Which discrete states have a continuum limit?

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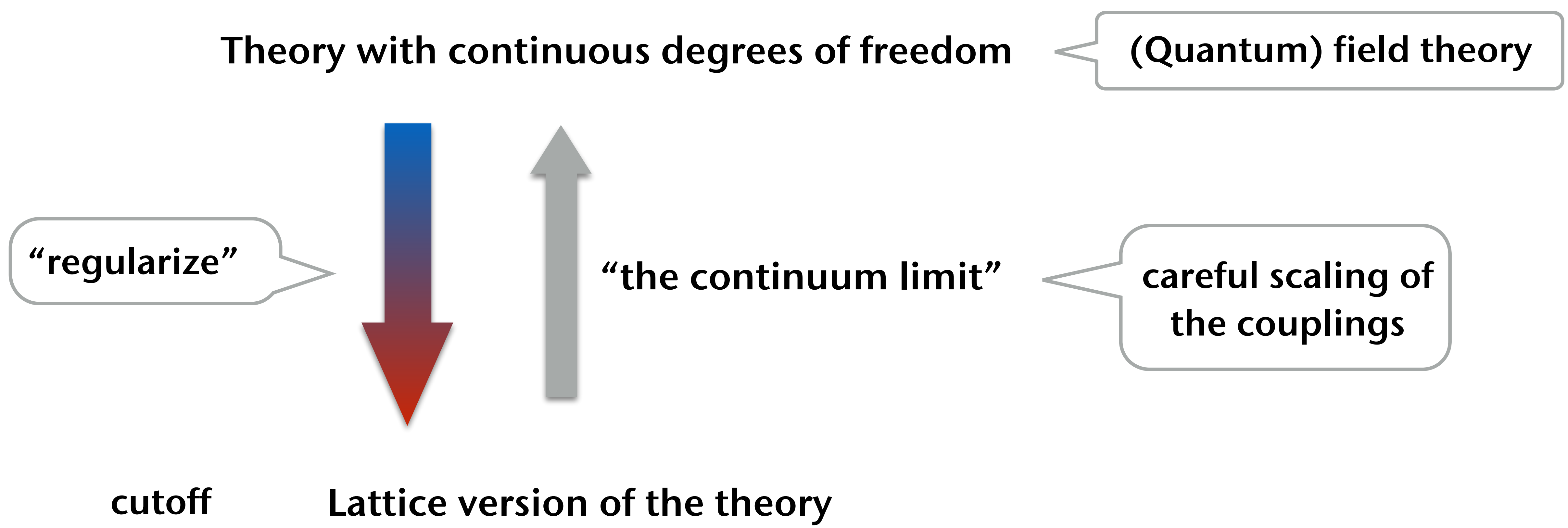
work in progress

Coogee Workshop, Sydney, Jan 22, 2015



Here: **Which discrete states are lattice versions of some continuum theory?**

Which discrete states have a continuum limit?

- In 1D, translational invariant
- Define continuum limit in 3 ways
- Characterize in terms of MPS

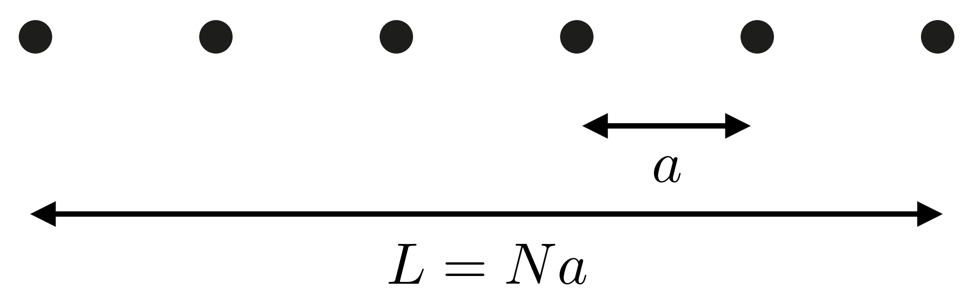
Outline

- **The setting**
- **Continuum limit 1** *The limit of the fine-graining process of the state*
- **Continuum limit 2** *The limit of the fine-graining process of the state at some coarse-grained level*
- **Continuum limit 3** *Continuum limit for expectation values*
- **Conclusions**

The setting

The state

We are given a state $|\psi\rangle = \{|\psi_N\rangle\}_{N=1}^{\infty}$ **with a certain lattice spacing** a

$|\psi_N\rangle$  $\in (\mathbb{C}^d)^{\otimes N}$

This state is

- **in 1 spatial dimension**
- **Translational invariant**
- **with Periodic Bounday Conditions**

If it is an MPS: $$|\psi_N\rangle = \mathcal{N}_N \sum_{i_1,\ldots,i_N=1}^{d} \mathrm{tr}(A_{i_1}\ldots A_{i_N})|i_1\ldots i_N\rangle$$

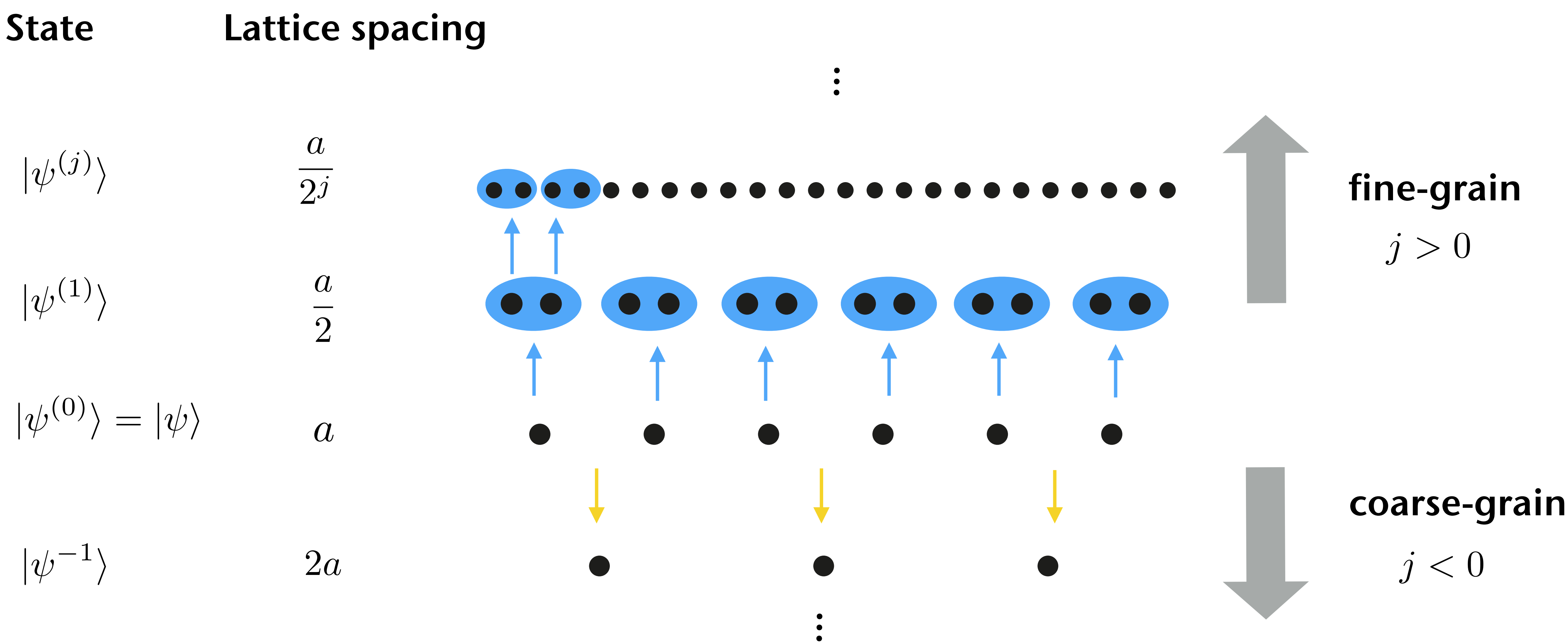
The state

Note: a state in the continuum $|\chi\rangle = \{|\chi_N\rangle\}_N$

where $|\chi_N\rangle \in \mathcal{F}_{[0,Na]}$

Fock space, where N **and** a **are defined in relation to some** $|\psi_N\rangle$

Change of scale



Note: the physical dimensions are the same throughout

Note: each $|\psi^{(j)}\rangle$ is translational invariant

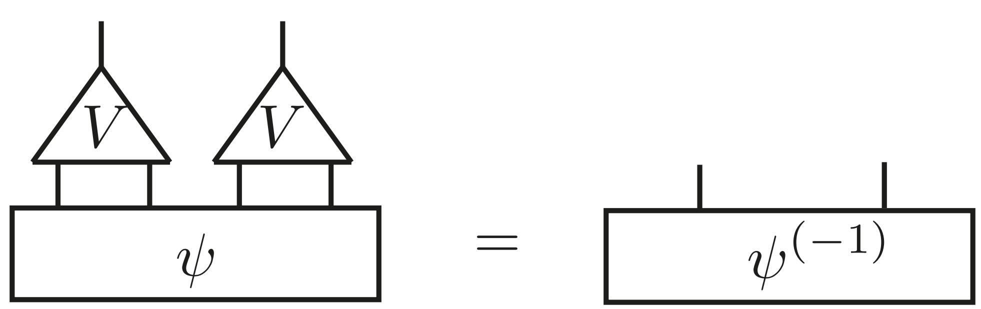
Coarse-grain

RG scale transformation to low energies

$|\psi^{(-1)}\rangle$ **is the coarse-grained version of** $|\psi\rangle$

if there is an isometry V **such that for all** N

$$V^{\otimes N}|\psi_{2N}\rangle = |\psi_N^{(-1)}\rangle$$



This is the RG flow considered by Verstraete, Rico, Latorre, Cirac & Wolf PRL 2005

Note: different notion that in recent work with Toby Cubitt

Fine-grain

RG scale transformation to high energies

$|\psi^{(1)}\rangle$ is the fine-grained version of $|\psi\rangle$

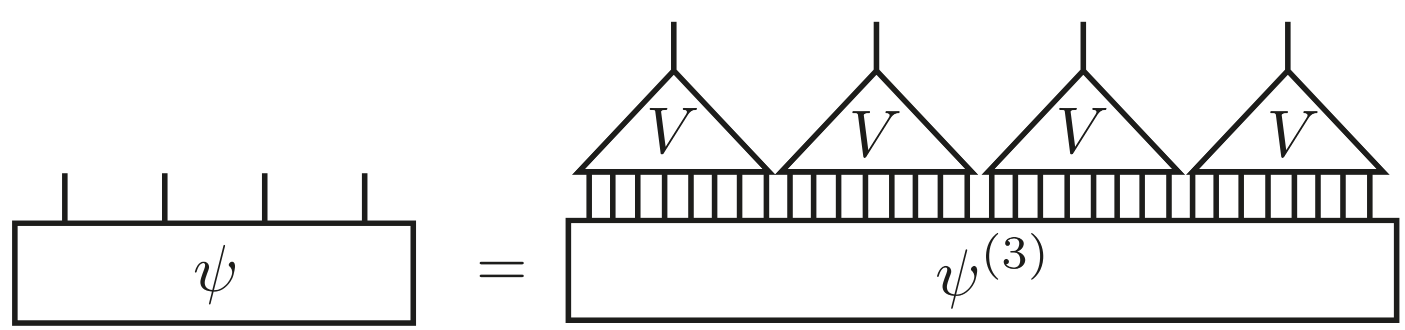
if $|\psi\rangle$ is the coarse-grained version of $|\psi^{(1)}\rangle$

Idea: $|\psi^{(1)}\rangle$ is describing the physics at half the lattice spacing

Note: Translational invariance at all scales

Note: This can be iterated:

Fine-grain 3 times:



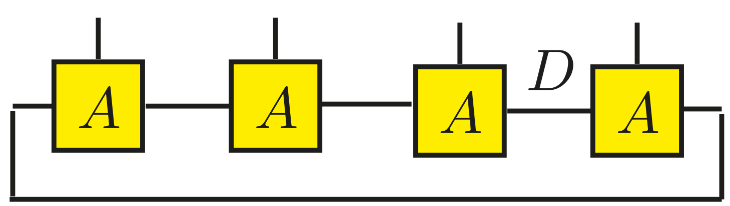
Tensor networks

We will characterise which states satisfy either definition for MPS

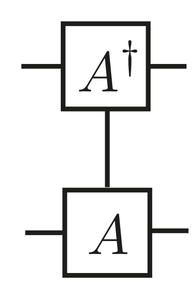
- **Matrix Product State:**

$$|\psi\rangle = \sum_{i_1 \ldots i_N} \mathrm{Tr}(A_{i_1} \ldots A_{i_N})|i_1 \ldots i_N\rangle$$

$|\psi\rangle =$



- **The transfer matrix of an MPS is:** $E =$



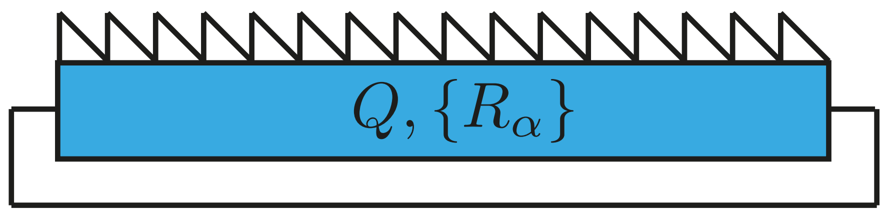
It is a quantum channel.

Tensor networks

- **Continuous MPS:**

$$|\psi\rangle = \mathrm{Tr}\{\mathcal{P}\exp[\int_0^L Q \otimes I + \sum_{\alpha=1}^{q} R_\alpha \otimes \hat{\psi}^\dagger(x)]\}|\Omega\rangle$$

$|\psi\rangle =$



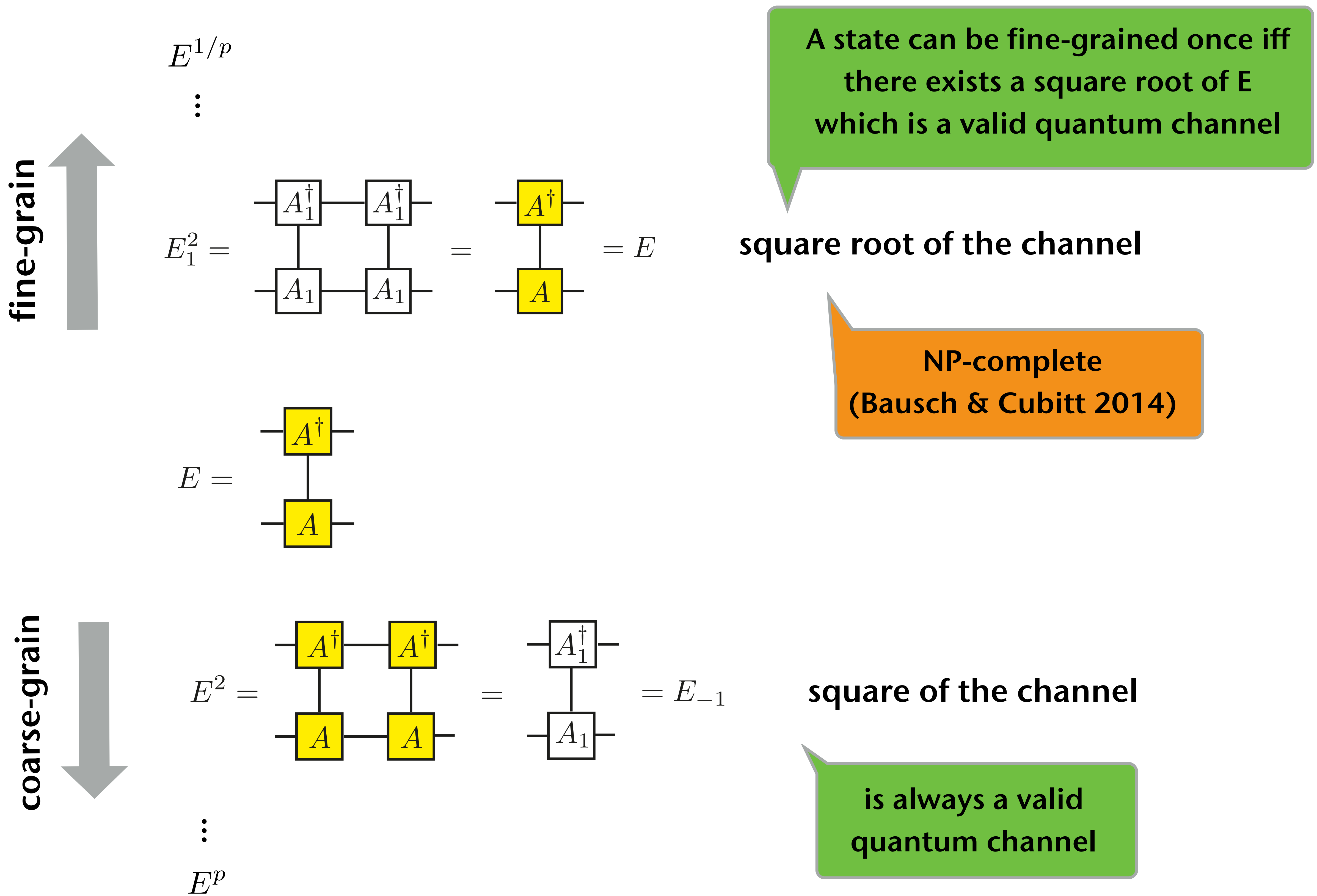
- **The transfer matrix of a cMPS is a Markovian quantum channel**

$$E(\rho) = e^{\mathcal{L}}(\rho)$$

Liouvillian of Lindblad form

$E^t = e^{t\mathcal{L}}$ **is a valid quantum channel for all** $t \in \mathbb{R}^+$

Coarse/fine-graining in terms of the transfer matrix:



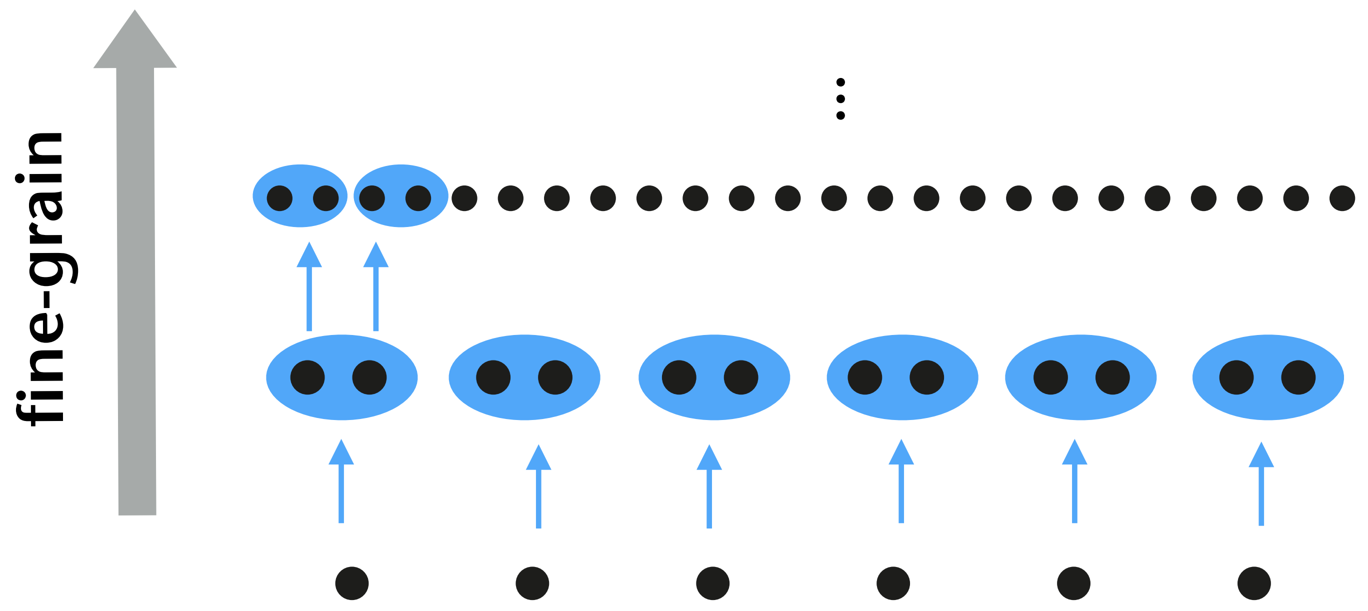
Continuum limit 1

the limit of the fine-graining process of the state

Definition

$|\psi\rangle$ has a continuum limit 1 if it can be fine-grained infinitely many times, and the limit converges.

Continuum limit 1



Characterisation

$|\psi\rangle$ has a CL1 if and only if

its transfer matrix is Markovian

Moreover the continuum limit is a cMPS with matrices given by the Hamiltonian and jump operators of the transfer matrix of $|\psi\rangle$

Determining if a quantum channel is Markovian is

- For fixed dimension: integer SDP (Wolf, Eisert, Cubitt, Cirac, PRL 2008)
- For variable dimension: NP-hard (Cubitt, Eisert, Wolf, CMP 2012)

Continuum limit 2

the limit of the fine-graining process of the state

at some coarse-grained level

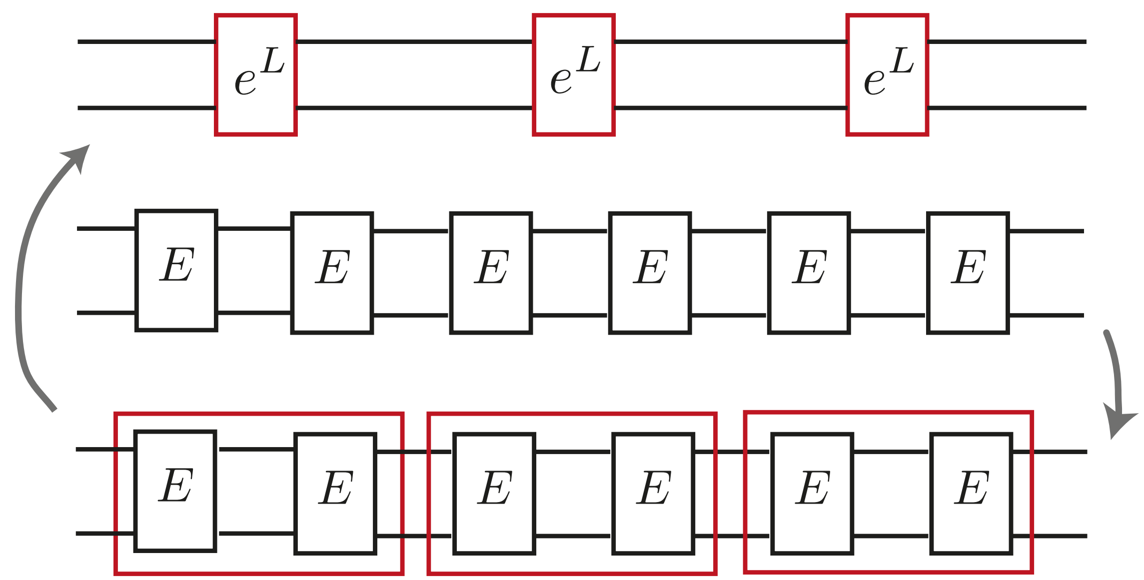
Definition

$|\psi\rangle$ **has a CL2 if it has a CL1**

after a finite number of coarse-graining steps

then fine-grain

to infinity



first coarse-grain

a finite number of times

Idea: capture states which have a finite periodicity.

Characterisation

Computational Complexity?

Characterisation: $|\psi\rangle$ has a CL2 iff there exists a $p \in \mathbb{N}$ such that E^p is Markovian

Result 1: The class of states with a CL2 is larger than that with CL1

- Example: Holevo channel (Qubit channel)

			State:
$E(\rho) = \frac{1}{3}\left(\rho^T + I\mathrm{Tr}(\rho)\right)$	Indivisible	$\longrightarrow$	Concatenated 0000s and 1111s
$E^2(\rho) = \frac{1}{9}\left(\rho + 4I\mathrm{Tr}(\rho)\right)$	Markovian	$\longrightarrow$	Essentially all 1111s

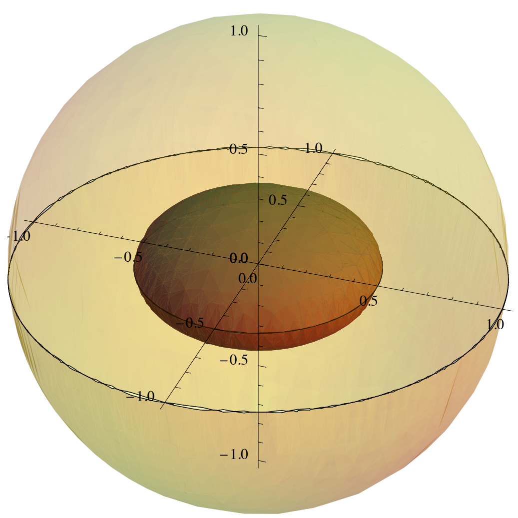
Every odd power is not Markovian, and every even power is Markovian.

Characterisation

Result 2: **Not all states have a CL2**

- **Example: Pancake channel** $E = \mathrm{diag}(1, a, a, a^2/2)$ **in the Pauli basis**

Image in the Bloch sphere:



Note: Qubit channel, unital, primitive

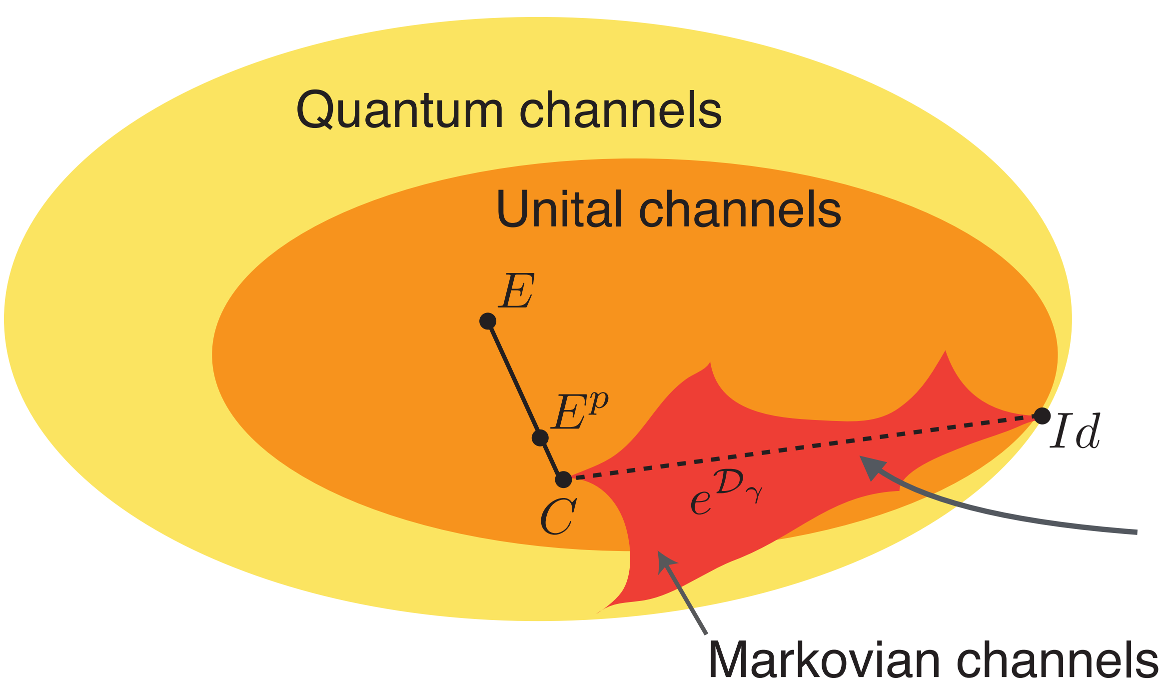
Wolf & Cirac , CMP 2008

Characterisation

Result 2: **Not all states have a CL2**

- **Example: Pancake channel** $E = \mathrm{diag}(1, a, a, a^2/2)$ **in the Pauli basis**

Note: arbitrarily close to the closure of Markovian channels



depolarizing channel with various strengths

Characterisation

Result 2: **Not all states have a CL2**

- **The set of states with this property has finite volume:**

The qubit unital channels with $E = 1 \oplus \Delta$ **in the Pauli basis**

with eigenvalues $\lambda_1, \lambda_2, \lambda_3$ **with** $0 < \lambda_3 < \lambda_1 \lambda_2$

satisfy that E^p **is not Markovian for any** p

Continuum limit 3

Expectation values coincide with those of a continuum theory,

at some coarse-grained level

Definition

$|\psi\rangle$ has a continuum limit 3 if there exists

a fixed $k \in \mathbb{N}$ ← level of coarse-graining

a state in the continuum $|\chi\rangle$

and an isometry $V : \mathcal{F}_{[0,ka]} \to \mathbb{C}^d$

such that for all N and for all observables,

it holds that $\langle\psi_N|O_1 \otimes \ldots O_N|\psi\rangle = \langle\chi_{N/k}|\tilde{O}_1 \ldots \tilde{O}_{N/k}|\chi_{N/k}\rangle$

where $\tilde{O}_i = V^\dagger O_i V$

Note: we can only access certain observables of the continuum theory

Note: isometries must only “refocus” the dimensions.

This restricts the bond dimension of $|\chi\rangle$

Characterisation

Result 3: If $|\psi\rangle$ is an MPS and it has a CL3, then it has a CL2.

(The definition is very strong, so it leaves no more room.)

Conclusions

Which discrete states have a continuum limit?

1D, Translationally invariant, MPS

- Continuum limit 1: the limit of the fine-graining process

All states whose transfer matrix is Markovian

The limit is a cMPS.

- Continuum limit 2: the limit of the fine-graining process of the state, at some coarse-grained level

All states whose transfer matrix to some power is Markovian

CL1 $\subset$ CL2 $\subset$ All states

- Continuum limit 3: Expectation values coincide with those of a continuum theory, at some coarse-grained level

CL3, and MPS = CL2