

# What is quantum field theory? A quantum information theorist's journey

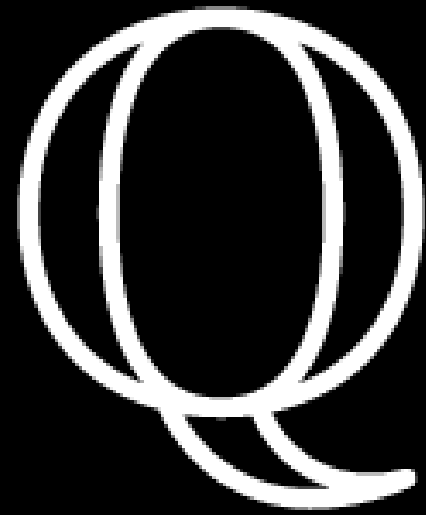
Cedric Beny and Tobias J. Osborne





QFT in 45  
minutes!





R

$$\sqrt{2} =$$

1

1.4

1.41

1.414

1.4142

1.41421

1.414214

1.4142136

1.41421356

1.414213562

...

$$(x_n)_{n \in \mathbb{N}}$$

where  $x_n \in \mathbb{Q}$

$$d(x, y) \equiv |x - y|$$



**Cauchy sequence:**

$$(x_n)_{n \in \mathbb{N}}$$

such that

$$\forall \epsilon \exists N(\epsilon), \forall m, n > N(\epsilon), d(x_m, x_n) < \epsilon$$

$\widehat{\mathbb{Q}}$  = set of Cauchy  
sequences

$$(x_n)_{n \in \mathbb{N}} \sim (y_n)_{n \in \mathbb{N}}$$



$$\lim_{n \rightarrow \infty} (x_n - y_n) = 0$$

$$\mathbb{R} \cong \widehat{\mathbb{Q}} / \sim$$

The background features a complex, fractal-like pattern of blue and white lines, creating a sense of depth and movement. A central, glowing blue sphere is the focal point, surrounded by a dark, semi-transparent rectangular area. The overall aesthetic is scientific and futuristic.

# Classical fields

Field theory is **hard**

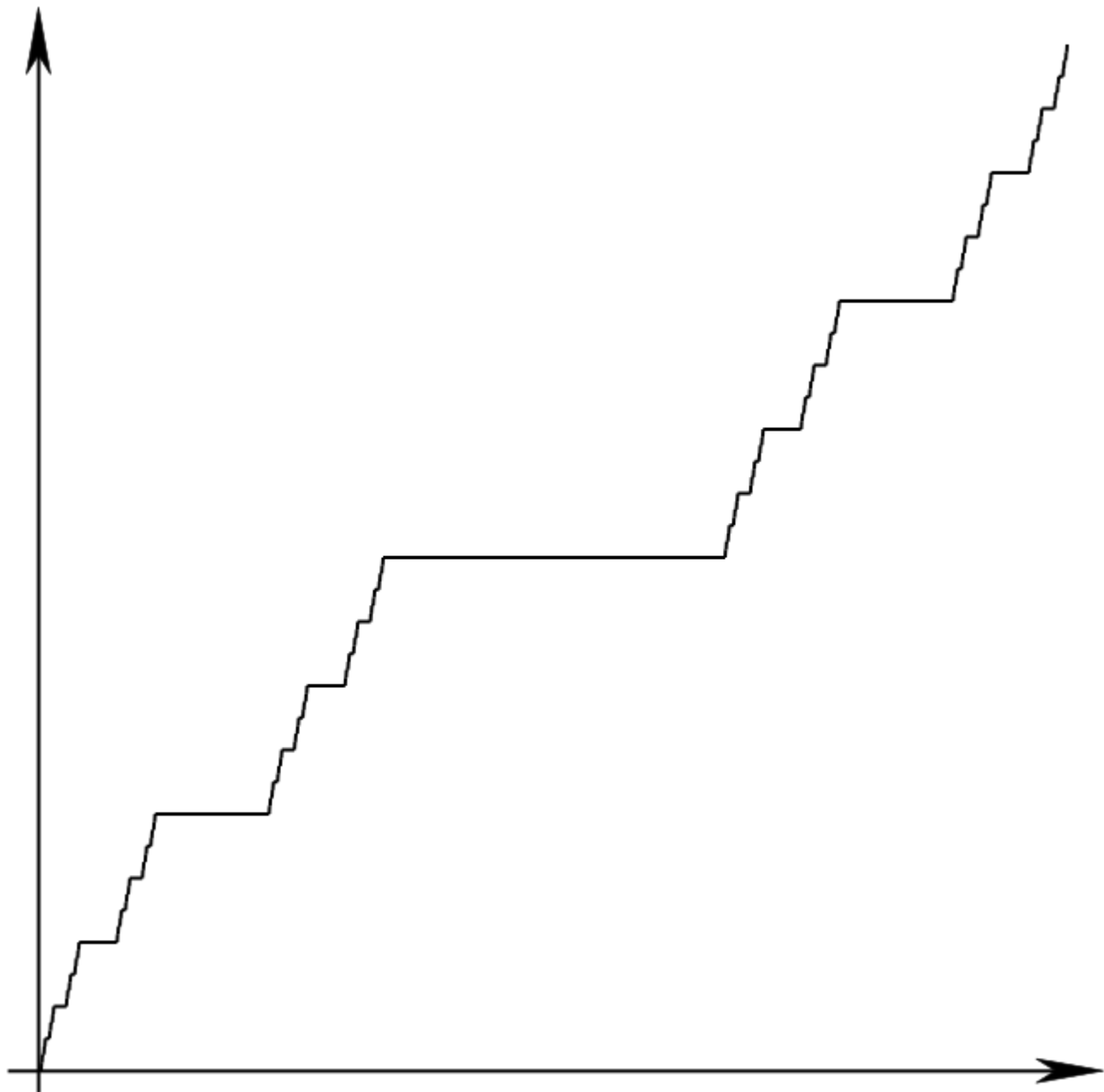


Classical approach: **calculus**

Pure field states are  
**continuous functions**

$$\phi : \mathbb{R}^d \rightarrow \mathbb{R}$$

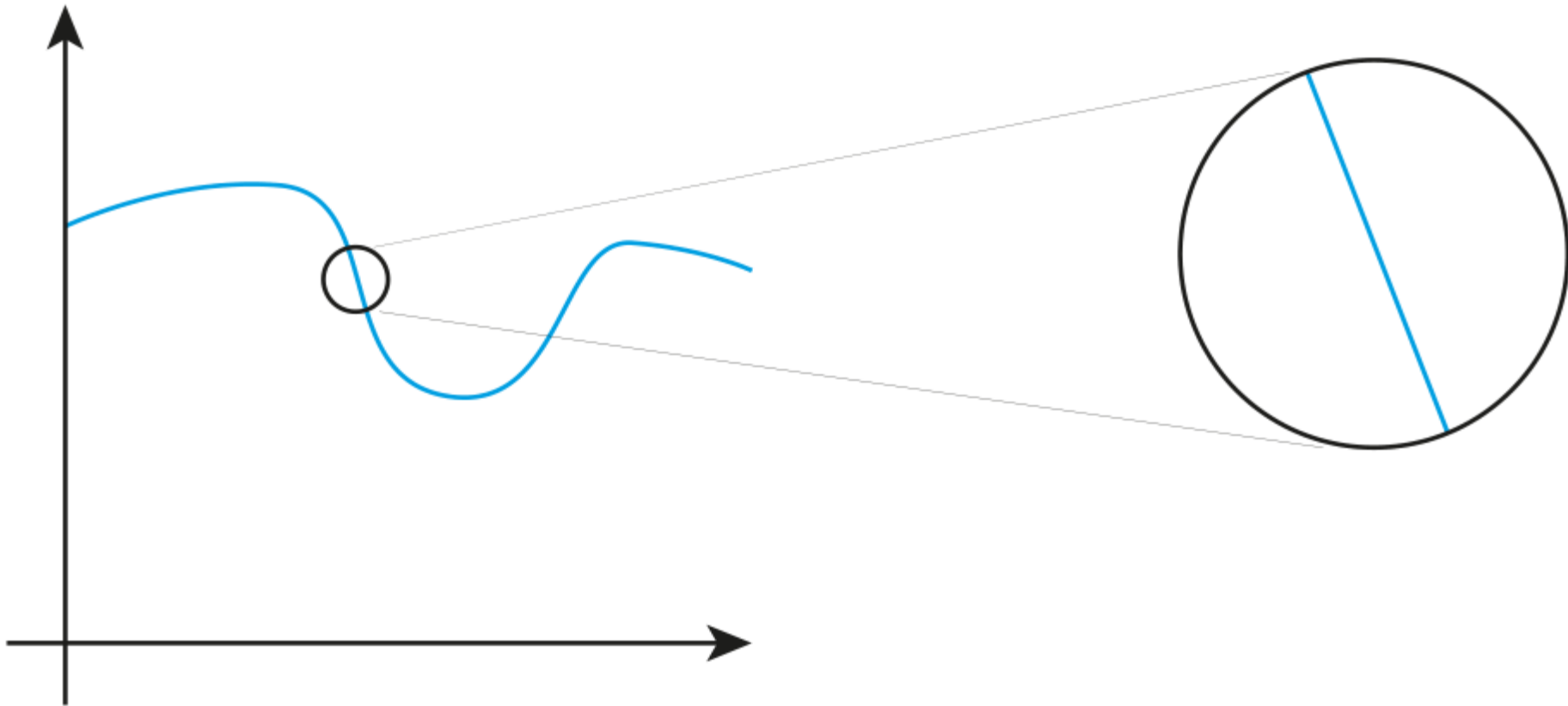




**Physical states** are  
solutions to DEs

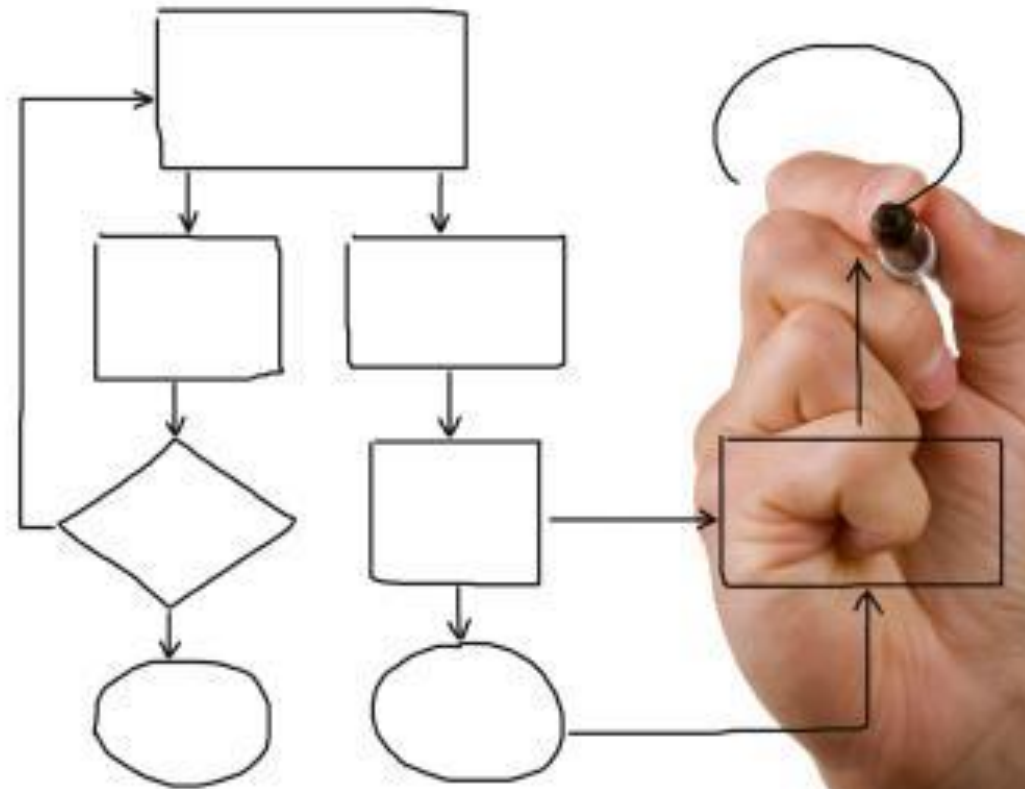
$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} + m^2 \phi = 0$$

**Physical states** are  
differentiable



Lagrangian is a **data structure**  
for equations of motion

$$\mathcal{L} = \partial_{\mu}\phi\partial^{\mu}\phi - m^2\phi^2$$



The background of the slide is a dense, repeating pattern of Feynman diagrams. These diagrams are drawn in black and light blue on a yellow background. They represent various particle interactions, including electron-electron scattering, electron-positron annihilation, and photon emission and absorption. The diagrams are scattered across the entire page, with some appearing larger and more prominent than others. A central white rounded rectangle contains the text 'Quantum fields'.

# Quantum fields

**Physicist:** put on hats!

$$\phi \mapsto \hat{\phi}$$



# Path integral

$$\mathcal{L} \mapsto \int \mathcal{D}\phi e^{iS[\phi]}$$









# PROBLEMS

NO MATTER HOW GREAT AND DESTRUCTIVE YOUR PROBLEMS MAY SEEM NOW,  
REMEMBER, YOU'VE PROBABLY ONLY SEEN THE TIP OF THEM.

... (Mukherjee, Cartwright, and Govindaraju) ...  
 ... of the local ridge orientations ...  
 ... The probability of a given  $r$  value ...  
 ... (block) is computed as the marginal den ...

$$p(r) = \frac{P(r, \theta)}{\int \int P(r, \theta) dr d\theta}$$

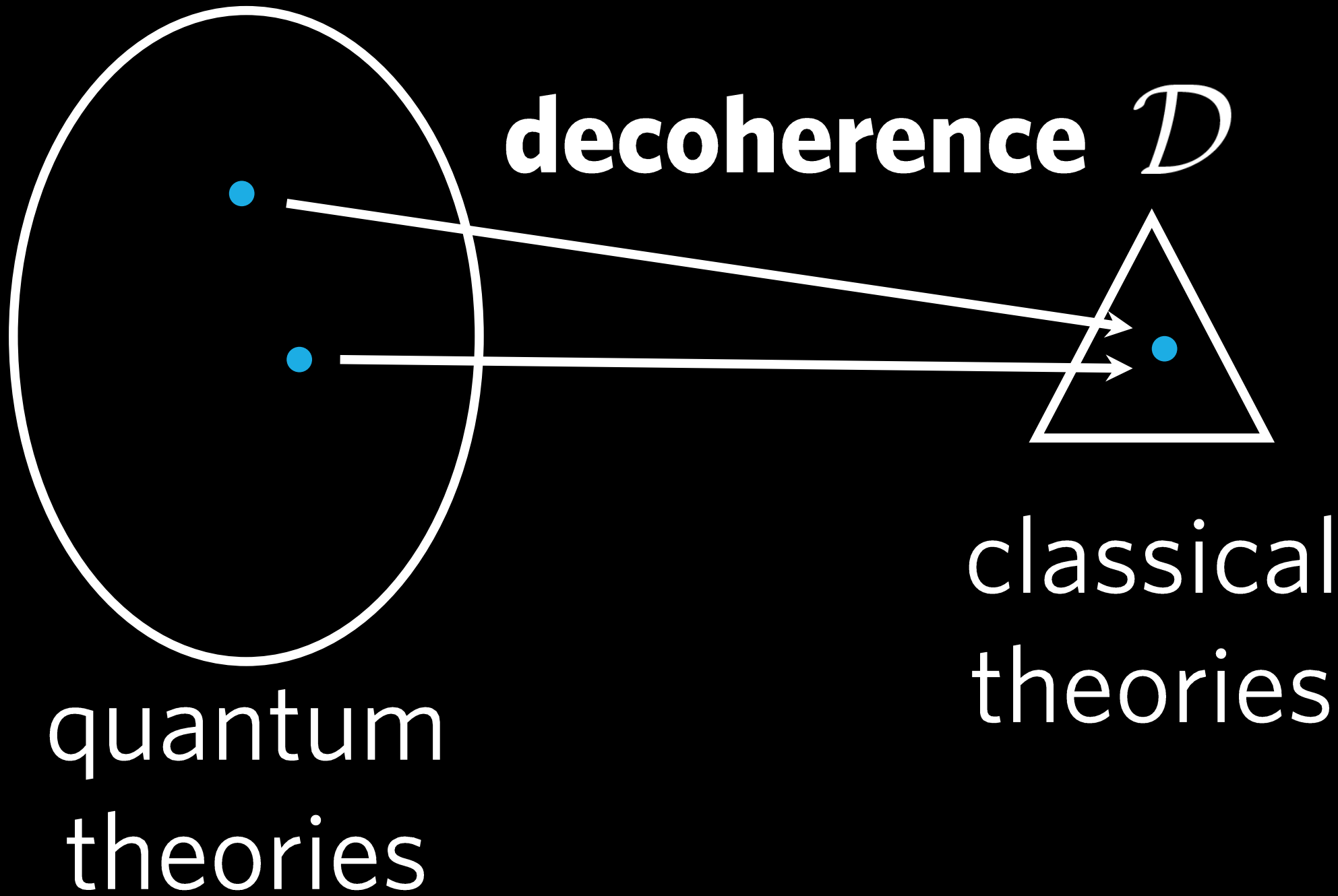
... for the block is estimated as:

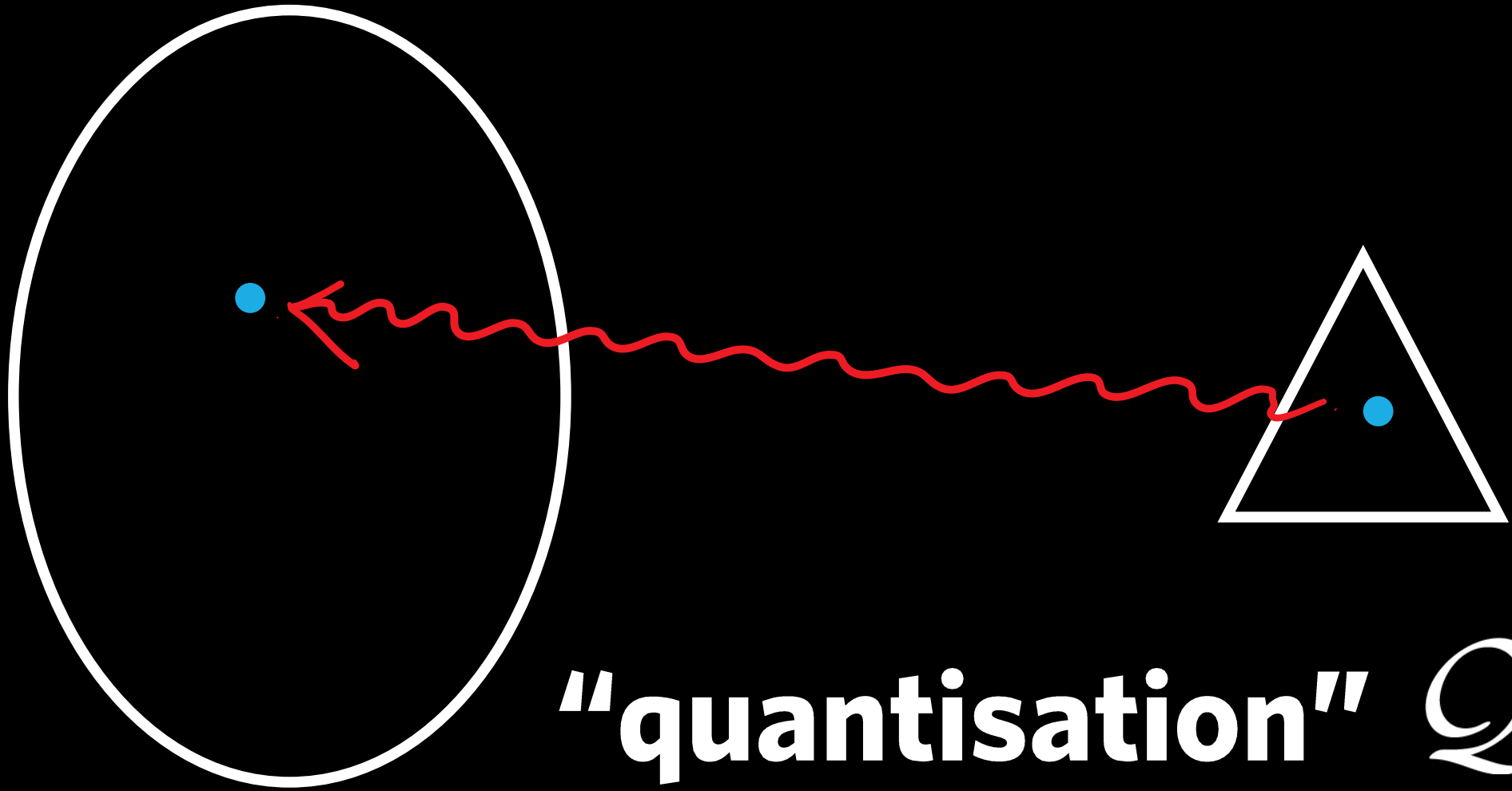
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$$p(r, \theta) = \frac{P(r, \theta)}{\int \int P(r, \theta) dr d\theta}$$

... for the block is estimated as:

Problem 1: quantisation is **not** a functor, it's an **inverse problem**





**"quantisation" Q**

**Quantisation:** quantum theory  
with desired **classical limit**

$$\mathcal{D} \circ \mathcal{Q}(\rho_{\text{cl}}) \equiv \rho_{\text{cl}}$$

Problem 2:  
**superpositions**

$$|\phi_1\rangle + |\phi_2\rangle + \dots$$

Principle to identify  
**physical states?**

Many attempts to deal  
with this:

**CQFT, AQFT, ...**



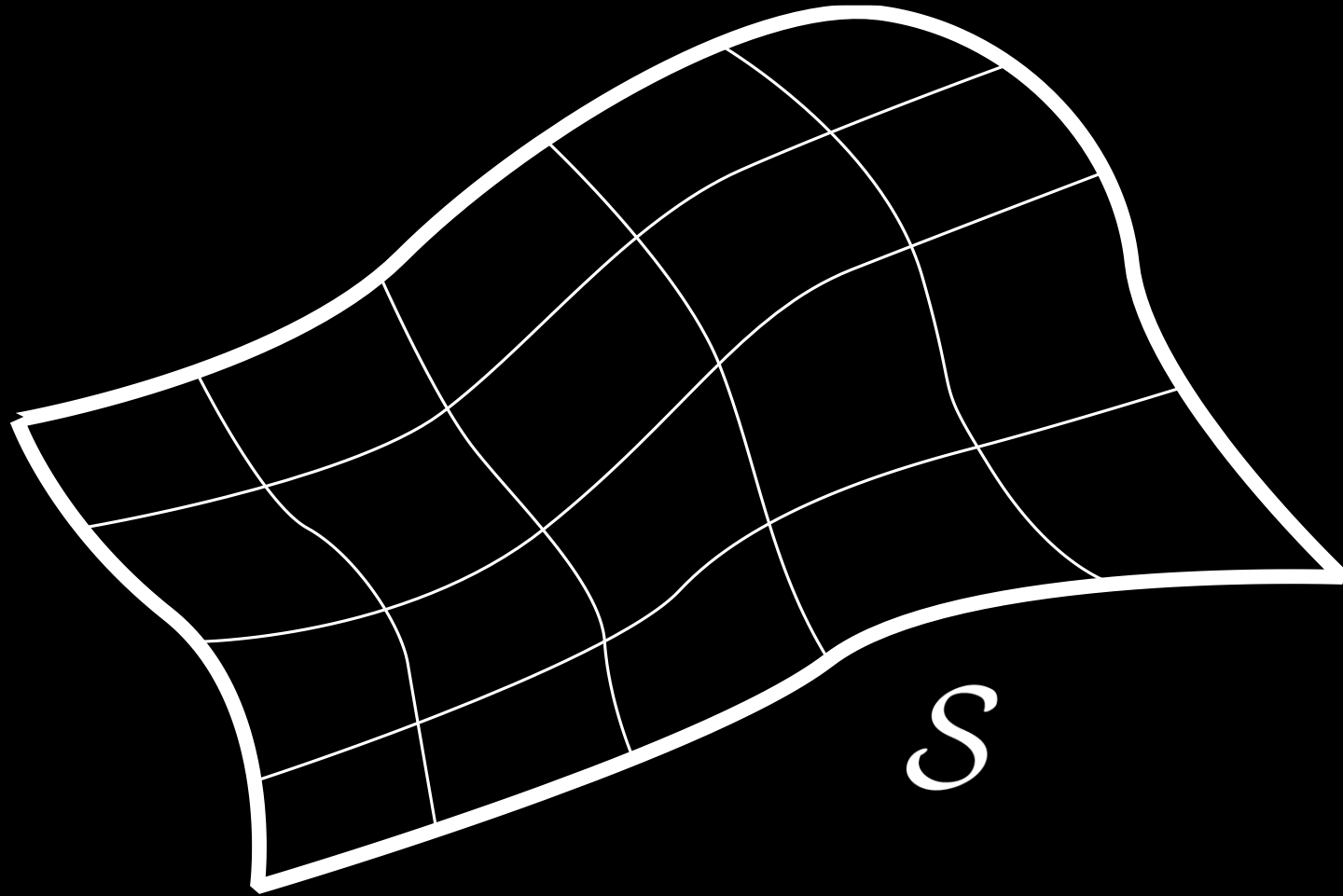
**WILSON**

**Wilson:** QFT is **effective**,  
not fundamental!



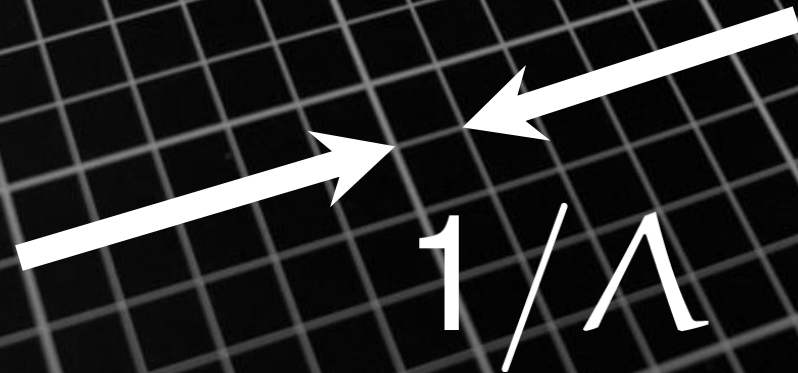
Wilsonian view of QFT

“space” of **all** theories (parametrised  
by lagrangians  $\mathcal{L}$ )

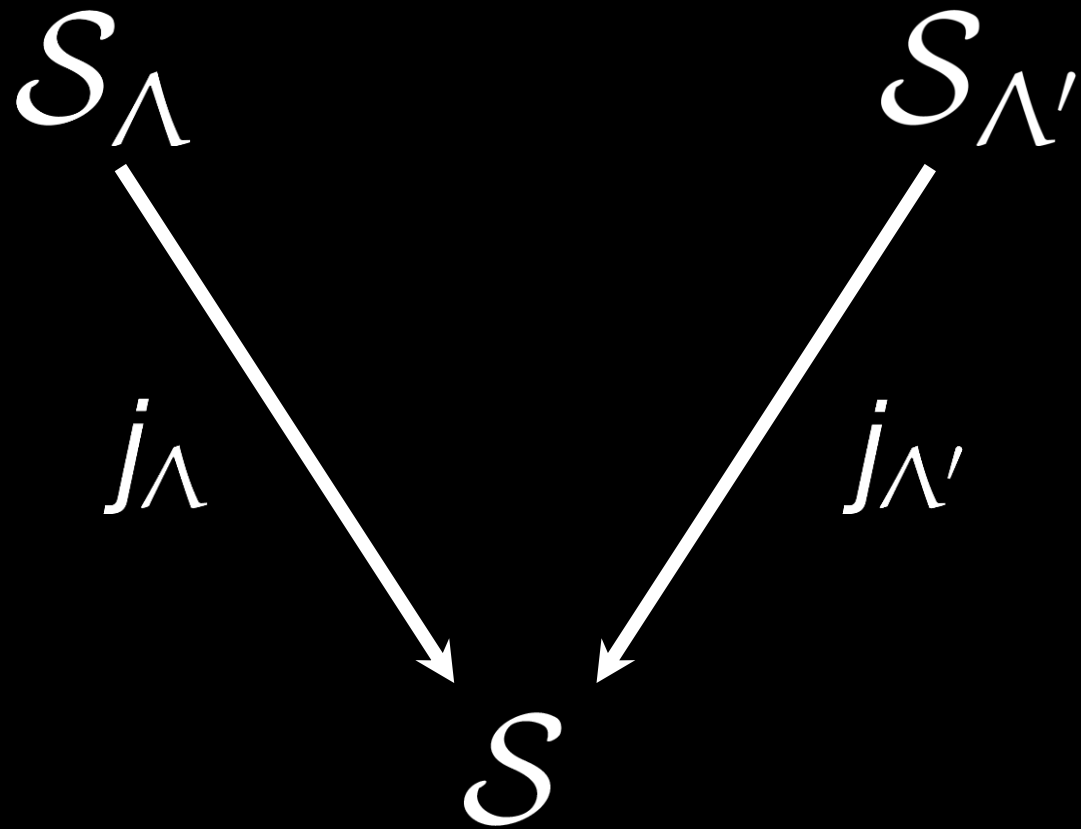


**Task:** find “submanifold” of **QFTs**

Space of regulated theories  $\mathcal{S}_\Lambda$



But  $\mathcal{S}_\Lambda$  aren't QFTs

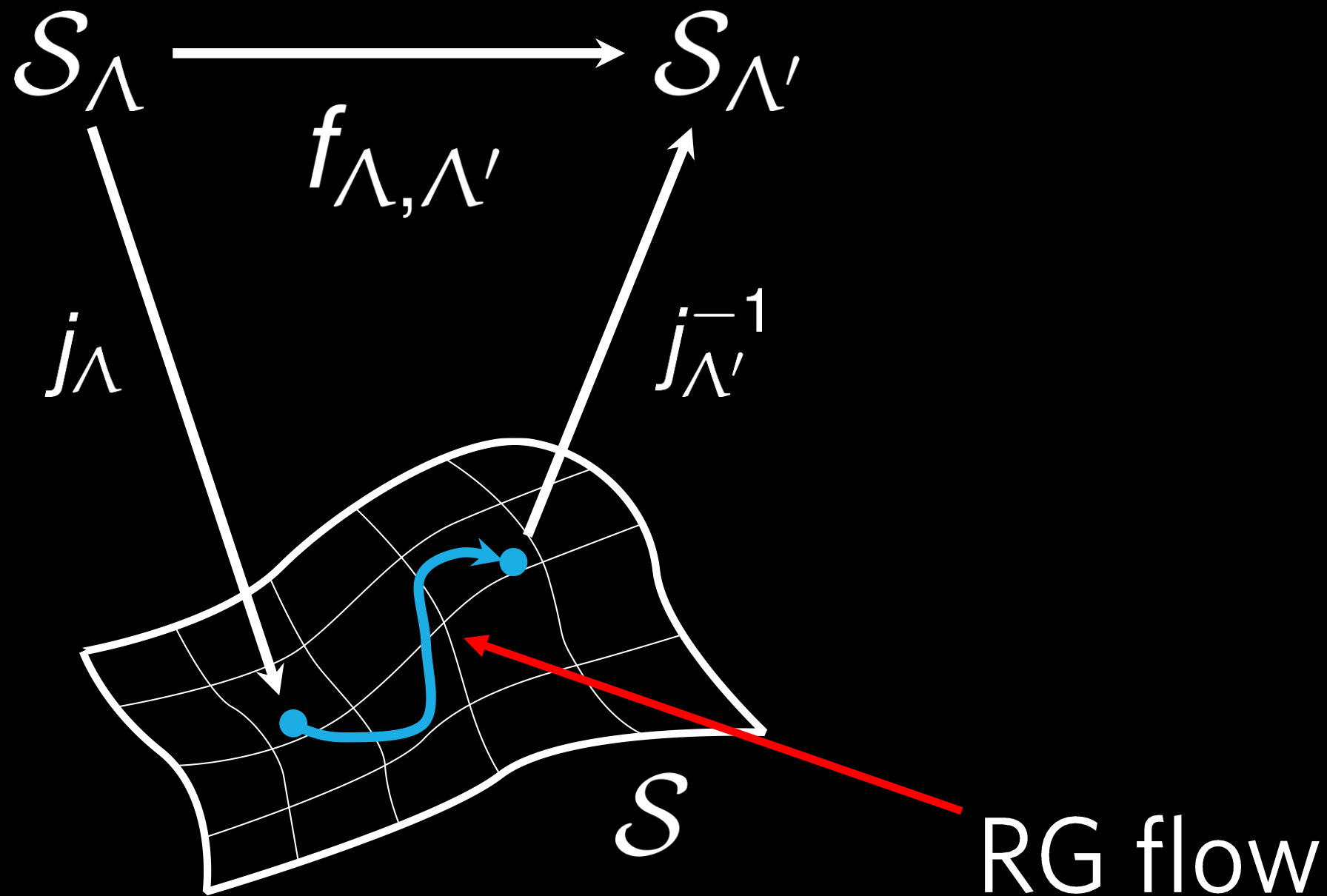




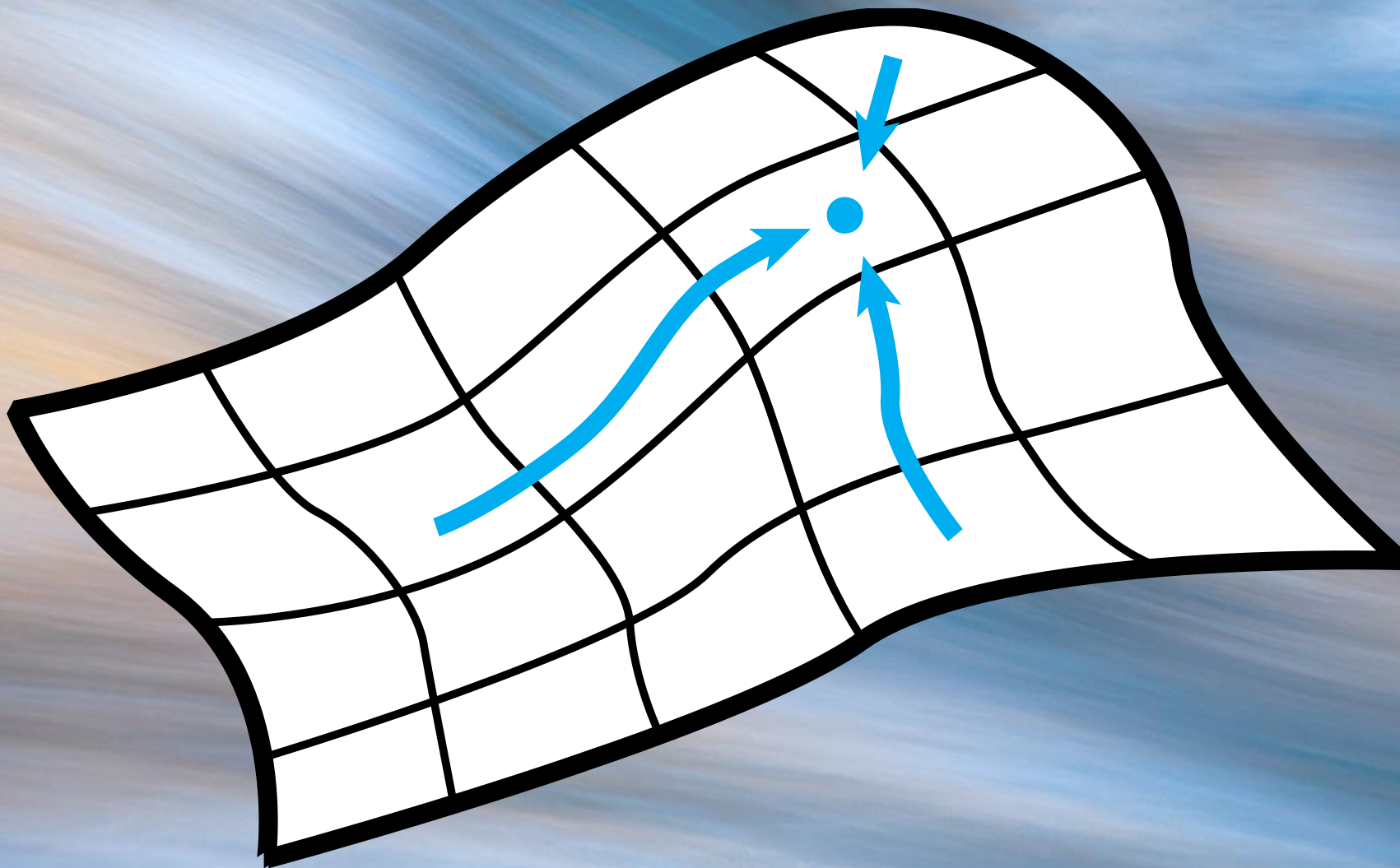
Change cutoff:  $\Lambda' \geq \Lambda$

$$\mathcal{S}_\Lambda \xrightarrow{f_{\Lambda, \Lambda'}} \mathcal{S}_{\Lambda'}$$

$f_{\Lambda, \Lambda'}$ : require **large-scale**  $n$ -pt  
correlation functions to be  
the **same**

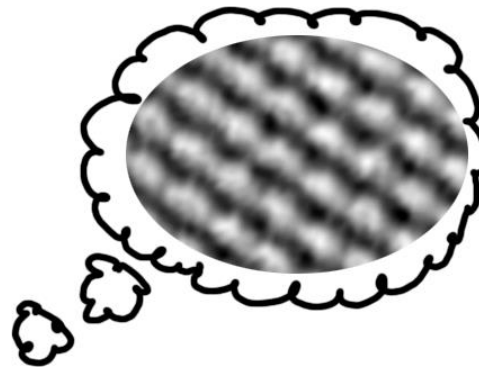
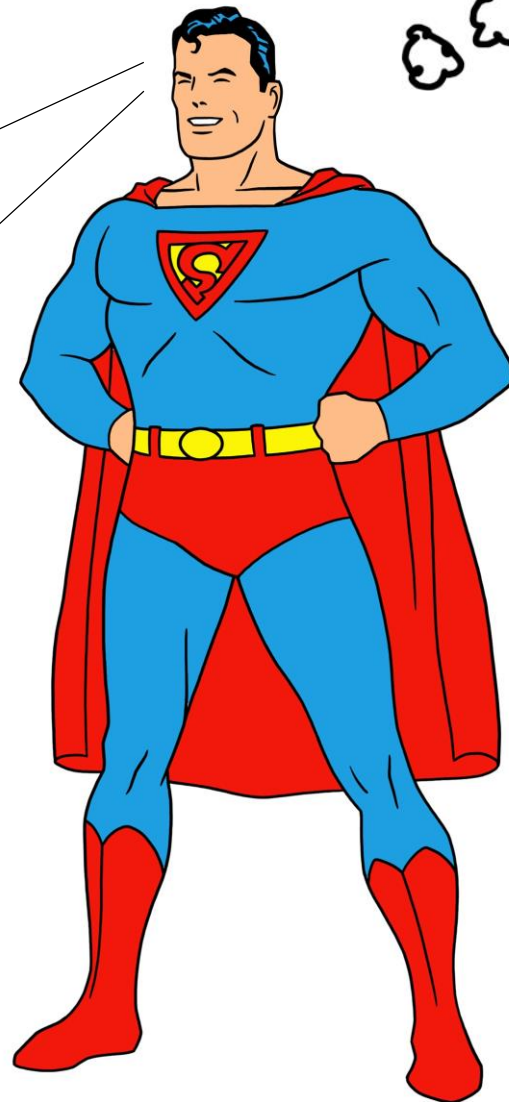
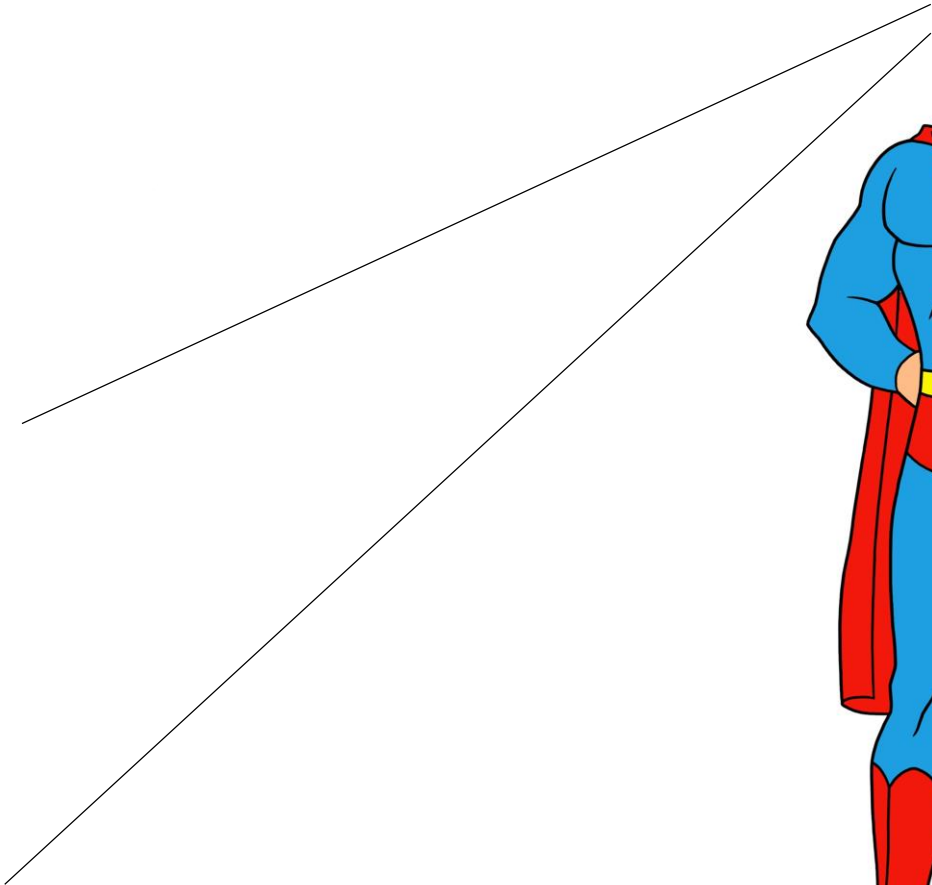


**Fixed points are QFTs**



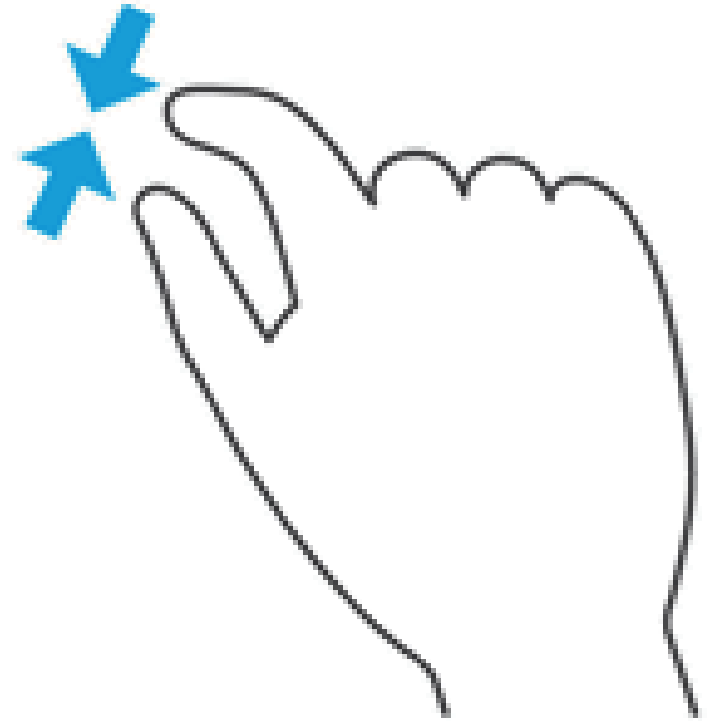
# Continuous limit:

Let  $\Lambda \rightarrow \infty$  while keeping large-scale  $n$ -pt correlation functions constant





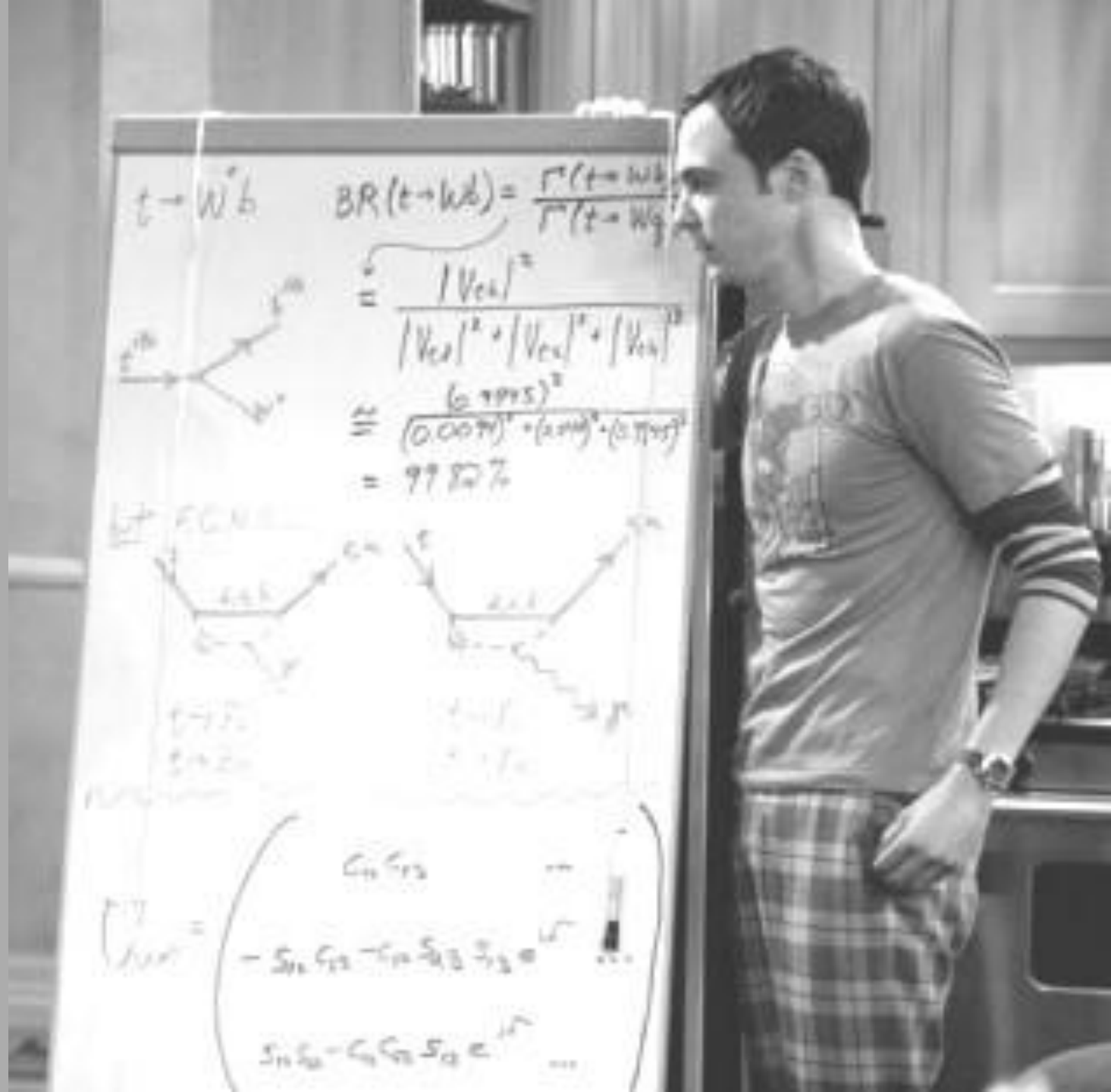
Zoom out  
= **fewer observables**



Pinch **to zoom**



Fewer  
observables  
= simpler  
hypothesis



Everything Looks Perfect from far away

oops



# Wilsonian formulation

1. Space of regulated theories
2. Distance measure
3. Large-scale observables

# Heisenberg picture

observables/effects:

$A$

(ordered unit space)



Quantum states are  
**preparations:**

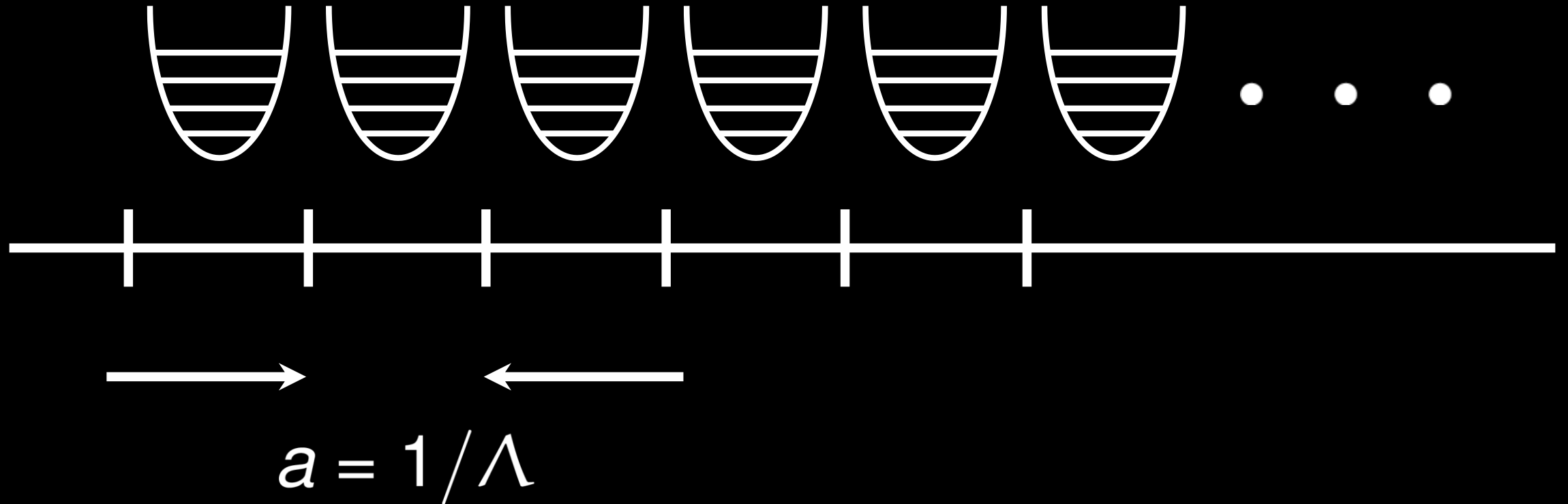
$$\omega : \mathcal{A} \rightarrow \mathbb{C}$$



$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m^2 \phi^2 - \frac{1}{4!} \lambda \phi^4$$

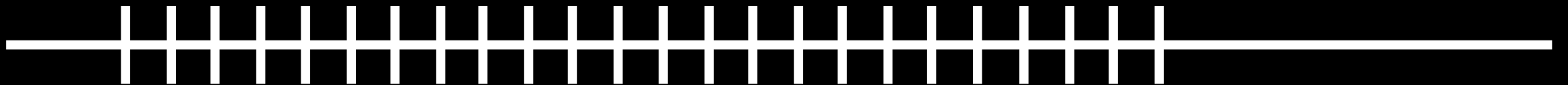
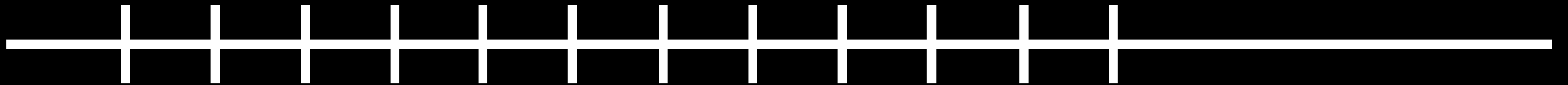


# 1. Space of regulated theories $\mathcal{A}_a$



$$[\hat{x}_{j,a}, \hat{p}_{k,a}] = i\delta_{j,k}$$

# 1. Space of regulated theories $\mathcal{A}_{\text{pre}}$





# 1. Space of regulated theories $\mathcal{A}_{\text{pre}}$

$$\mathcal{A}_{\text{pre}} \equiv \bigoplus_{N=0}^{\infty} \mathcal{A}_{\frac{1}{2^N}}$$

Can measure at resolution  $1/2^N$

**or**

at resolution  $1/2^{N+1}$

**or**

**...**

**etc**



$$\hat{X}_{j, \frac{1}{2N}} \sim \frac{1}{\sqrt{2}} \left( \hat{X}_{2j, \frac{1}{2N+1}} + \hat{X}_{2j+1, \frac{1}{2N+1}} \right)$$

# 1. Space of regulated theories

$$\mathcal{A}_{\text{reg}} \equiv \left( \bigoplus_{N=0}^{\infty} \mathcal{A}_{\frac{1}{2N}} \right) / \sim$$

## 2. Distance measure

$$D(\rho_1, \rho_2)^2 = 2(1 - \sqrt{F(\rho_1, \rho_2)})$$

# 3. Large-scale observables



A background of scattered white 3D block letters on a dark surface. The letters are of various sizes and orientations, creating a sense of depth and abundance. A semi-transparent white rounded rectangle is overlaid in the center, containing the text.

**Too many observables!**

*A*reg



*A*reg

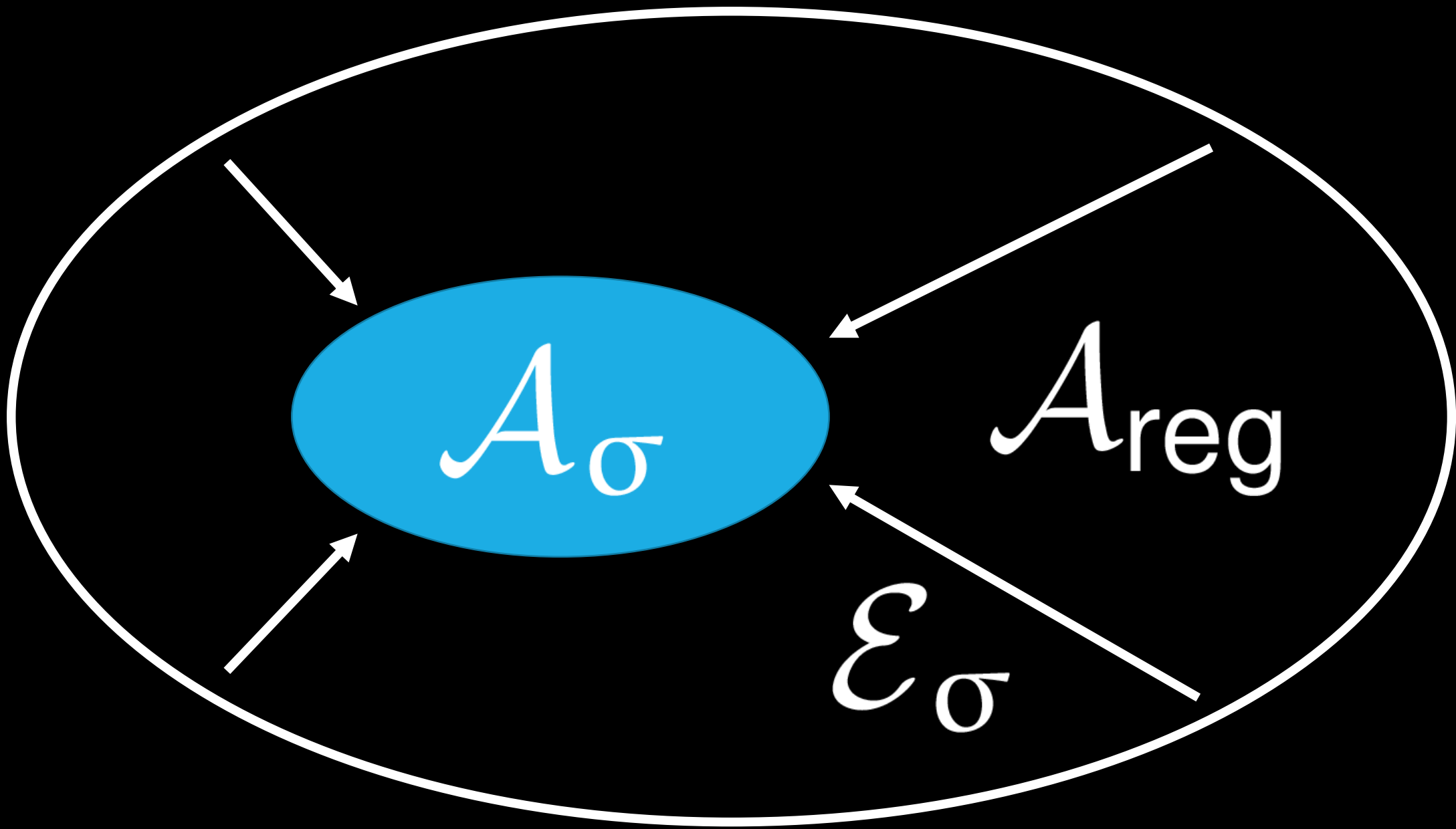
$$\mathcal{E}_\sigma : A_{\text{reg}} \rightarrow A_{\text{reg}}$$

$\mathcal{E}_\sigma$  : convolve with  
gaussian

$$\mathcal{E}_\sigma(\hat{X}_{j,a}) = \sum_{k,b} g_{j,k;\sigma}(b-a)\hat{X}_{k,b}$$

$$\mathcal{A}_\sigma \equiv \mathcal{E}_\sigma(\mathcal{A}_{\text{reg}})$$

**Observables at scale  $\sigma$**



$A_\sigma$

$A_{\text{reg}}$

$\epsilon_\sigma$

$$D_{\sigma}(\rho_1, \rho_2) \equiv D(\mathcal{E}_{\sigma}^*(\rho_1), \mathcal{E}_{\sigma}^*(\rho_2))$$

**Large-scale distinguishability  
metric**

$$\mathcal{S}(\mathcal{A}_{\text{reg}}) \equiv \{f : \mathcal{A}_{\text{reg}} \rightarrow \mathbb{C} \mid f(\mathcal{A}_{\text{reg}}^+) \geq 0\}$$

Regulated QFT states



$\mathcal{S}(A_{\text{reg}})$  is full of holes!

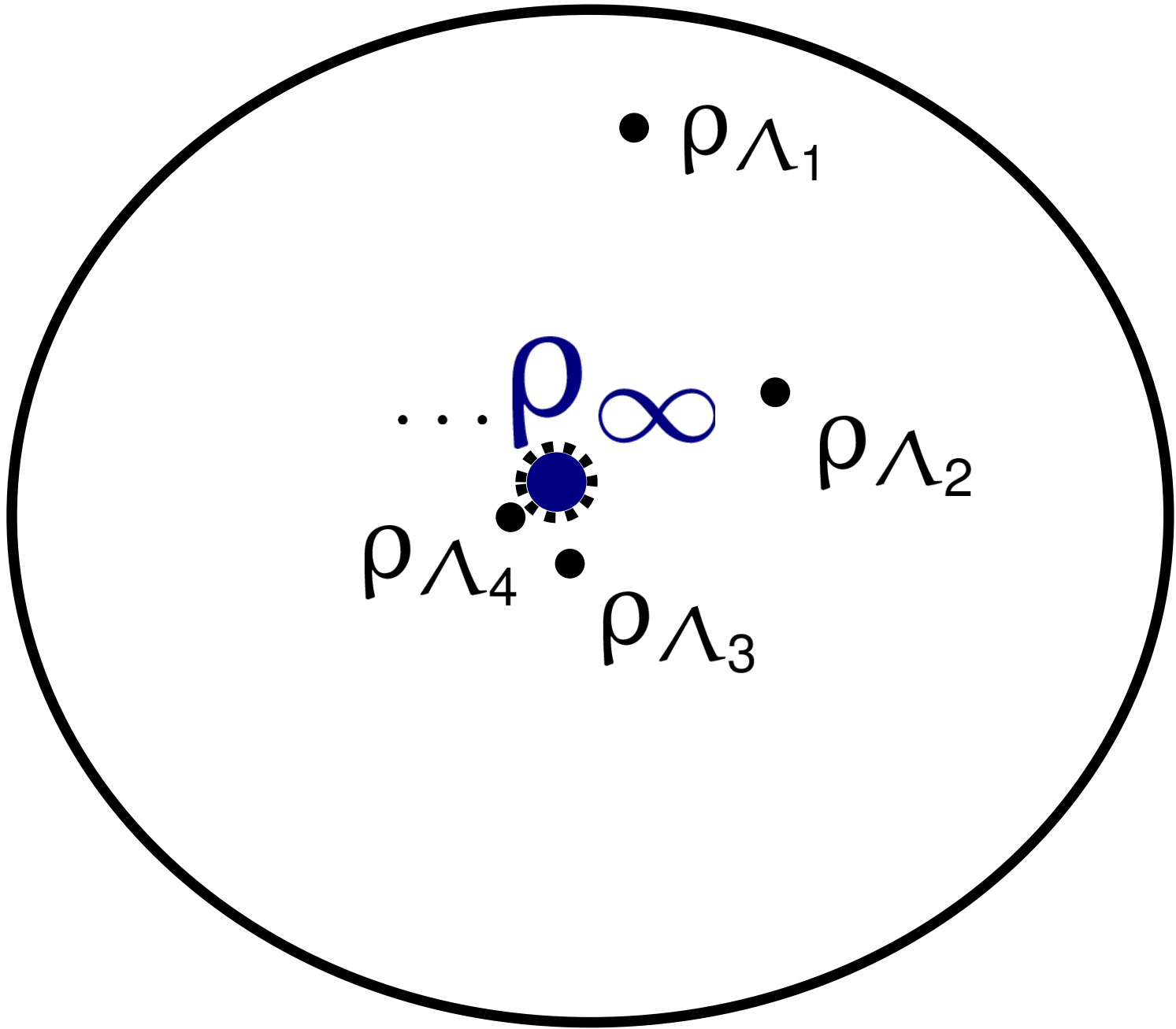
We don't have a quantum  
field state yet

**Quantum field state:**  
Cauchy sequence  
of regulated field states

$$\rho_{\text{field}} \equiv (\rho_{\Lambda_1}, \rho_{\Lambda_2}, \rho_{\Lambda_3}, \dots)$$

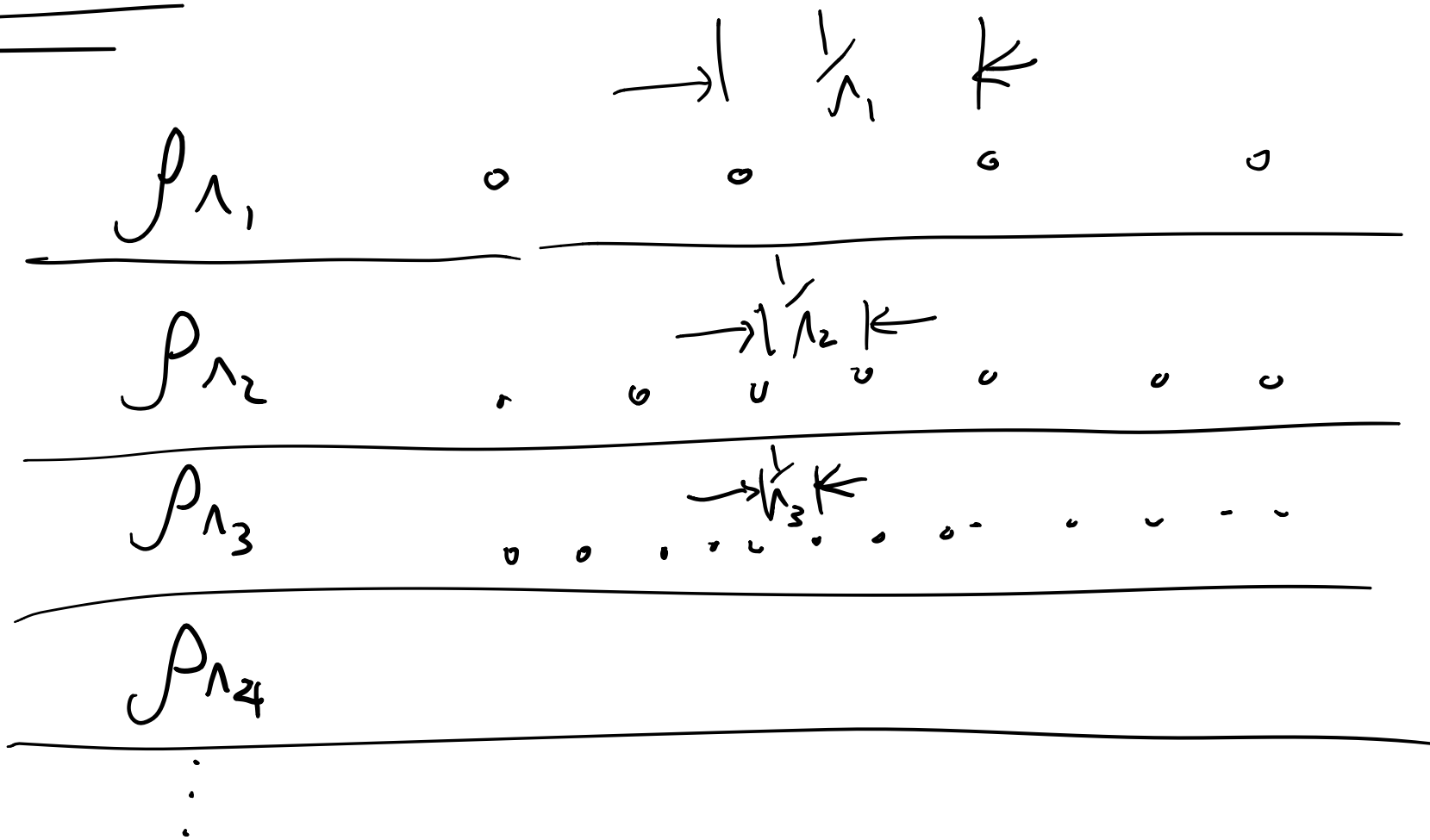
$$\forall \epsilon, \exists N(\epsilon), \forall j, k > N(\epsilon)$$

$$D_{\sigma}(\rho_{\Lambda_j}, \rho_{\Lambda_k}) < \epsilon$$



# Quantum field state:

$\beta_{\text{field}} \equiv$



such that

$$D_0(P_{\Lambda_j}, P_{\Lambda_k}) \rightarrow 0$$

$$S_{\text{field}} \equiv \hat{S}(A_{\text{reg}}) / \sim$$



**Building Cauchy sequences**



$$\hat{H}_N(\mu_N, \lambda_N) = a_N \sum_{j \in \mathbb{Z}} \frac{\hat{p}_j^2}{2a_N^2} + \frac{(\hat{x}_j - \hat{x}_{j+1})^2}{2a_N^2} + \frac{\mu_N^2}{2} \hat{x}_j^2 + \frac{\lambda_N}{4!} \hat{x}_j^4$$

$$a_N = \frac{1}{2^N}$$

$$\rho_N \sim \rho(\mu_N, \lambda_N, \beta_N) \equiv \frac{e^{-\beta_N \hat{H}_N(\mu_N, \lambda_N)}}{\mathcal{Z}_N}$$

How must  $\mu_N, \lambda_N, \beta_N$  **run**  
with  $N$  to make

$$\mathcal{E}_\sigma^*(\rho_N)$$

a Cauchy sequence?

**Nobody  
knows**

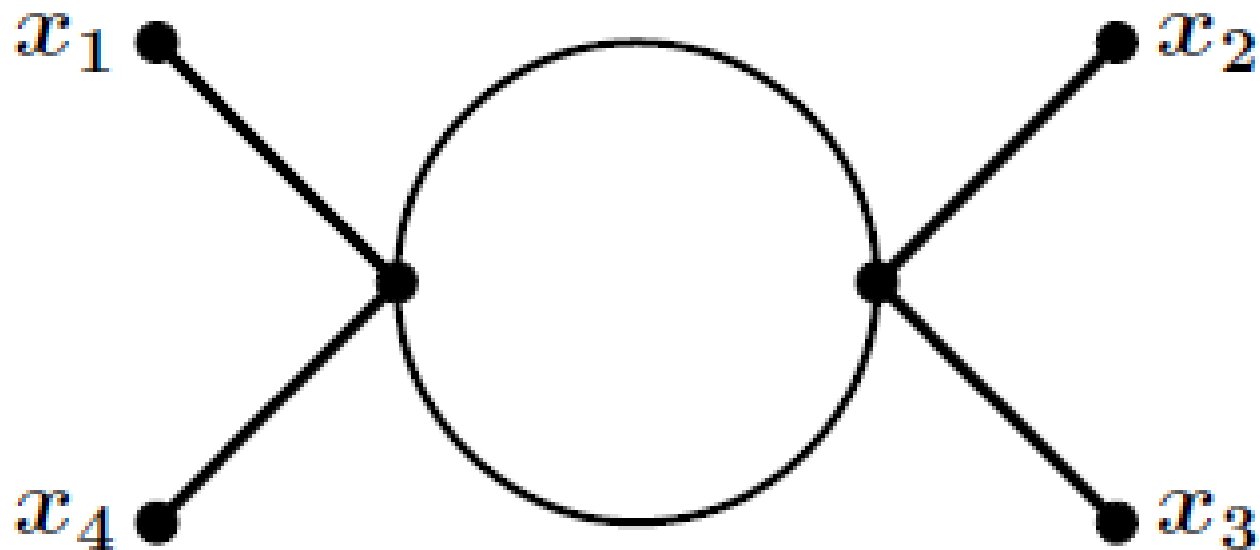




CLOSE ENOUGH

$$\rho_N = \rho_N^{(0)} + \rho_N^{(1)} \lambda_N + \rho_N^{(1)} \frac{\lambda_N^2}{2!} + \dots$$

(3+1)D:



$$\lambda_{N+1} - \lambda_N \approx \frac{3}{16\pi^2} \lambda_N^2$$

**RG flow equation:**  
recipe to construct  
Cauchy sequences  
of states





