Single Shot Fault Tolerance

I don't know much about it but I don't think it works

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Département de Physique
Université de Sherbrooke

Coogee Quantum Information Theory Workshop

Outline

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An $[n, k, d]$ binary linear codes

- An $(n-k) \times n$ parity check matrix H with binary entries.
- Codewords are the n-bit strings x which live in the kernel of H:

$$\mathcal{C} = \left\{ x \in \{0,1\}^n : Hx = 0 \right\}$$

  (arithmetic is binary throughout).
- Assuming that the rows of H are linearly independent, there are 2^k strings in $\mathcal{C}$.
  - The code encodes k bits.
- The shortest non-zero string in $\mathcal{C}$ has weight d, and this defines the minimum distance of the code.
- A bit flip error on codeword x produces $y = x + e$.
- The syndrome of y is $s \equiv Hy = H(x+e) = He$.
- The syndrome $s = He$ gives us partial information about e and decoding consists in inferring e from this partial information.

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Repetition code $[7, 1, 7]$

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Check nodes
Bit nodes

- Is 6×7 so $k = 1$
- $C = \{0000000, 1111111\}$.
- We see that $d = 7$.

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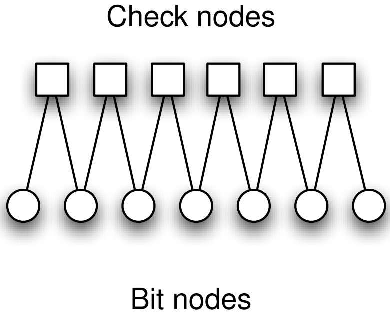

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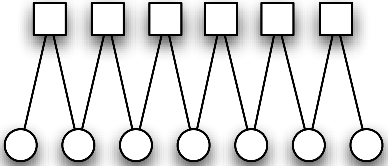

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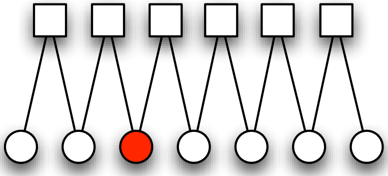

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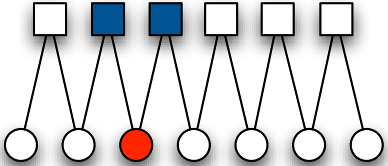

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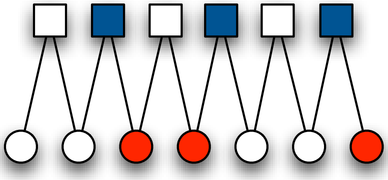

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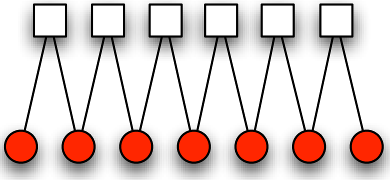

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Hamming code $[7, 4, 3]$

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- Is 3×7 so $k = 4$
- $C = \mathrm{span}\{1110000, 1001100, 0101010, 1101001\}$.
- We see that $d = 3$.

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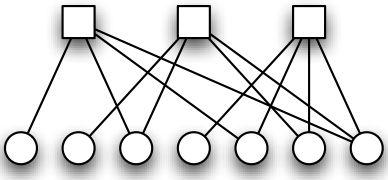

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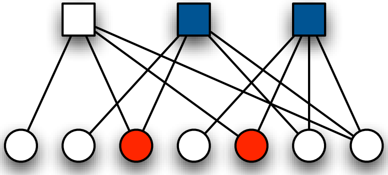

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Generating matrix

- For the Hamming code, we had $\mathcal{C} = \mathrm{span}\{1110000, 1001100, 0101010, 1101001\}$.
- We can arrange these into the columns of a generating matrix

$$G = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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Sparse codes

- A sparse code, or low density parity check (LDPC) code is one whose parity check matrix is row and column sparse.
  - There are no more than λ_r nonzero entries per row.
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- It's easy to recognize an LDPC code from its Tanner graph.
- In a quantum setting, where extracting a syndrome implies actually measuring the corresponding checks, low weight is a blessing!

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Random codes

- Define the binary entropy $h(p) = -p \log p - (1-p)\log(1-p)$. (Logarithm is base 2)
- Pick the $(n-k) \times k$ binary entries of H at random.
- With very high probability
  - The $n-k$ rows of H are linearly independent, so the code encodes k bits.
  - The code can 'correct' typical bit-flip errors provided the bit-flip probability p obeys $\frac{k}{n} < 1 - h(p)$.
- By 'correct', we mean that typical errors all have different syndromes, so they can in principle be uniquely identified from their syndrome.
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Syndrome errors

- For the purpose of this talk, fault tolerance will refer to something very simple:

  The syndrome bits are not reliable.

  - Bits/qubits are subjected to errors with error-rate p.
  - The syndrome s is measured on the corrupted string.
  - The syndrome bits are subjected to a bit-flip noise with rate q.
  - We will consider the 'symmetric' case $p = q$ for simplicity.
- This is ofter referred to as the 'phenomenological noise model'.
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- Repeated syndrome measurements is a good way to cope with syndrome errors.
- We can't just naively take the majority vote among the syndrome bits since additional errors can occur in between syndrome measurements, so they are not meant to agree even in the absence of syndrome errors.
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Outline

Is it possible to do fault-tolerant quantum error correction without repeating the syndrome measurements?

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$$H = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

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- In general, $H \rightarrow (I|H)$.
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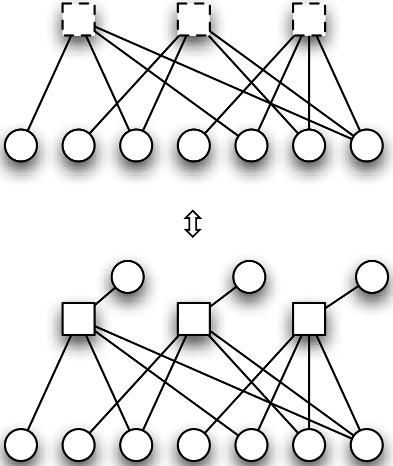

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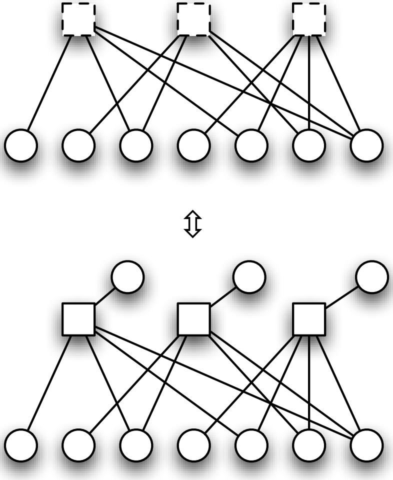

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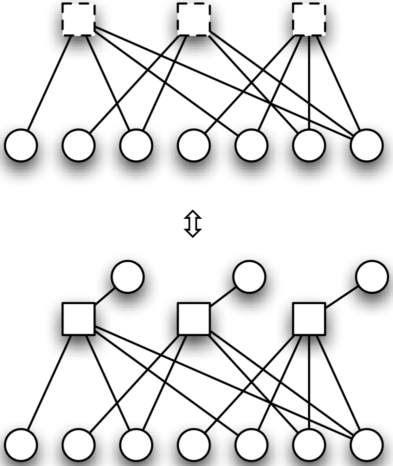

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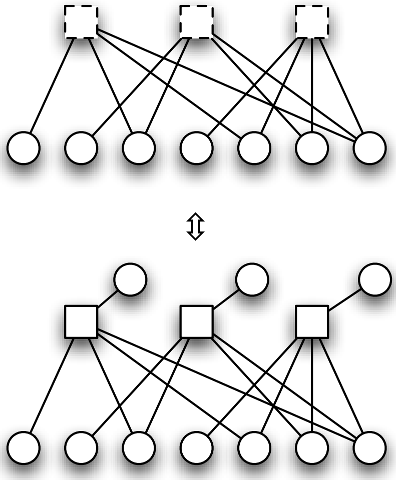

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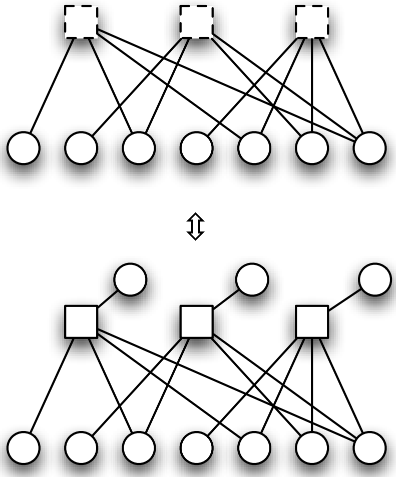

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Normal form

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- Another way of deriving this upper bound is to say that
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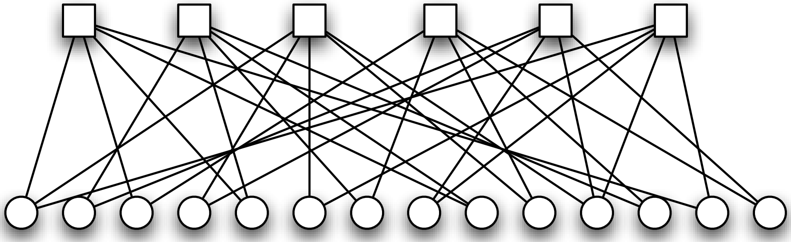
Sparse codes cannot be single-shot FT



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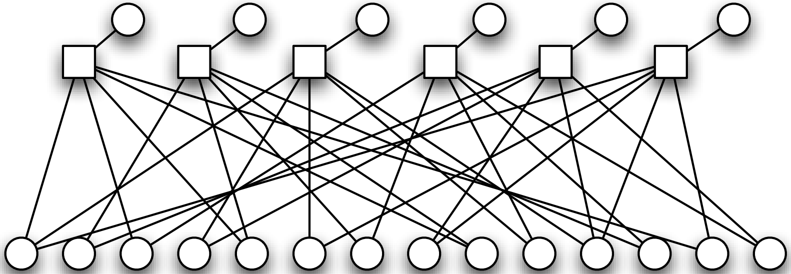

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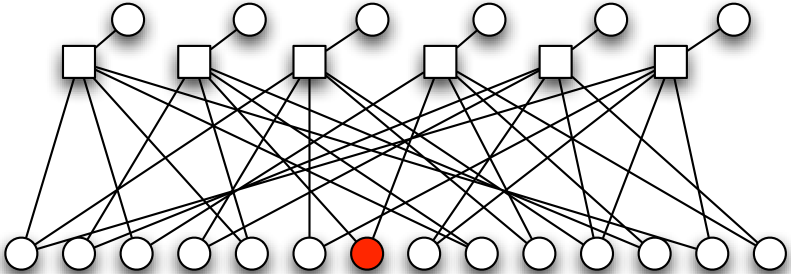

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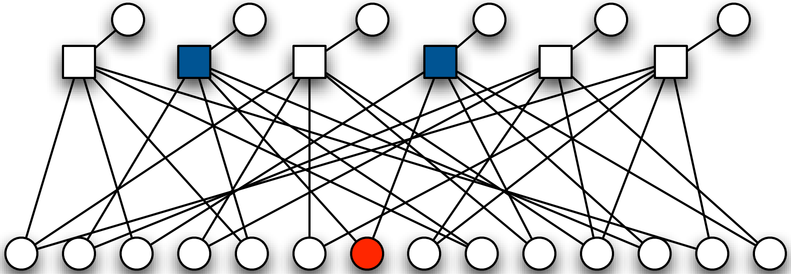

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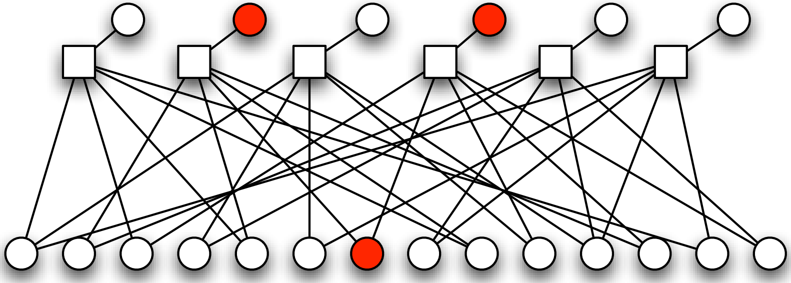

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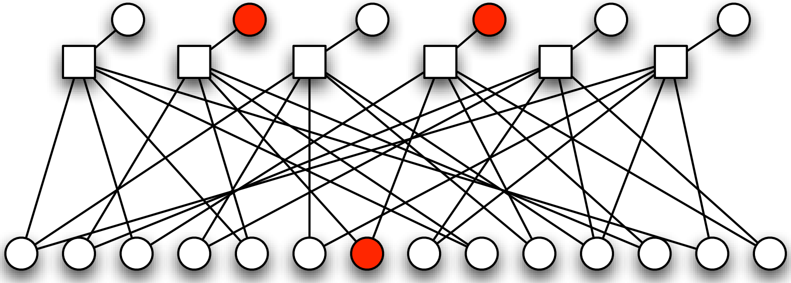

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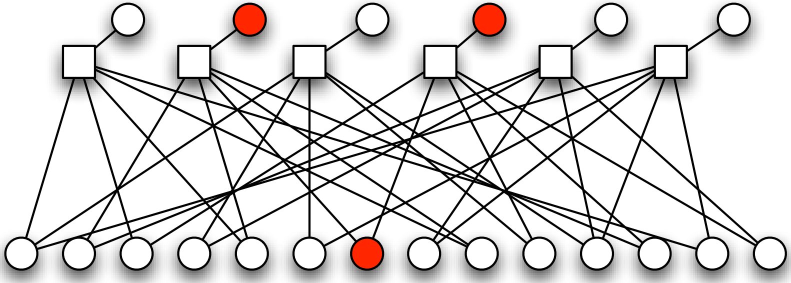

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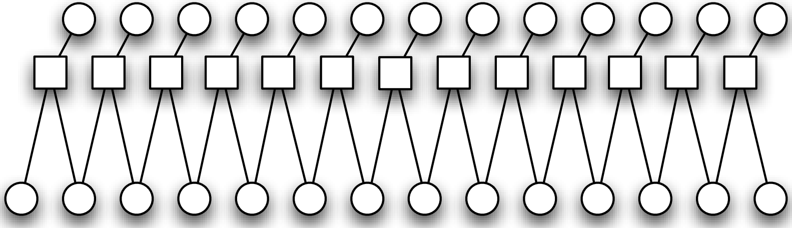

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Self-correction requires $|s(E)| > |E|^\delta$ for $\delta > 0$?

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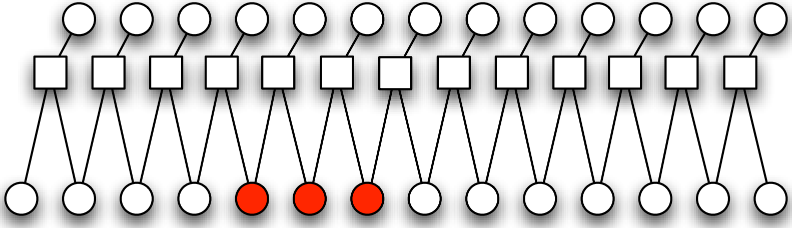

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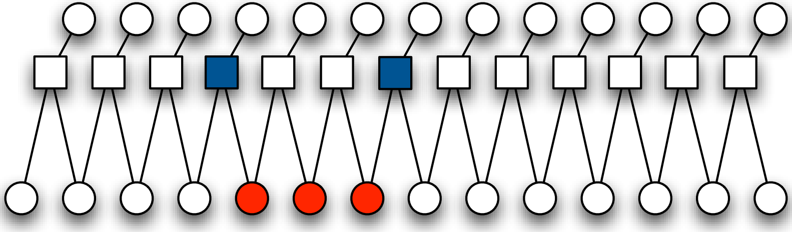

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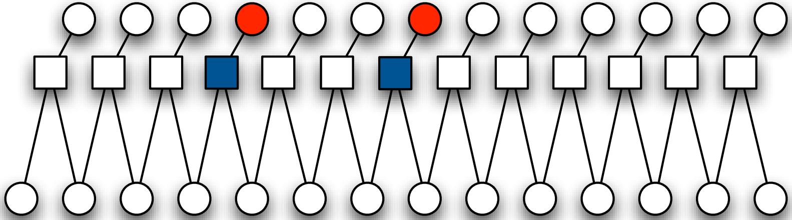

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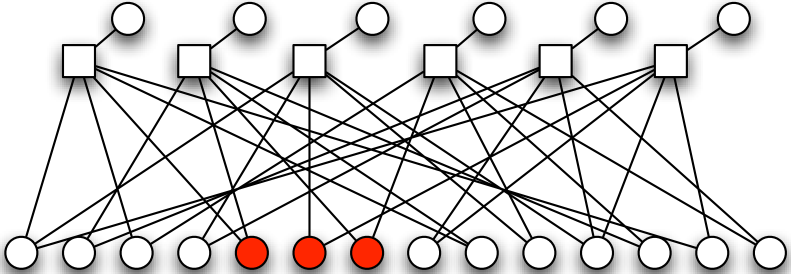

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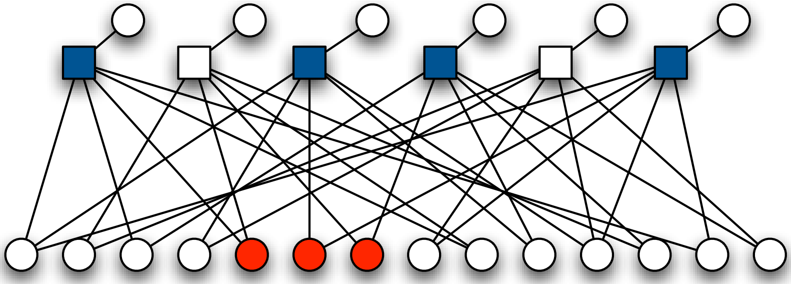

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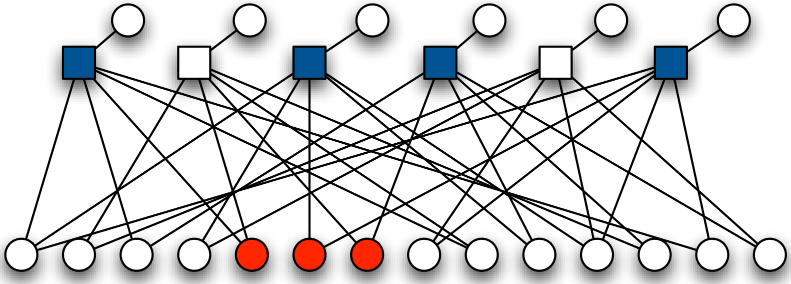

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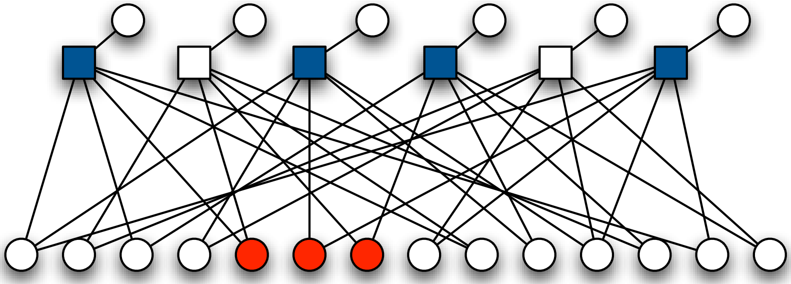

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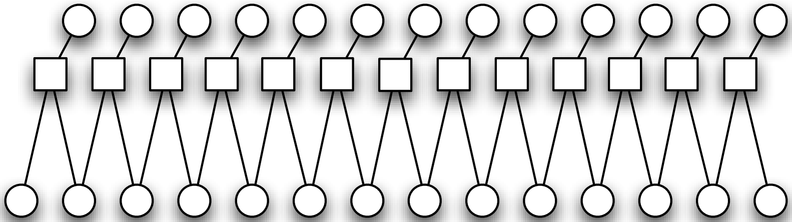
Outline

Bounded error growth



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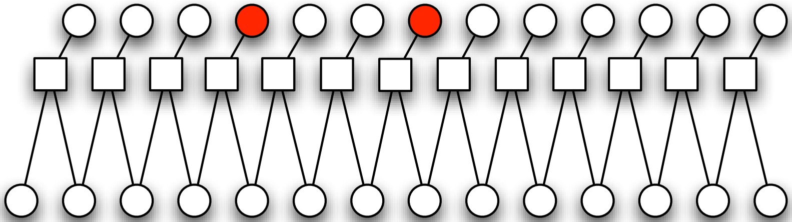

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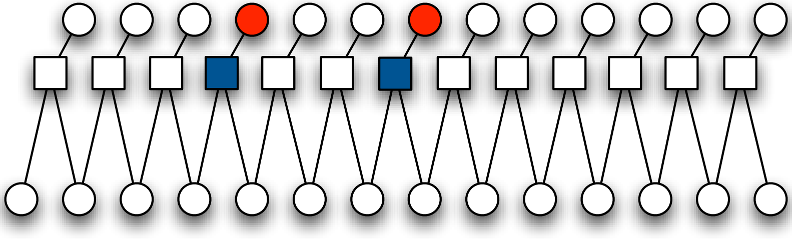

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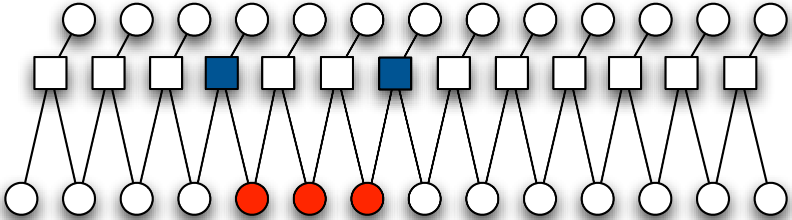

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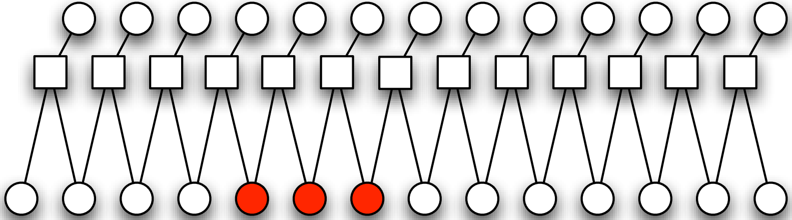

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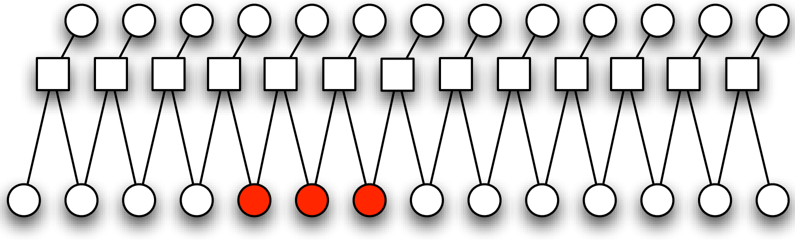

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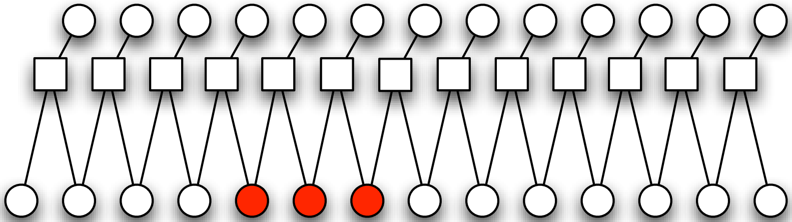

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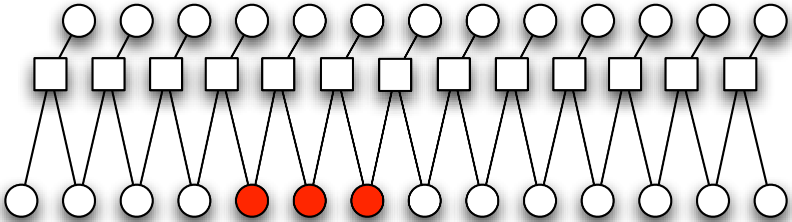

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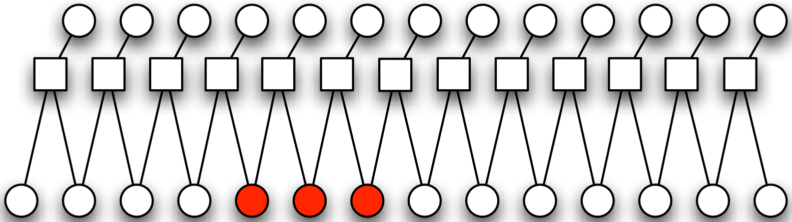

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2D Ising ferromagnet

- Error correction has increased the error from $E \sim L$ to $EC(E) \sim L^2$.
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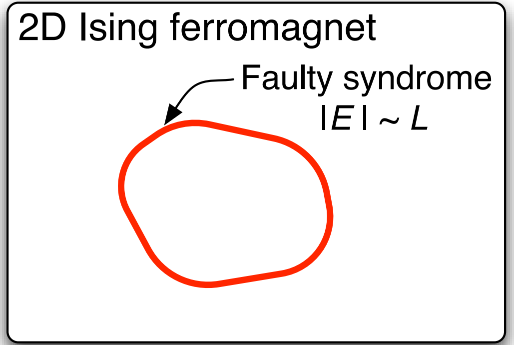

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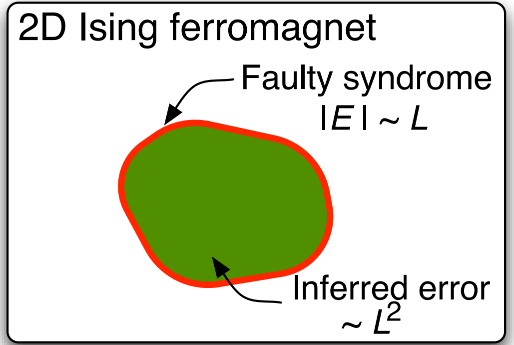

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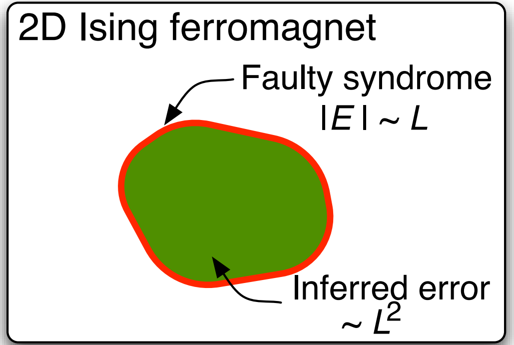

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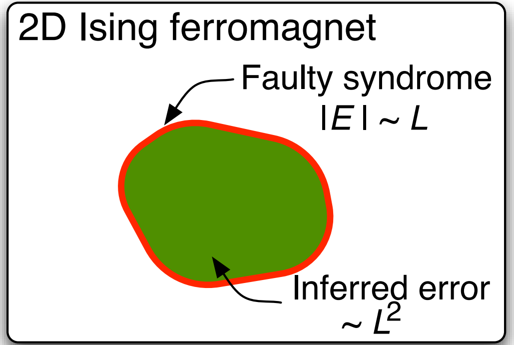

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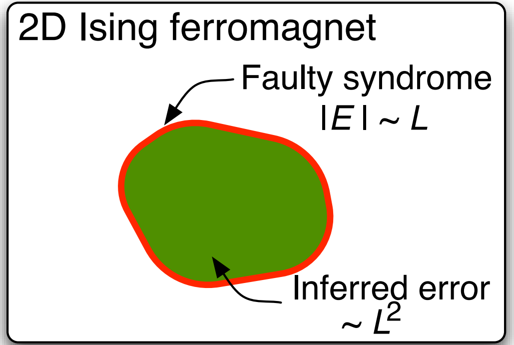

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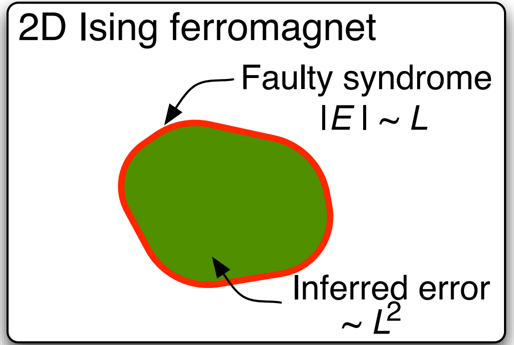

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Bombin's definition

- $\tau =$ Syndrome temperature, how much it's confined.
- $\epsilon =$ Logical error rate.
- $\eta =$ Syndrome error rate.

$$EC_\eta \circ \mathcal{E}_{\tau,\epsilon} \sim \mathcal{E}_{\tau',\epsilon+\delta}$$

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I need to sober up to think about that one!

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- Strict single-shot error correction with a phenomenological noise model is possible using random codes.
  - Derived a lower bound on achievable rate.
  - Upper bound depends on what counts as single-shot.
  - Phenomenological noise model not justified in this setting.
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