What is quantum field theory? A quantum information theorist's journey

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QFT in 45 minutes!

Q

R

$$\sqrt{2} = \begin{array}{l} 1 \\ 1.4 \\ 1.41 \\ 1.414 \\ 1.4142 \\ 1.41421 \\ 1.414214 \\ 1.4142136 \\ 1.41421356 \\ 1.414213562 \\ \ldots \end{array}$$

$$(x_n)_{n\in\mathbb{N}}$$

where $x_n \in \mathbb{Q}$

$$d(x, y) \equiv |x - y|$$

Cauchy sequence:

$$(x_n)_{n \in \mathbb{N}}$$

such that

$$\forall \epsilon \exists N(\epsilon), \forall m, n > N(\epsilon), d(x_m, x_n) < \epsilon$$

$\widehat{\mathbb{Q}}$ = set of Cauchy sequences

$$(x_n)_{n\in\mathbb{N}} \sim (y_n)_{n\in\mathbb{N}}$$

$$\Leftrightarrow$$

$$\lim_{n\to\infty}(x_n - y_n) = 0$$

$$\mathbb{R} \equiv \widehat{\mathbb{Q}} / \sim$$

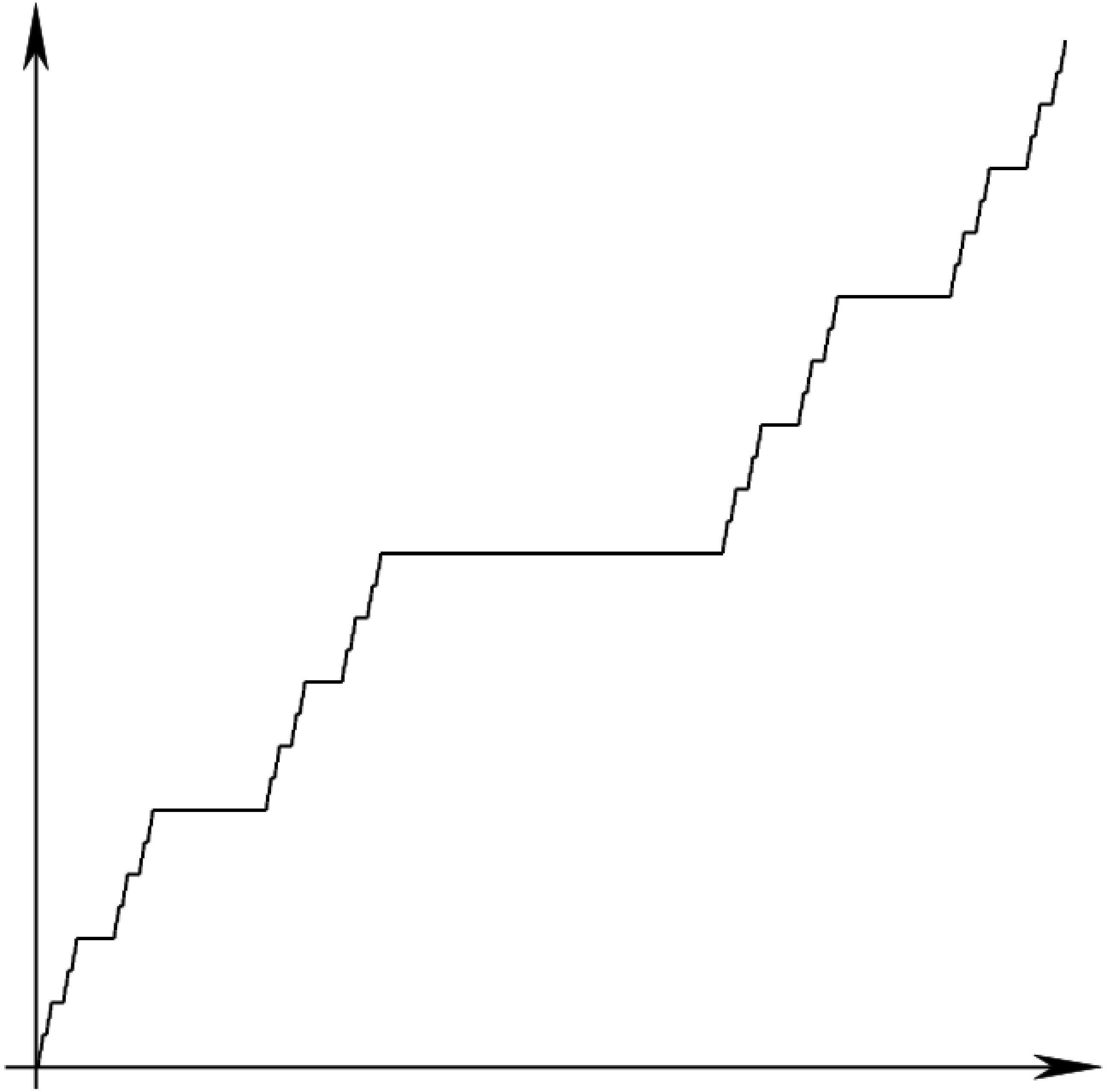
Classical fields

Field theory is **hard**

Classical approach: **calculus**

Pure field states are **continuous functions**

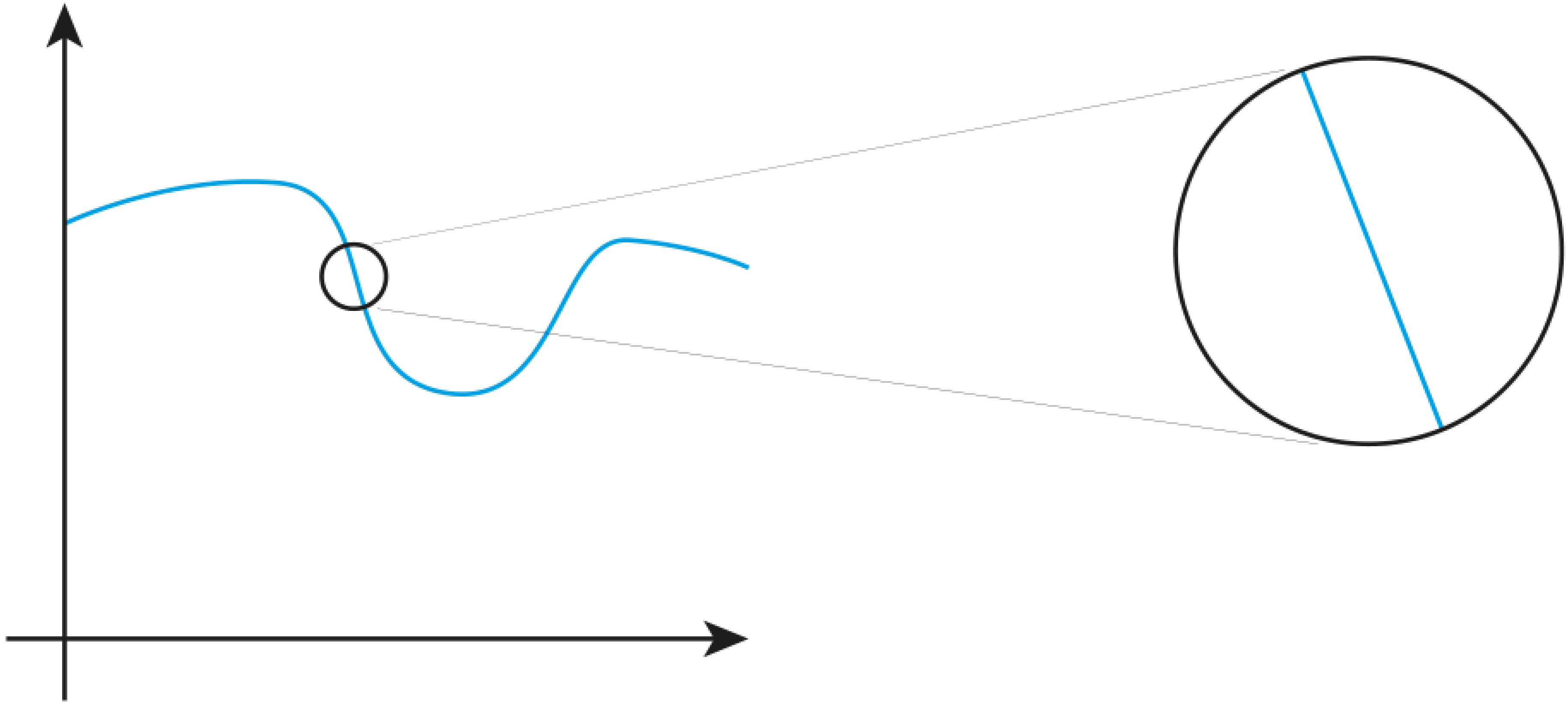
$$\phi : \mathbb{R}^d \to \mathbb{R}$$



Physical states are solutions to DEs

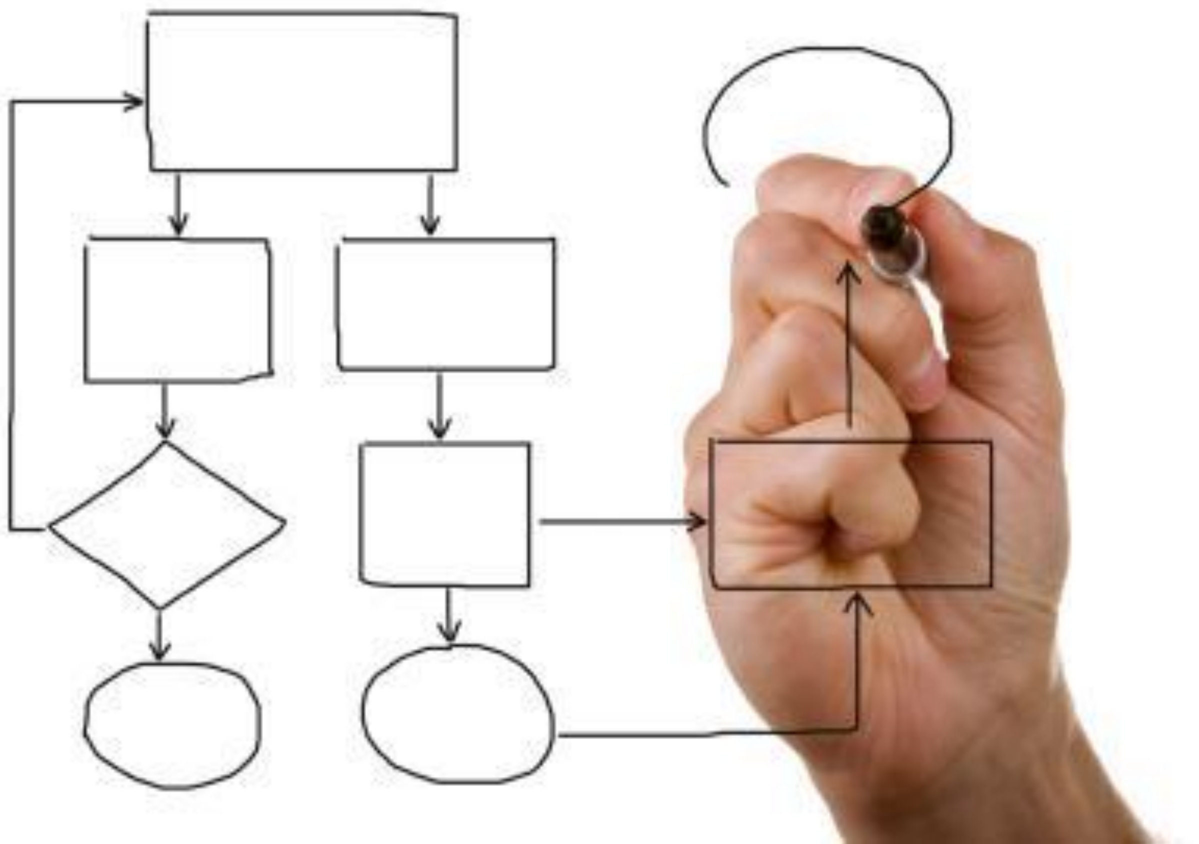
$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} + m^2\phi = 0$$

Physical states are differentiable

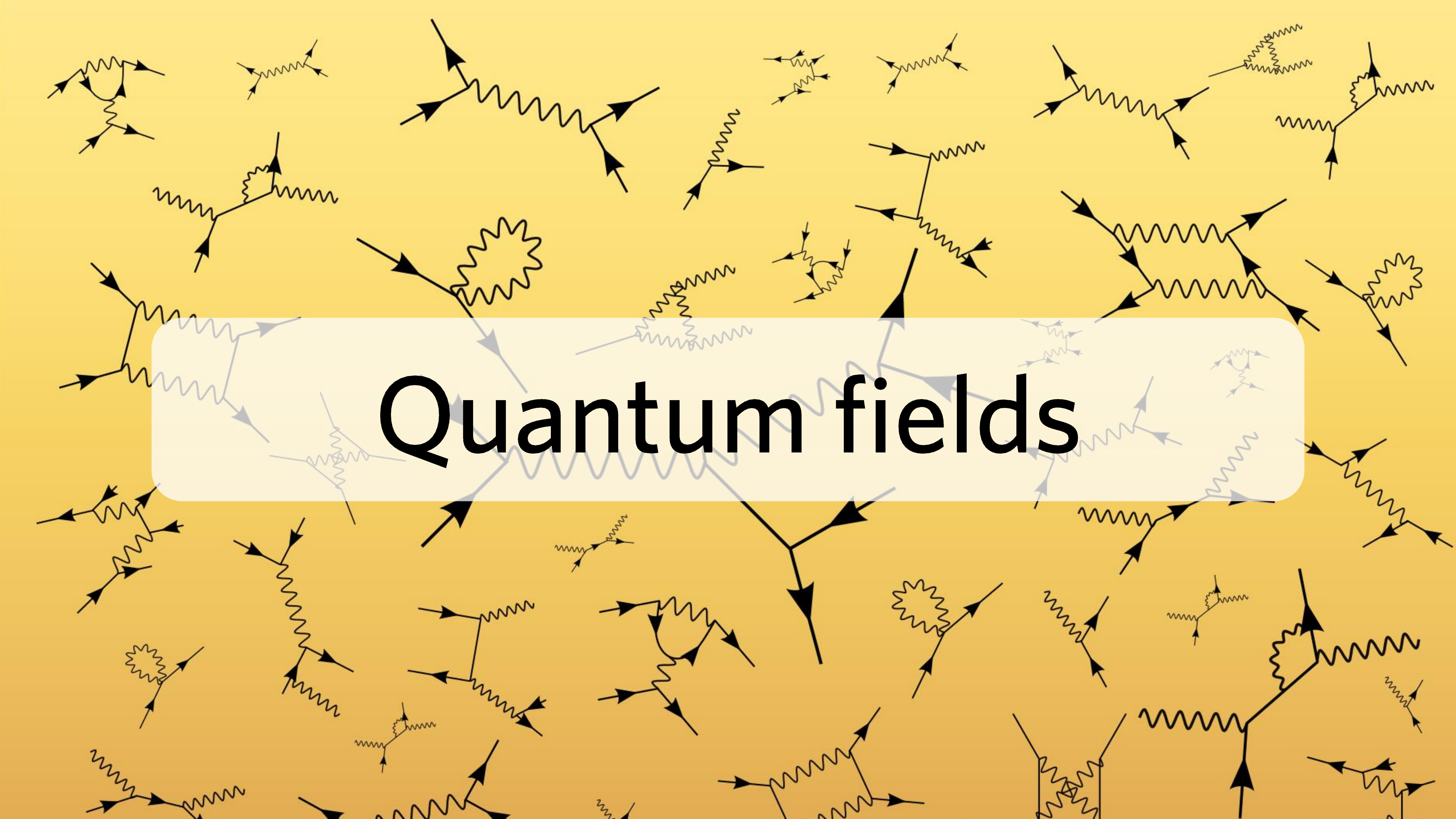


Lagrangian is a **data structure** for equations of motion

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2$$



Quantum fields



Physicist: put on hats!

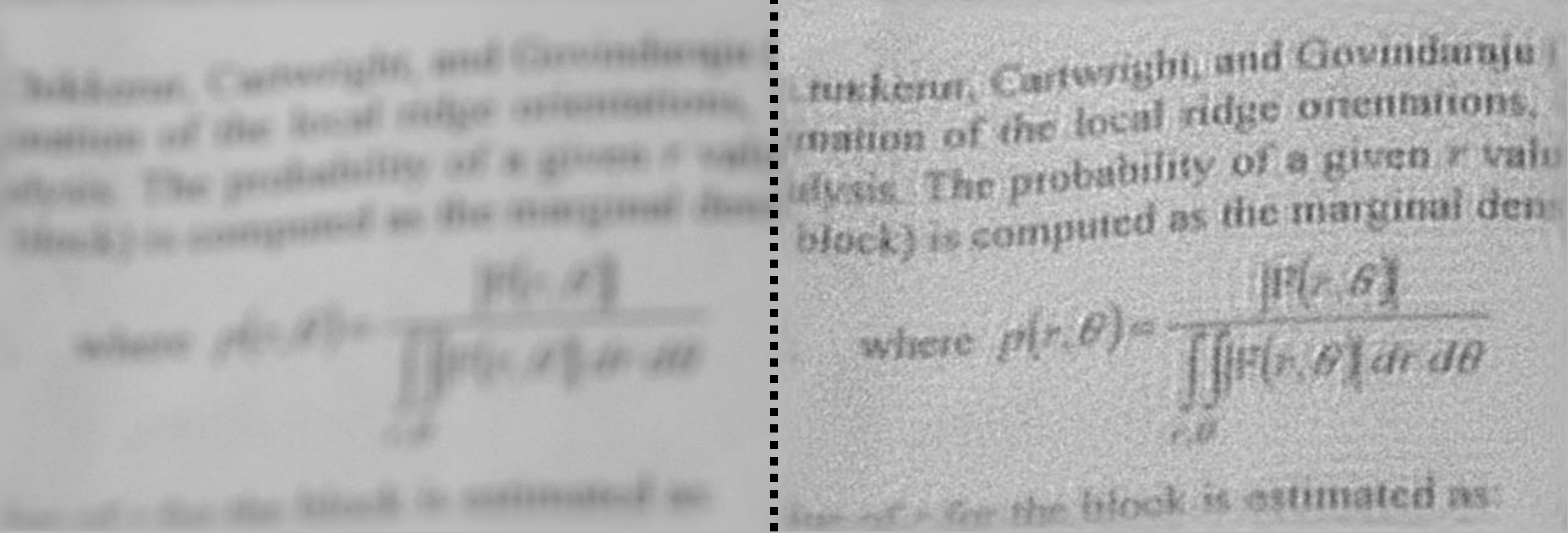
$$\phi \mapsto \widehat{\phi}$$

Path integral

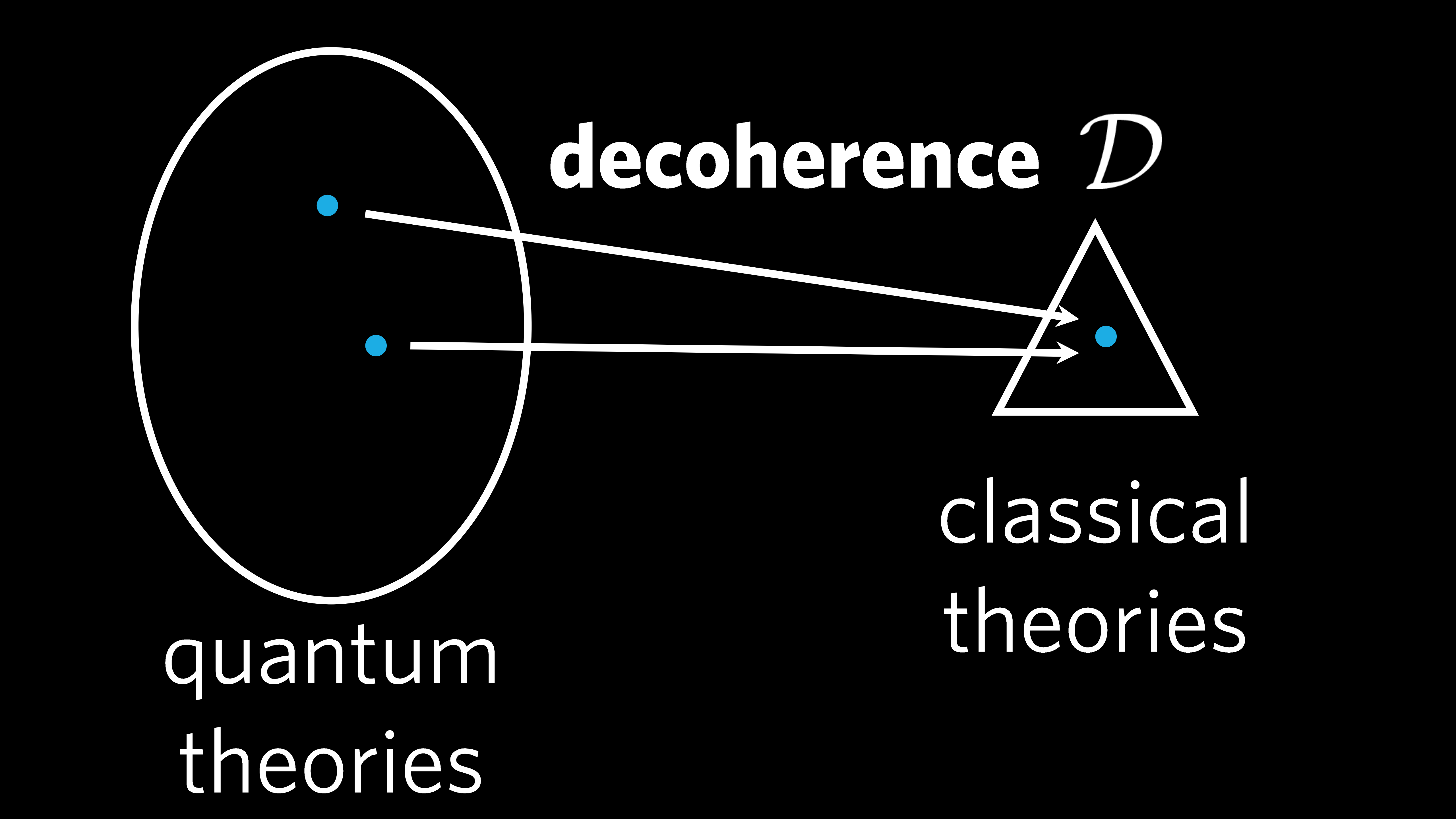
$$\mathcal{L} \mapsto \int \mathcal{D}\phi \; e^{iS[\phi]}$$

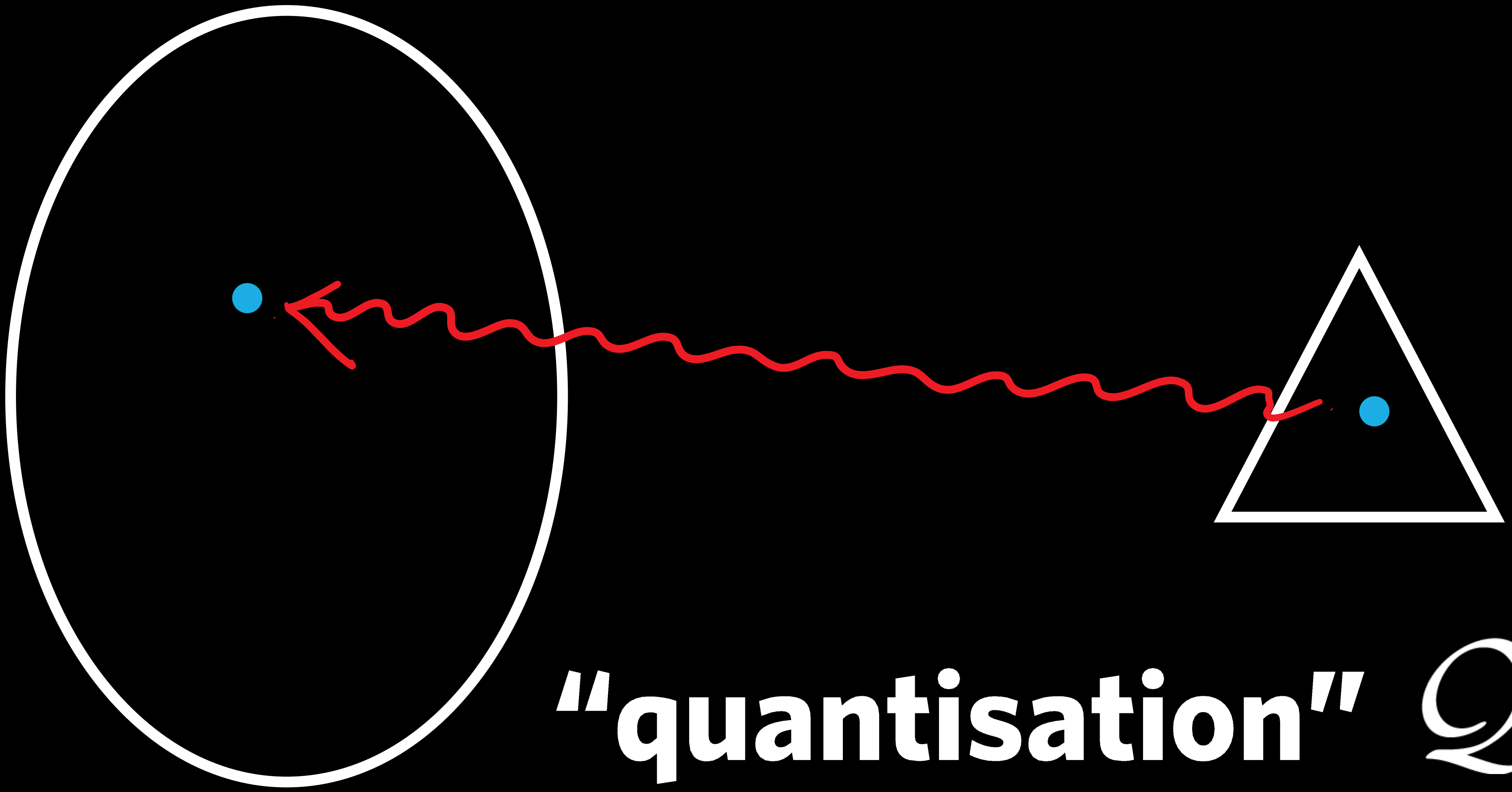
PROBLEMS

No Matter how Great and Destructive your Problems May Seem Now, Remember, You've Probably only Seen the Tip of Them.



Problem 1: quantisation is **not** a functor, it's an **inverse problem**


decoherence $\mathcal{D}$
classical theories
quantum theories



“quantisation” $\mathcal{Q}$

Quantisation: quantum theory with desired **classical limit**

$$\mathcal{D} \circ \mathcal{Q}(\rho_{\mathrm{cl}}) \equiv \rho_{\mathrm{cl}}$$

Problem 2: **superpositions**

$$|\phi_1\rangle + |\phi_2\rangle + \cdots$$

Principle to identify
physical states?

Many attempts to deal with this:

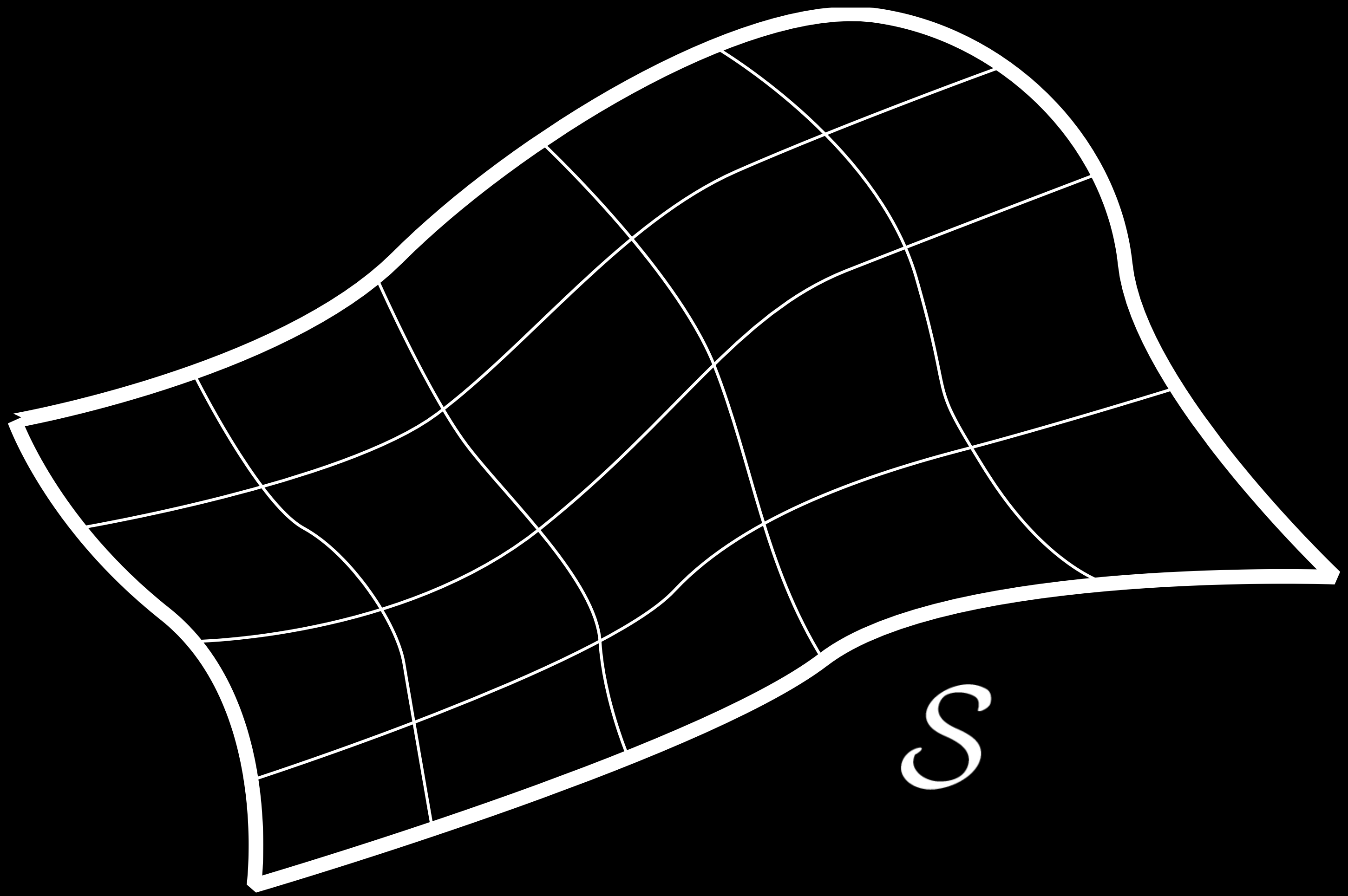
CQFT, AQFT, …

WILSON

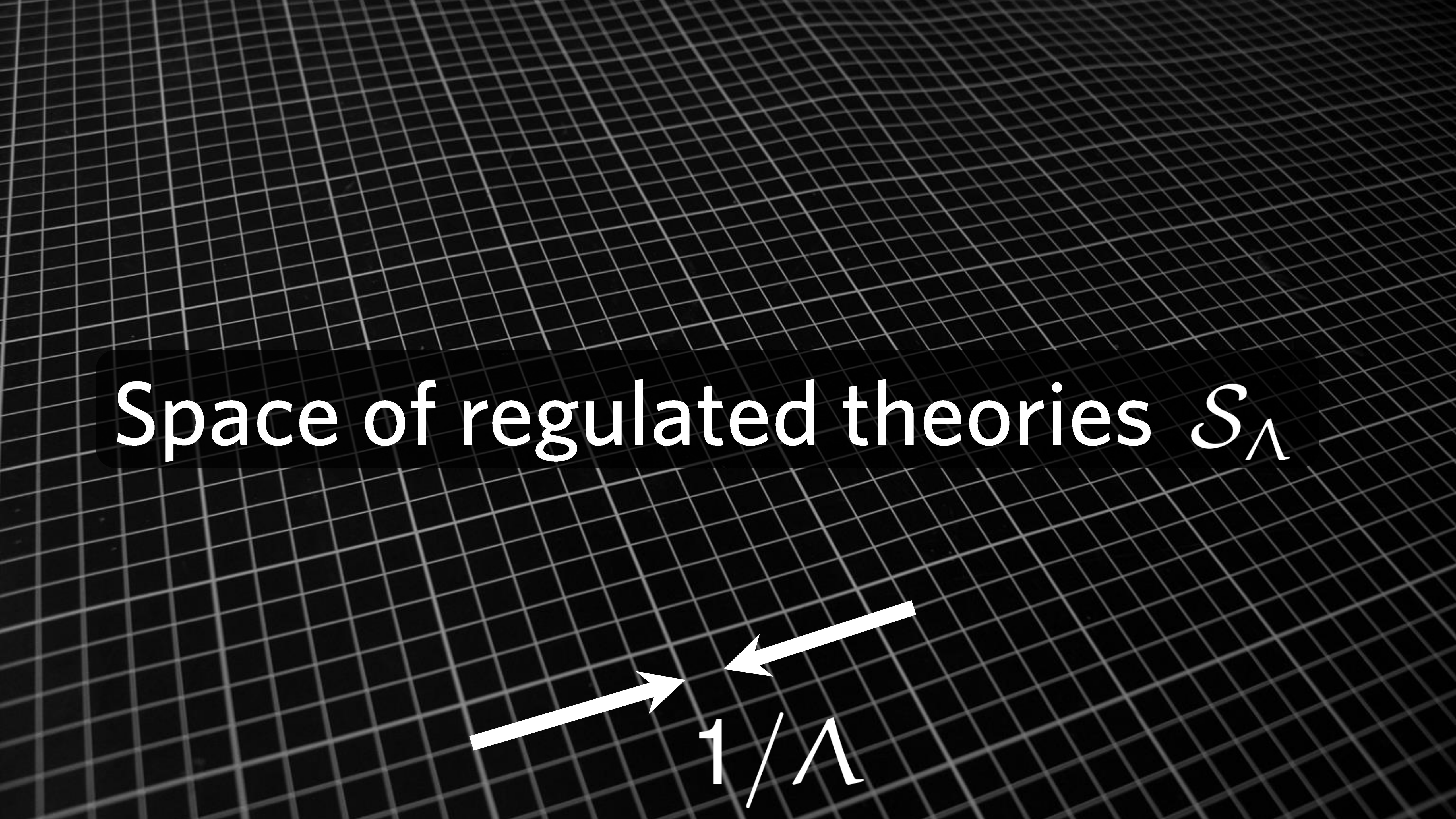
Wilson: QFT is **effective**,
not fundamental!

Wilsonian view of QFT

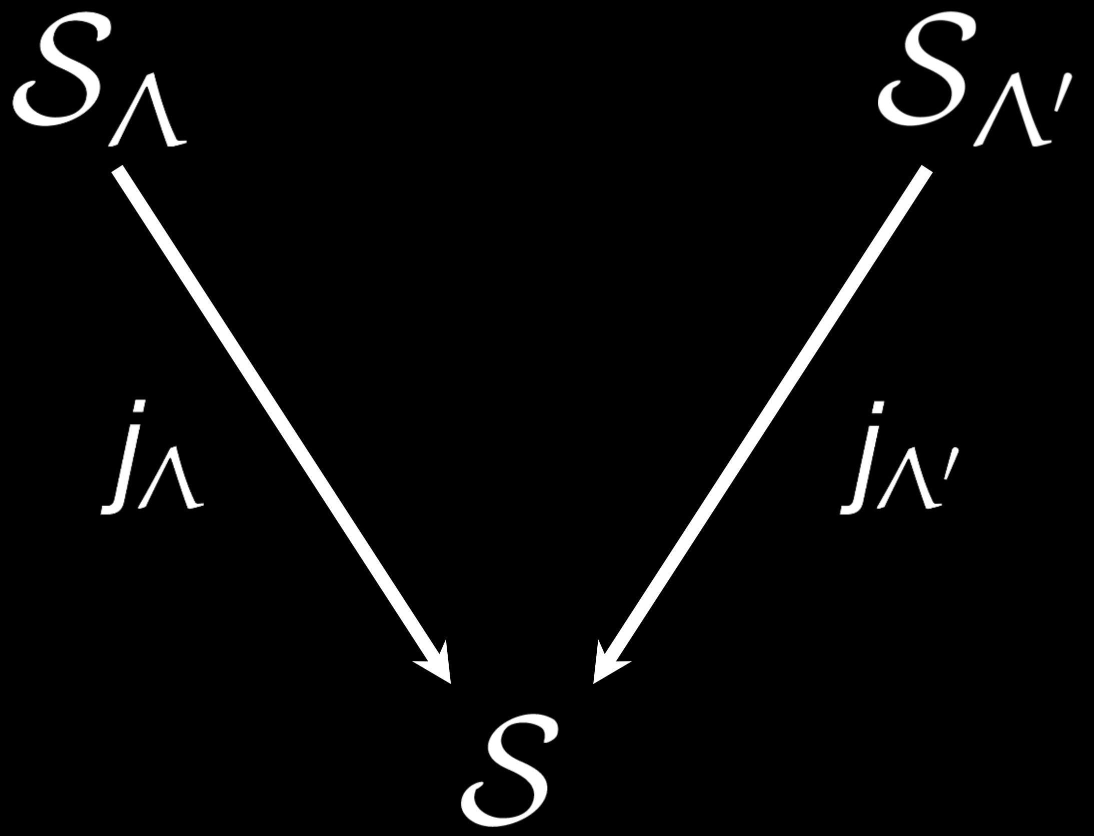
“space” of **all** theories (parametrised by lagrangians $\mathcal{L}$)



Task: find “submanifold” of **QFT**s


Space of regulated theories $\mathcal{S}_\Lambda$
$1/\Lambda$

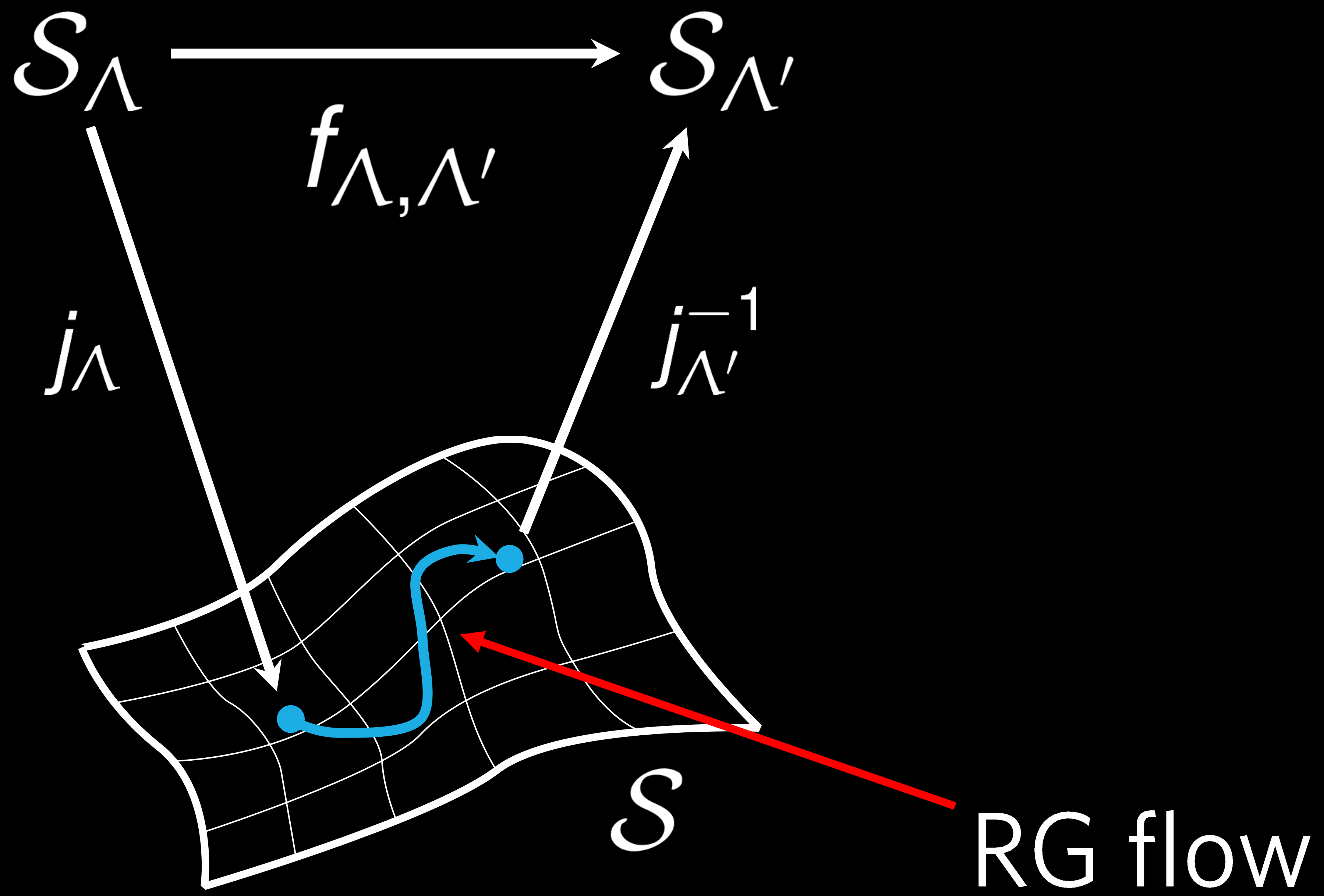
But $\mathcal{S}_\Lambda$ aren't QFTs


$\mathcal{S}_\Lambda$
$\mathcal{S}_{\Lambda'}$
j_Λ
$j_{\Lambda'}$
$\mathcal{S}$

Change cutoff: $\Lambda' \geq \Lambda$

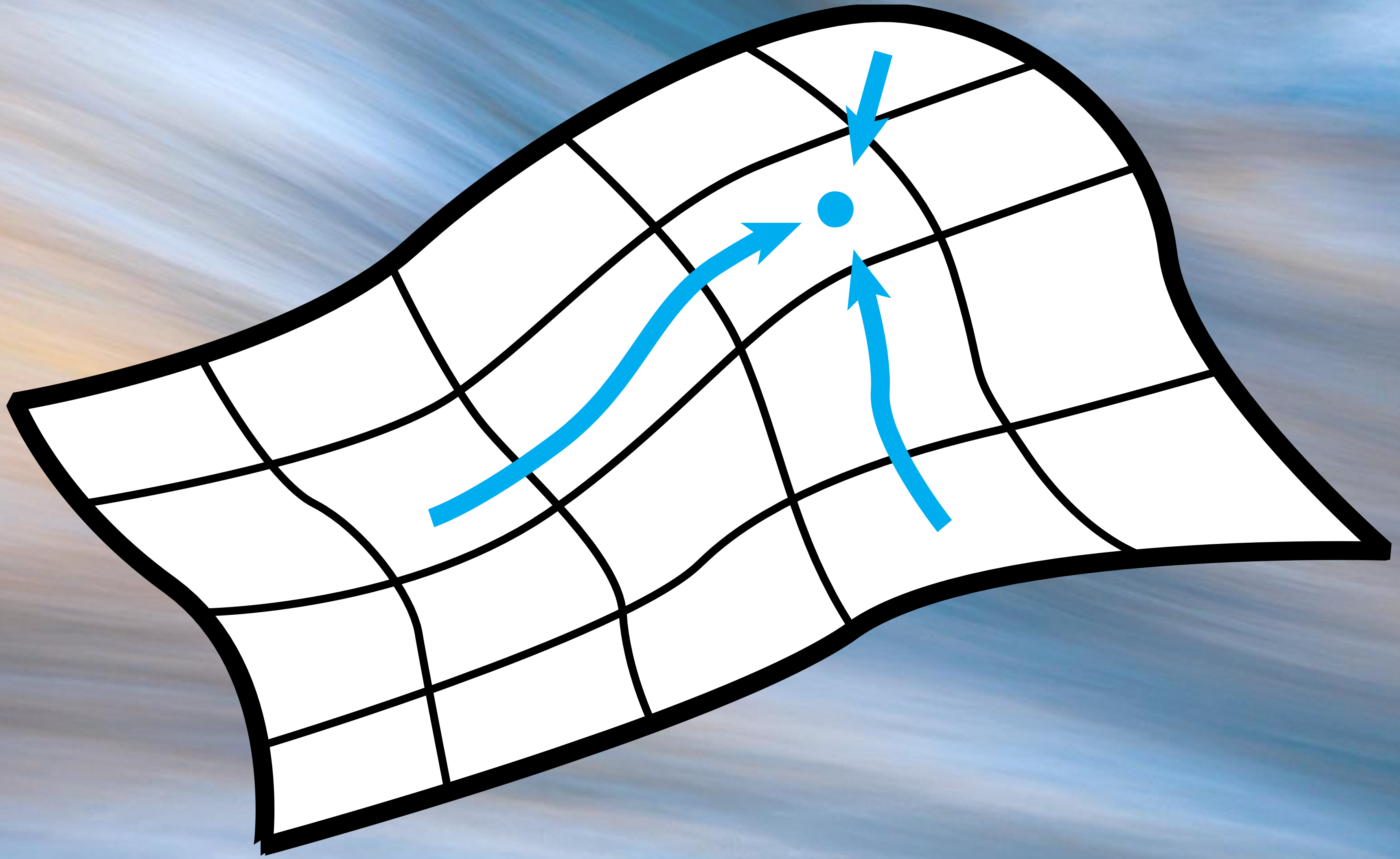
$$\mathcal{S}_\Lambda \xrightarrow[f_{\Lambda,\Lambda'}]{} \mathcal{S}_{\Lambda'}$$

$f_{\Lambda,\Lambda'}$: require **large-scale** *n*-pt correlation functions to be the **same**



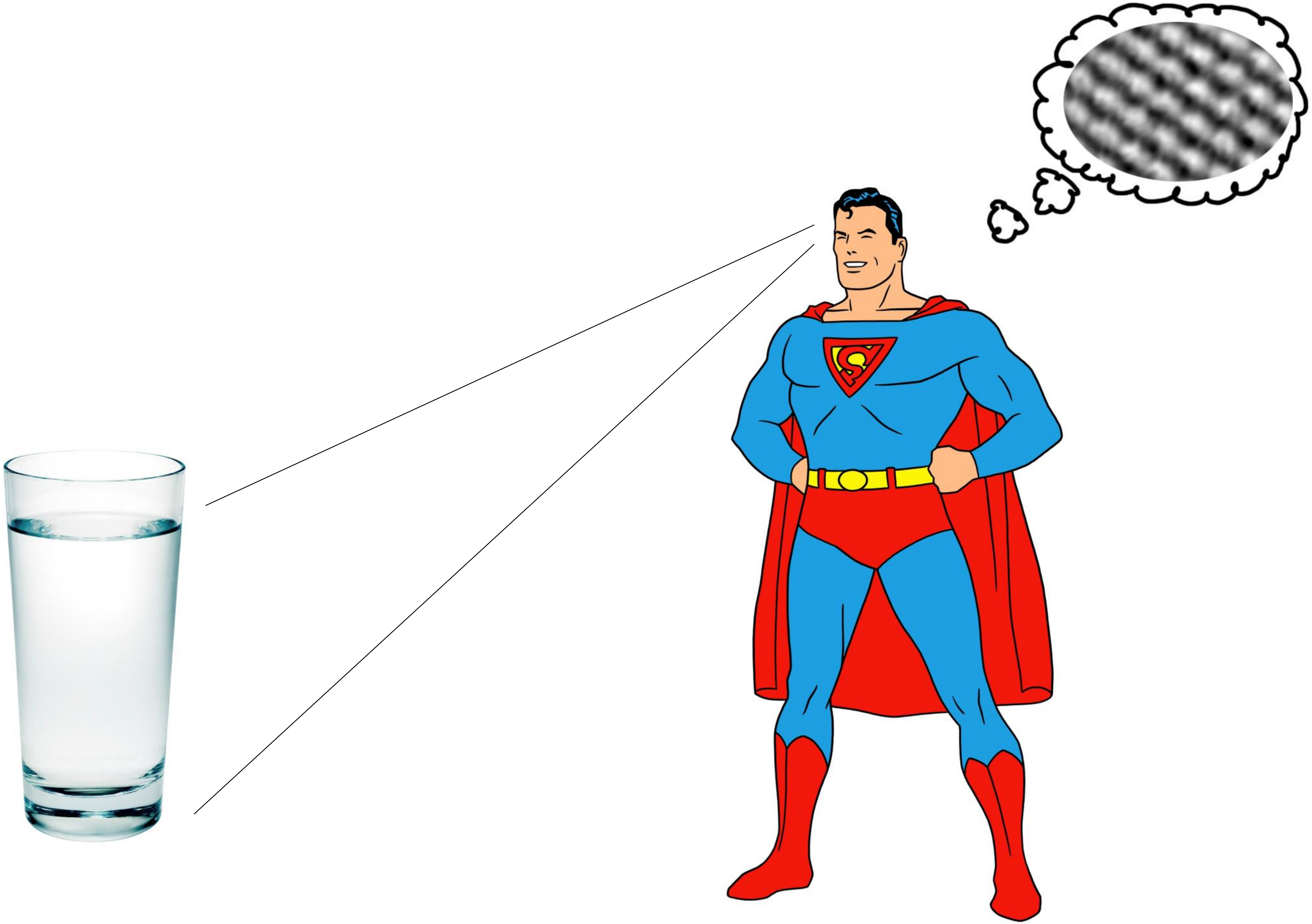
Costello, K. (2011), *Renormalization and effective field theory.*

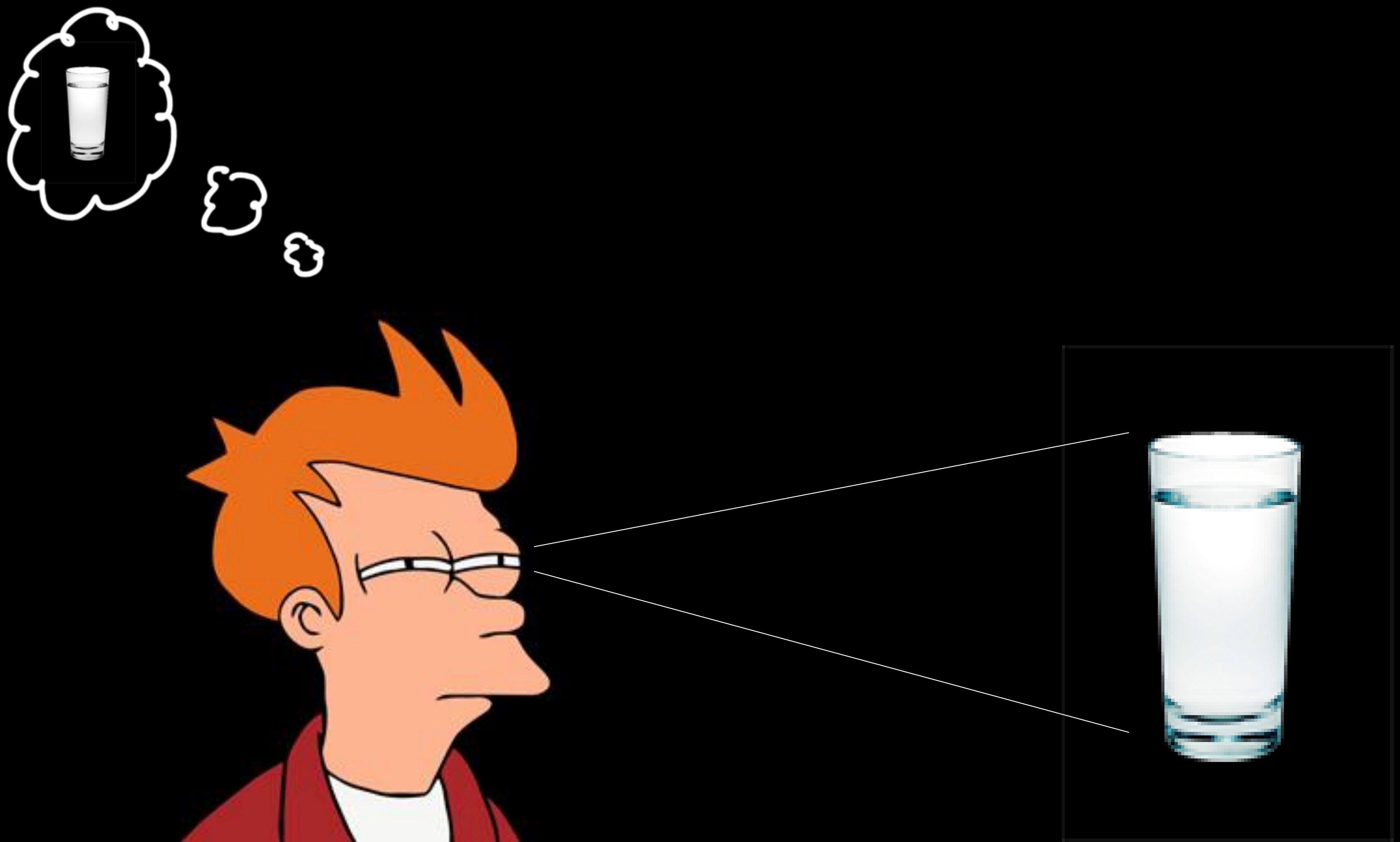
Fixed points are QFTs



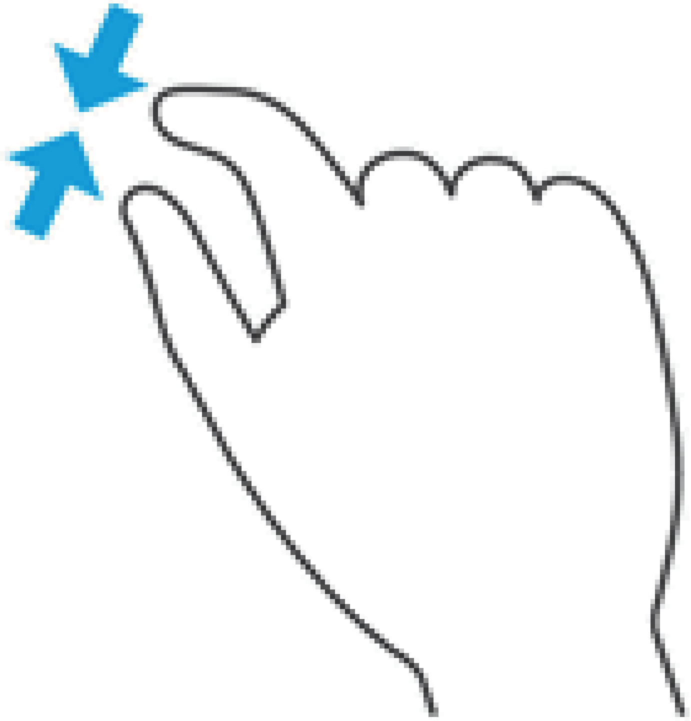
Continuous limit:

Let $\Lambda \to \infty$ while keeping large-scale *n*-pt correlation functions constant



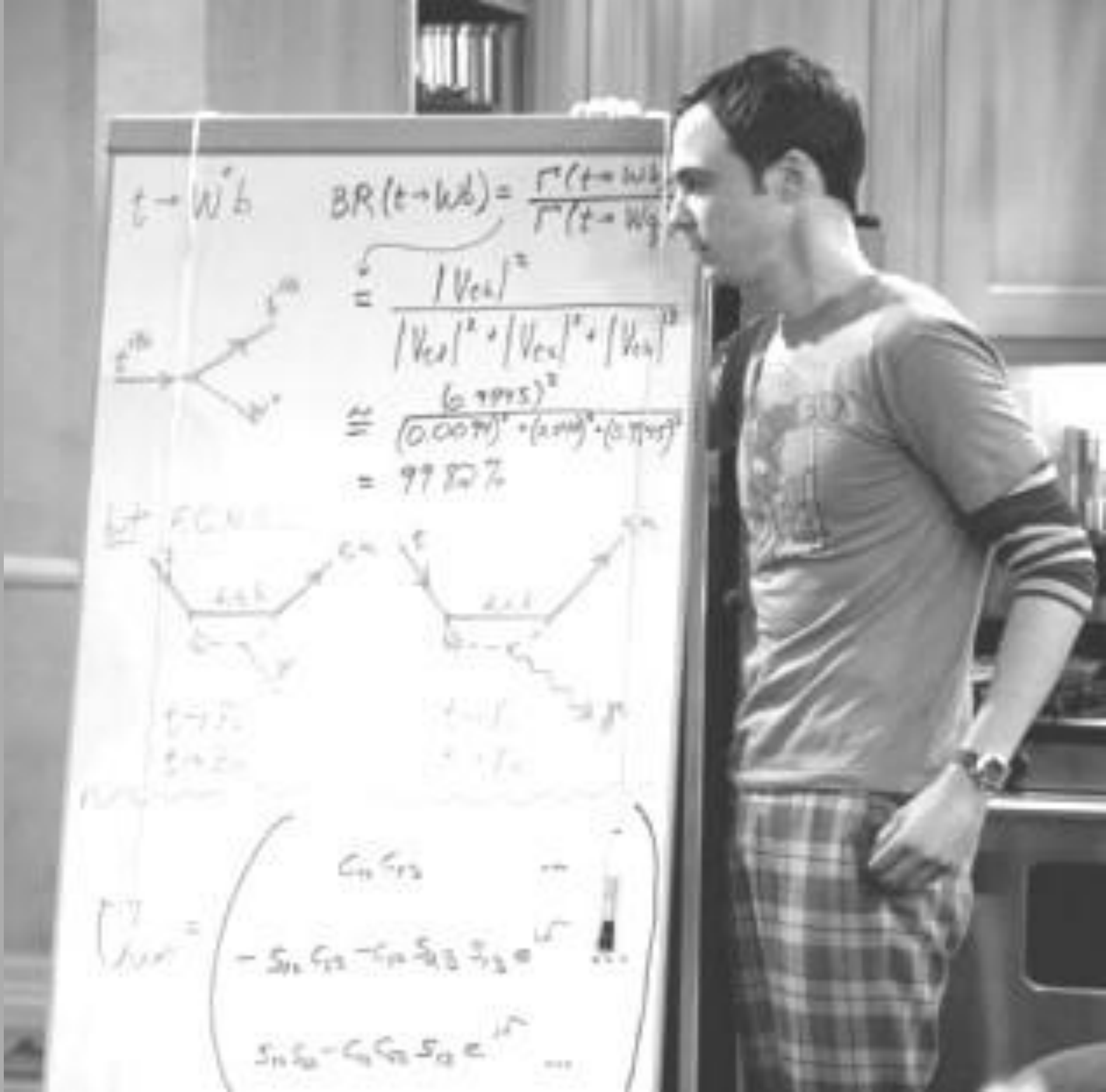


Zoom out
= fewer observables



Pinch **to zoom**

Fewer observables **= simpler hypothesis**




Everything Looks Perfect from far away
oops.

Wilsonian formulation

1. Space of regulated theories
2. Distance measure
3. Large-scale observables

Heisenberg picture

observables/effects:

$$\mathcal{A}$$

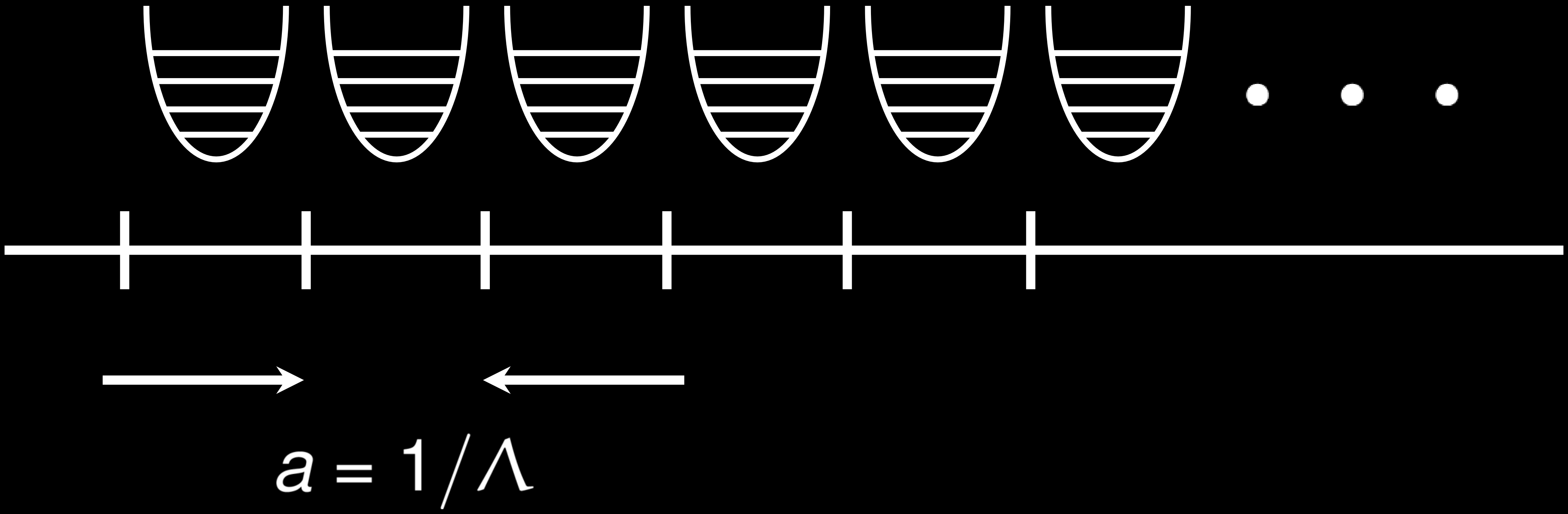
(ordered unit space)

Quantum states are **preparations:**

$$\omega : \mathcal{A} \to \mathbb{C}$$

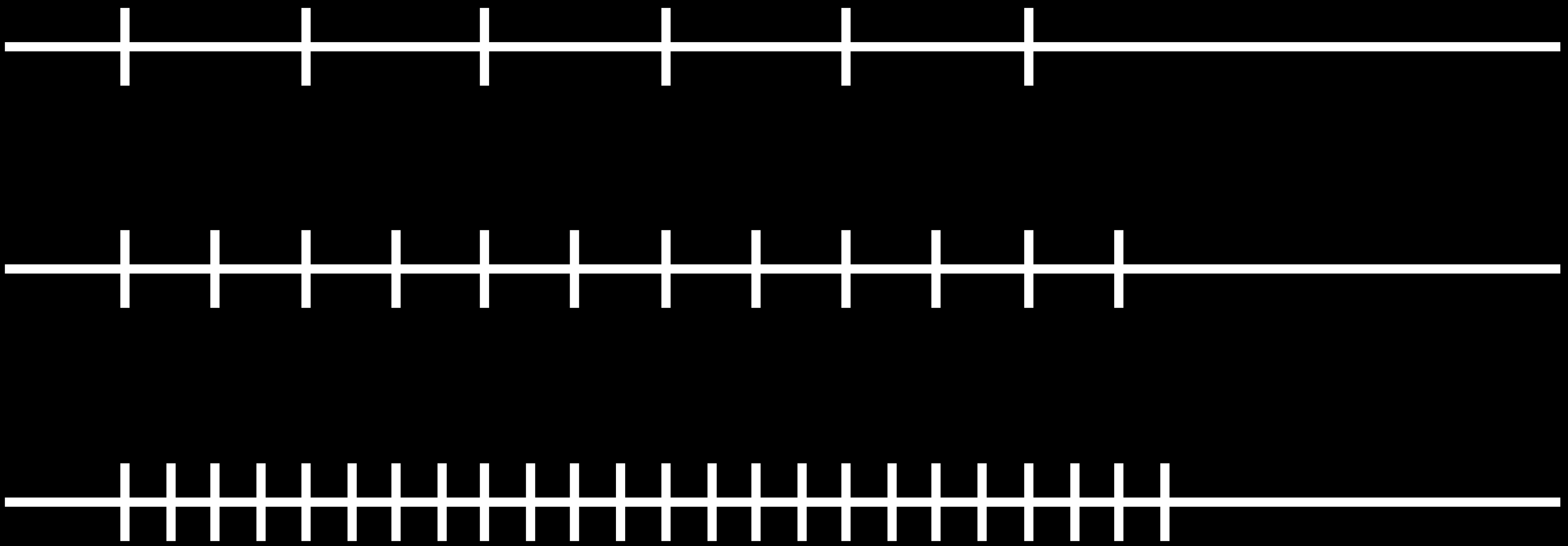

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$

1. Space of regulated theories $\mathcal{A}_a$



$$[\widehat{x}_{j,a}, \widehat{p}_{k,a}] = i\delta_{j,k}$$

1. Space of regulated theories $\mathcal{A}_{\text{pre}}$



1. Space of regulated theories $\mathcal{A}_{\text{pre}}$

$$\mathcal{A}_{\text{pre}} \equiv \bigoplus_{N=0}^{\infty} \mathcal{A}_{\frac{1}{2^N}}$$

Can measure at resolution $1/2^N$

or

at resolution $1/2^{N+1}$

or

...

etc

$$\hat{x}_{j,\frac{1}{2^N}} \sim \frac{1}{\sqrt{2}}\left(\hat{x}_{2j,\frac{1}{2^{N+1}}} + \hat{x}_{2j+1,\frac{1}{2^{N+1}}}\right)$$

1. Space of regulated theories

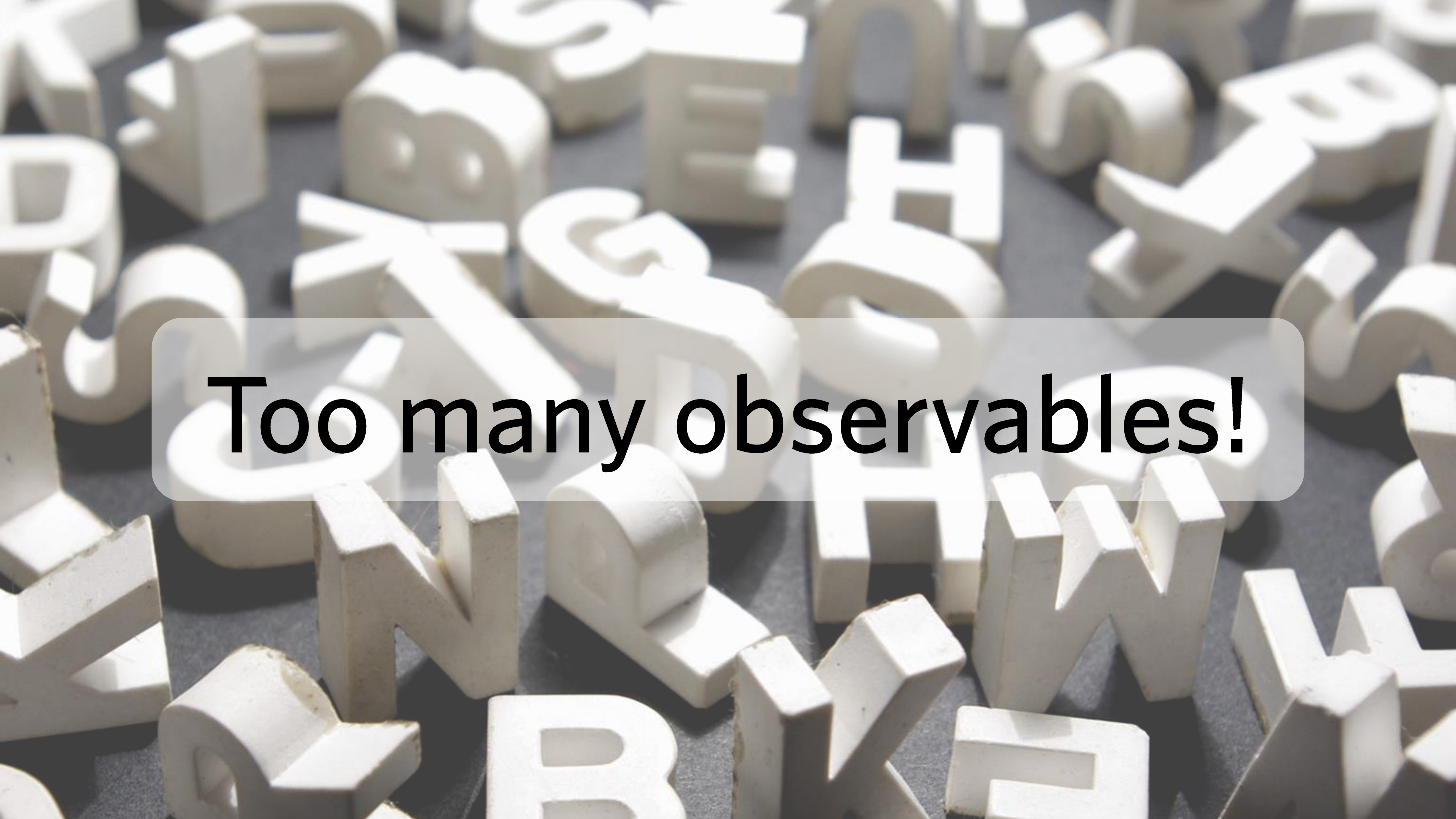
$$\mathcal{A}_{\text{reg}} \equiv \left(\bigoplus_{N=0}^{\infty} \mathcal{A}_{\frac{1}{2^N}} \right) \Big/ \sim$$

2. Distance measure

$$D(\rho_1, \rho_2)^2 = 2(1 - \sqrt{F(\rho_1, \rho_2)})$$

3. Large-scale observables

?


Too many observables!

$$\mathcal{A}_{\text{reg}}$$

Areg

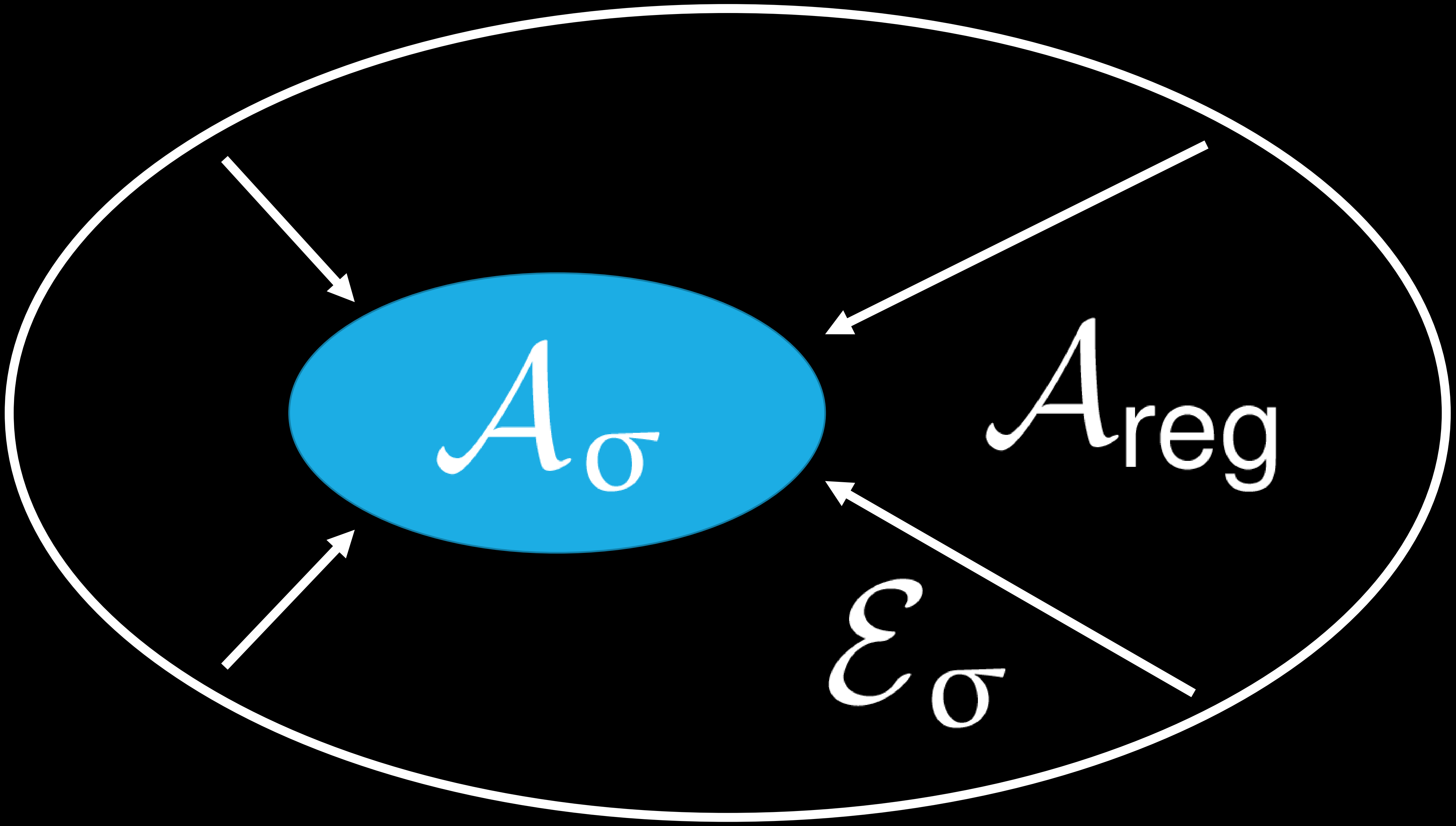
$$\mathcal{E}_\sigma : \mathcal{A}_{\mathrm{reg}} \to \mathcal{A}_{\mathrm{reg}}$$

$\mathcal{E}_\sigma$: convolve with gaussian

$$\mathcal{E}_\sigma(\widehat{x}_{j,a}) = \sum_{k,b} g_{j,k;\sigma}(b-a)\widehat{x}_{k,b}$$

$$\mathcal{A}_\sigma \equiv \mathcal{E}_\sigma(\mathcal{A}_{\mathrm{reg}})$$

Observables at scale σ



$\mathcal{A}_\sigma$
$\mathcal{A}_{reg}$
$\mathcal{E}_\sigma$

$$D_\sigma(\rho_1, \rho_2) \equiv D(\mathcal{E}_\sigma^*(\rho_1), \mathcal{E}_\sigma^*(\rho_2))$$

Large-scale distinguishability metric

$$\mathcal{S}(\mathcal{A}_{\mathsf{reg}}) \equiv \{f : \mathcal{A}_{\mathsf{reg}} \to \mathbb{C} \mid f(\mathcal{A}^{+}_{\mathsf{reg}}) \geq 0\}$$

Regulated QFT states


$\mathcal{S}(\mathcal{A}_{\mathrm{reg}})$ is full of holes!

We don’t have a quantum field state yet

Quantum field state:

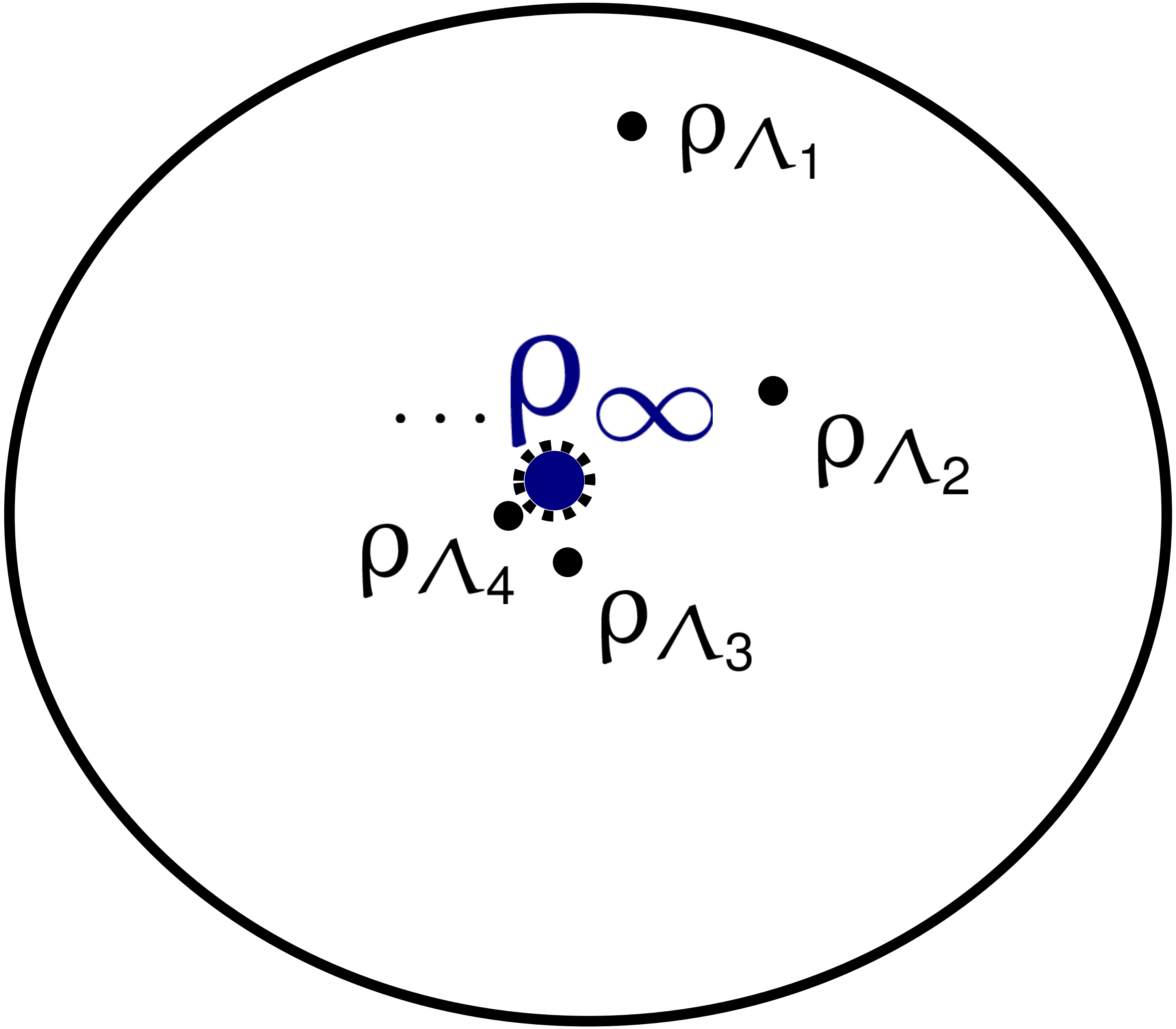
Cauchy sequence

of regulated field states

$$\rho_{\text{field}} \equiv (\rho_{\Lambda_1}, \rho_{\Lambda_2}, \rho_{\Lambda_3}, \ldots)$$

$$\forall \epsilon, \exists N(\epsilon), \forall j, k > N(\epsilon)$$

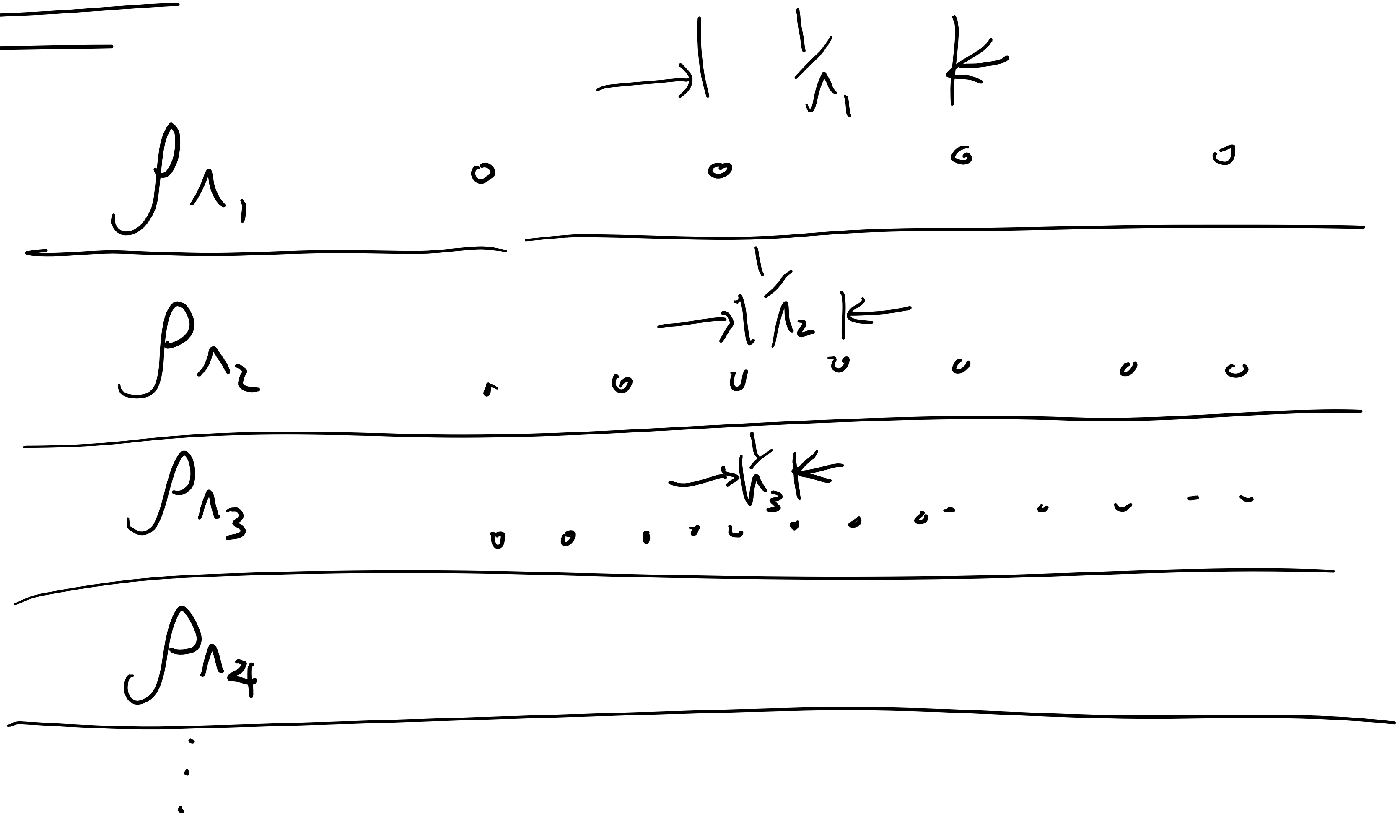
$$D_\sigma(\rho_{\Lambda_j}, \rho_{\Lambda_k}) < \epsilon$$



ρ_{Λ_1}
ρ_∞
ρ_{Λ_2}
ρ_{Λ_4}
ρ_{Λ_3}

Quantum field state:

$\rho_{field} \equiv$



such that

$$D_\sigma(\rho_{\Lambda_j}, \rho_{\Lambda_k}) \text{ "}\rightarrow\text{" } 0$$

$$\mathcal{S}_{\text{field}} \equiv \widehat{\mathcal{S}}(\mathcal{A}_{\text{reg}})/\sim$$

Building Cauchy sequences

$$\widehat{H}_N(\mu_N, \lambda_N) = a_N \sum_{j\in\mathbb{Z}} \frac{\widehat{p}_j^2}{2a_N^2} + \frac{(\widehat{x}_j - \widehat{x}_{j+1})^2}{2a_N^2} + \frac{\mu_N^2}{2}\widehat{x}_j^2 + \frac{\lambda_N}{4!}\widehat{x}_j^4$$

$$a_N = \frac{1}{2^N}$$

$$\rho_N \sim \rho(\mu_N, \lambda_N, \beta_N) \equiv \frac{e^{-\beta_N \widehat{H}_N(\mu_N, \lambda_N)}}{\mathcal{Z}_N}$$

How must $\mu_N, \lambda_N, \beta_N$ **run** with N to make

$$\mathcal{E}^*_\sigma(\rho_N)$$

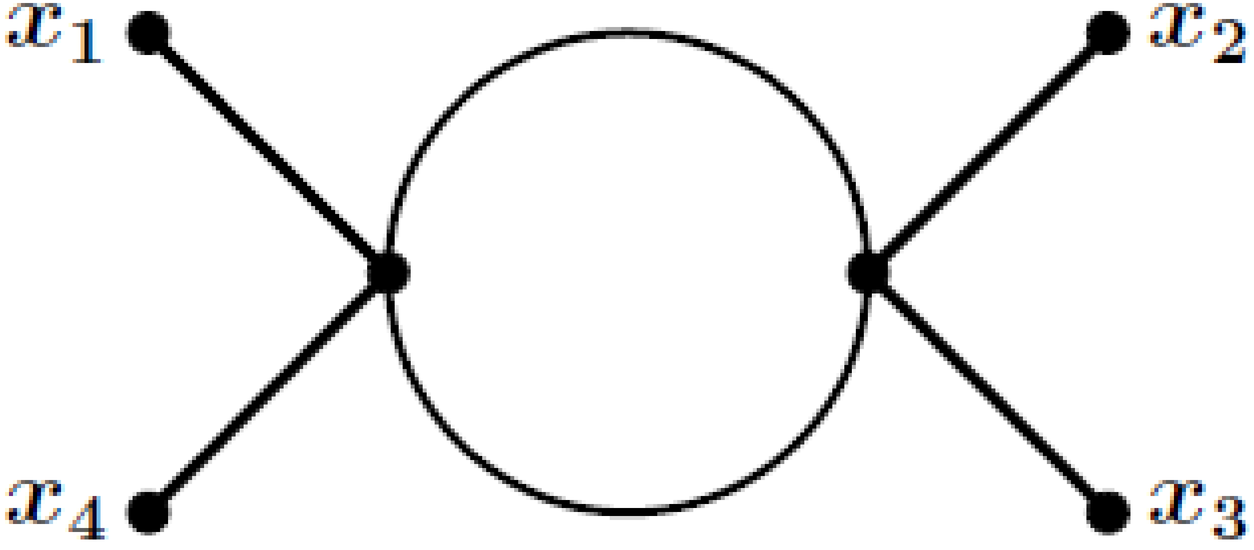
a Cauchy sequence?

Nobody knows

CLOSE ENOUGH

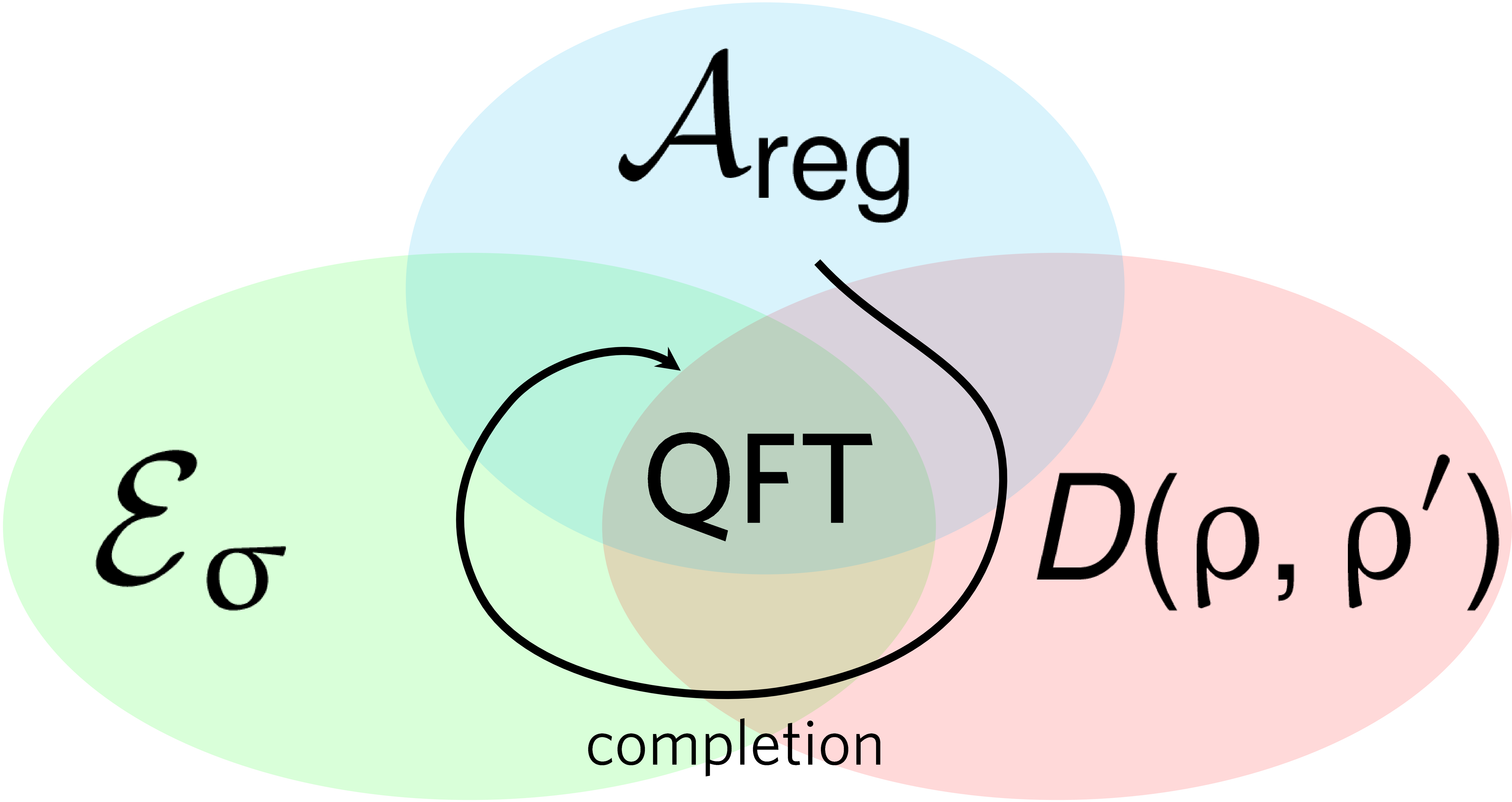
$$\rho_N = \rho_N^{(0)} + \rho_N^{(1)} \lambda_N + \rho_N^{(1)} \frac{\lambda_N^2}{2!} + \cdots$$

(3+1)D:



$$\lambda_{N+1} - \lambda_N \approx \frac{3}{16\pi^2}\lambda_N^2$$

RG flow equation:
recipe to construct
Cauchy sequences
of states



$\mathcal{A}_{\mathrm{reg}}$
$\mathcal{E}_{\sigma}$
QFT
$D(\rho, \rho')$
completion