The Niels Bohr International Academy

UNIVERSITY OF COPENHAGEN

# NON-EQUILIBRIUM PHASE TRANSITIONS

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Coogee 2015

CARLSBERGFONDET

VILLUM FONDEN

# OUTLINE

Part 1: i) Some notions of non-equilibrium phase transitions
ii) Directed percolation universality class
iii) Self-organized criticality
iv) connections to QIT

Part 2: A topological dynamical phase transition

# SETTING

Lindblad master equation:

$$\mathcal{L}(\rho) = -i[H,\rho] + \sum_j K_j \rho K_j^\dagger - \frac{1}{2}\{K_j^\dagger K_j, \rho\}_+$$

We will mostly focus on classical systems

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Asynchronous cellular automata

Pick a site a random, and update

Detailed balance

$$P(x,y)\pi(y) = P(y,x)\pi(x)$$

Examples:

Gibbs samplers, conways game of life, etc

# SETTING

Lindblad master equation:

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A non-equilibrium phase transition, is a sudden change in the steady state properties of a non-reversible markovian dissipative system

~~Detailed balance~~ $P(x,y)\pi(y) = P(y,x)\pi(x)$

Examples:

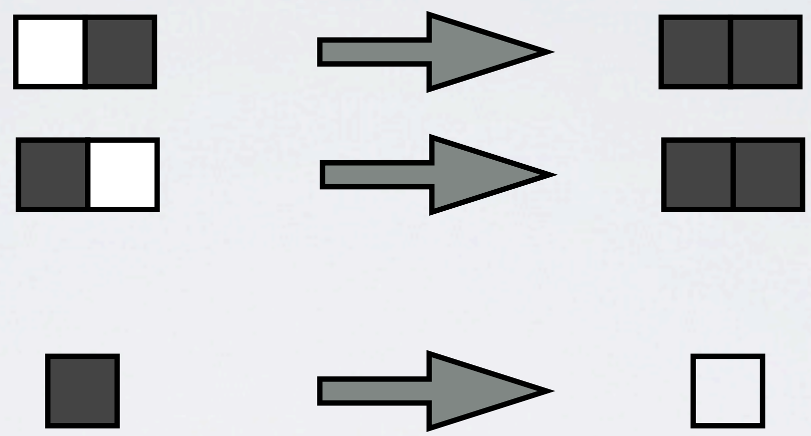
Gibbs samplers, conways game of life, etc

# DIRECTED PERCOLATION

**The contact process:** inspired by population dynamics (1D)

infection of healthy individuals

sick individuals becoming healthy



asynchronous updates!

rate $\kappa$

phase transition at $\kappa \approx 2.7$

Non-integrable model; critical value not rational

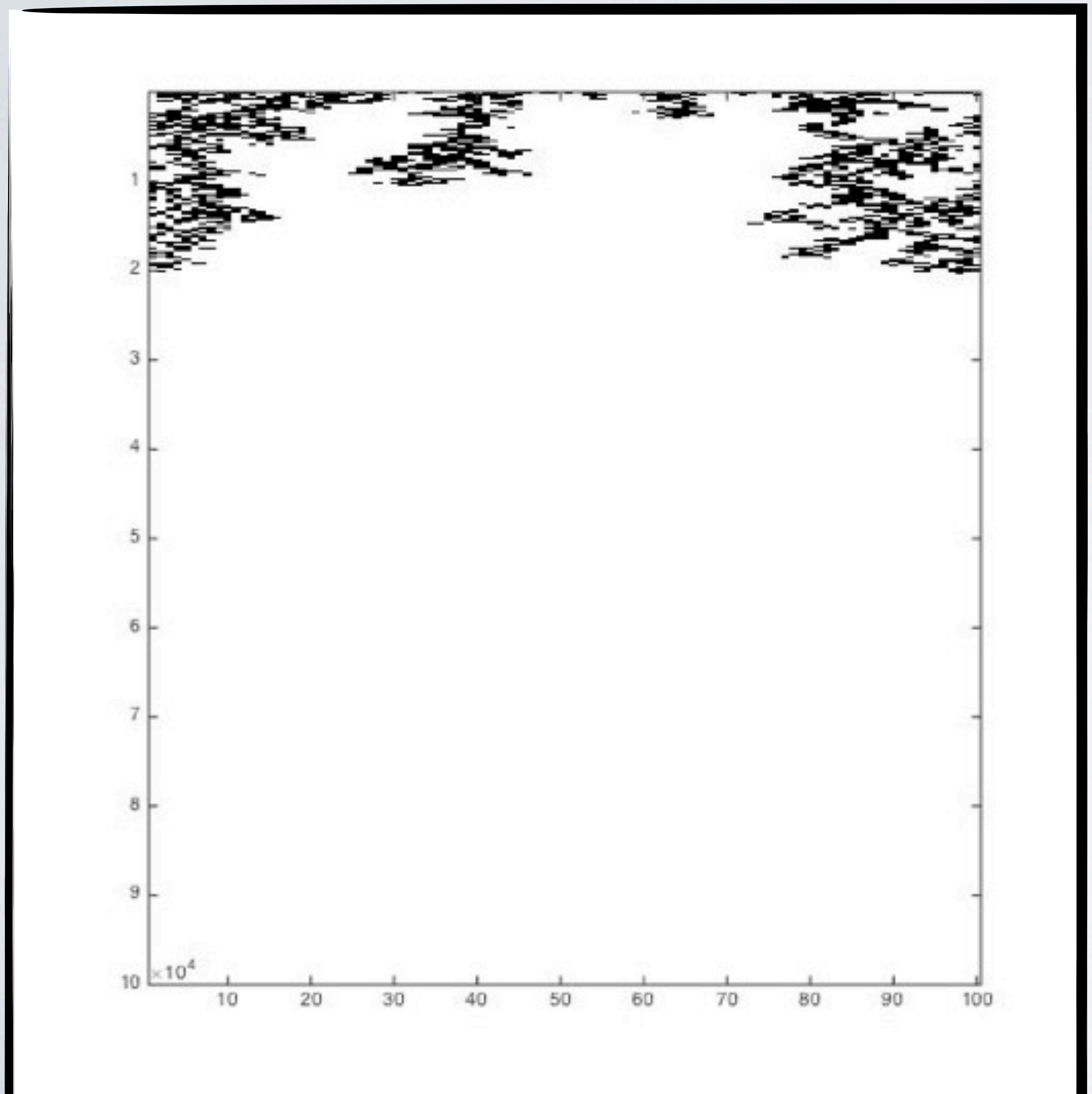
two regimes

trivial regime -> one stationary state, gapped generator

non-trivial regime -> two stationary states, non-gapped generator

# DIRECTED PERCOLATION

**trivial phase** $\kappa < \kappa_c$

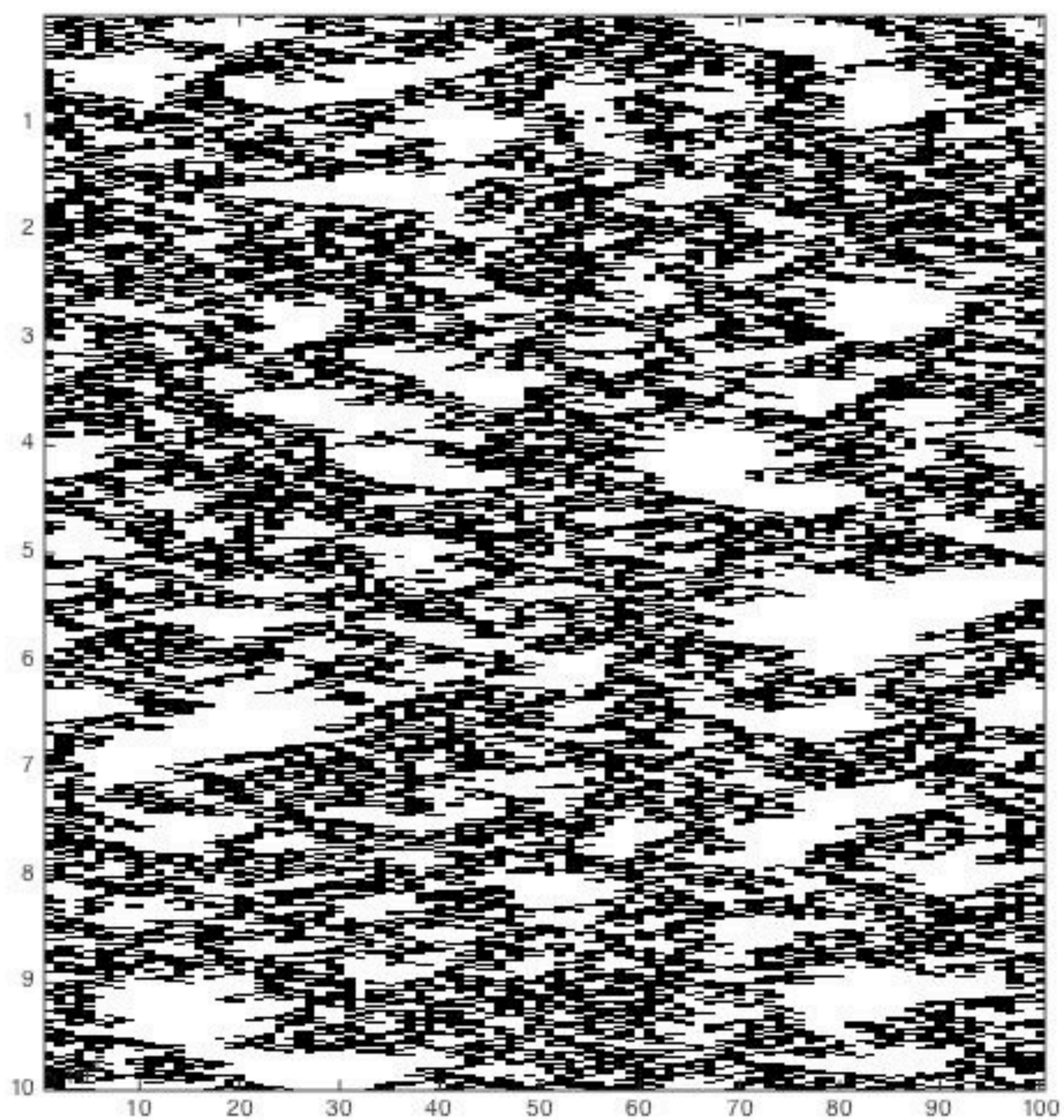



one stationary state (all 0)

O(log(N)) mixing

# DIRECTED PERCOLATION

## "symmetry broken" phase $\kappa > \kappa_c$



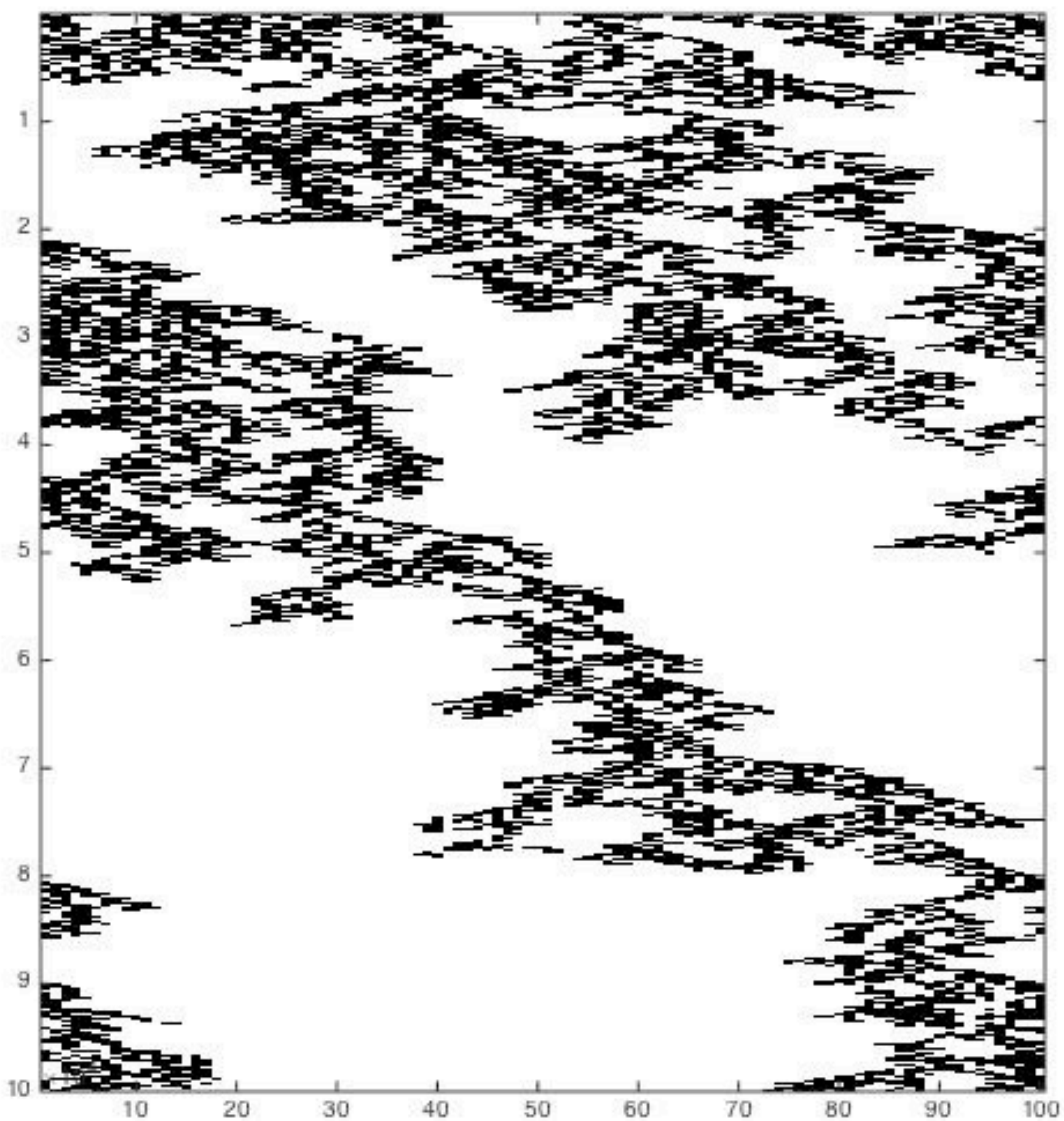

two stationary states in the thermodynamic limit

one stable one unstable

mixing time O(N) to the metastable state

# DIRECTED PERCOLATION

**critical phase** $\kappa = \kappa_c$




one stationary state for finite systems

polynomial mixing time

long range correlations (poly scaling)

# CRITICAL EXPONENTS

**scale invariance at criticality**

Equilibrium phase transitions

$$|m| \sim (T_c - T)^\beta$$ order parameter: magnetization

$$\xi \sim |T_c - T|^{-\nu}$$ correlation length

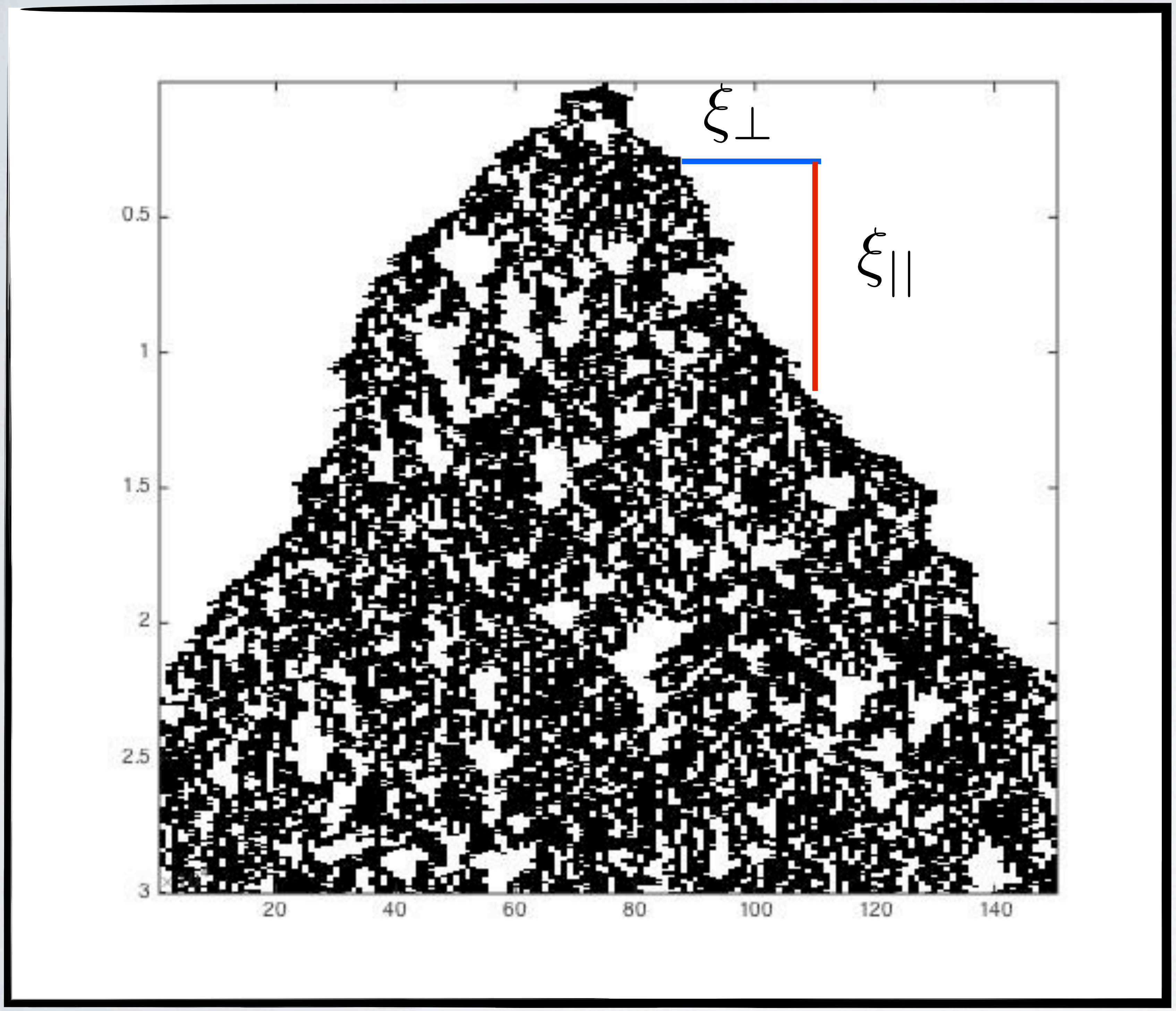
Non-equilibrium phase transitions: two correlation lengths

$$\rho \sim (\kappa - \kappa_c)^\alpha$$ order parameter: density of active sites

$$\xi_\perp \sim |\kappa - \kappa_c|^{-\nu_\perp}$$ spacial correlation length

$$\xi_{||} \sim |\kappa - \kappa_c|^{-\nu_{||}}$$ temporal correlation length

# DIRECTED PERCOLATION




**Directed percolation universality class!**

The Ising model of non-equilibrium phase transitions

non-integrable

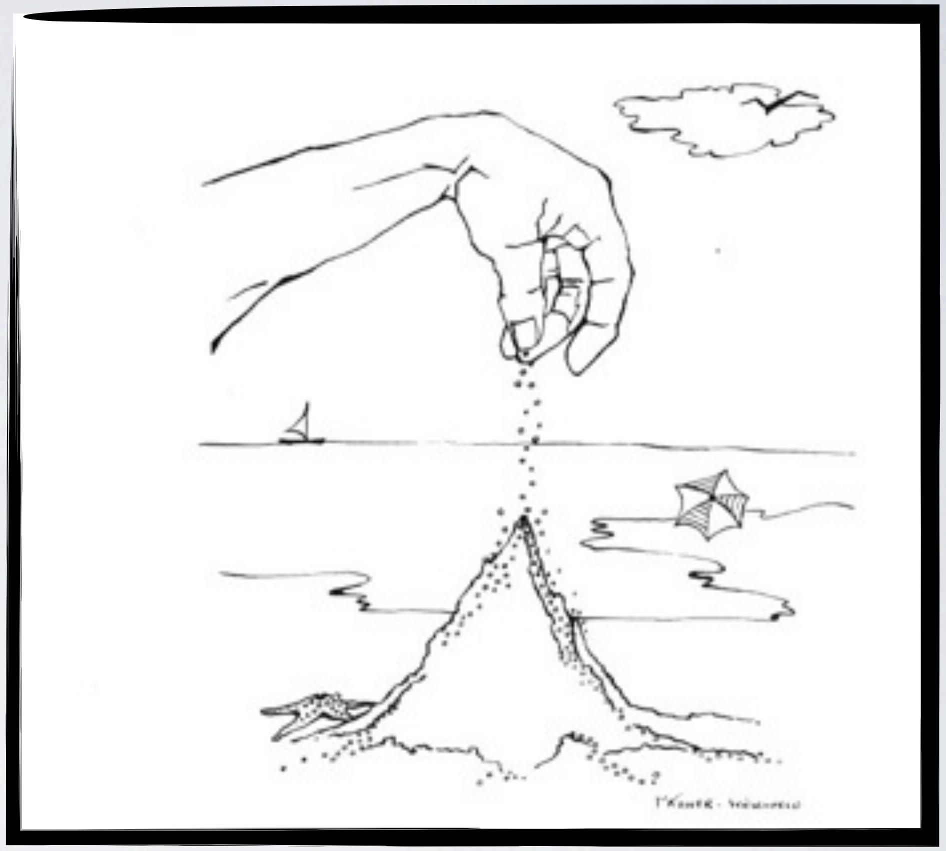
# SELF-ORGANIZED CRITICALITY

## The sandpile model:

drop a grain of sand at random on one of N sites

if the number of sand grains exceeds a given threshold (>d), then distribute one sand grain to each neighbor

continue until there is no more hopping in the system



**power law correlations**

No fine tuned parameter

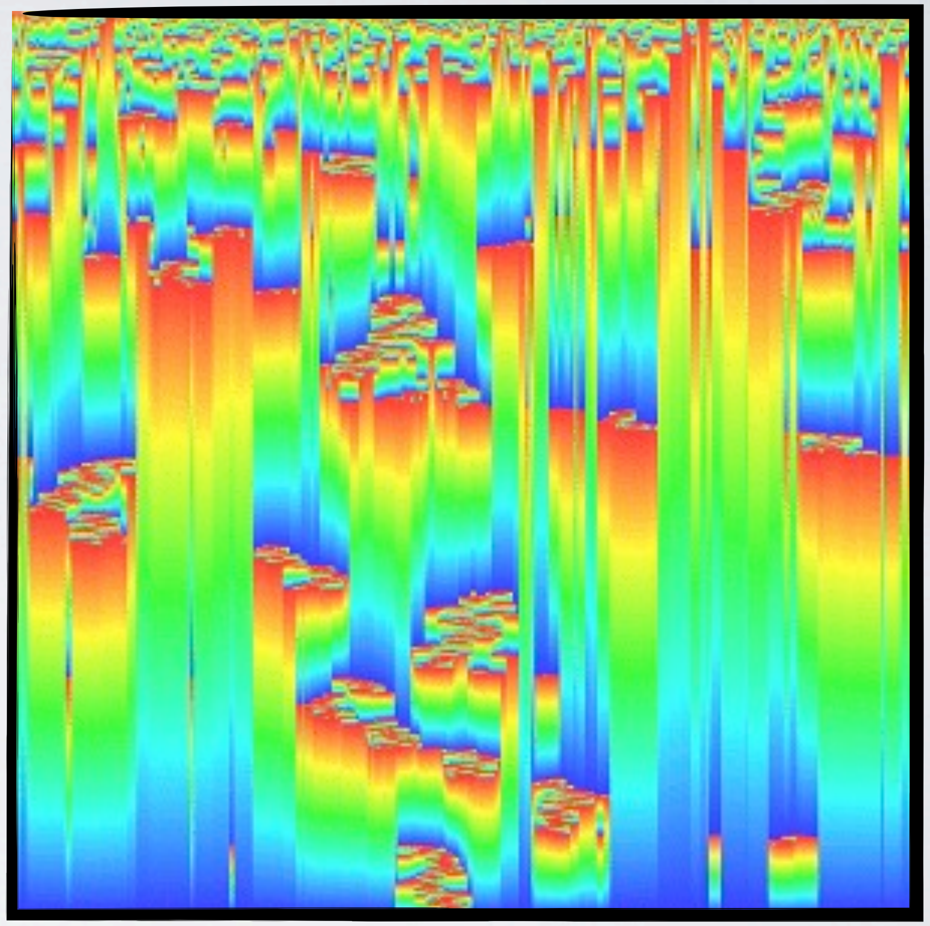
# SELF-ORGANIZED CRITICALITY

## The evolution model:

each site takes values between [0,1]

pick the site with the lowest value

replace its value with a random number between [0,1], do the same with its neighbors



**power law correlations**

No fine tuned parameter

# SELF-ORGANIZED CRITICALITY

| | Gapped | Bounded | local |
|---|---|---|---|
| Critical DP | ✗ | ✓ | ✓ |
| Sandpile | ✓ | ✗ | ✓ |
| Evolution | ✓ | ✓ | ✗ |

Reminiscent of the area law!

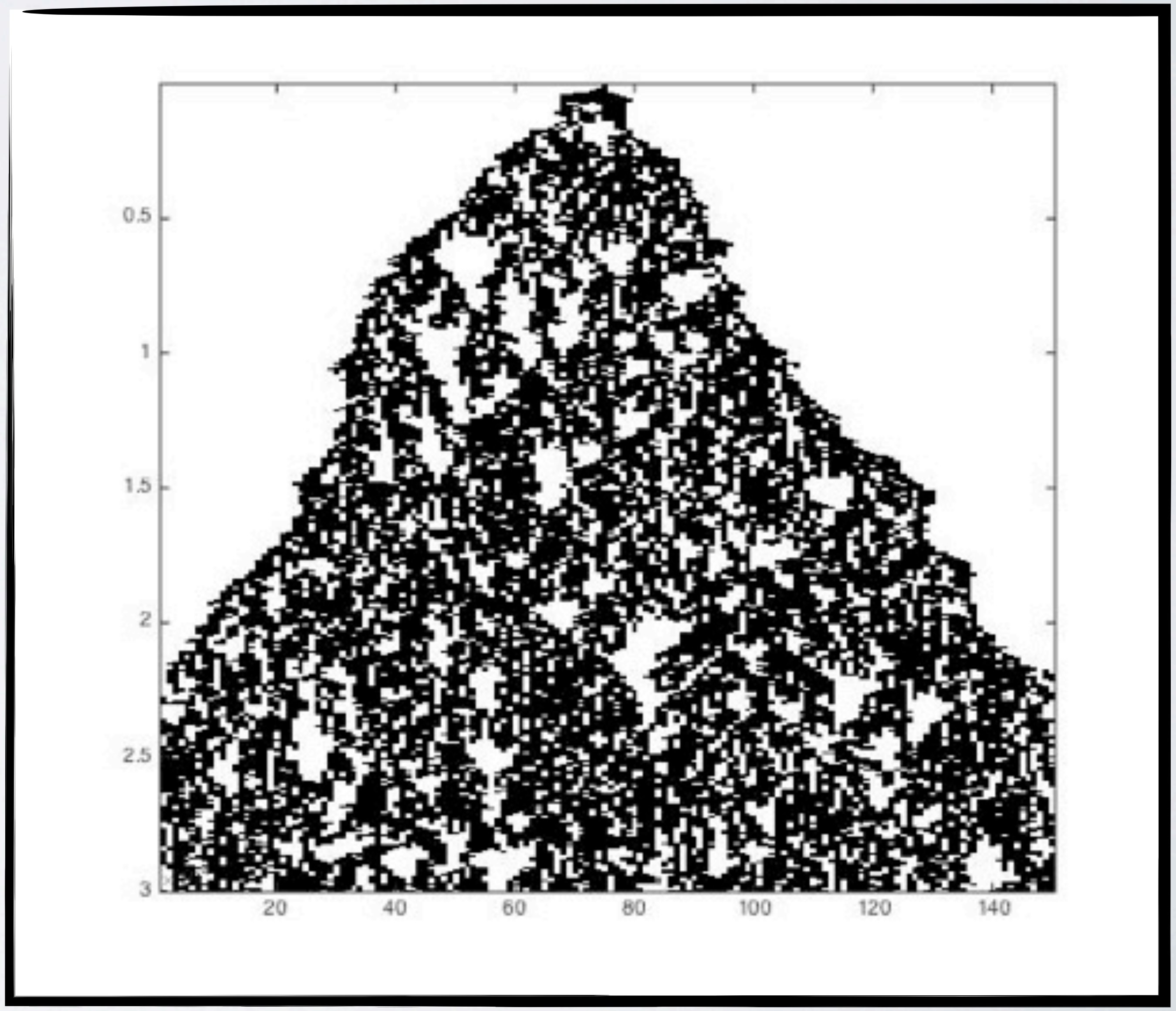
# AREA LAW CONJECTURE

What I want: gapped | bounded | local | well defined in thermo limit

→ **Area Law**

Why we should expect a counterexample:

- unique ground state with mutual info growing as volume
- intuition: gapped liouvillians can take time N to relax to equilibrium
- enough time to drag correlations across the system
- characteristic "Lieb-Robinson velocity"
- provides an easy counter-example to the stability conjecture for gapped liouvillians



# Counter-example to the generalized area-law conjecture

right of center  [e][x] → [x][e]

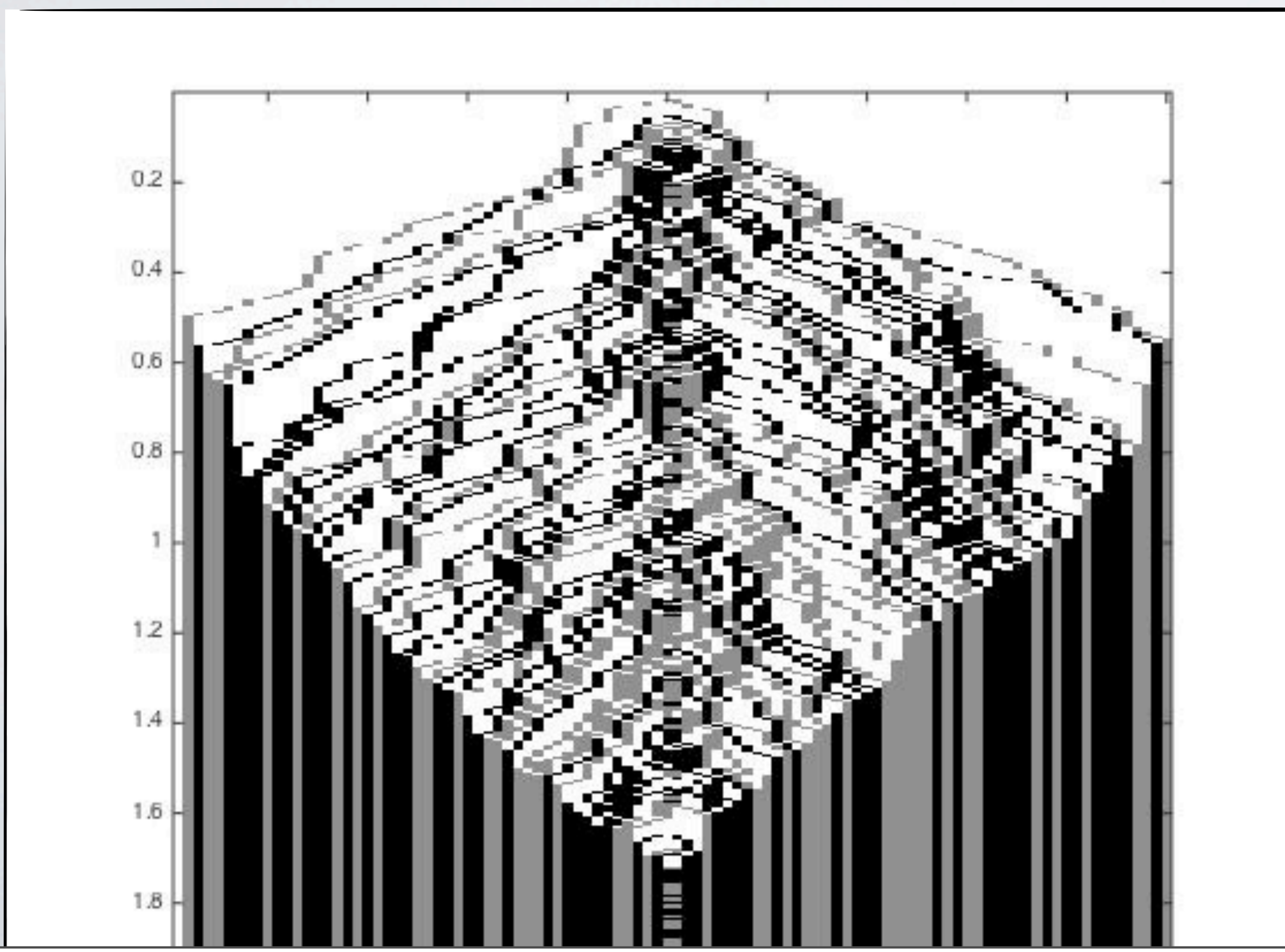
left of center  [x][e] → [e][x]

$x \in \{0, 1\}$

at center  [?][?] → [x][x]

high rate

x selected randomly from {0,1}

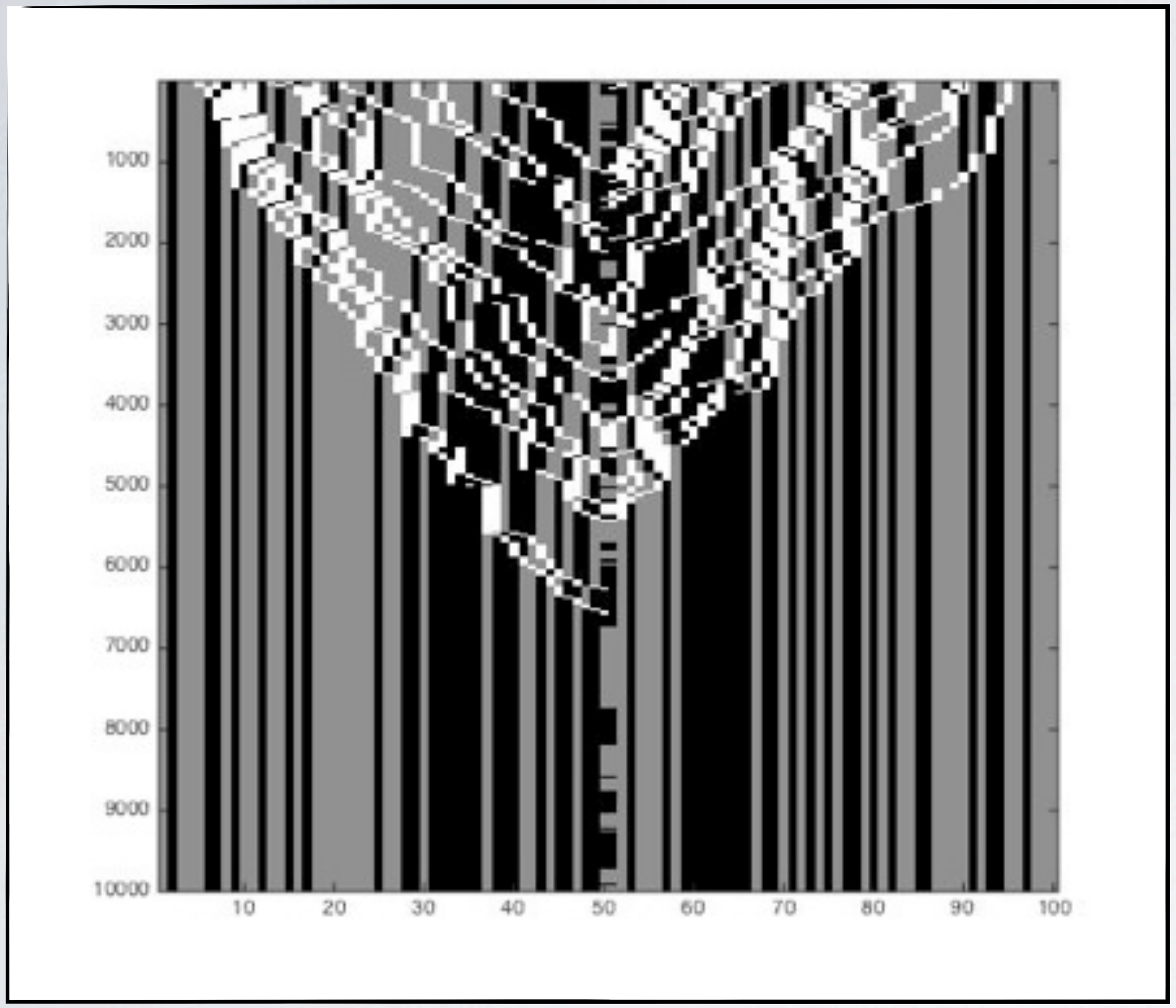



System is gapped!

If we prepare singlet state at the center, the steady state is has entanglement growing linearly

Local bounded update rules

# Counter-example to the generalized area-law conjecture




example is not perfect, non-unique stationary state

obvious improvement is to use an initialization gadget

quick fix: place e at each end with a certain probability

then the stationary state is unique

**But the Gap closes!**

Still new ideas needed to find a fully satisfactory example

Perhaps it provides insight to the closed system problem?

Dissipative computation with a gap!

*Toby!*

# INTERESTING QUESTIONS

Open system area law violation

Does the area law hold with detailed balance?

Understand the role of reversibility better

Topological non-equilibrium phase transitions?

Anything really quantum (or useful) here?

# PART II

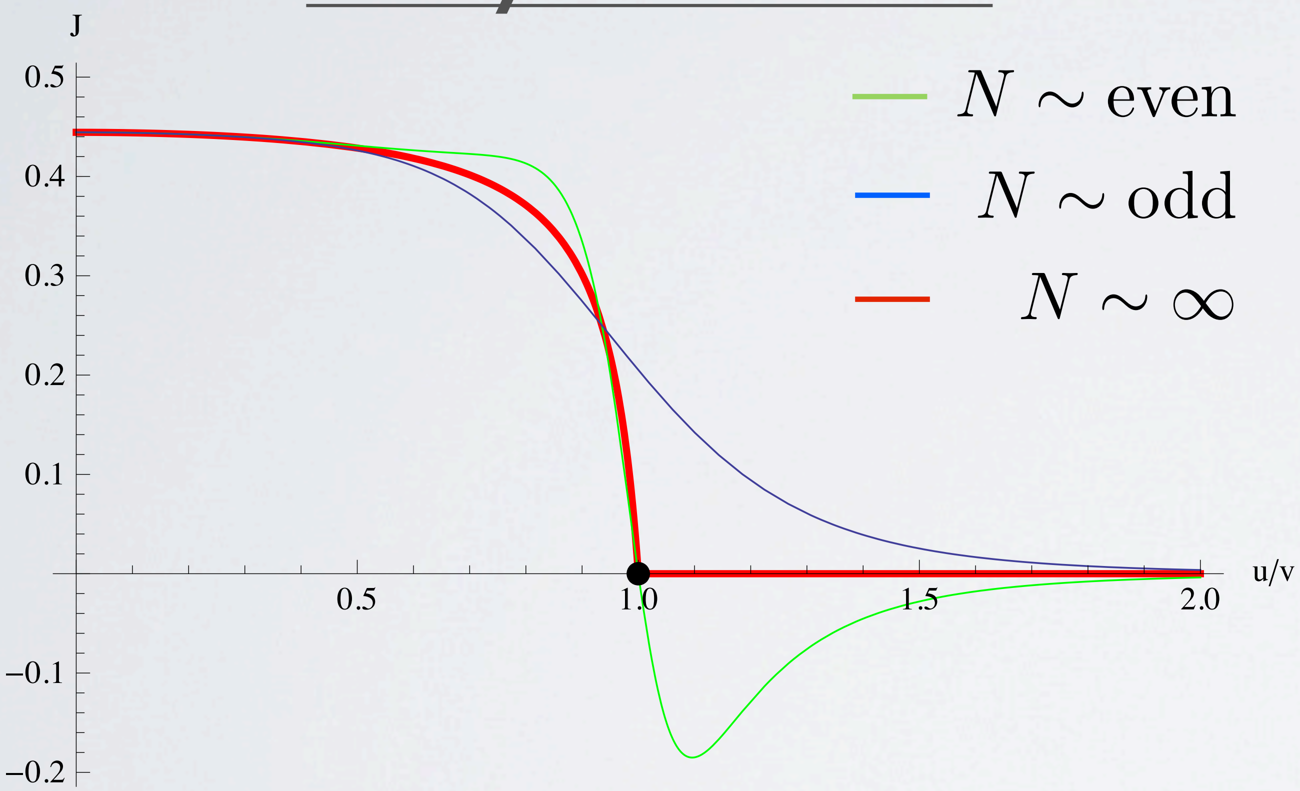
## A topological dynamical phase transition

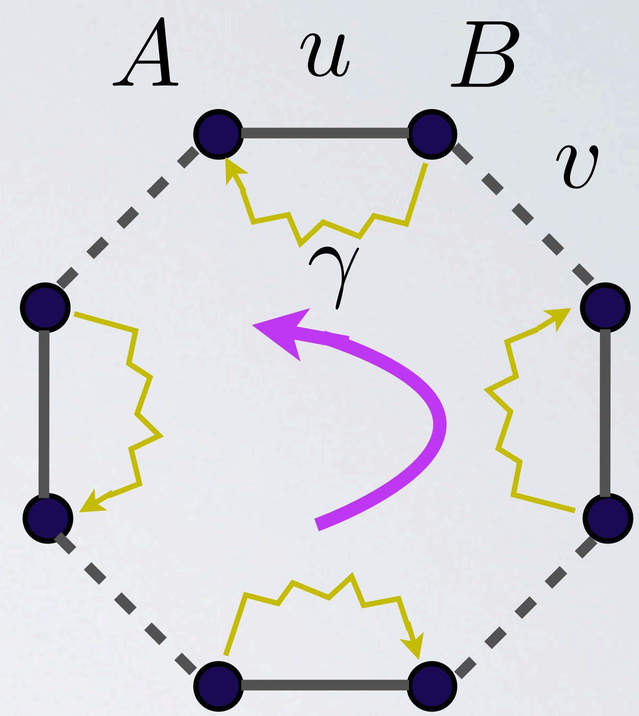
Mark Rudner

# CRITICALITY

Does this system exhibit critical behavior?

**Steady state current:**




No current when u>v

Gap closes at u=v

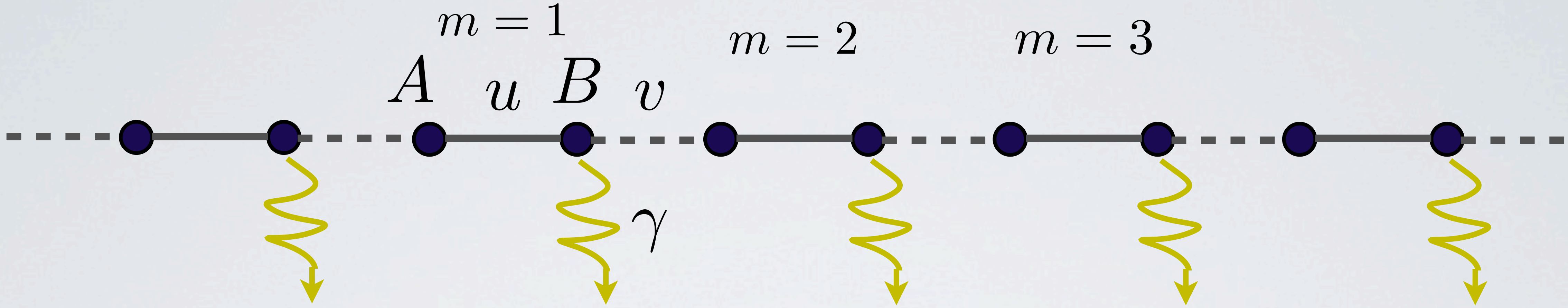
Transition insensitive to:

hopping disorder

even time dependent

$\gamma$ independent

# PHYSICS BEHIND IT




Perform trials: start at a given block, and keep track of what block you exit at.

Note: you always exit!

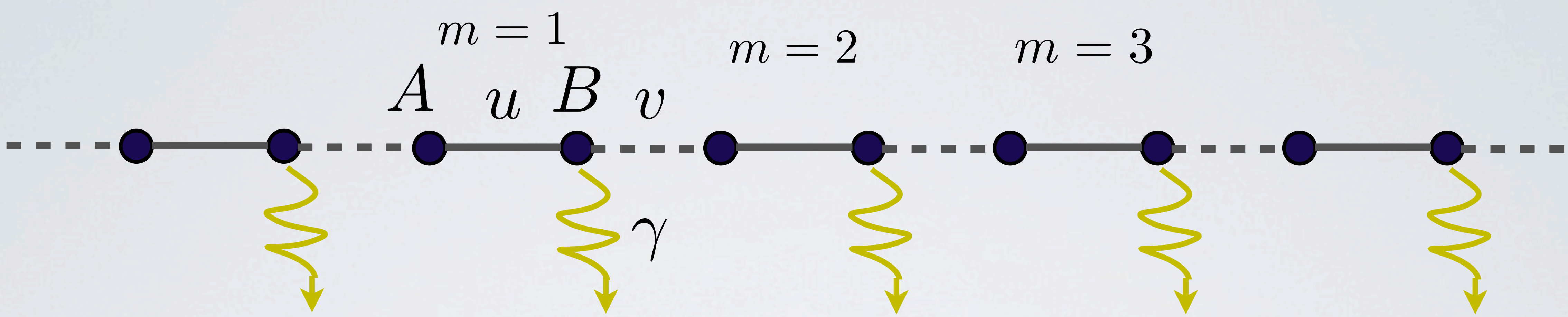
Estimate the average exit distance:

$$\langle \Delta m \rangle = \sum_m m P_m$$

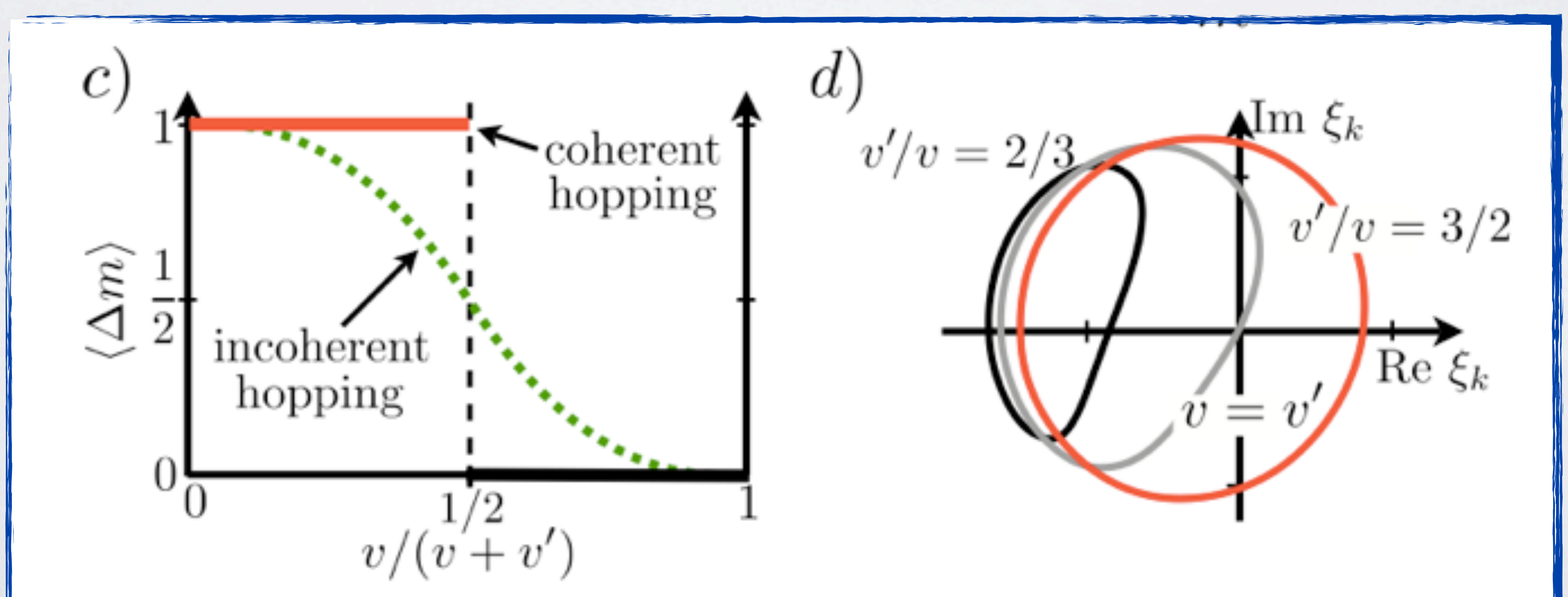
$$\langle \Delta m \rangle = \begin{cases} 1, & u < v \\ 0, & u > v. \end{cases}$$

No dependence on $\gamma$ !

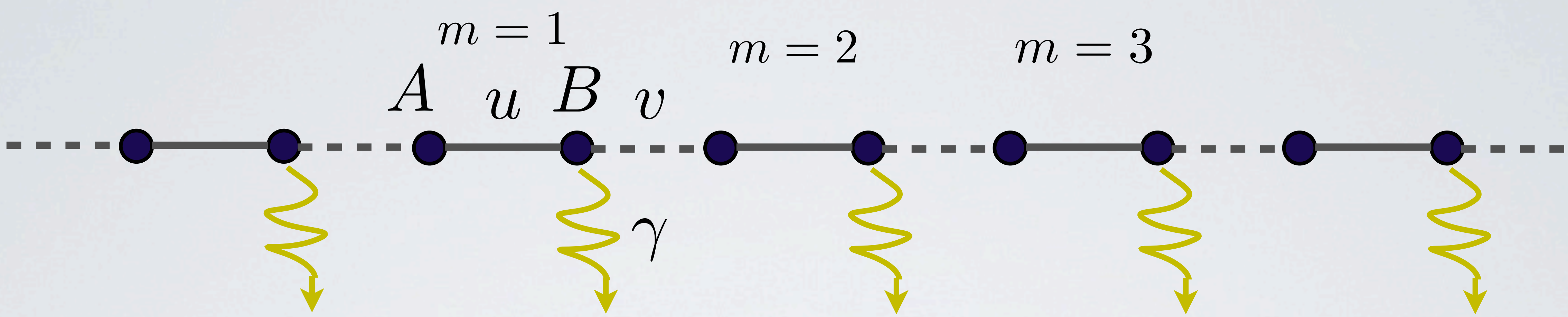
# TOPOLOGICAL TRANSITION




The transition is **topological**, average distance travelled can be reduced to an integral over the Brillouin zone: just gives a winding number



# TOPOLOGICAL TRANSITION



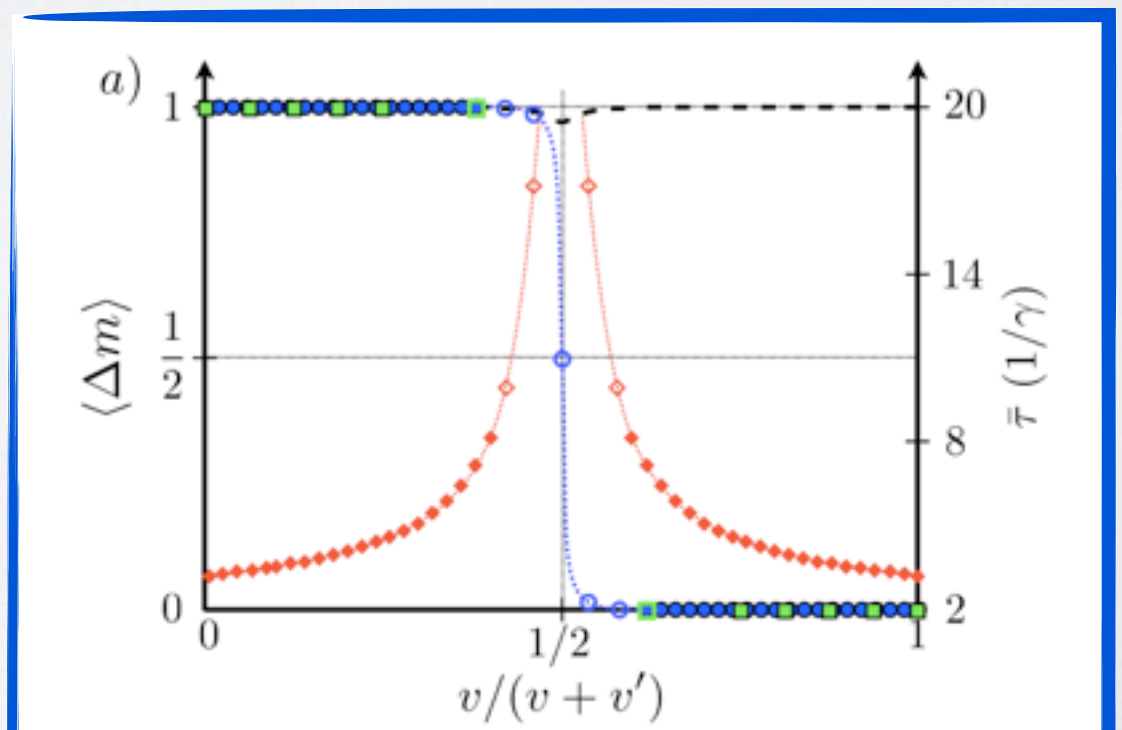

Current ~ average distance (in cells) travelled per run over survival time

$$J \approx \frac{\langle \Delta m \rangle}{\tau}$$

$$\tau \to \infty, \quad \text{as} \quad u \approx v$$

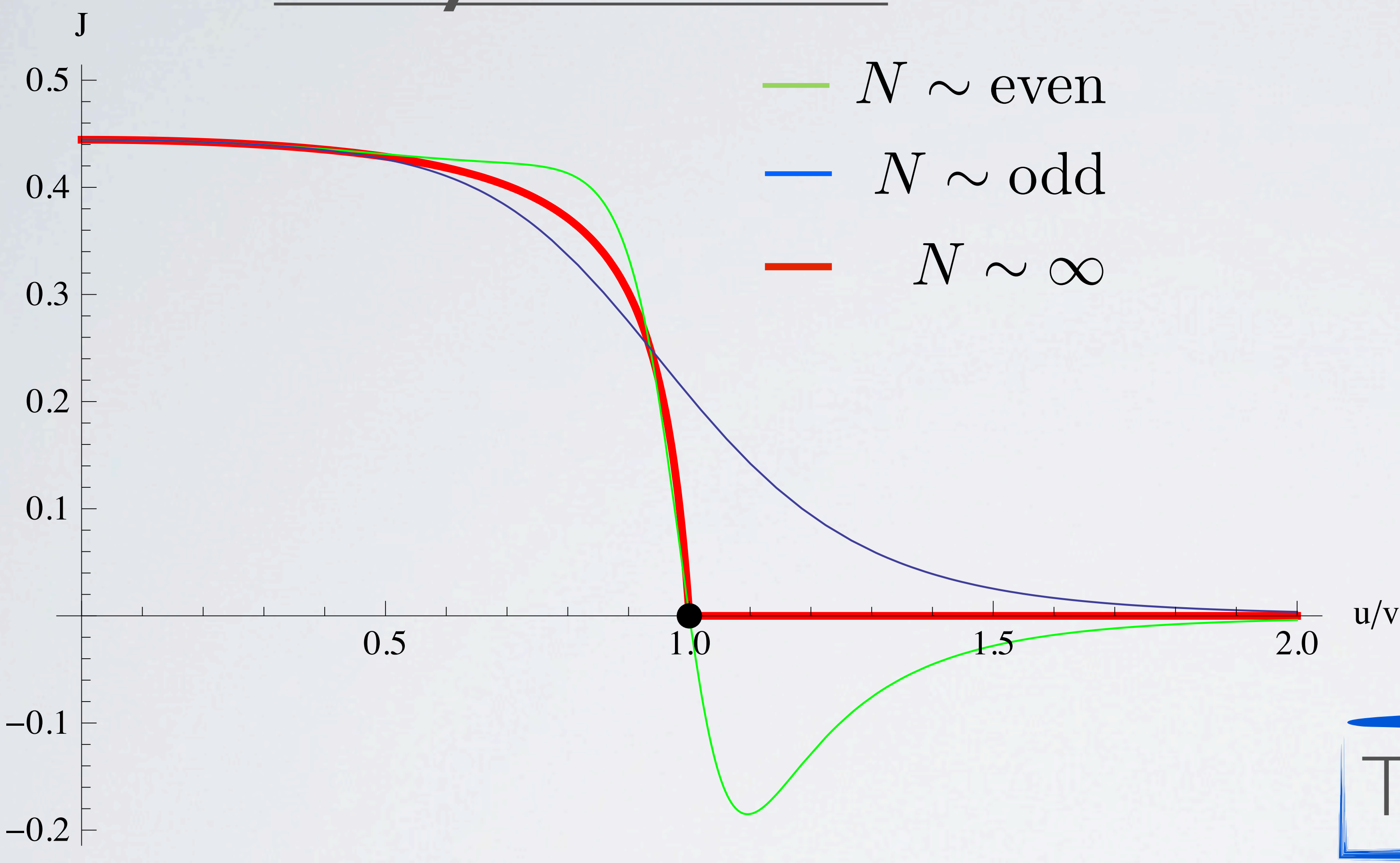
Divergence of $\tau$ suggests a Dark state at u=v

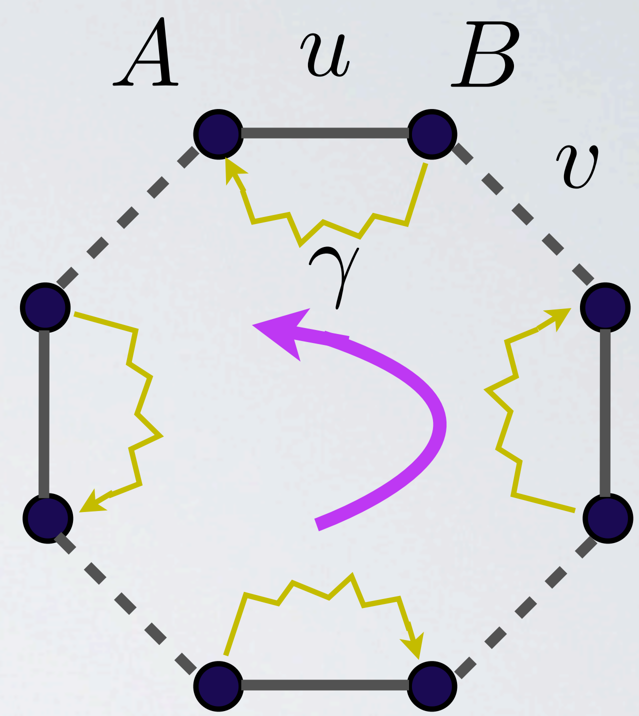
Higher moments can be calculated using counting statistics



# RESULTS

**Steady state current:**




No current when u>v

Gap closes at u=v

Weird finite size effects

Transition insensitive to:

on-site energy fluctuations

even time dependent

$\gamma$ independent

# TAKE HOME MESSAGES

Dissipation can cause exotic behavior

Very unexplored territory

New type of topological phenomena?

Can provide insight into close system problems?

# Thank you for your attention!