Coogee'14 – Sydney Quantum Information Workshop

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On the shape of entanglement

Yangang Chen

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

outline

Yangang Chen

- Introduction
- Shape of entanglement in systems of free fermions
    - ground state entanglement
    - entanglement after a quantum quench
- Beyond free fermions

outline

- **Introduction**

- Shape of entanglement in systems of free fermions

    - ground state entanglement

    - entanglement after a quantum quench

- Beyond free fermions

Scaling of entanglement in many-body systems

How does entropy scale with L?

- generic state of $\mathcal{H}^A \otimes \mathcal{H}^B$

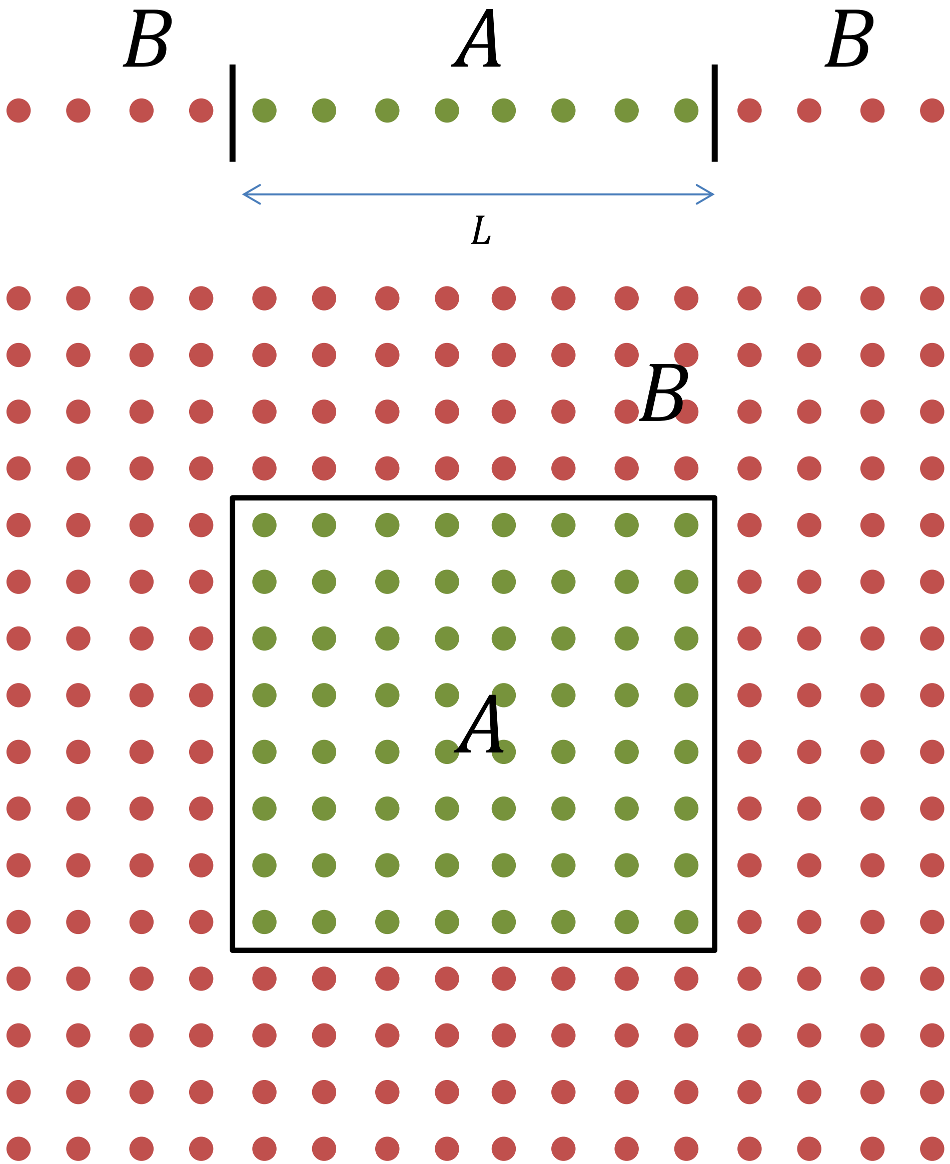
$$S(A) \sim |A| = L^D$$

bulk law

- ground state of a local Hamiltonian

$$S(A) \sim |\partial A| = L^{D-1}$$

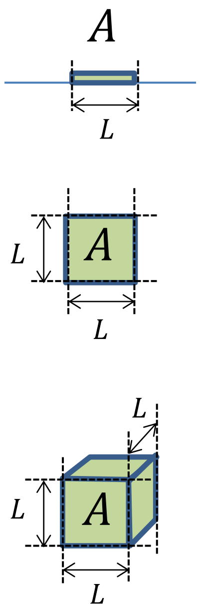
boundary law



D=1

D=2

D=3



boundary law
for entanglement entropy

$$S(A) \sim |\partial A| \sim L^{D-1}$$

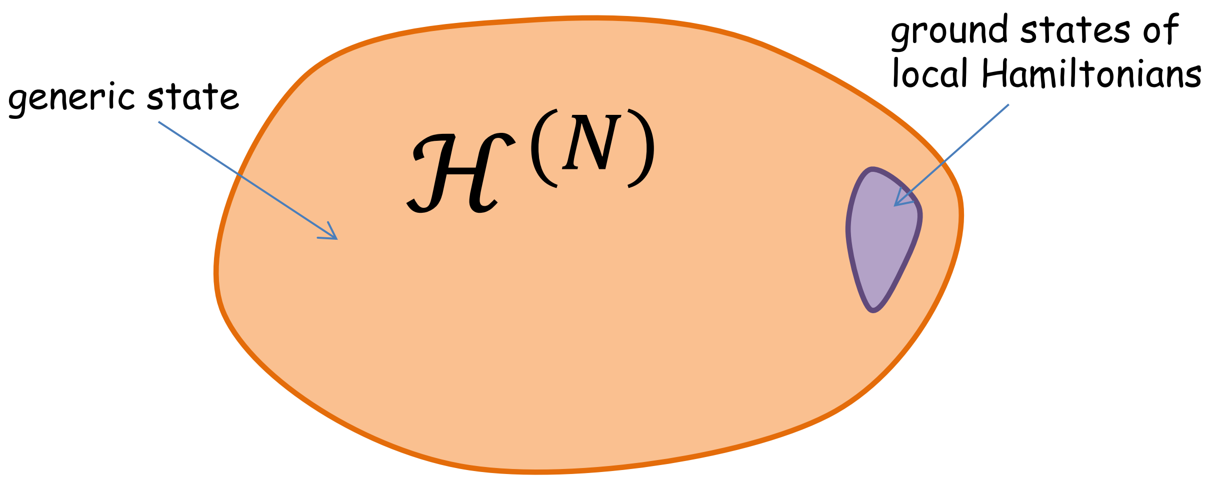
instead of bulk law

$$S(A) \sim |A| \sim L^{D}$$

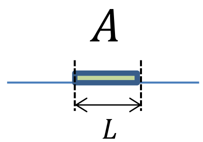
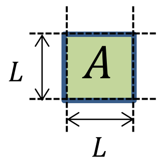
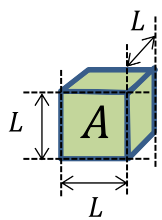
sometimes, logarithmic corrections

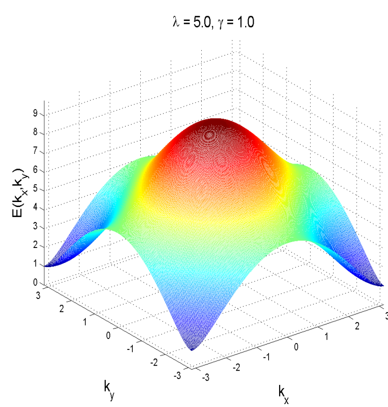
$$S(A) \sim L^{D-1}\log(L)$$

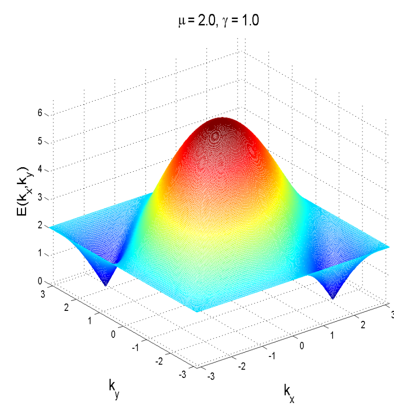
Ground states of local Hamiltonians are special/non-generic states

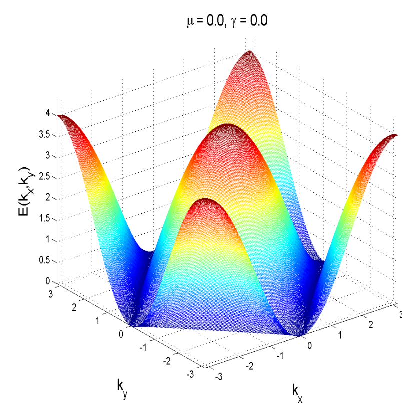


Scaling of entanglement in many-body ground states (complete list?)

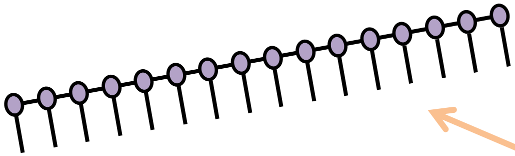
		gapped system	gapless system	
			$D - D_\Gamma \geq 2$	$D - D_\Gamma = 1$
$D = 1$		const.	-	$\log(L)$
$D = 2$		L	L	$L\log(L)$
$D = 3$		L^2	L^2	$L^2\log(L)$

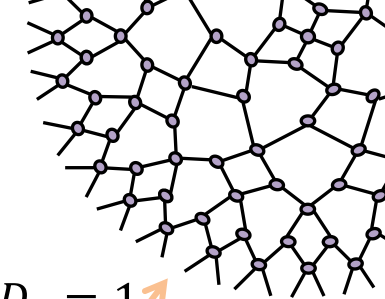




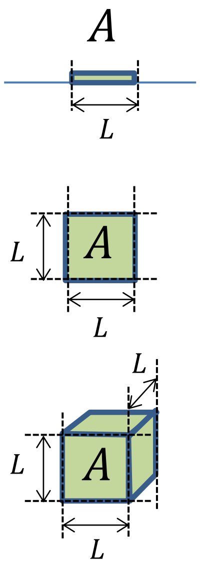


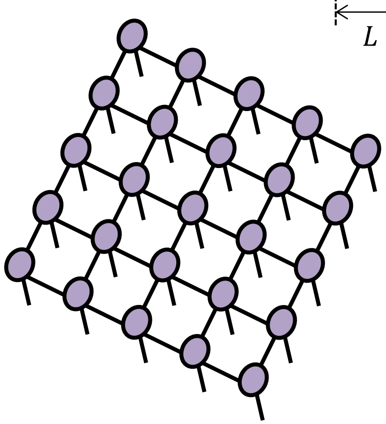
APPLICATION: tensor network states

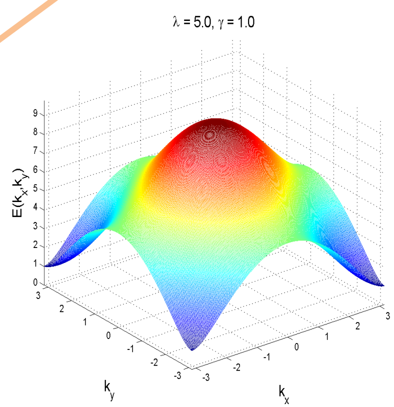


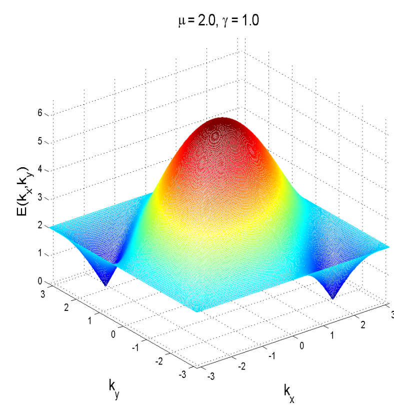


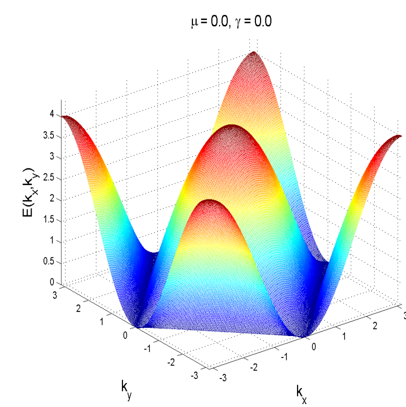
	gapped system	gapless system: $D - D_\Gamma \geq 2$	gapless system: $D - D_\Gamma = 1$
$D = 1$	MPS const.	-	MERA $\log(L)$
$D = 2$	PEPS L	MERA L	branching MERA $L \log(L)$
$D = 3$	PEPS L^2	MERA L^2	branching MERA $L^2 \log(L)$



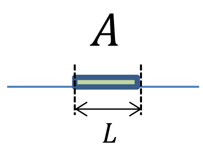
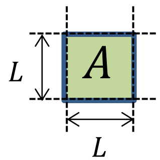








APPLICATION: quantum criticality and topological order

		gapped system	gapless system
			$D - D_\Gamma = 1$
$D = 1$		-	$\log(L)$
$D = 2$		L	

- quantum criticality:

$$S(L) \approx \frac{c}{3} \log(L)$$

c central charge

- topological order:

$$S(L) \approx aL - \log(D)$$

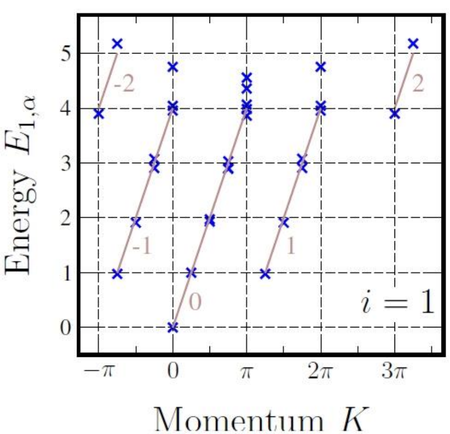
D total quantum dimension

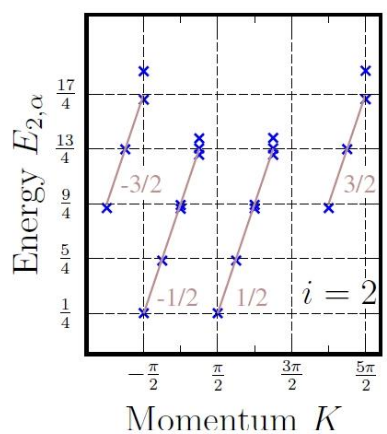
Beyond entanglement entropy of a block?

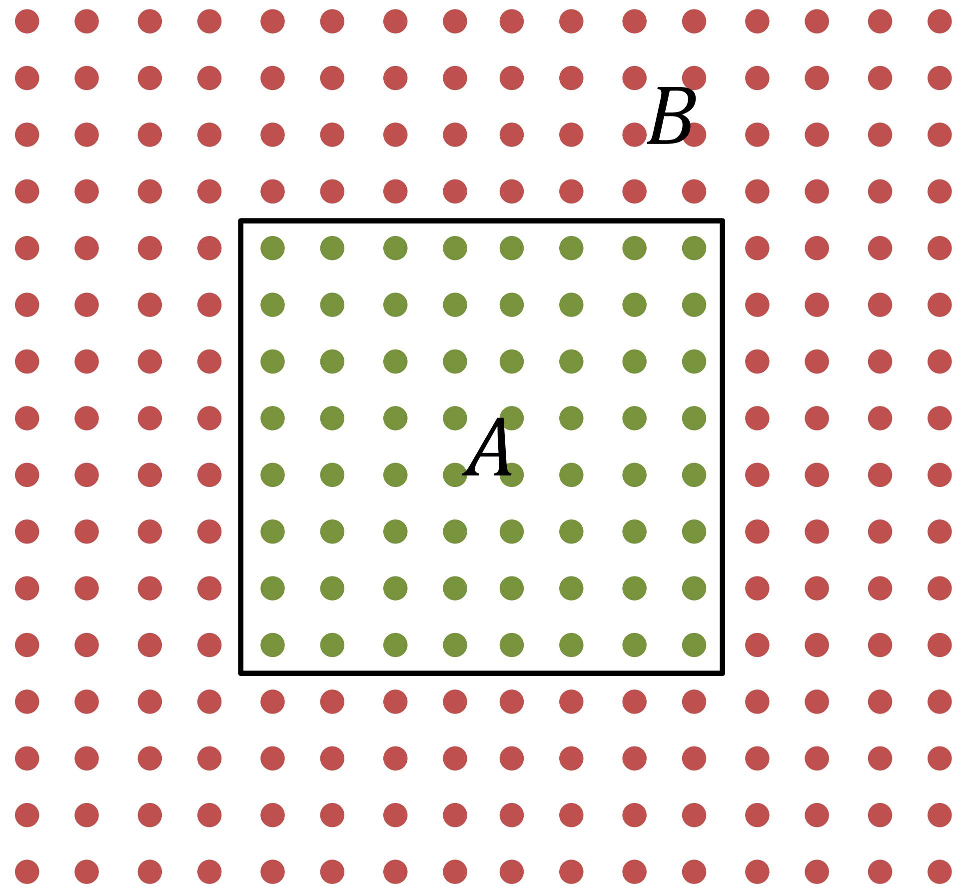
- **entanglement spectrum**

$$\rho^A = \sum_\alpha p_\alpha \, |\Psi_\alpha\rangle\langle\Psi_\alpha|$$

$$p_\alpha = e^{-E_\alpha}$$





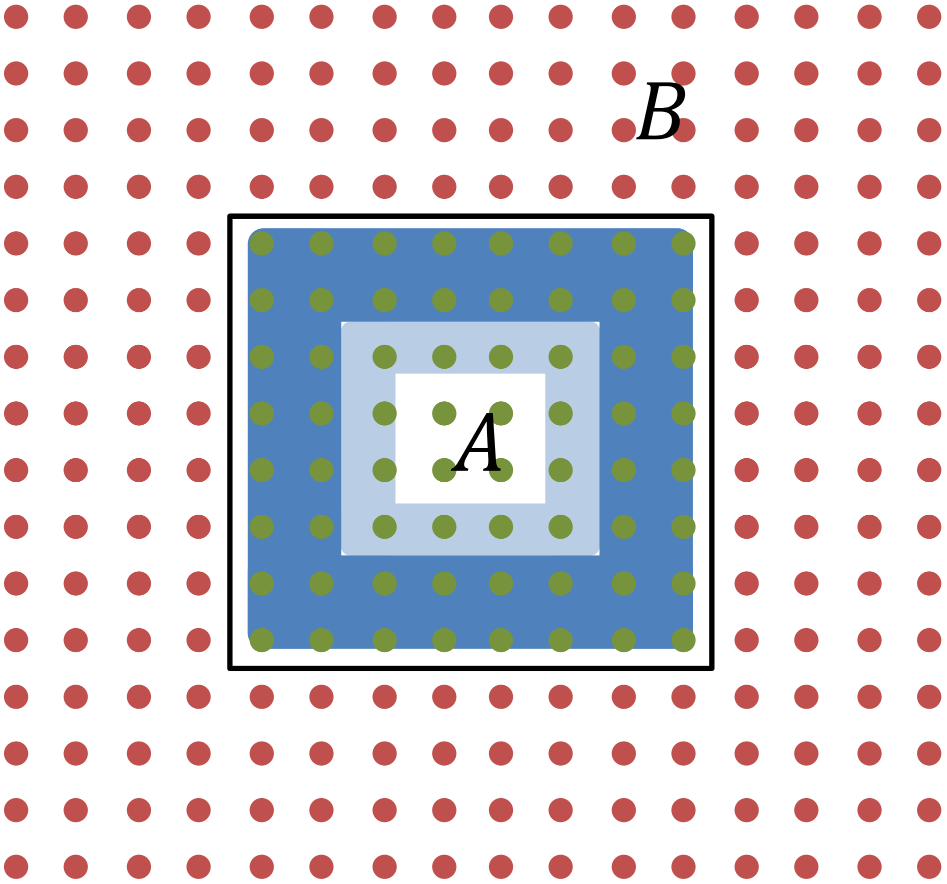


In chiral topological order, entanglement spectrum related to spectrum of the boundary theory

Beyond entanglement entropy of a block?

- **entanglement shape**

Where are the entangled degrees of freedom?



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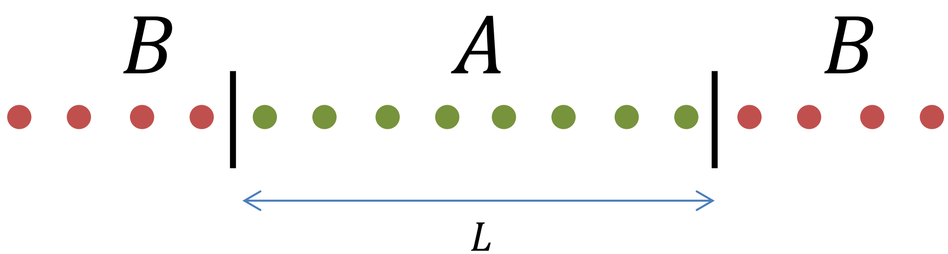
Entanglement entropy in free fermions

Hamiltonian

$$H = \sum_i (a_i^\dagger a_{i+1} + h.c.) + \gamma \sum_i (a_i a_{i+1} + h.c.) + \mu \sum_i a_i^\dagger a_i$$

↓

ground state $|\Psi\rangle$

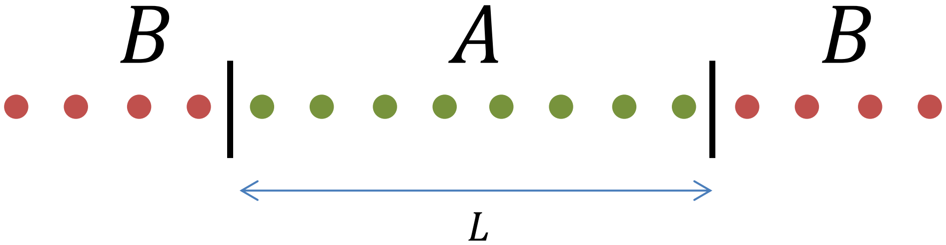


↓

reduced density matrix ρ^A ⟷ Γ^A correlation matrix

$$(\Gamma^A)_{ij} \equiv \langle \Psi | a_i^\dagger a_j | \Psi \rangle$$

* more generally, majorana fermion formalism if particle number is not preserved



reduced density matrix $\rho^A \longleftrightarrow \Gamma^A$ correlation matrix

$$(\Gamma^A)_{ij} \equiv \langle\Psi|a_i^\dagger a_j|\Psi\rangle$$

Hermitian matrix

$$\Gamma^A = U \begin{bmatrix} q_1 & & \\ & \ddots & \\ & & q_L \end{bmatrix} U^\dagger$$

$$\langle\Psi|b_m^\dagger b_n|\Psi\rangle = \delta_{mn} q_m$$

$$a_i = \sum_m U_{im} b_m$$

Entropy $S^A = \sum_m S_m = \sum_m [-q_m \log q_m - (1 - \mathrm{q_m}) \log(1 - \mathrm{q_m})]$

Entropy of site i $\quad s_i \equiv \sum_m |U_{im}|^2 S_m$

$$\sum_i |U_{im}|^2 = 1$$

$$\sum_i s_i = \sum_{im} |U_{im}|^2 S_m = \sum_m S_m = S^A$$

* For majorana modes (only fermion parity conservation)

$$\Gamma^A = O\left(\oplus_m \begin{bmatrix} 0 & \nu_m \\ -\nu_m & 0 \end{bmatrix}\right) O^T$$

$$\begin{bmatrix} c_{2i-1} \\ c_{2i} \end{bmatrix} = \sum_m \begin{bmatrix} O_{2i-1,2m-1} & O_{2i-1,2m} \\ O_{2i,2m-1} & O_{2i,2m} \end{bmatrix} \begin{bmatrix} d_{2m-1} \\ d_{2m} \end{bmatrix}$$

Entropy

$$S^A = \sum_m S_m = \sum_m -\left[\frac{1+\nu_m}{2}\log\left(\frac{1+\nu_m}{2}\right) + \frac{1-\nu_m}{2}\log\left(\frac{1-\nu_m}{2}\right)\right]$$

Entropy of site i

$$s_i \equiv \sum_m p_{im} S_m$$

$$\sum_i s_i = \sum_{im} p_{im} S_m = \sum_m S_m = S^A$$

$$p_{im} \equiv \frac{1}{2}\left\{\left|O_{2i-1,2m-1}\right|^2 + \left|O_{2i-1,2m}\right|^2 + \left|O_{2i,2m-1}\right|^2 + \left|O_{2i,2m}\right|^2\right\} \qquad \sum_i p_{im} = 1$$

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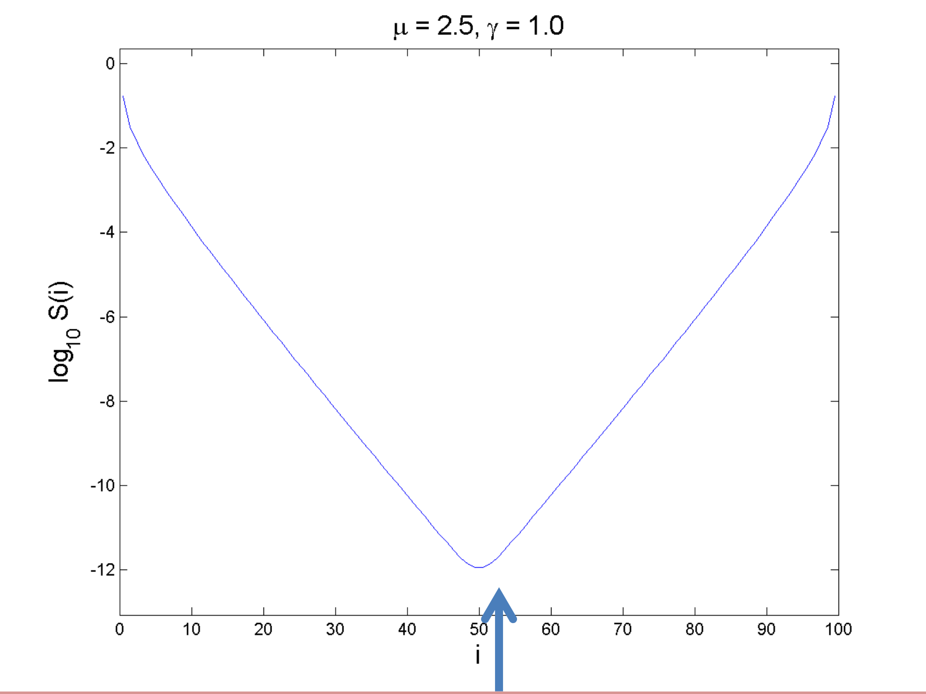
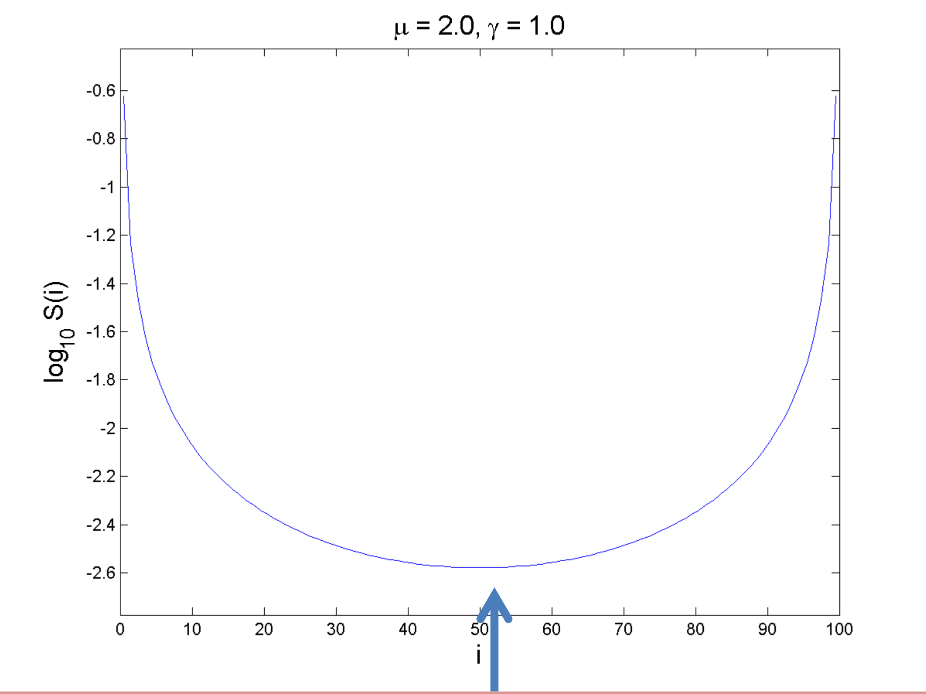
$$H = \sum_i (a_i^\dagger a_{i+1} + h.c.) + \gamma \sum_i (a_i a_{i+1} + h.c.) + \mu \sum_i a_i^\dagger a_i$$

- infinite chain
- finite block (100 sites)

gapped



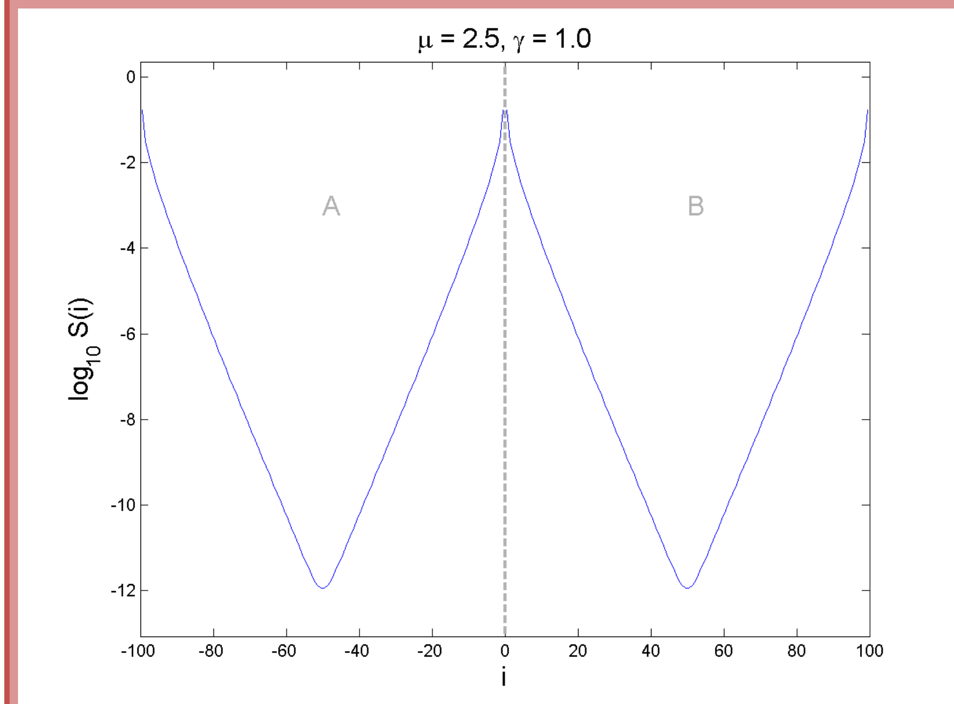
gapless



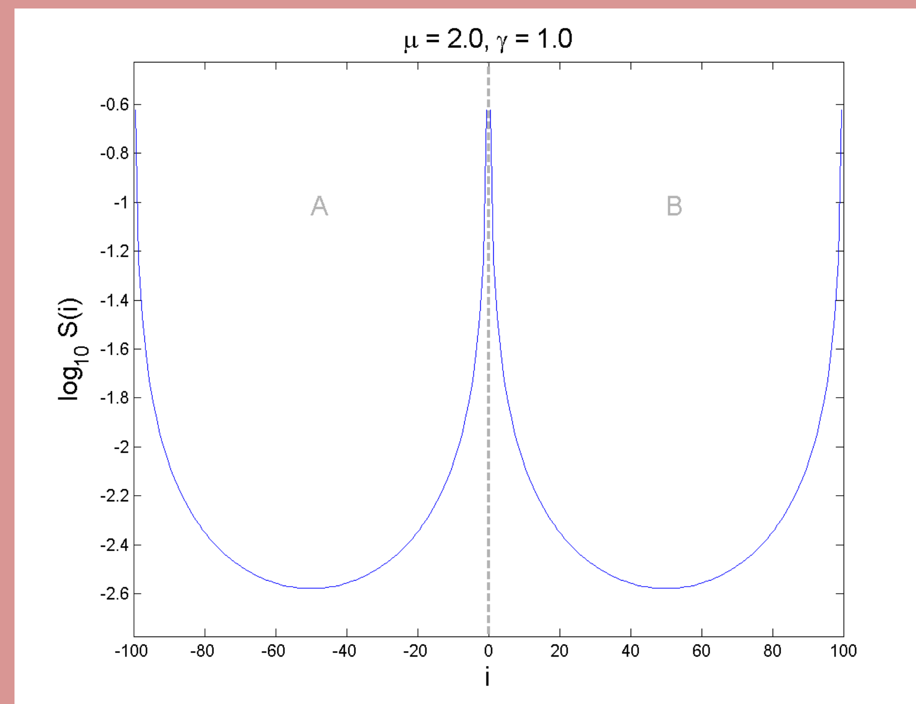
$$H = \sum_i (a_i^\dagger a_{i+1} + h.c.) + \gamma \sum_i (a_i a_{i+1} + h.c.) + \mu \sum_i a_i^\dagger a_i$$

- finite periodic chain (200 sites)
- region A (B) has 100 sites

gapped



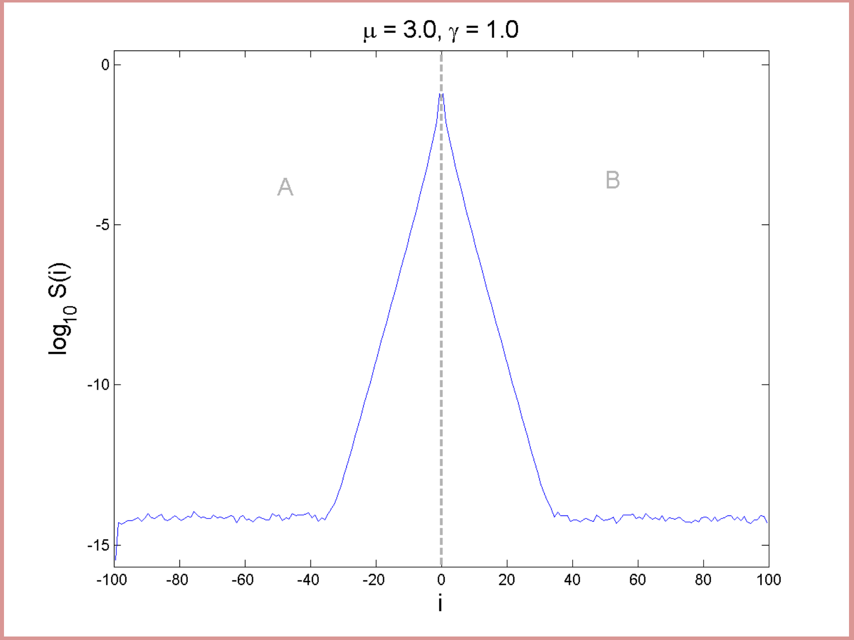
gapless



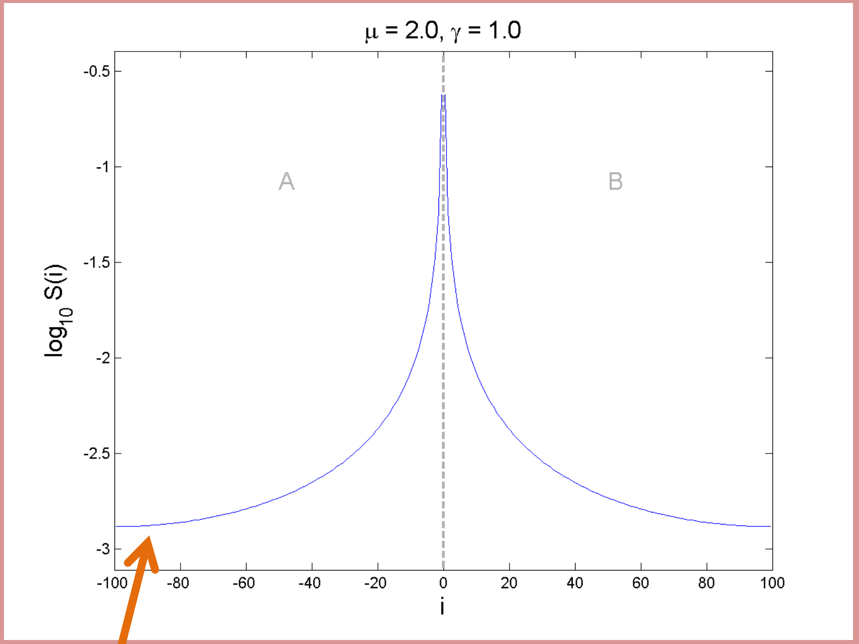
$$H = \sum_i (a_i^\dagger a_{i+1} + h.c.) + \gamma \sum_i (a_i a_{i+1} + h.c.) + \mu \sum_i a_i^\dagger a_i$$

- finite open chain (200+ sites)
- region A (B) has 100 sites

gapped



gapless



this tail explains divergence of
entanglement of half an infinite chain at criticality

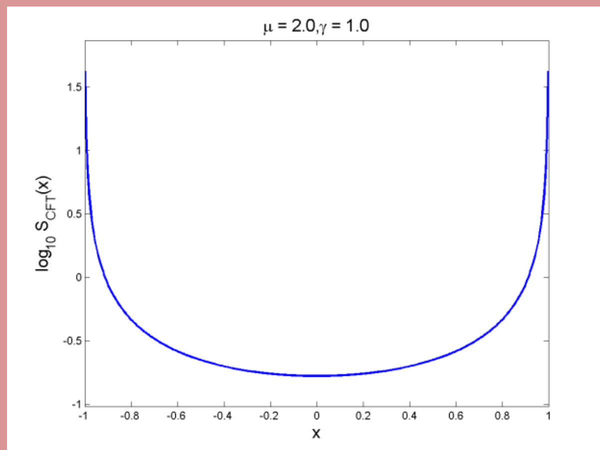
$$H = \sum_i (a_i^\dagger a_{i+1} + h.c.) + \gamma \sum_i (a_i a_{i+1} + h.c.) + \mu \sum_i a_i^\dagger a_i$$

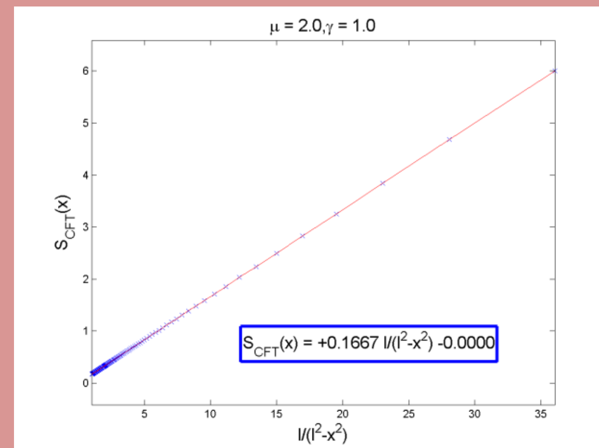
- infinite chain, gapless
- finite block $[-l, l]$
- CFT prediction (Rob Myers)

$$s_x = \frac{c}{3} \frac{l}{l^2 - x^2}$$

c central charge

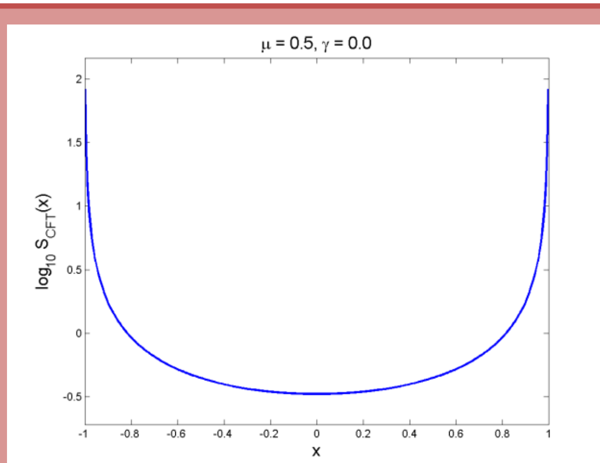
critical Ising model

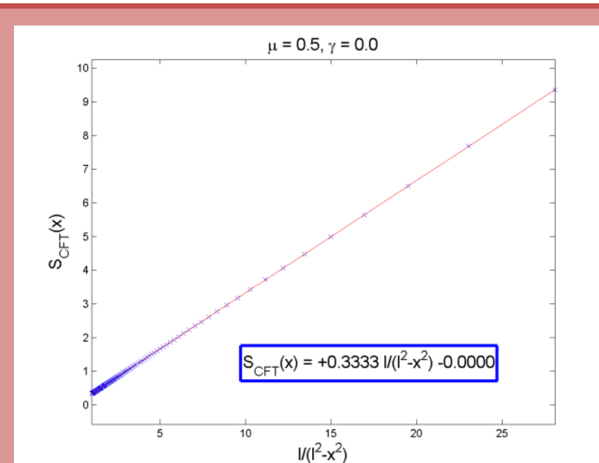




$c = 1/2$

critical XX model

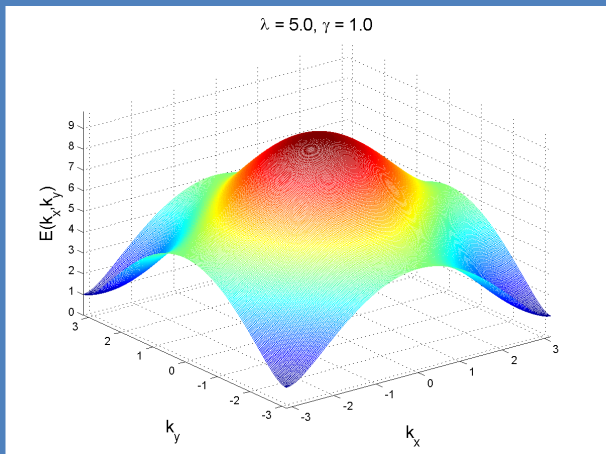


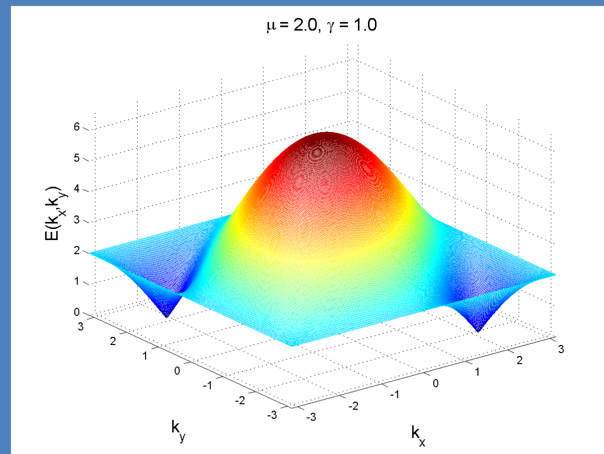


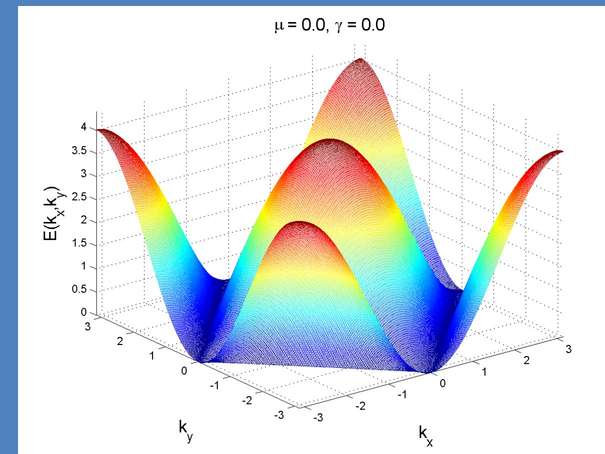
$c = 1$

2D

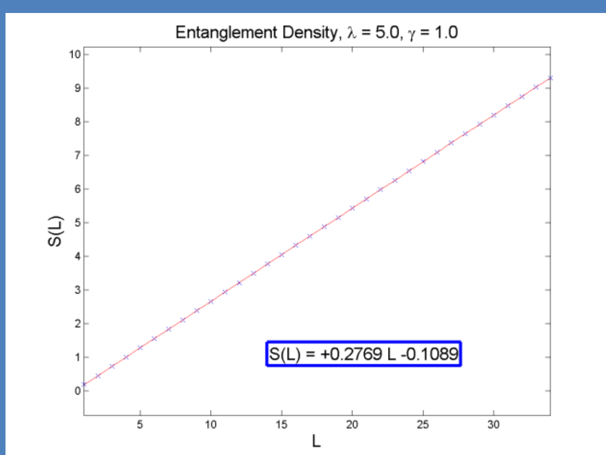
$$H = \sum_{<i,i>} (a_i^\dagger a_j + h.c.) + \gamma \sum_{<i,j>} (a_i a_j + h.c.) + \mu \sum_i a_i^\dagger a_i$$

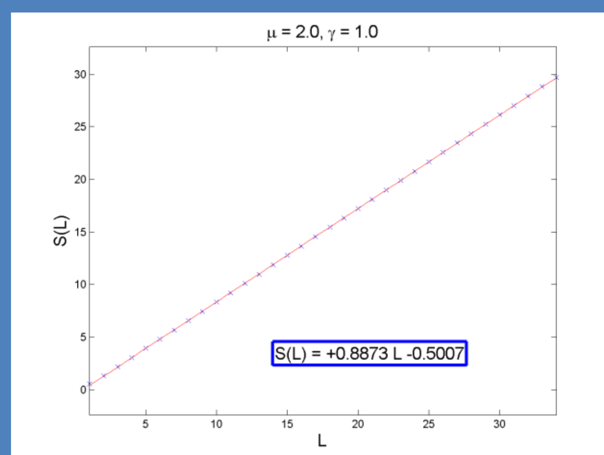


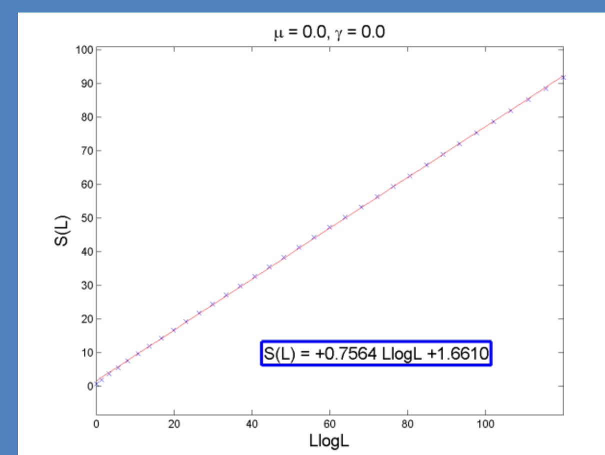




gapped | gapless | gapless II





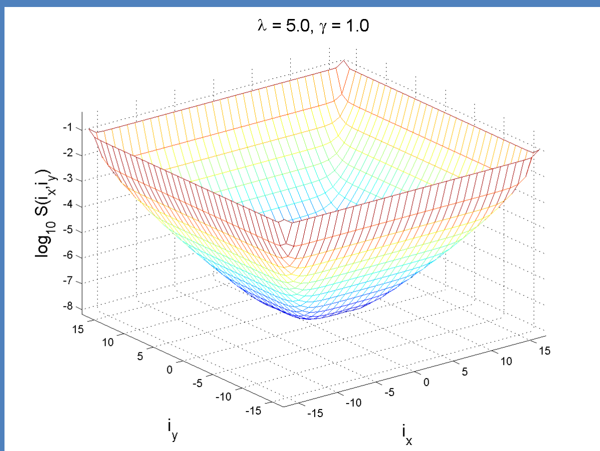


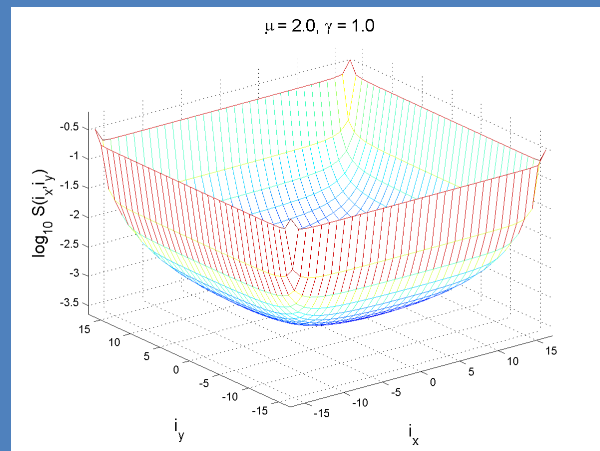
$s \approx L$ | $s \approx L$ | $s \approx L \log(L)$

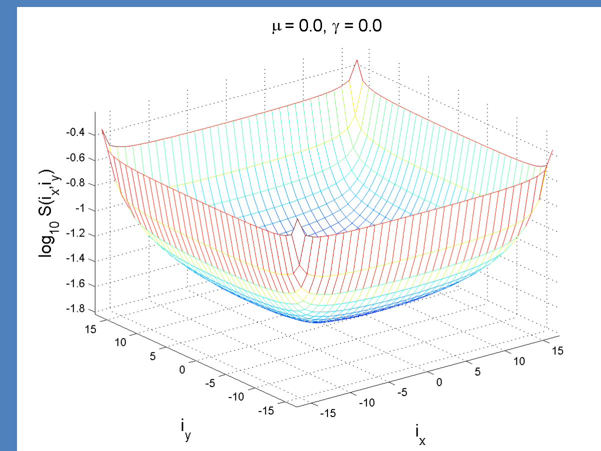
gapped

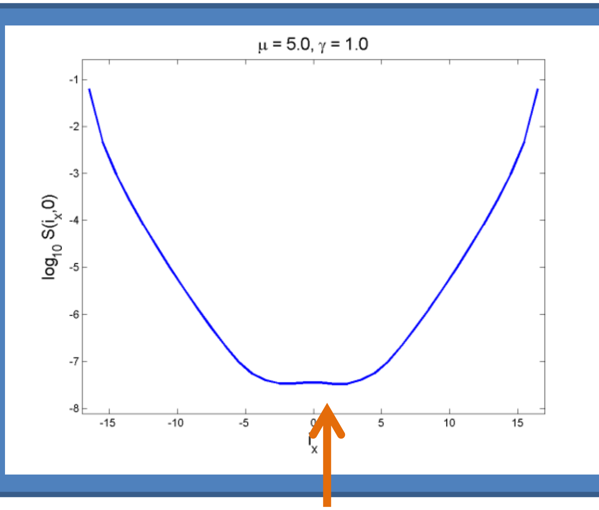
gapless

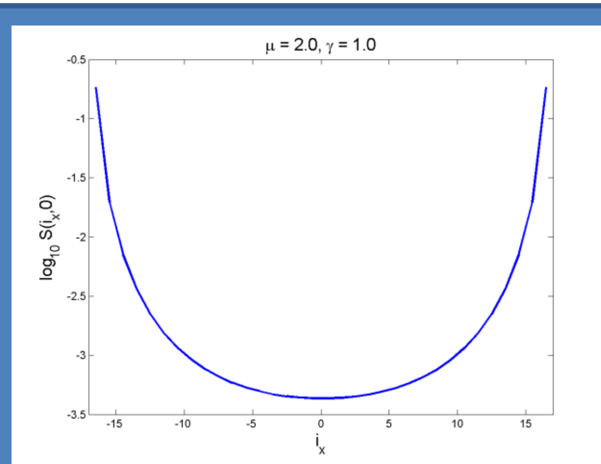
gapless II

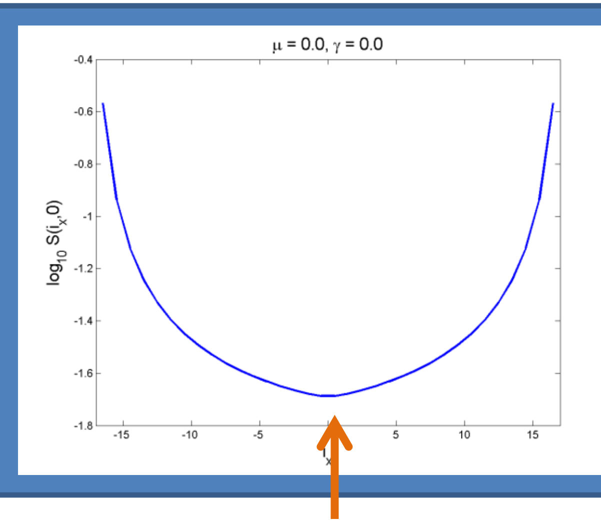












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  - ground state entanglement
  - **entanglement after a quantum quench**
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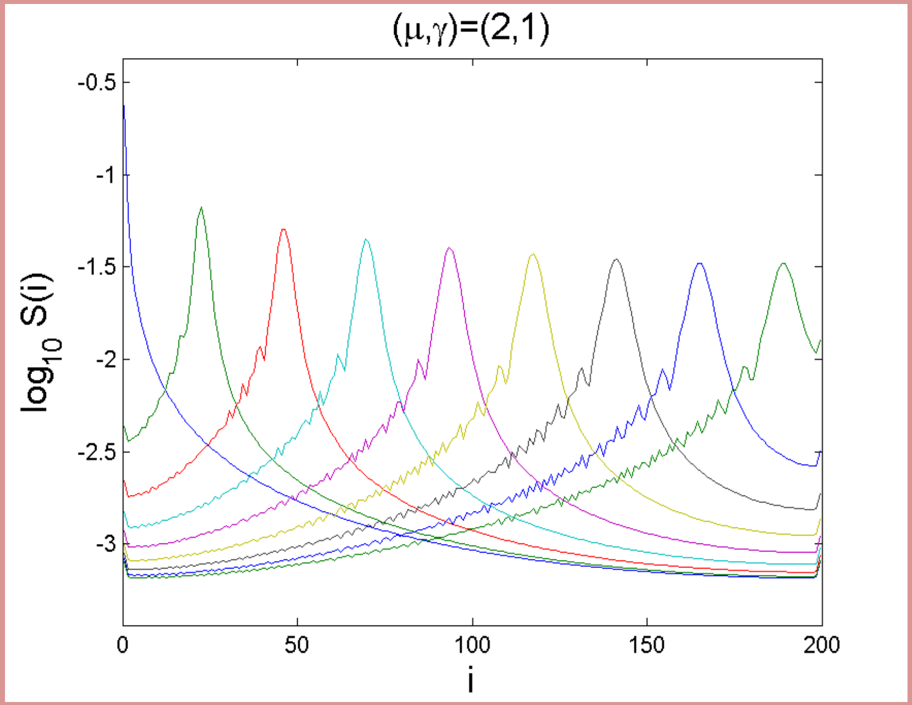
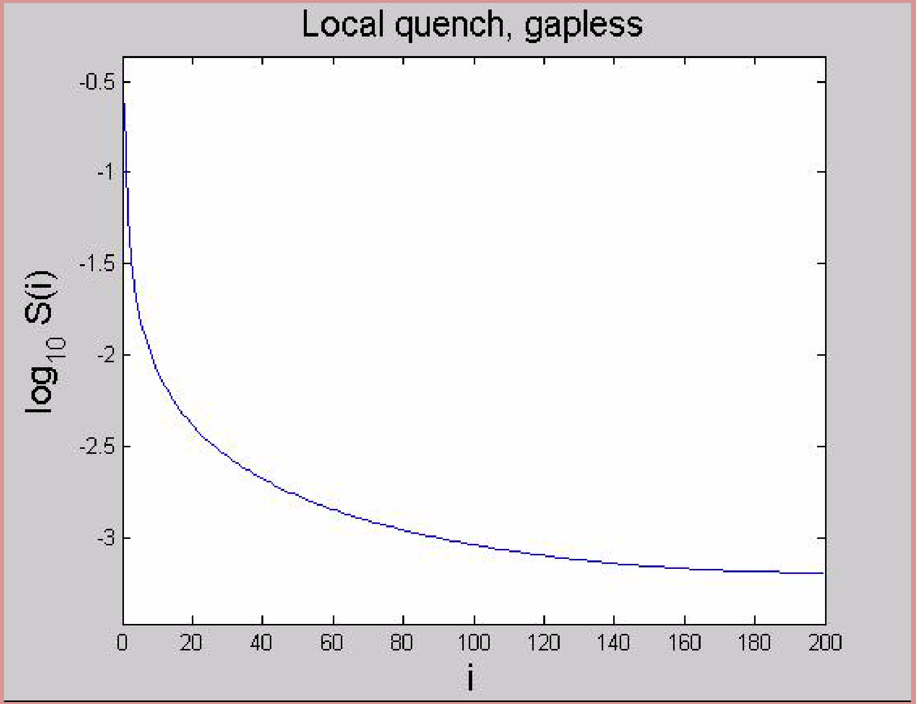
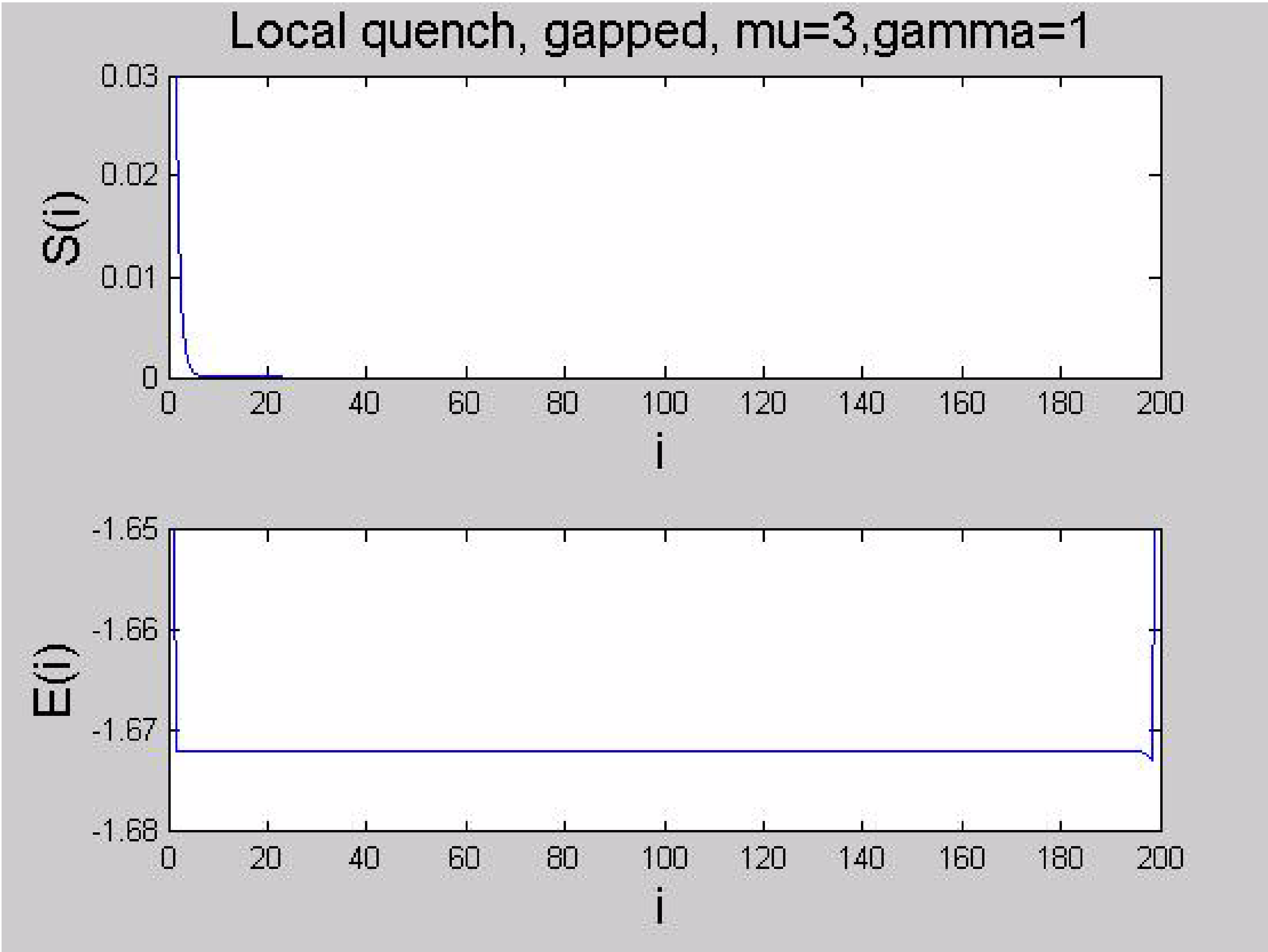
$$H = \sum_i (a_i^\dagger a_{i+1} + h.c.) + \gamma \sum_i (a_i a_{i+1} + h.c.) + \mu \sum_i a_i^\dagger a_i$$

- gapless system (200+200 sites)
- local quench -- disconnect A and B





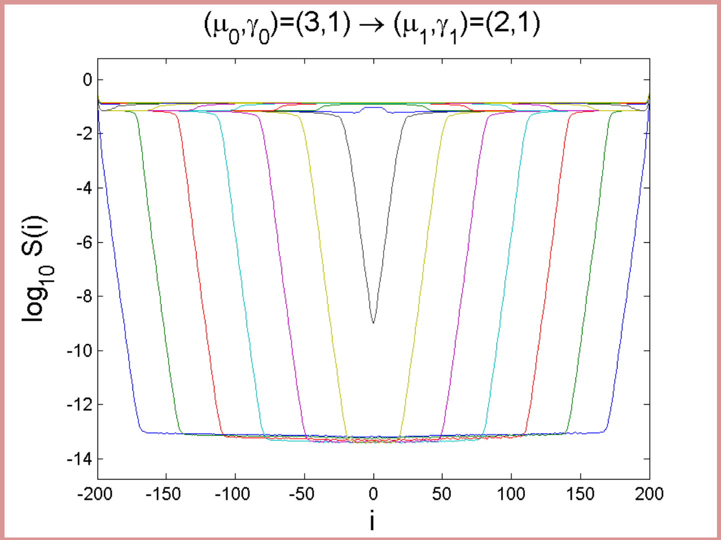


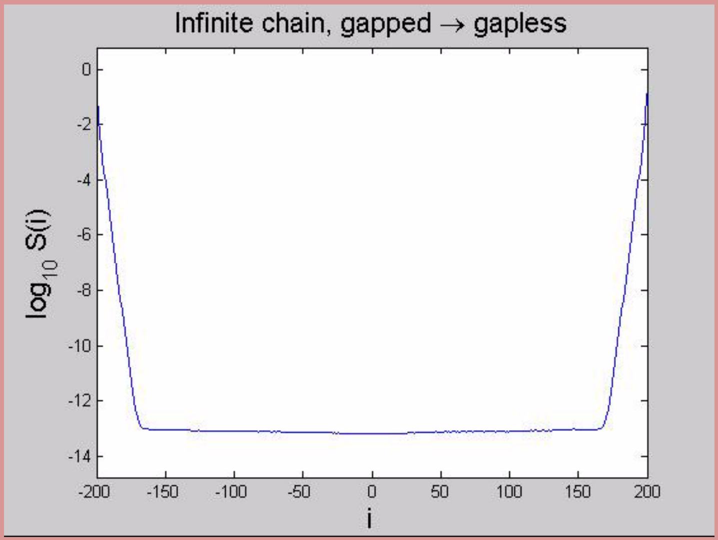
Detect low energy decoupling (e.g. spin charge separation) without knowing what degrees of freedom separate.

Global quench

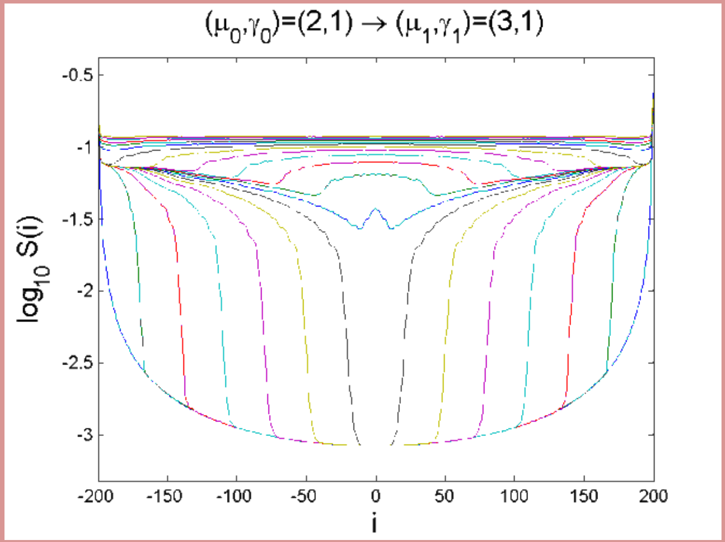
$$H = \sum_i (a_i^\dagger a_{i+1} + h.c.) + \gamma \sum_i (a_i a_{i+1} + h.c.) + \mu \sum_i a_i^\dagger a_i$$

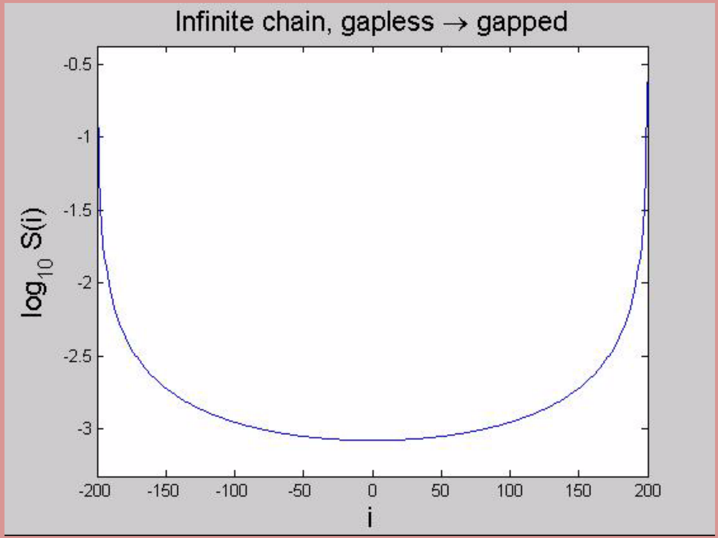
- gapped to gapless





- gapless to gapped





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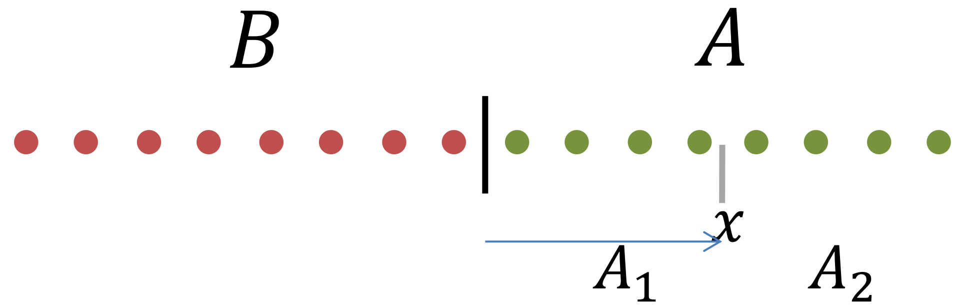
several possible generalizations

$$|\Psi\rangle = \sum_\alpha \sqrt{p_\alpha}|\psi_\alpha^A\rangle|\psi_\alpha^B\rangle$$



- Holevo's χ quantity

$$\chi(A_2) \equiv S\left(\sum_\alpha p_\alpha \rho_\alpha^{A2}\right) - \sum_\alpha p_\alpha S(\rho_\alpha^{A2})$$

- mutual information

$$I(A_1B) \equiv S(A_1) + S(B) - S(A_1B) = S(A_1) + S(A_1A_2) - S(A_2)$$

- negativity

$$E_N(A_1|B) \equiv \log tr([\rho^{A_1B}]^{T_B})$$

define entanglement shape through subtraction or derivative:

$$\chi(x) - \chi(x+1) \qquad I(x+1) - I(x) \qquad E_N(x+1) - E_N(x)$$

in a CFT:

$$\frac{dI}{dx} = \frac{c}{3}\frac{1}{x}$$

in a CFT:

$$\frac{dE_N}{dx} = \frac{c}{4}\frac{1}{x}$$

summary

Yangang Chen

- Shape of entanglement in systems of free fermions
    - ground state entanglement
    - entanglement after a quantum quench
- Beyond free fermions

THANK YOU!