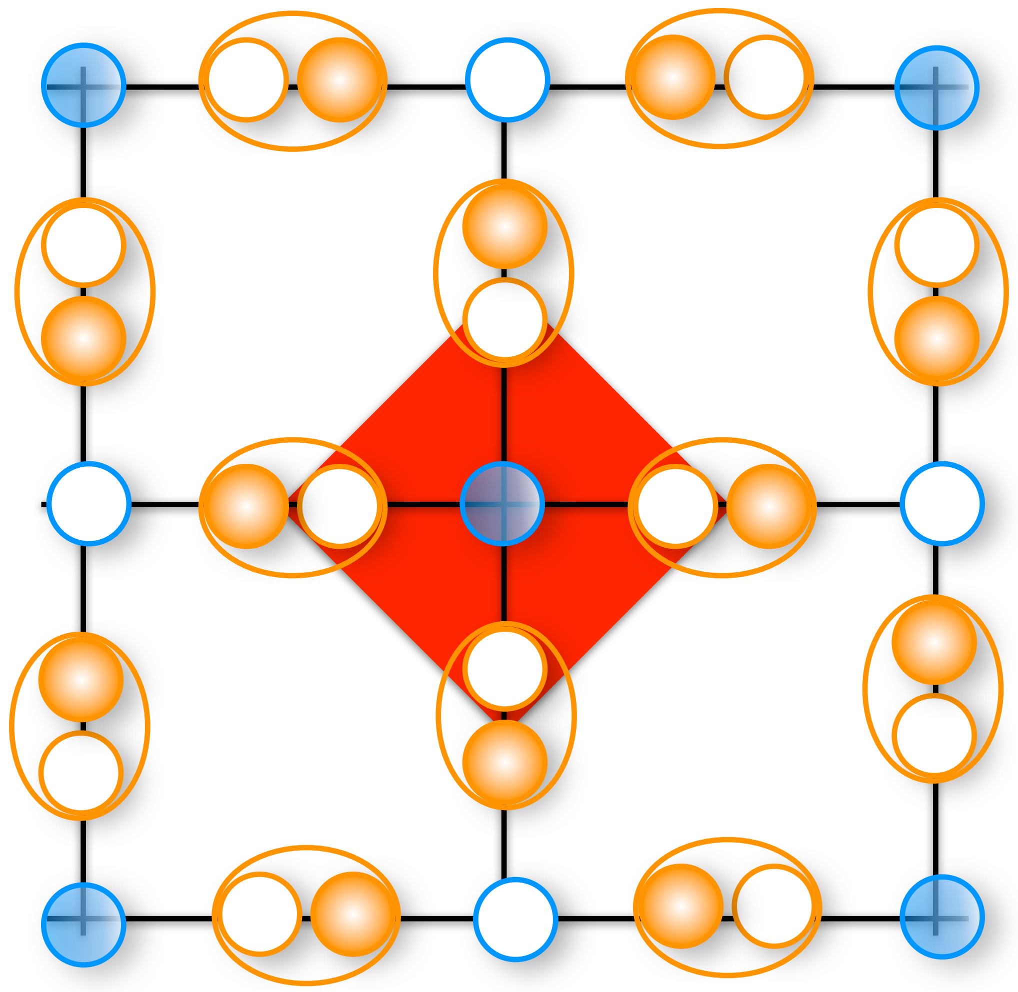
Spin representation

$$N_{x,y} = c_y^\dagger c_y + c_x^\dagger c_x$$

$$E_{x,y} \equiv S_{x,y}^{(3)} = \tfrac{1}{2}\left[n_y - n_x\right] = \frac{N_{x,y}}{2} - n_x = n_y - \frac{N_{x,y}}{2}$$



Tensor networks and gauge symmetry

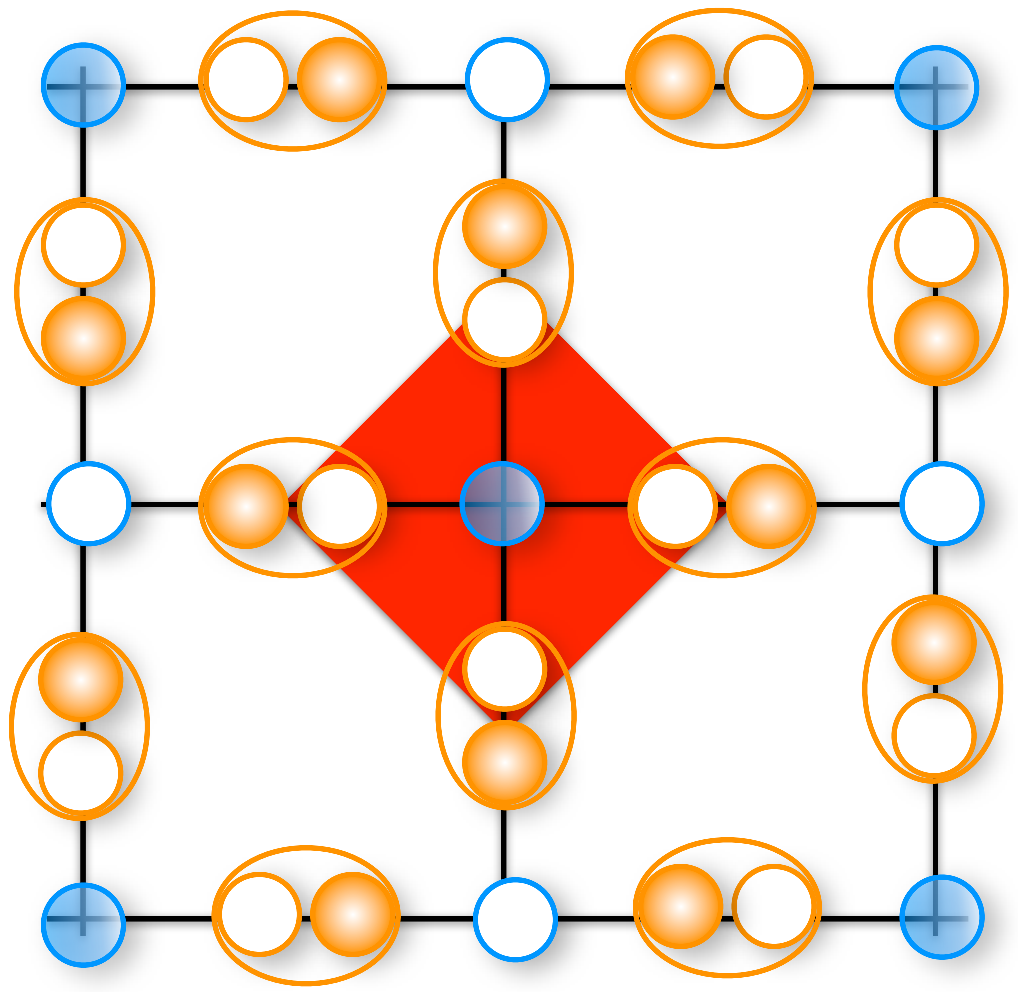


$$G_{\text{vert}}|s_{\text{vert}}\rangle = 0$$

Gauge generator

$$|s_{\text{vert}}\rangle = \sum_{n_c,n_\psi} A^{(s_{\text{vert}})}_{n_c,n_\psi}|n_c,n_\psi\rangle$$

Tensor networks and gauge symmetry



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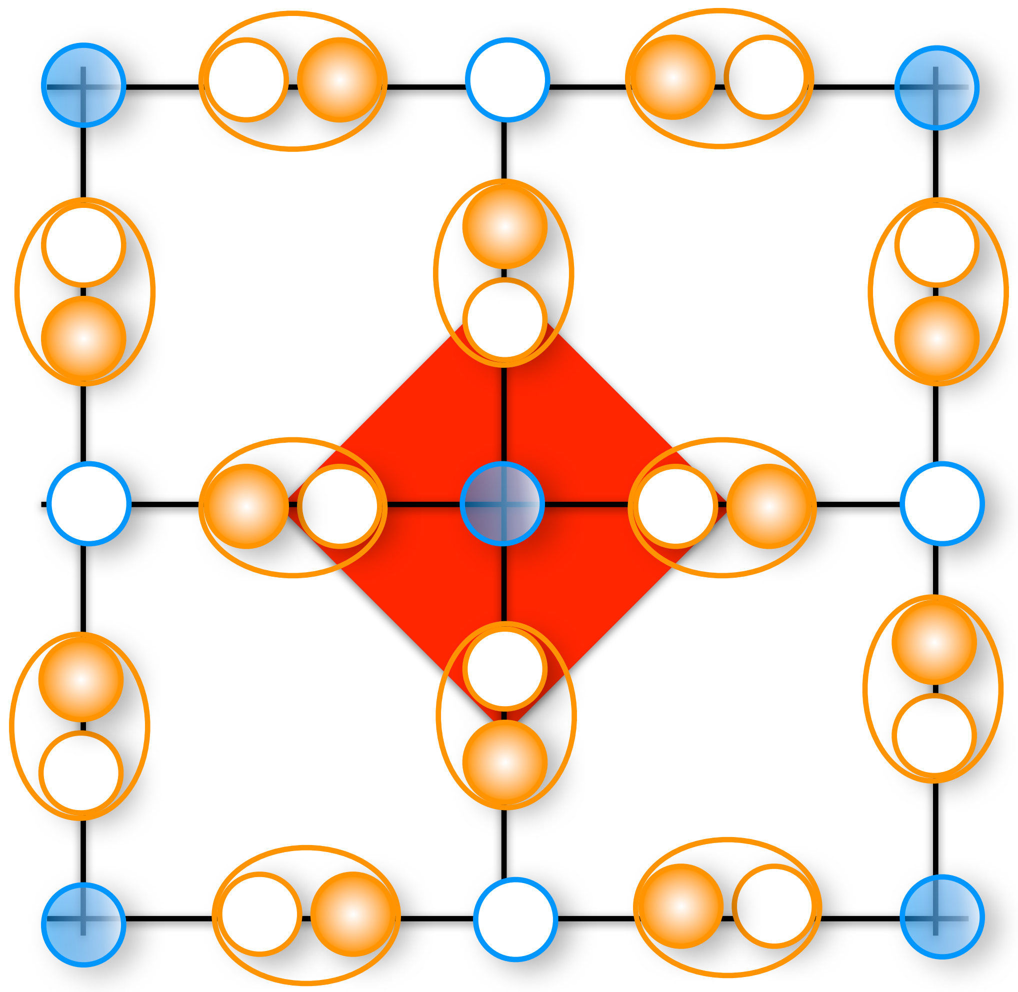
$$|s_{\rm vert}\rangle = \sum_{n_c, n_\psi} A^{(s_{\rm vert})}_{n_c, n_\psi} |n_c, n_\psi\rangle$$

$$N_{x,y} = n_y + n_x$$

Gauge generator

Spin representation

Tensor networks and gauge symmetry



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$$N_{x,y} = n_y + n_x$$

Gauge generator

Spin representation

Exact description of the gauge invariant subspace with tensor network states

$$|\text{phys}\rangle = \sum_{s_1, \cdots, s_x, \cdots} a(s_1, \cdots, s_x, \cdots) \text{Tr}\left[A^{(s_1)} \ldots A^{(s_x)} \ldots\right] |s_1, \cdots, s_x, \cdot\rangle$$

Schwinger model: QED in (1+1)-d

Hamiltonian: staggered fermions in 1D coupled to quantum link spin S

$$H = \frac{g^2}{2}\sum_x ({S^z}_{x,x+1})^2 - t\sum_x \left[\psi_x^\dagger S^+_{x,x+1}\psi_{x+1} + \mathrm{h.c.}\right] + m\sum_x (-1)^x \psi_x^\dagger \psi_x$$

electric term — interaction term — staggered fermions

Schwinger model: QED in (1+1)-d

<u>Hamiltonian:</u> staggered fermions in 1D coupled to quantum link spin S

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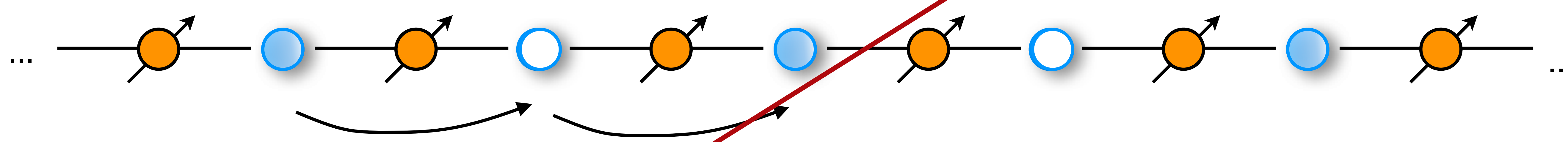
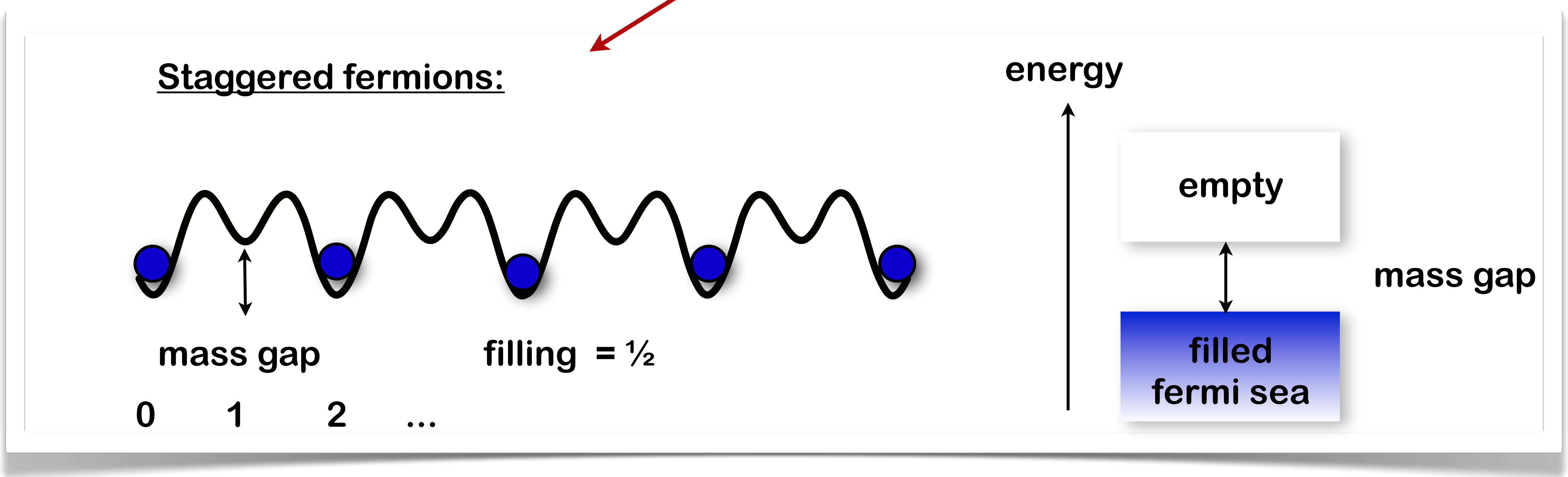
electric term | interaction term | staggered fermions





Phenomenology

Phase diagram: QED link model in (1+1)-d

Spin-1/2 representation

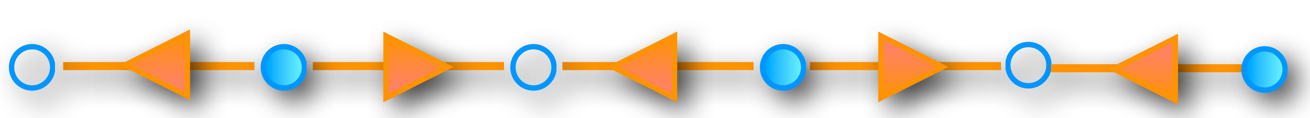
Vacuum (reference) state

$$H = \mu \sum_x (-1)^x \, \psi_x^\dagger \psi_x$$

Staggered mass term

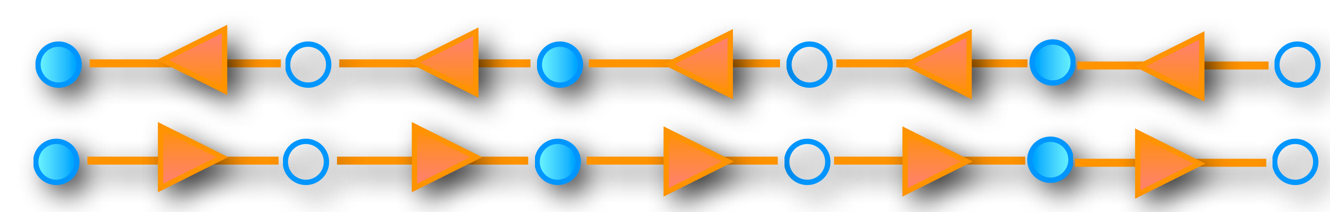
Total electric flux

$\mu < 0 :$ $\quad \mathcal{E} = \sum_x E_{x,x+1} = 0$



$2x-1 \quad 2x \quad 2x+1$

$\mu > 0 :$ $\quad \mathcal{E} = \sum_x E_{x,x+1} \neq 0$



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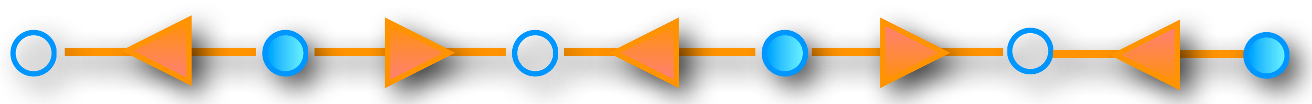

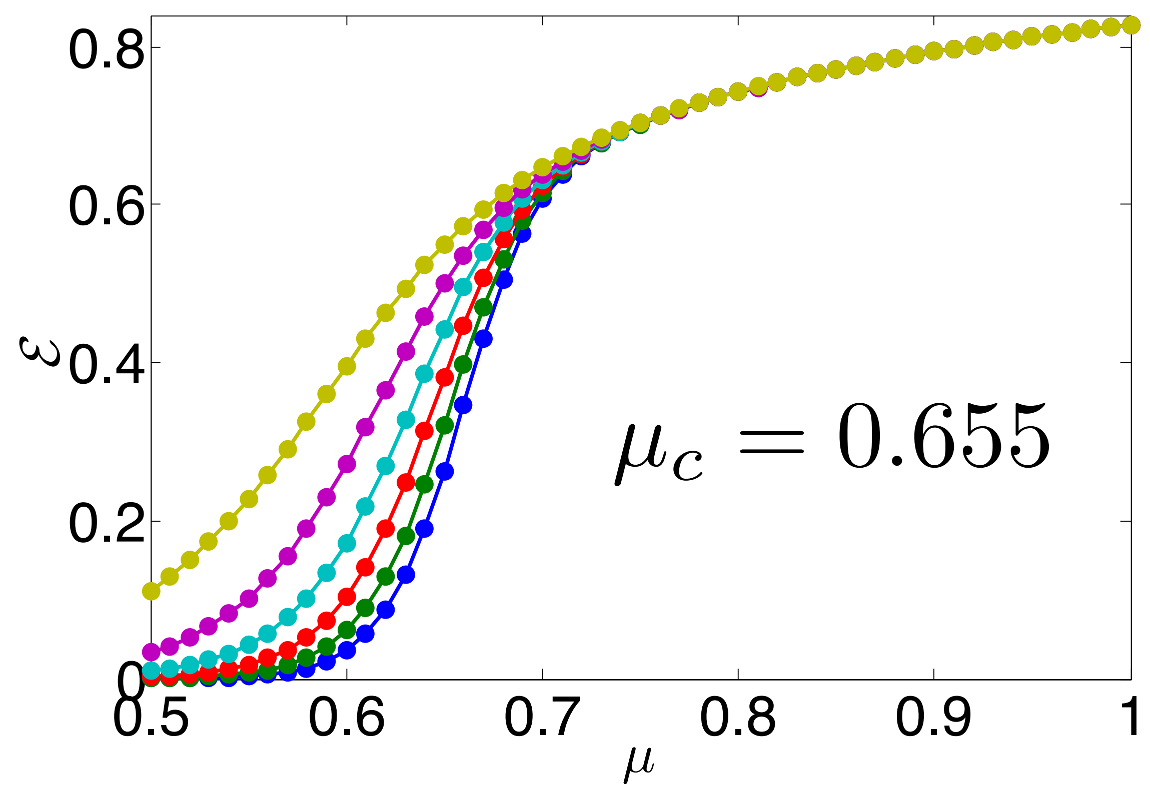

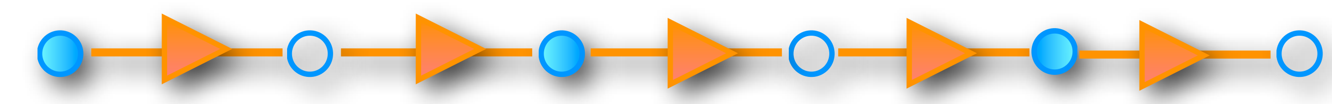

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Total electric flux

Phenomenology

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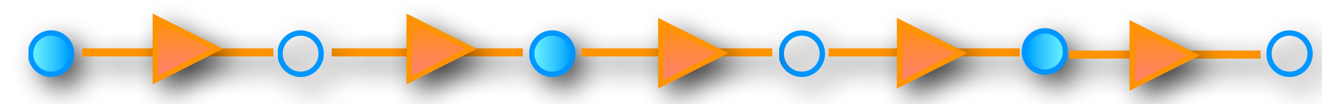

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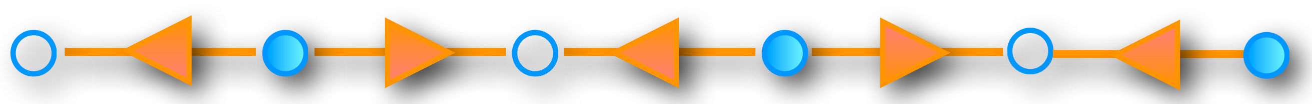
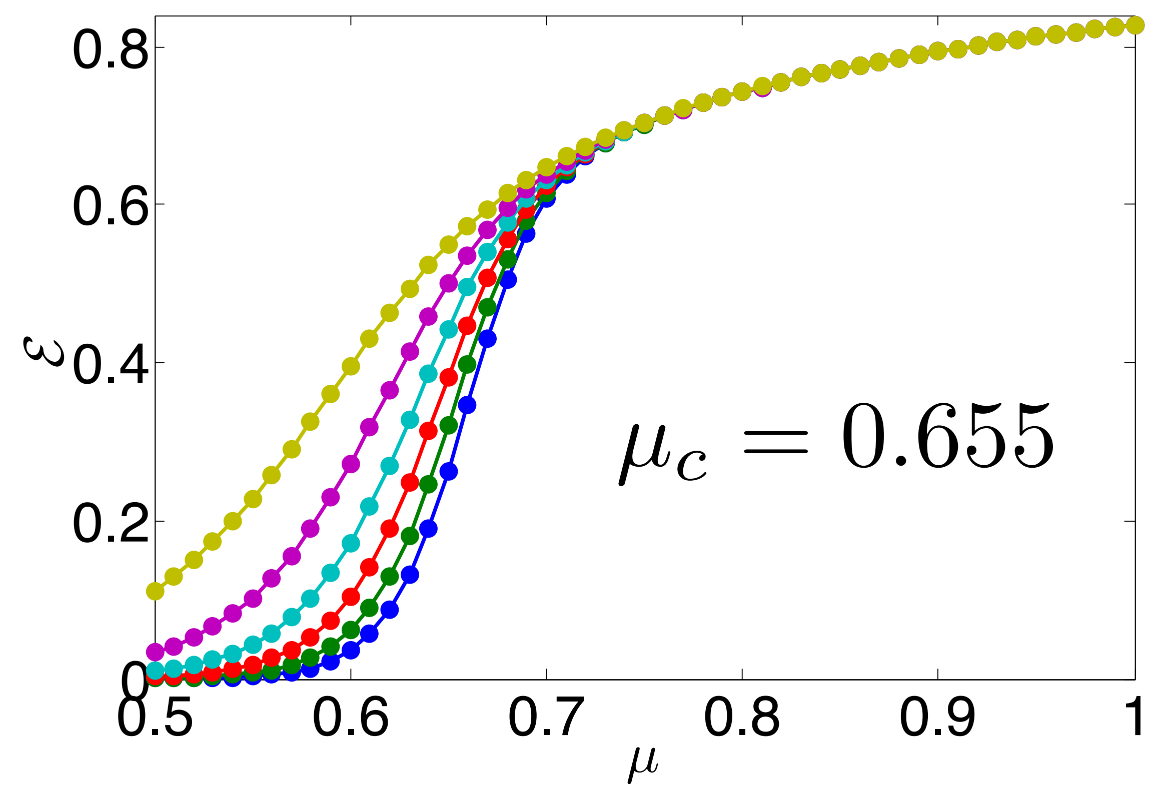
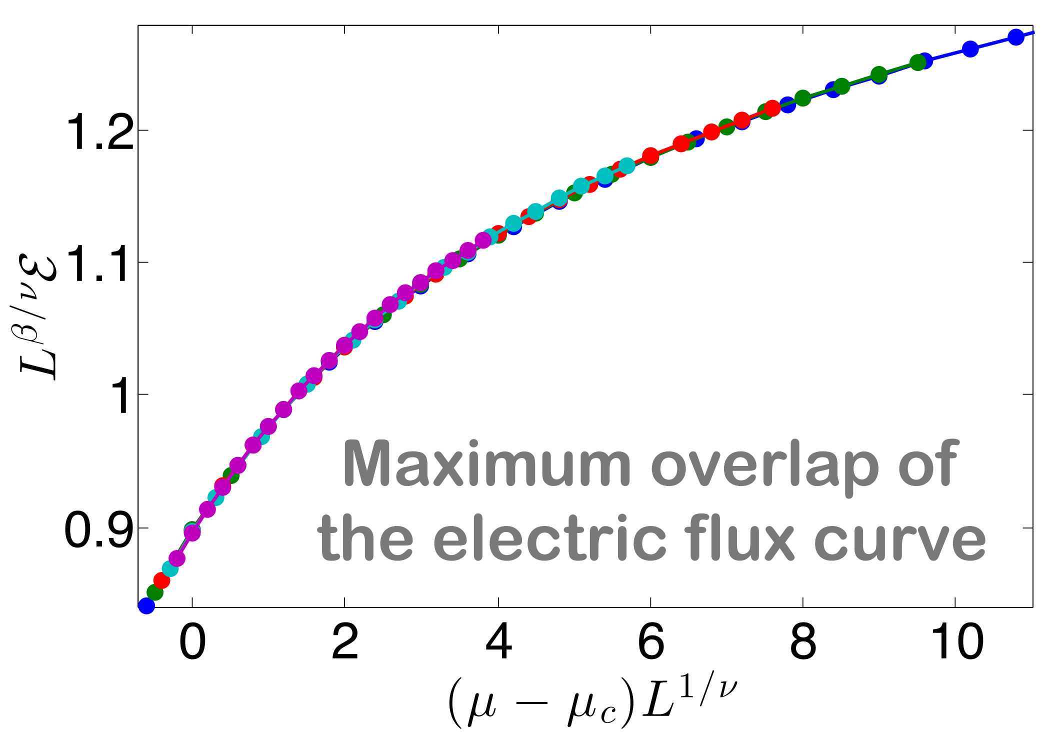


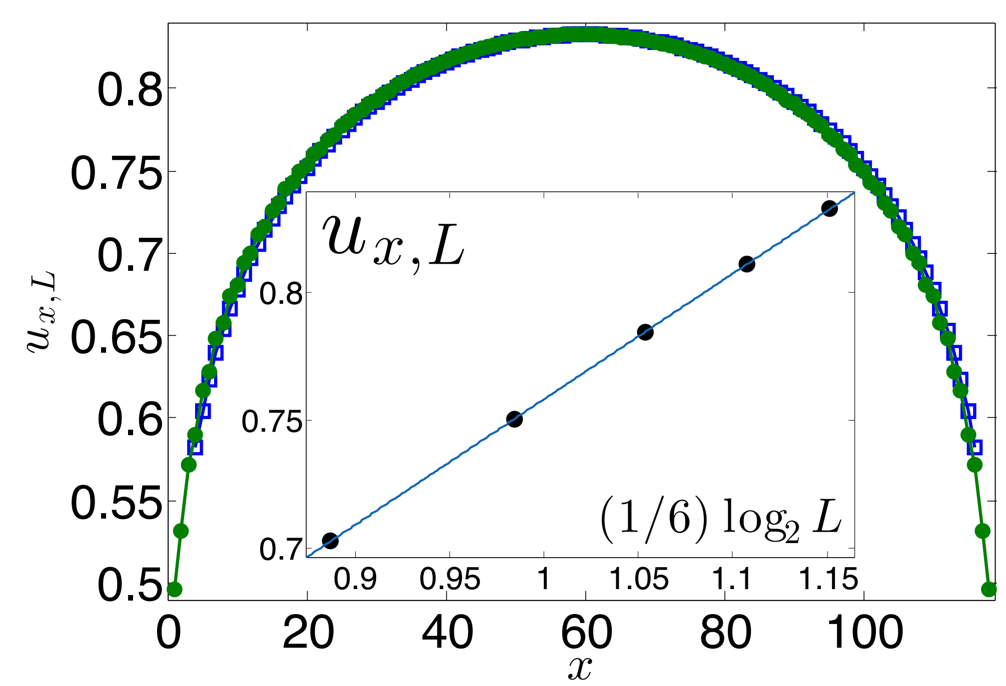


Second order phase transition: Parity and charge conjugation spontaneously broken (Ising universality class)

Critical exponents:

$$\beta = \frac{1}{8} \qquad \nu = 1$$





Central charge:

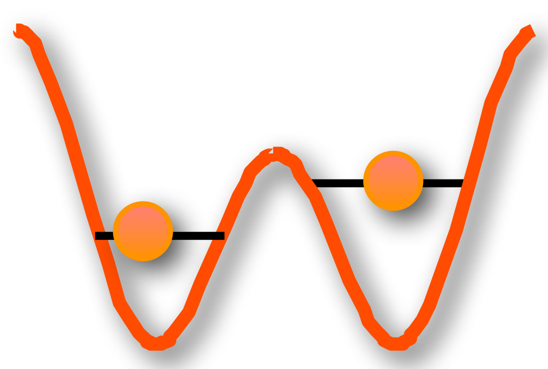
$$c = \frac{1}{2}$$

Atomic implementation

$$H = \frac{g^2}{2}\sum_x (S^z{}_{x,x+1})^2 - t\sum_x \left[\psi_x^\dagger S^+_{x,x+1}\psi_{x+1} + \text{h.c.}\right] + m\sum_x (-1)^x \psi_x^\dagger \psi_x$$

Building blocks:

Gauge field: bosonic double-well potentials



$$\begin{aligned} H = \quad & -t_B \textstyle\sum_{x\text{odd}} c^\dagger_{1,x} c_{1,x+1} + \frac{3g^2}{8}\left(n^2_{1,x+1} + n^2_{1,x}\right) + \text{h.c.} \\ & -t_B \textstyle\sum_{x\text{even}} c^\dagger_{2,x} c_{2,x+1} + \frac{3g^2}{8}\left(n^2_{2,x+1} + n^2_{2,x}\right) + \text{h.c.} \end{aligned}$$

Atomic implementation

$$H = \frac{g^2}{2}\sum_x (S^z{}_{x,x+1})^2 - t\sum_x \left[\psi_x^\dagger S^+_{x,x+1}\psi_{x+1} + \text{h.c.}\right] + m\sum_x (-1)^x \psi_x^\dagger \psi_x$$

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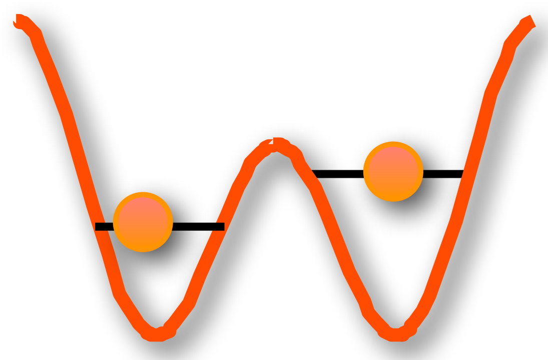

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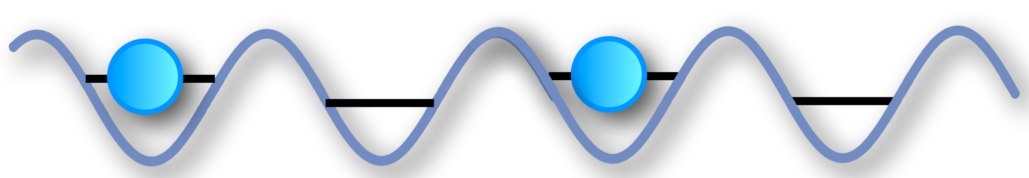
Fermionic field: fermionic optical super-lattice



$$H = -t_F \sum_x \psi_x^\dagger \psi_{x+1} + \text{h.c.} + m\sum_x (-1)^x \psi_x^\dagger \psi_x$$

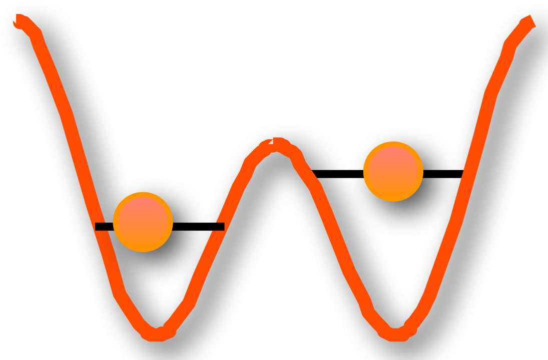
Atomic implementation

$$H = \frac{g^2}{2}\sum_{x,i}\left(\frac{n_{i,x+1}-n_{i,x}}{2}\right)^2 + m\sum_x (-1)^x\,\psi_x^\dagger\psi_x - t\sum_{x,i}\left[\psi_x^\dagger c_{i,x} c_{i,x+1}^\dagger \psi_{x+1} + \text{h.c.}\right]$$

electric term staggered fermions interaction term

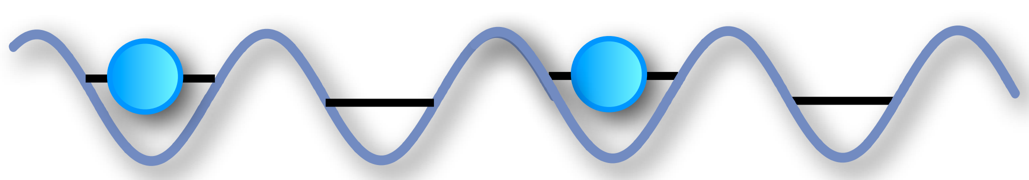
Building blocks:

Gauge field: bosonic double-well potentials



$$H = \begin{array}{l} -t_B \sum_{x\text{odd}} c_{1,x}^\dagger c_{1,x+1} + \frac{3g^2}{8}\left(n_{1,x+1}^2 + n_{1,x}^2\right) + \text{h.c.} \\ -t_B \sum_{x\text{even}} c_{2,x}^\dagger c_{2,x+1} + \frac{3g^2}{8}\left(n_{2,x+1}^2 + n_{2,x}^2\right) + \text{h.c.} \end{array}$$

Fermionic field: fermionic optical super-lattice



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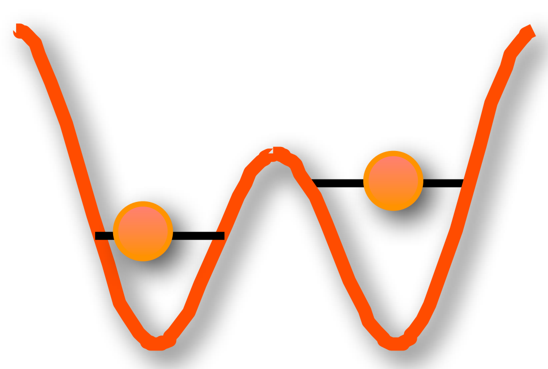
electric term　　staggered fermions　　interaction term

Building blocks:

Gauge field: bosonic double-well potentials



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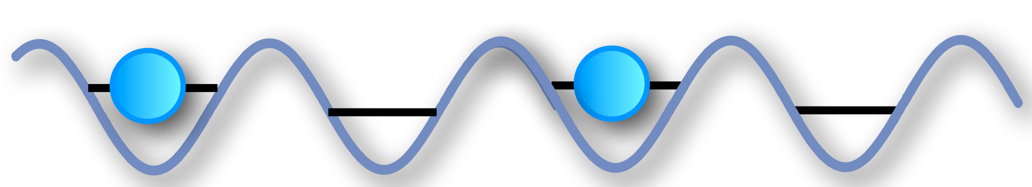
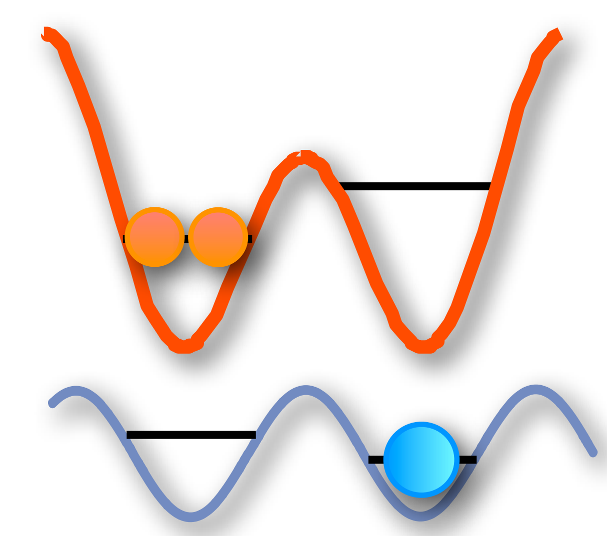
Fermionic field: fermionic optical super-lattice



$$H=-t_F\sum_x\psi_x^\dagger\psi_{x+1}+\mathrm{h.c.}+m\sum_x(-1)^x\,\psi_x^\dagger\psi_x$$



Gauge condition: Local Bose-Fermi interaction

$$G_x=n_{\psi,x}+n_{1,x}+n_{2,x}-2S+\frac{1}{2}\left[(-1)^x-1\right]\qquad H=\Delta\sum_x G_x^2\qquad |\Delta|\gg|t_B|,|t_F|$$

Atomic implementation

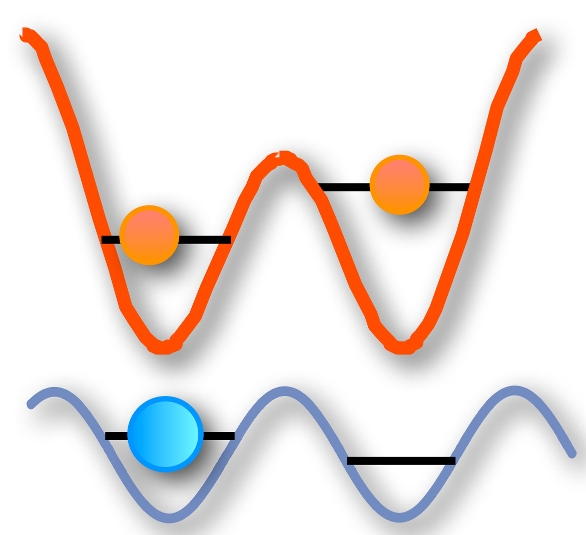
Hopping is a perturbation:

$$H_{\text{pert}} = -t_B \sum_{x \text{ odd}} c_{1,x}^\dagger c_{1,x+1} - t_B \sum_{x \text{ even}} c_{2,x}^\dagger c_{2,x+1} - t_F \sum_x \psi_x^\dagger \psi_{x+1} + \text{h.c.}$$

Gauss' law constraint the biggest energy scale Δ:

$$\Delta H_0 = \begin{pmatrix} \begin{bmatrix} 0 \end{bmatrix} & & \\ & \begin{bmatrix} \Delta \end{bmatrix} & \\ & & \ddots \end{pmatrix}$$

Starting with a "physical" configuration



Atomic implementation

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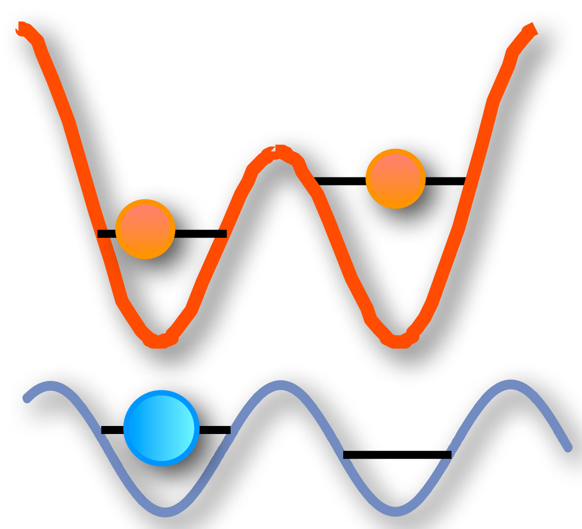
The action of the perturbation



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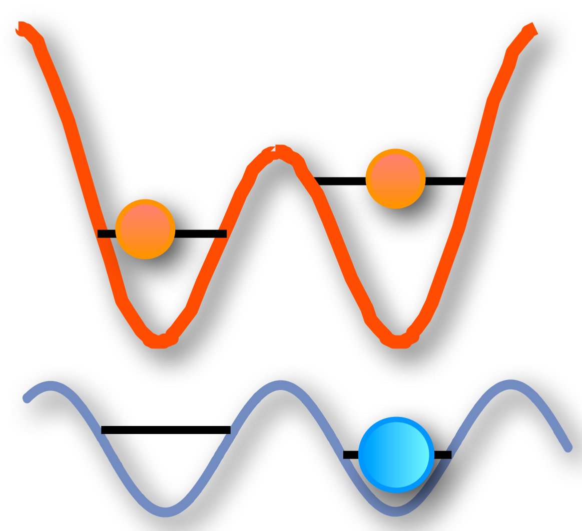

takes the state out of the "physical" Hilbert space

Atomic implementation

Hopping is a perturbation:

$$H_{\text{pert}} = -t_B \sum_{x \text{ odd}} c^\dagger_{1,x} c_{1,x+1} - t_B \sum_{x \text{ even}} c^\dagger_{2,x} c_{2,x+1} - t_F \sum_x \psi^\dagger_x \psi_{x+1} + \text{h.c.}$$

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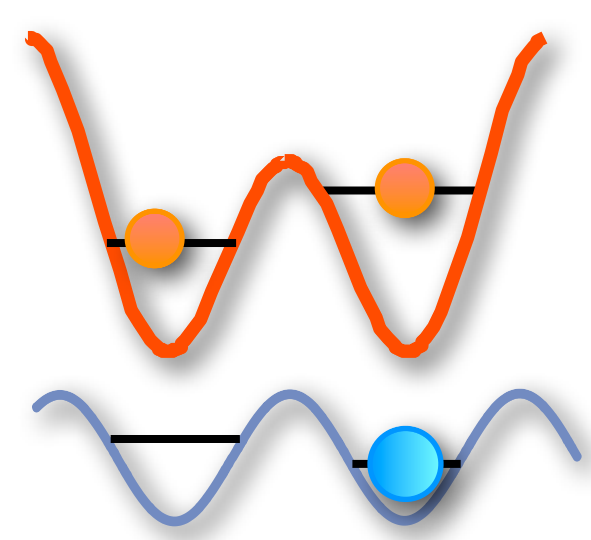
at second order in perturbation theory



Atomic implementation

Hopping is a perturbation:

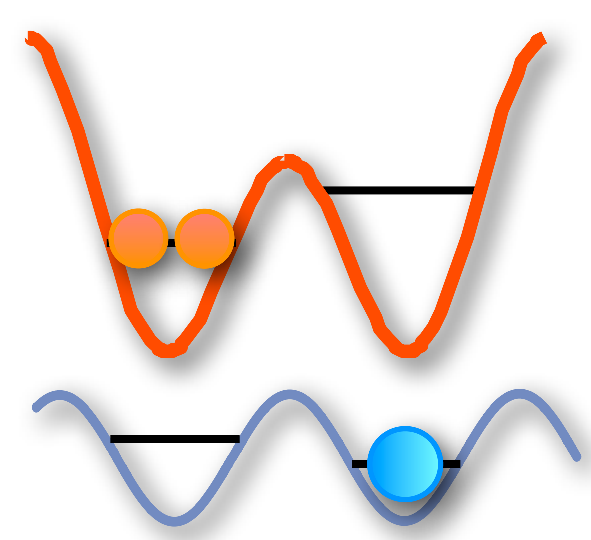
$$H_{\text{pert}} = -t_B \sum_{x \text{ odd}} c^\dagger_{1,x} c_{1,x+1} - t_B \sum_{x \text{ even}} c^\dagger_{2,x} c_{2,x+1} - t_F \sum_{x} \psi^\dagger_x \psi_{x+1} + \text{h.c.}$$

Gauss' law constraint the biggest energy scale Δ:

$$t_{\text{eff}} \sim \frac{t_F t_B}{\Delta}$$

$$\Delta H_0 = \begin{pmatrix} \begin{bmatrix} 0 \end{bmatrix} & & \\ & \begin{bmatrix} \Delta \end{bmatrix} & \\ & & \ddots \end{pmatrix}$$

at second order in perturbation theory



Atomic implementation

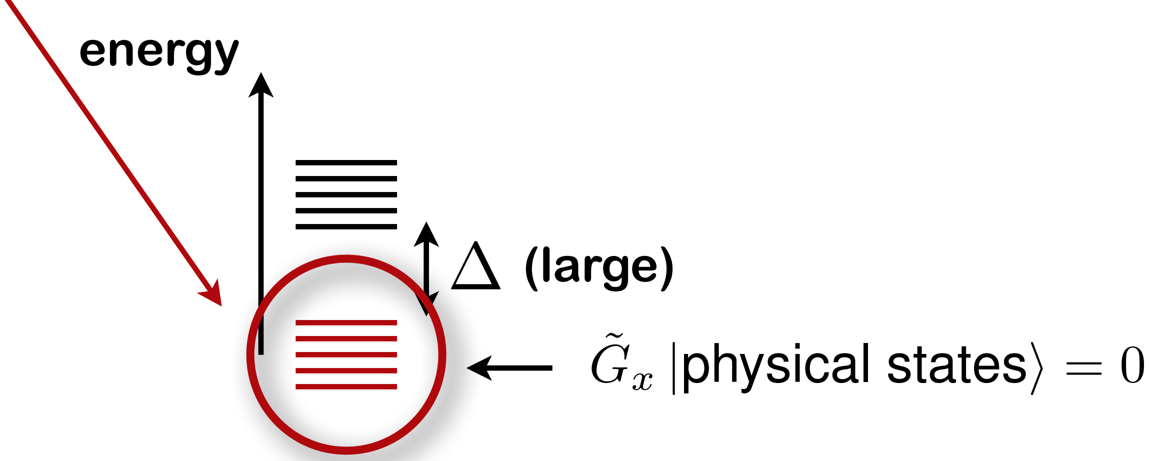
Emergent lattice gauge theory

$$H = \frac{g^2}{2}\sum_x E^2_{x,x+1} - t\sum_x \left[\psi^\dagger_x U_{x,x+1}\psi_{x+1} + \text{h.c.}\right] + m\sum_x (-1)^x \psi^\dagger_x \psi_x$$

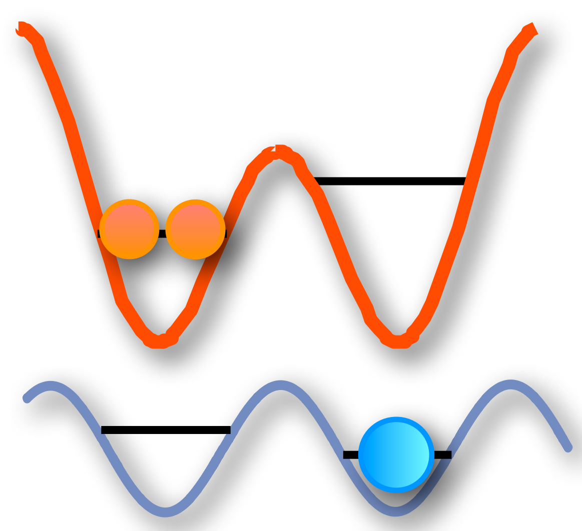
Gauss' law constraint the biggest energy scale Δ:

$$t_{\text{eff}} \sim \frac{t_F t_B}{\Delta}$$

$$\Delta H_0 = \begin{pmatrix} [0] & & \\ & [\Delta] & \\ & & \ddots \end{pmatrix}$$



at second order in perturbation theory



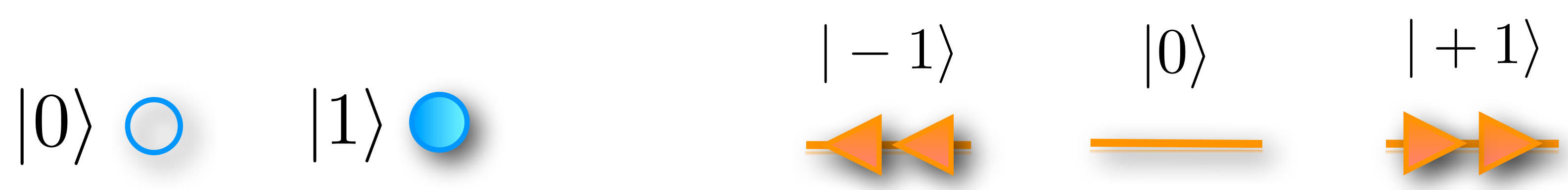
Bose + Fermi Hubbard model

$$H_{\text{microscopic}} = \Delta \sum_x G_x^2 + \cdots$$

Phenomenology

Confinement and string breaking: QED in (1+1)-d (Schwinger model)

Spin-1 representation



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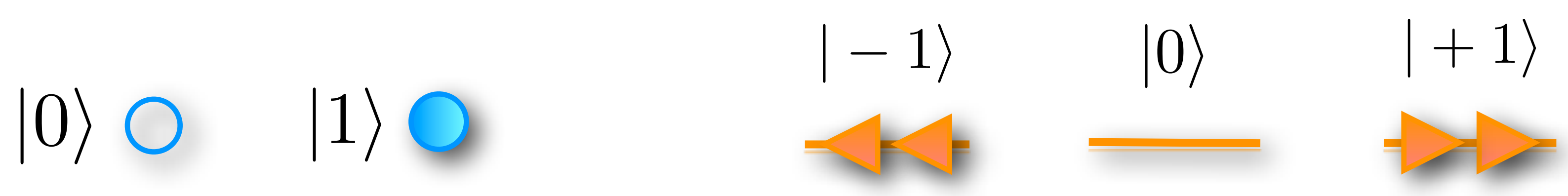
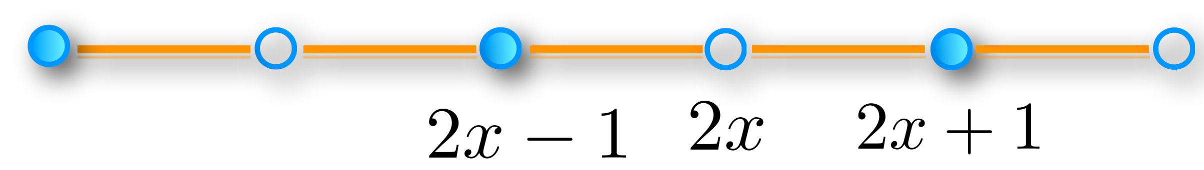


Vacuum (reference) state

$$H = \frac{g^2}{2} \sum_{\langle x,y \rangle} \left(S^{(3)}_{x,y} \right)^2 + m \sum_x (-1)^x \, \psi^\dagger_x \psi_x$$



Phenomenology

Confinement and string breaking: QED in (1+1)-d (Schwinger model)

Spin-1 representation

$|0\rangle$ $|1\rangle$

$|-1\rangle$ $|0\rangle$ $|+1\rangle$

Creating a quark - antiquark pair:

$$\psi^\dagger_{2x} S^+_{2x,2x+1} \psi_{2x+1}$$

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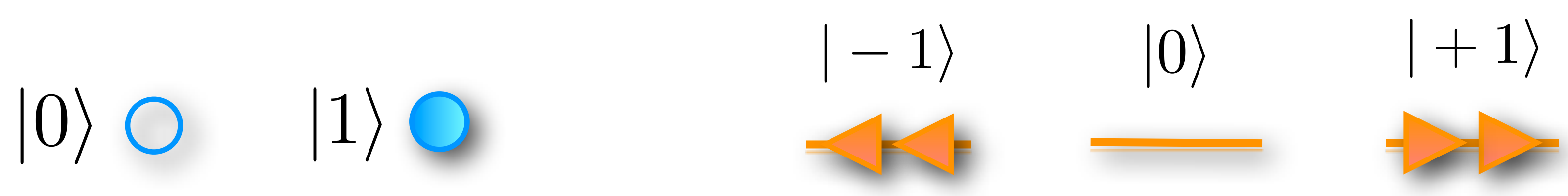

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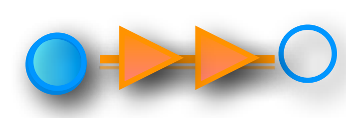

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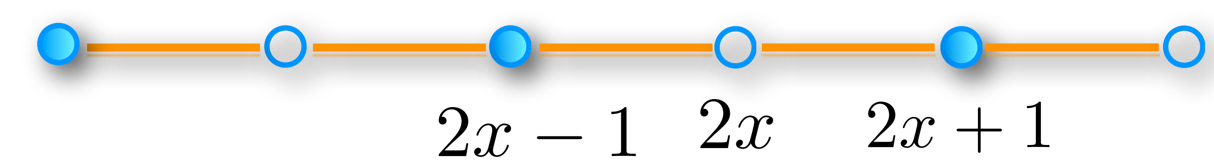

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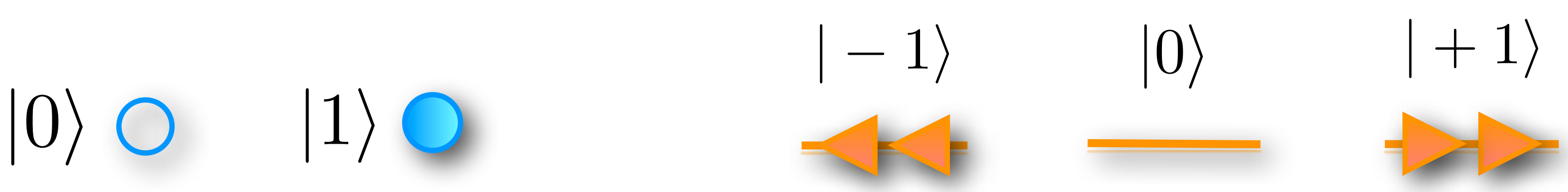

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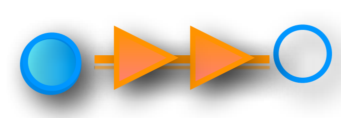

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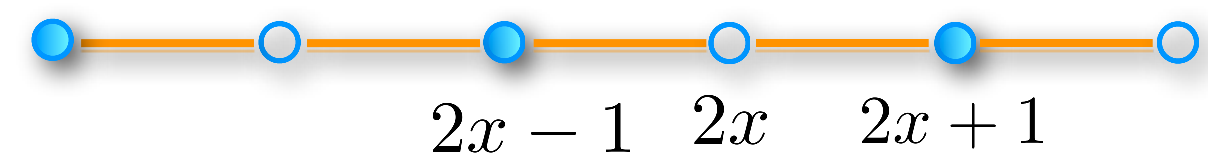

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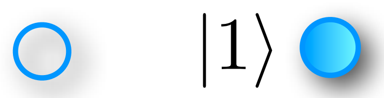
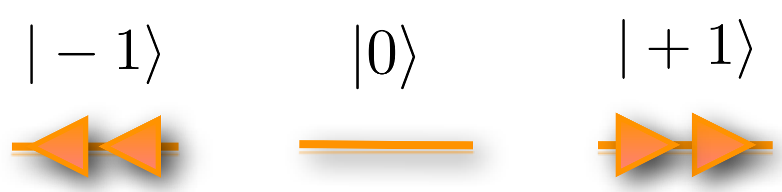
$|0\rangle$ $|1\rangle$ 



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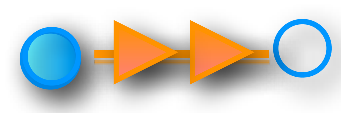

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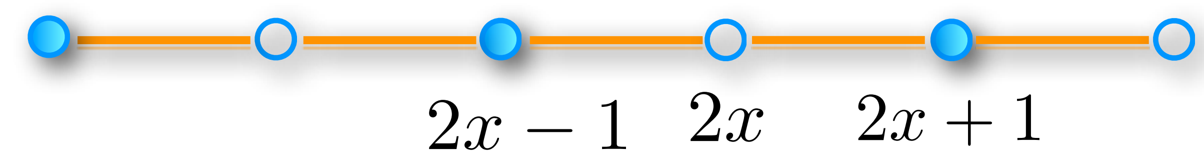


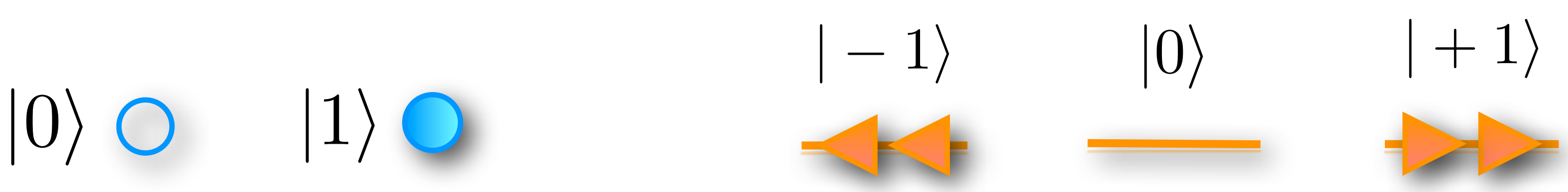
$$E_{\text{string}} - E_0 = \frac{g^2}{2}(L-1)$$

Confinement

Phenomenology

Confinement and string breaking: QED in (1+1)-d (Schwinger model)

Spin-1 representation



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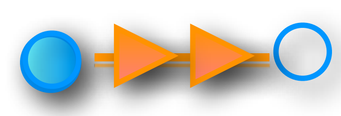

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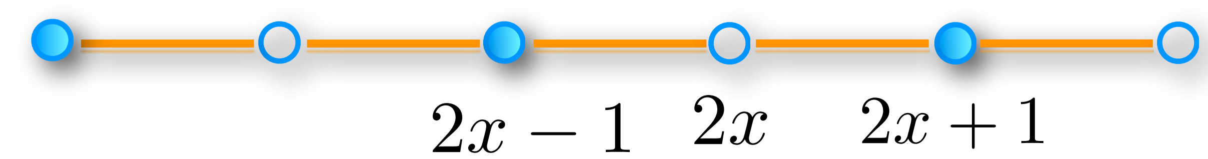
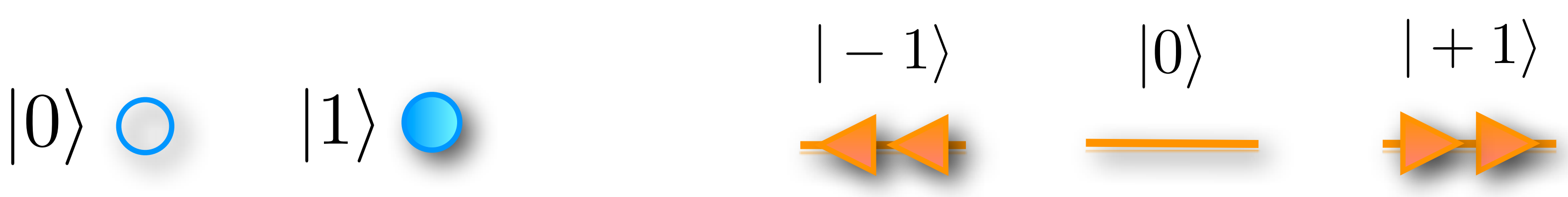


Phenomenology

Confinement and string breaking: QED in (1+1)-d (Schwinger model)

Spin-1 representation



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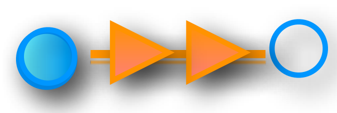

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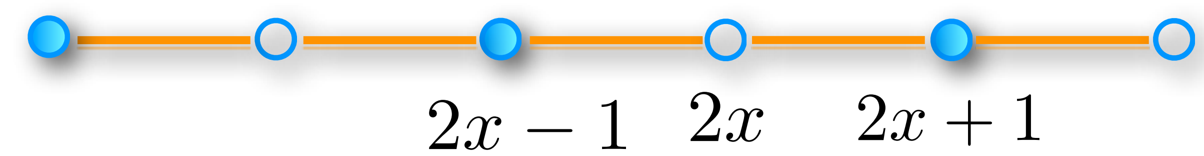


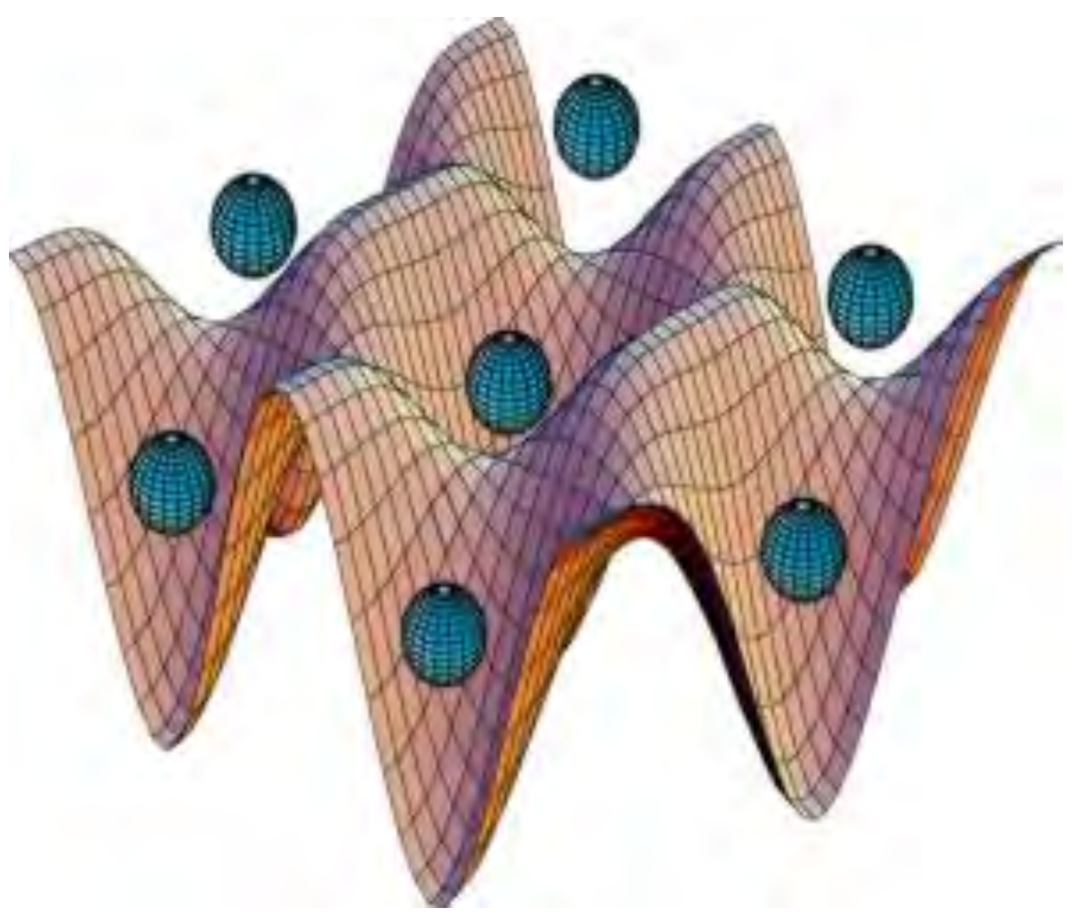
String breaking and hadronization

$$E_{\text{meson}} - E_0 = g^2 + 2m$$

$$L_c = 3 + \frac{4m}{g^2}$$

Observability of phenomena

Preparation of many body states (Mott phase)



Greiner et al. (2002)
Joerdens et al. (2008)
Schneider et al. (2008)

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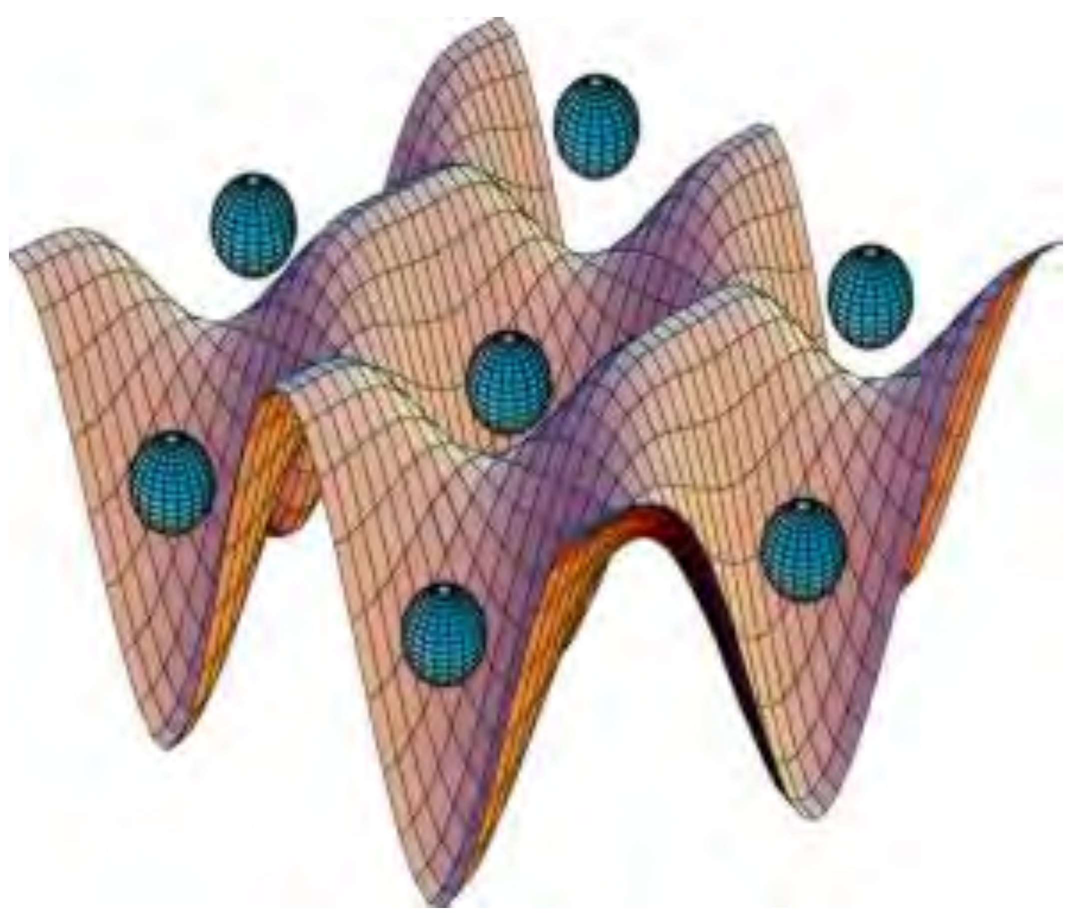

Greiner et al. (2002)
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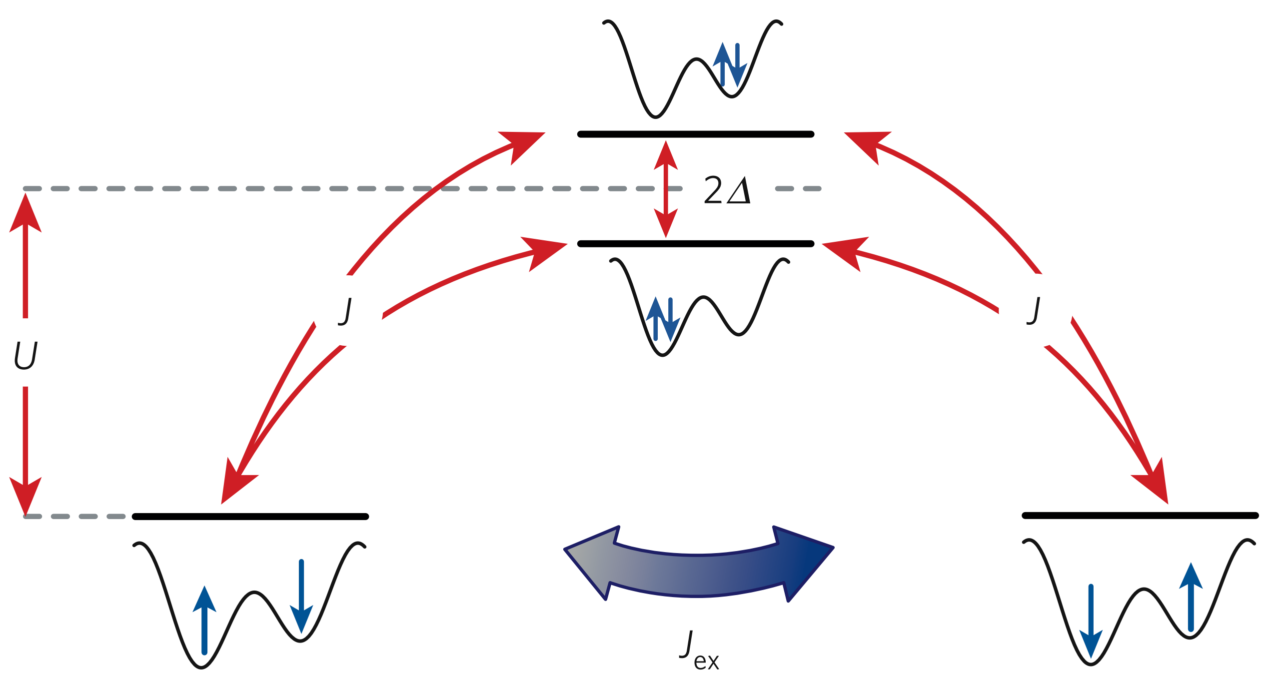
Evolution (Super-exchange)



Anderlini et al. (2007)
Trotzky et al. (2008)

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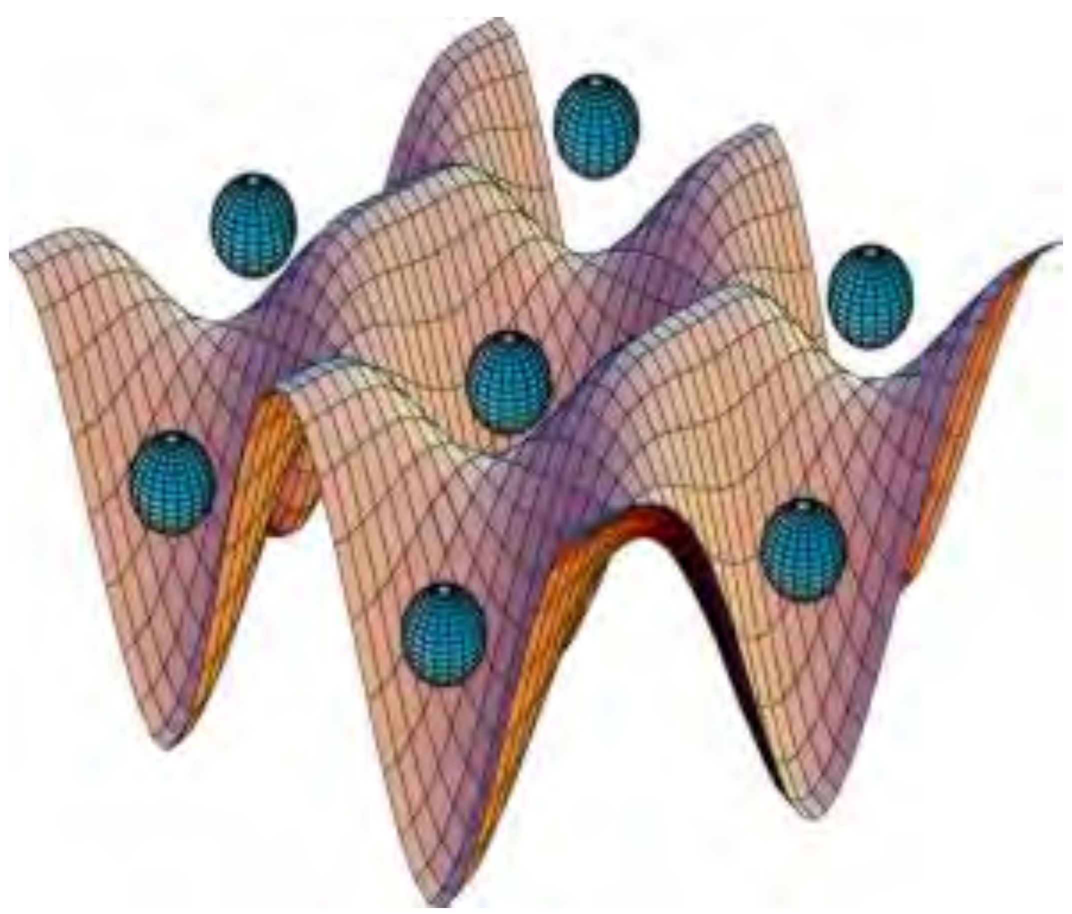

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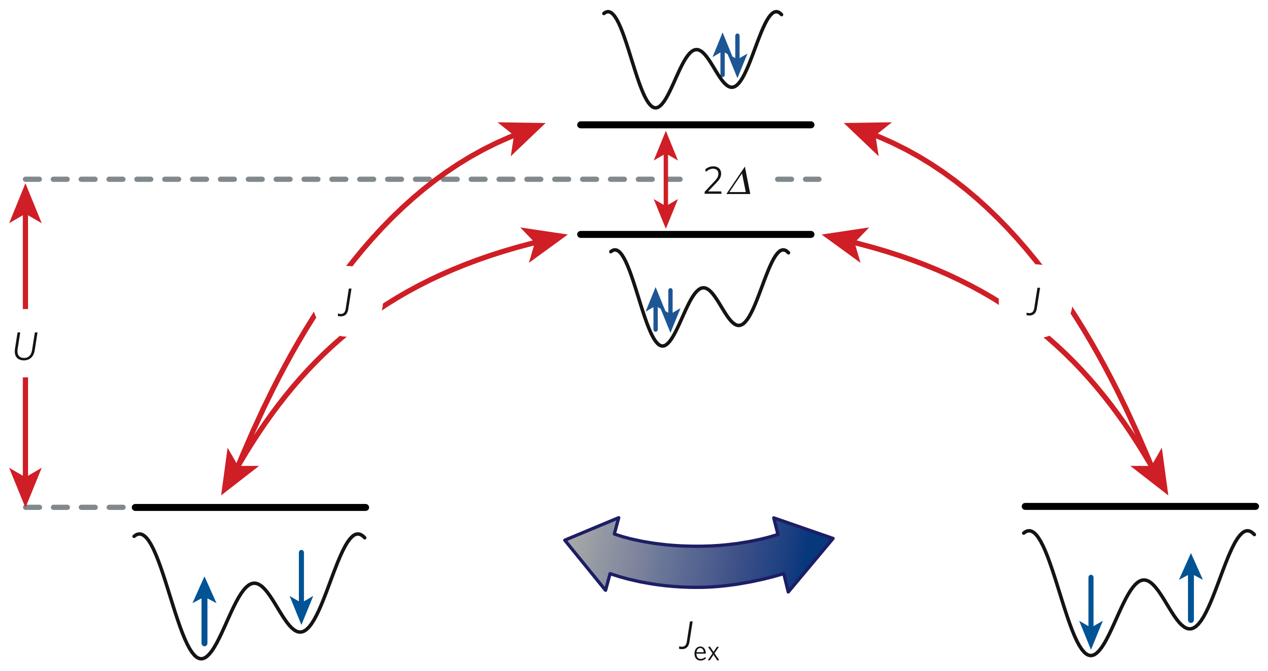
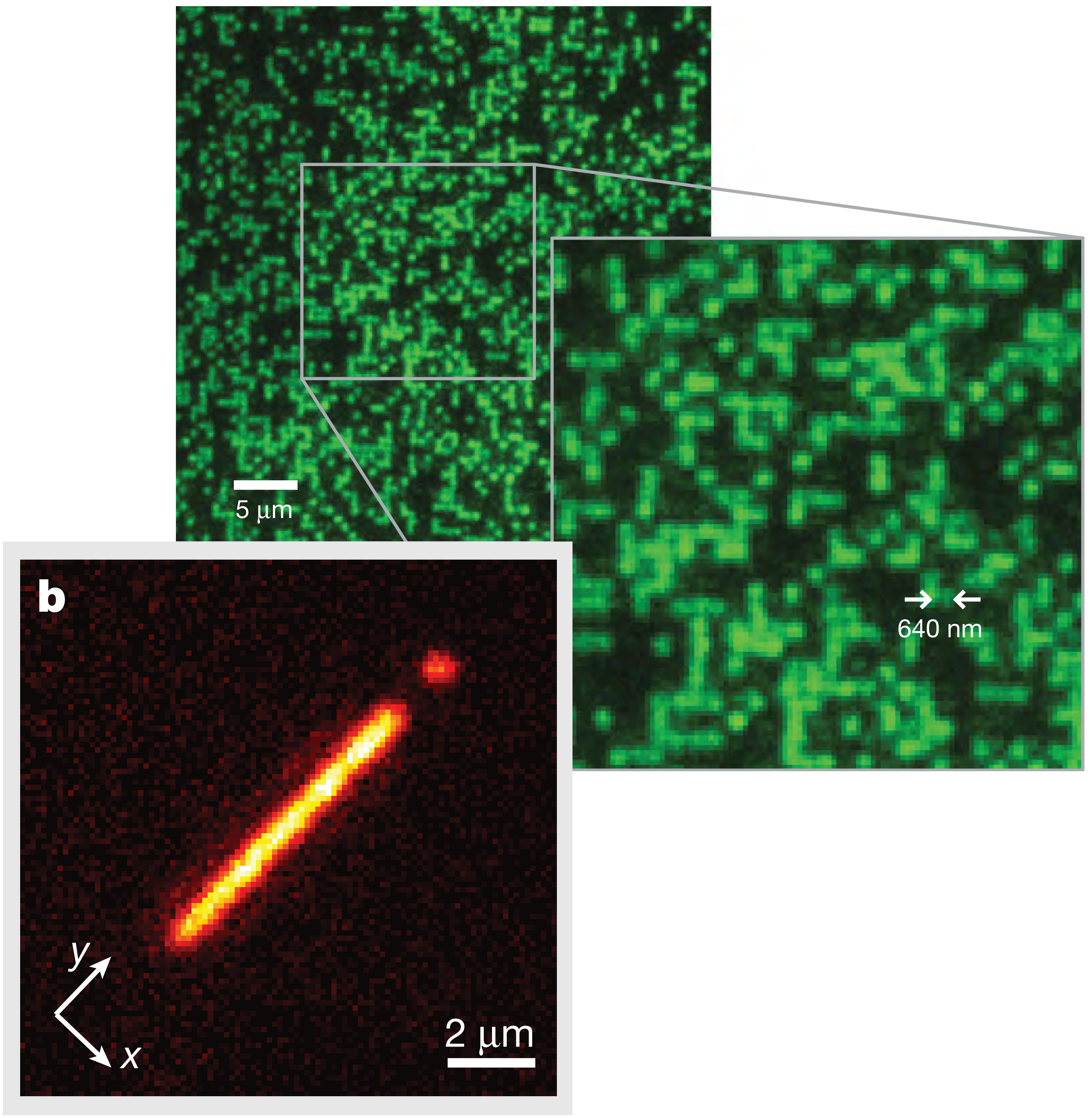
Schneider et al. (2008)

Evolution (Super-exchange)



Anderlini et al. (2007)
Trotzky et al. (2008)

Detection (Single-site fluorescence)



Bakr et al. (2010)
Weitenberg et al. (2011)

Conclusions and Outlook

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- We have characterized the phase diagram of a (1+1)-d QLM version of the Schwinger model, finding a phase transition between a CP ordered to a CP disordered phase described by the Ising universality class.

- We have seen how to implement in an AMO setup this U(1) QLM which is a relevant explame as its implementation in cold atom gases can be foreseen in the next years.

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