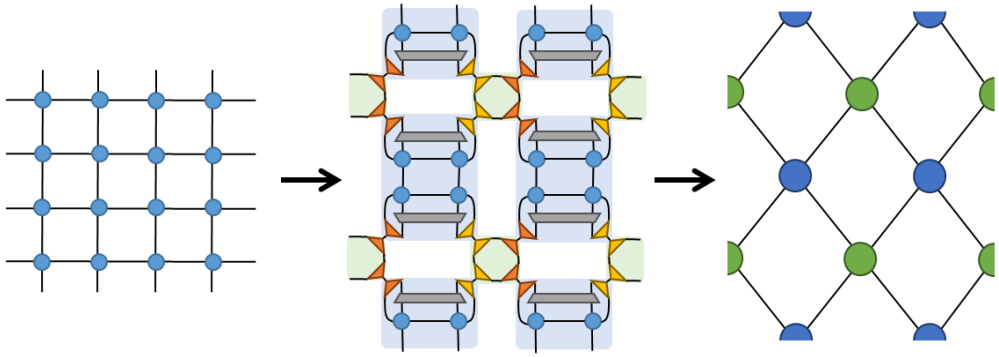
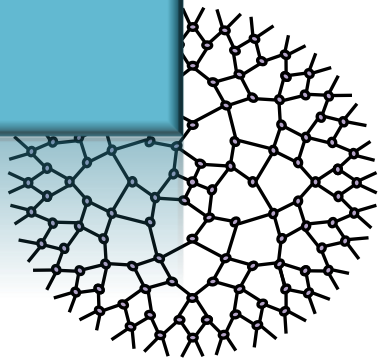


Tensor network renormalization



Guifre Vidal

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

SIMONS FOUNDATION

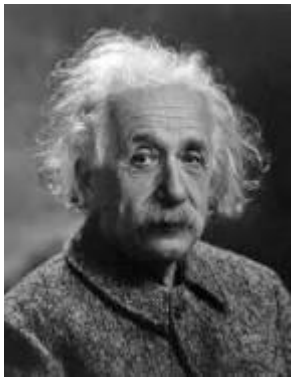
JOHN TEMPLETON FOUNDATION

In collaboration with
GLEN EVENBLY

IQIM Caltech → UC Irvine

Quantum Mechanics 1920-1930

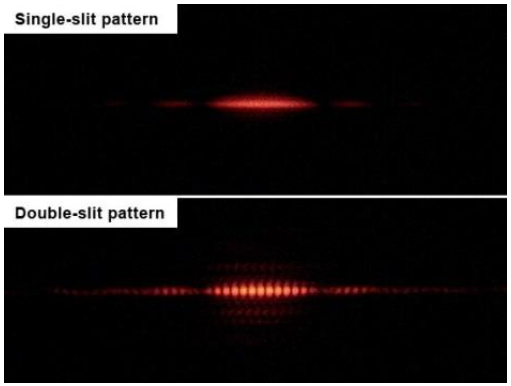
Niels Bohr



Albert Einstein

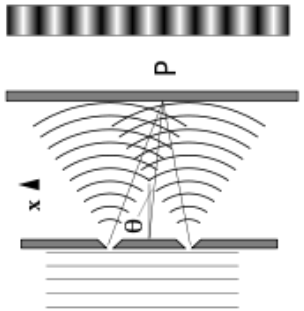
Wolfgang Pauli

Paul Dirac



$$i\hbar \frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

Schrodinger equation



Erwin Schrodinger

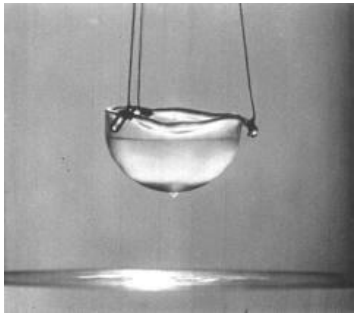
Enrico Fermi

Werner Heisenberg

Richard Feynman

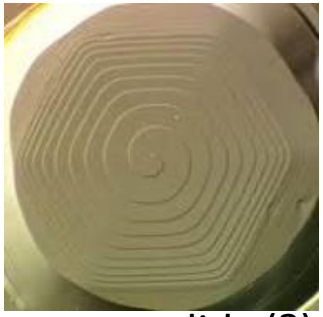
Exotic phases of quantum matter:

Collective quantum many-body phenomena

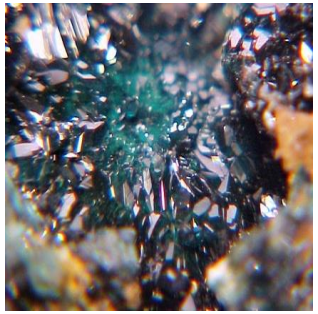


superfluids

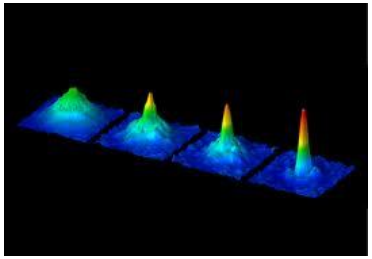
superconductors



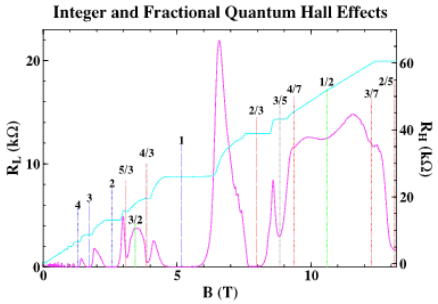
supersolids (?)



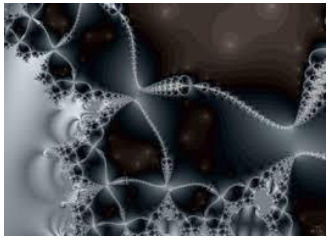
spin liquids



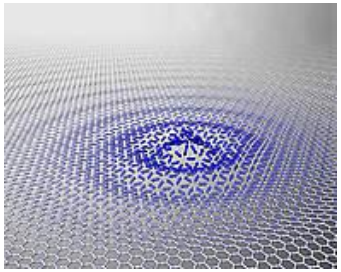
Bose-Einstein condensation



fractional quantum Hall effect

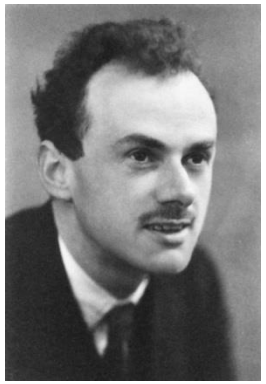


quantum criticality



topological order

There is a problem…



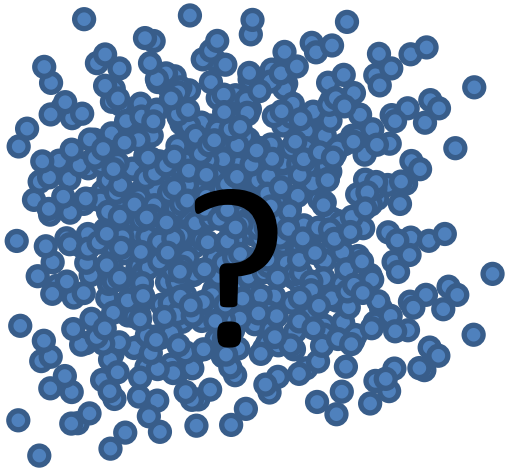
Paul Dirac

"The fundamental laws necessary for the mathematical treatment of a large part of physics and the whole of chemistry are thus completely known,

and the difficulty lies only in the fact that application of these laws leads to equations that are too complex to be solved."

$$i\hbar \frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

Schrodinger equation



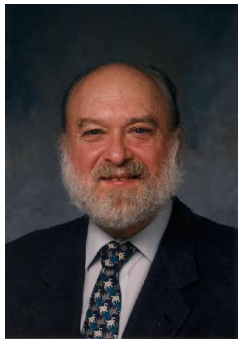
Emergence:

long-distance physics is often radically different from short-distance physics

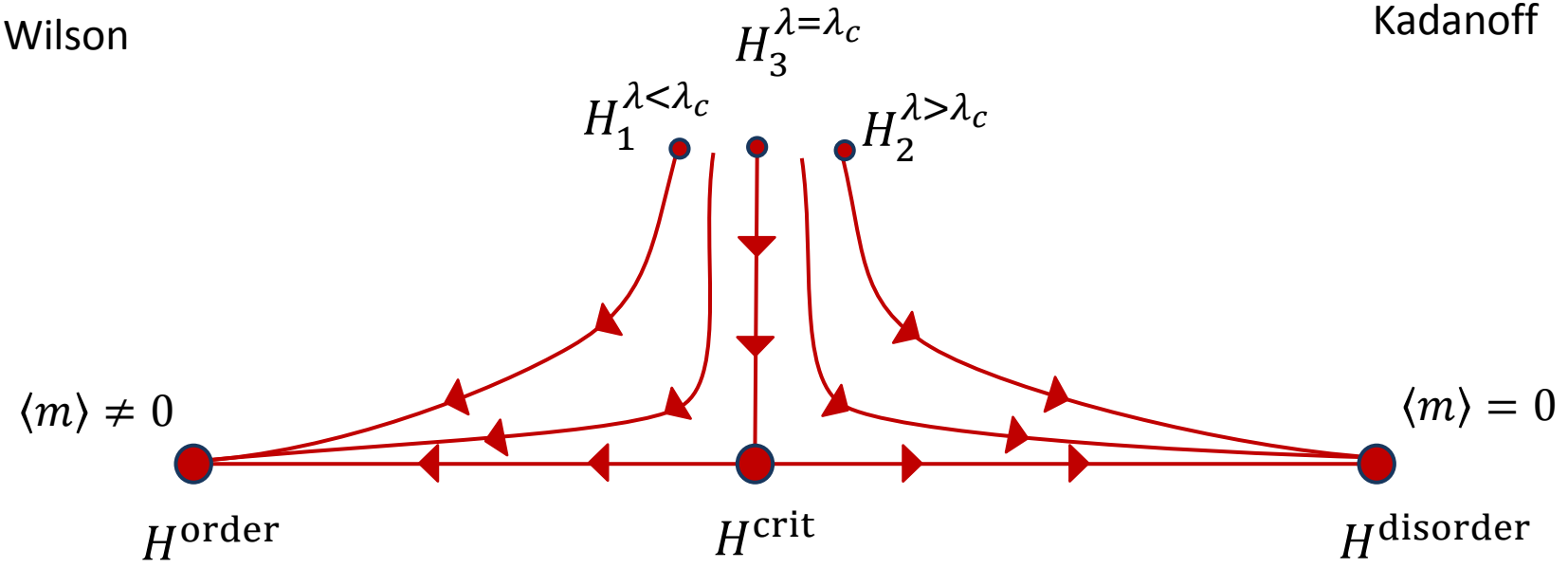
… and there is a solution: the Renormalization Group

Kenneth Wilson

Renormalization group flow:



Leo Kadanoff

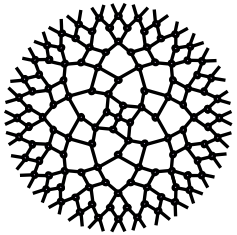


… and there is a solution: the Renormalization Group

Nice! But how do we do this in practice?

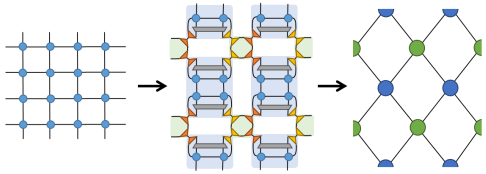
Outline of this talk

- entanglement renormalization (old stuff)



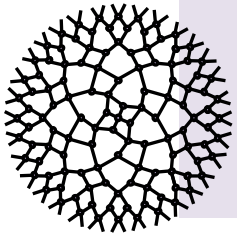
Real space RG transformation for
→ ground state wave-functions
→ Hamiltonians

- tensor network renormalization (new stuff)



- Real space RG of Euclidean path integral — arXiv:1412.0732
- TNR → MERA (+ thermal states!) — arXiv:1501.xxx
- Theory of minimal updates — arXiv:1501.yyy
- Conformal transformations — arXiv:1501.zzz

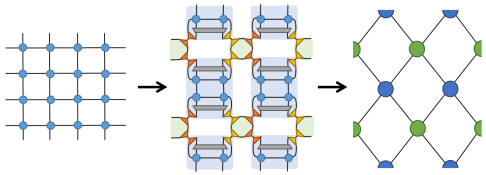
Outline



- entanglement renormalization (old stuff)

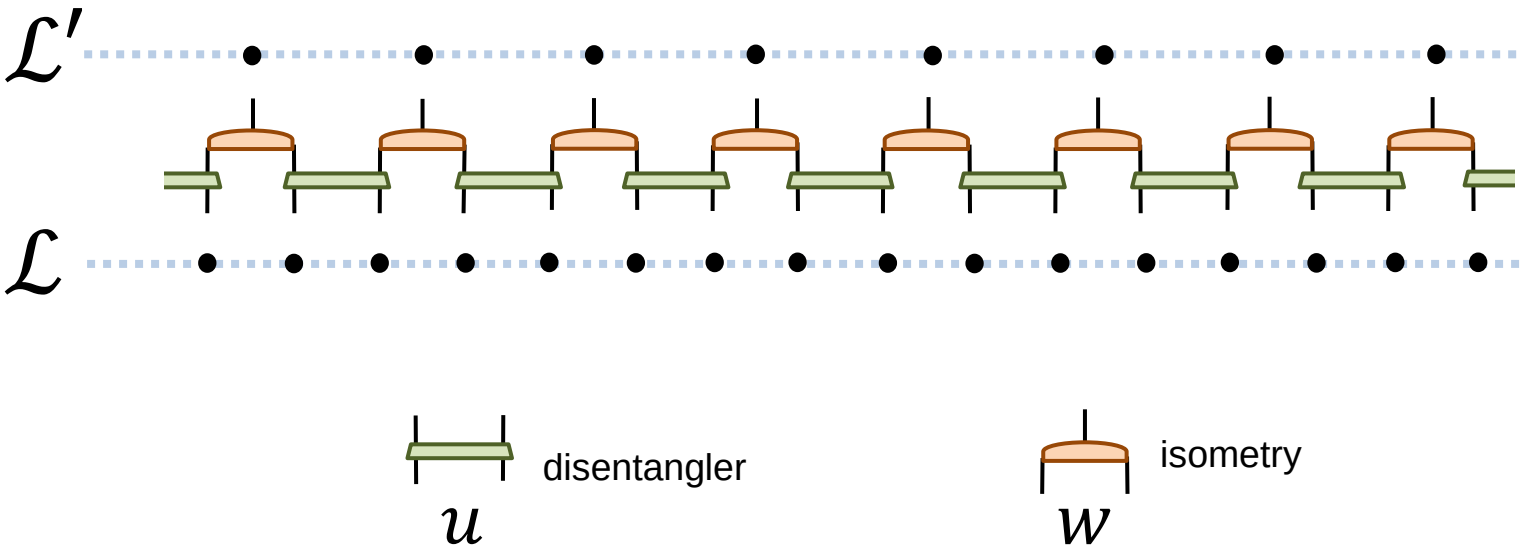
  Real space RG transformation for
  → ground state wave-functions
  → Hamiltonians

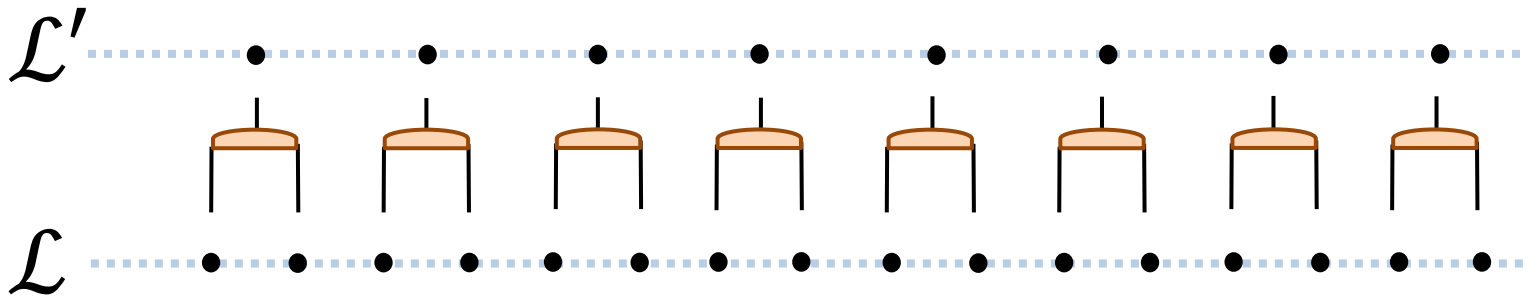
- tensor network renormalization (new stuff)



- Real space RG of Euclidean path integral — arXiv:1412.0732
- TNR → MERA (+ thermal states!) — arXiv:1501.xxx
- Theory of minimal updates — arXiv:1501.yyy
- Conformal transformations — arXiv:1501.zzz

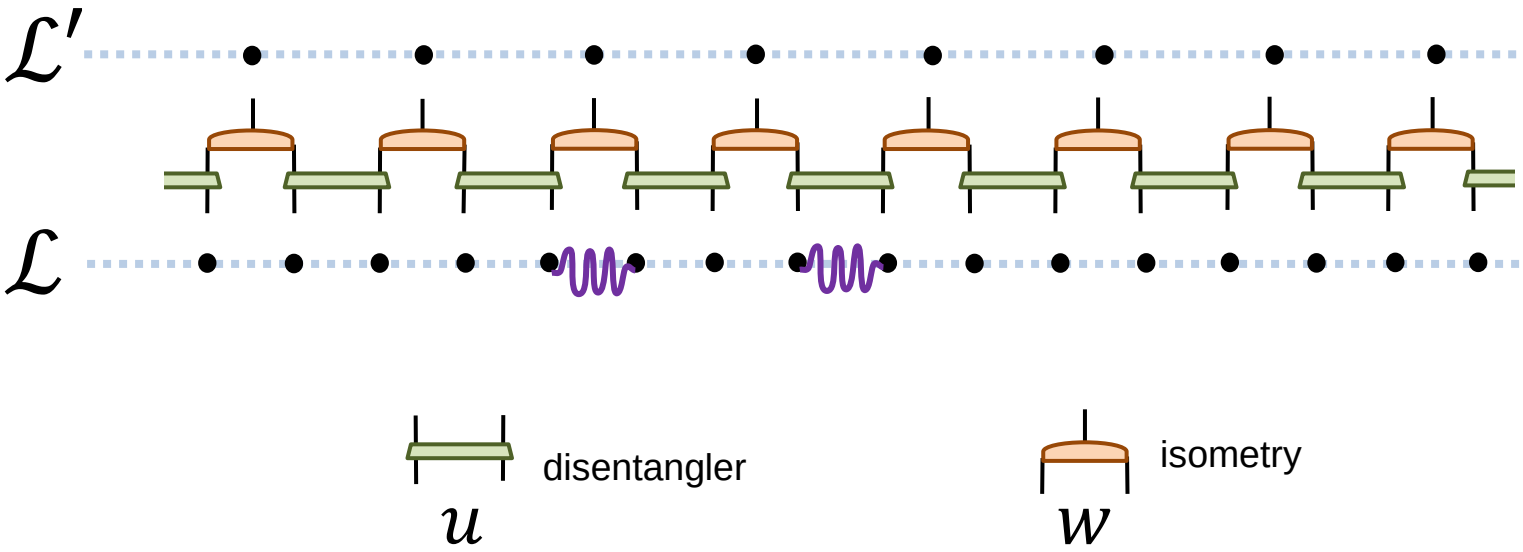
Entanglement renormalization (old stuff)



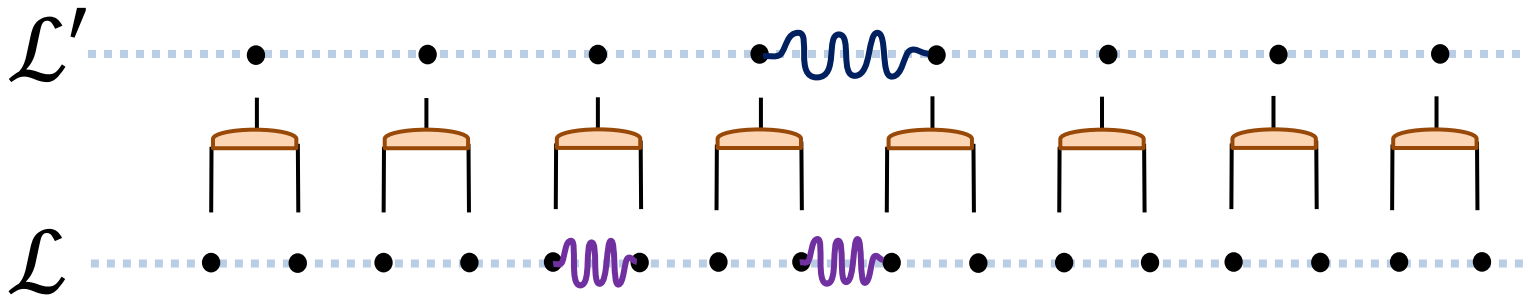


Entanglement renormalization (old stuff)

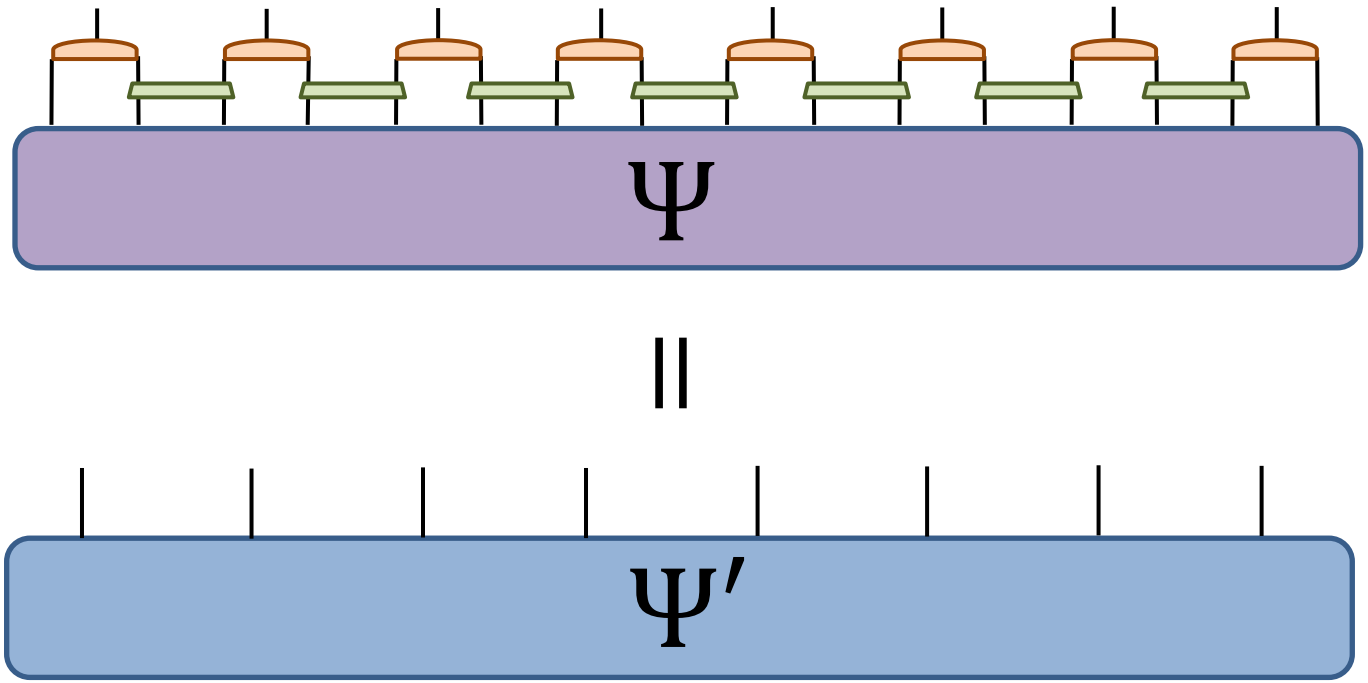
- removal of all short-range entanglement



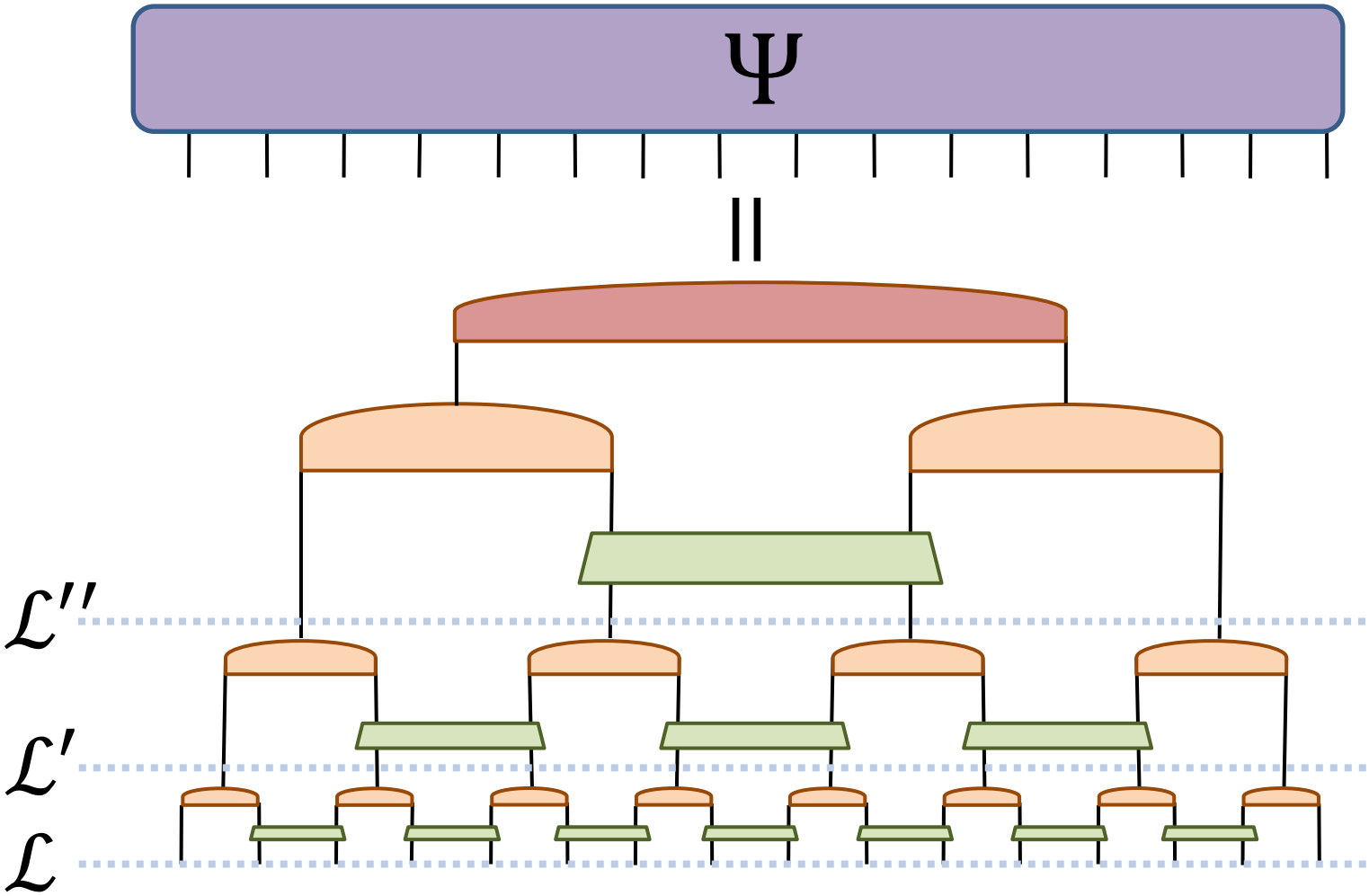
- some short-range entanglement is not removed



RG transformation on ground state wave-functions



RG transformation on ground state wave-functions

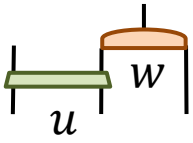


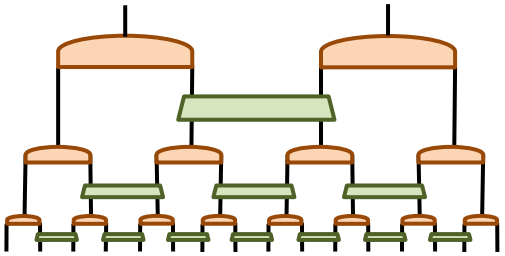
Multi-scale entanglement renormalization ansatz (MERA)

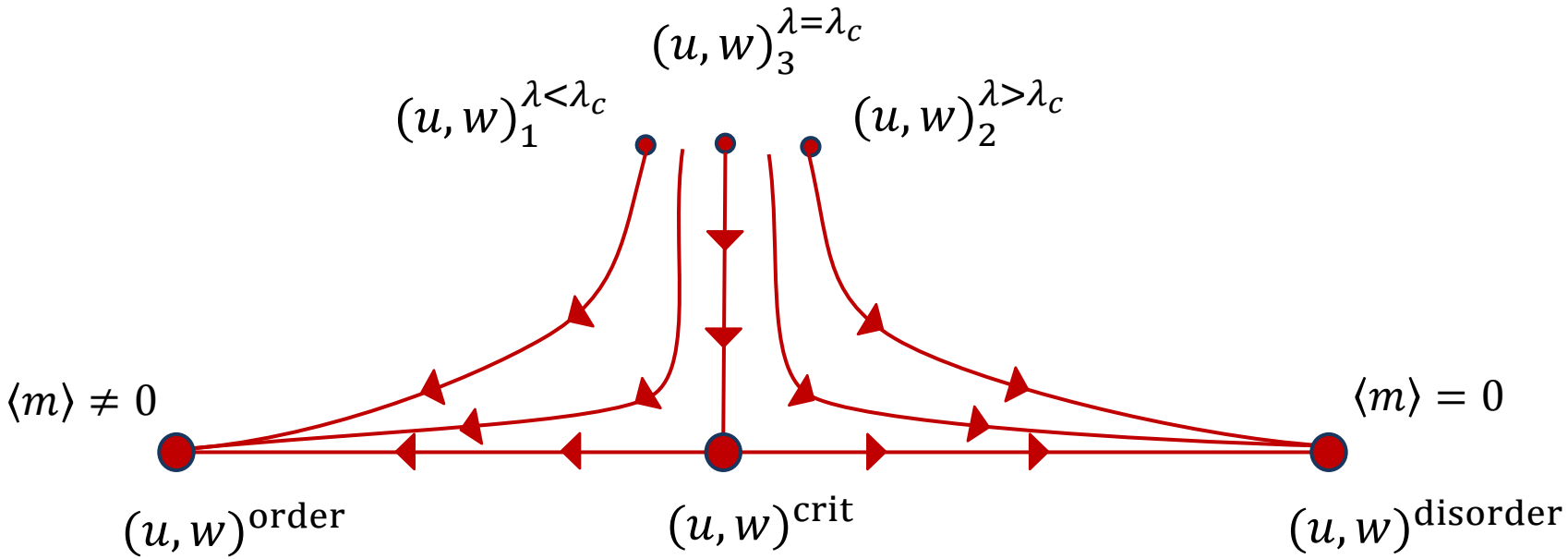
- topological order
- quantum criticality
- holography

RG flow in the space of wave-functions,

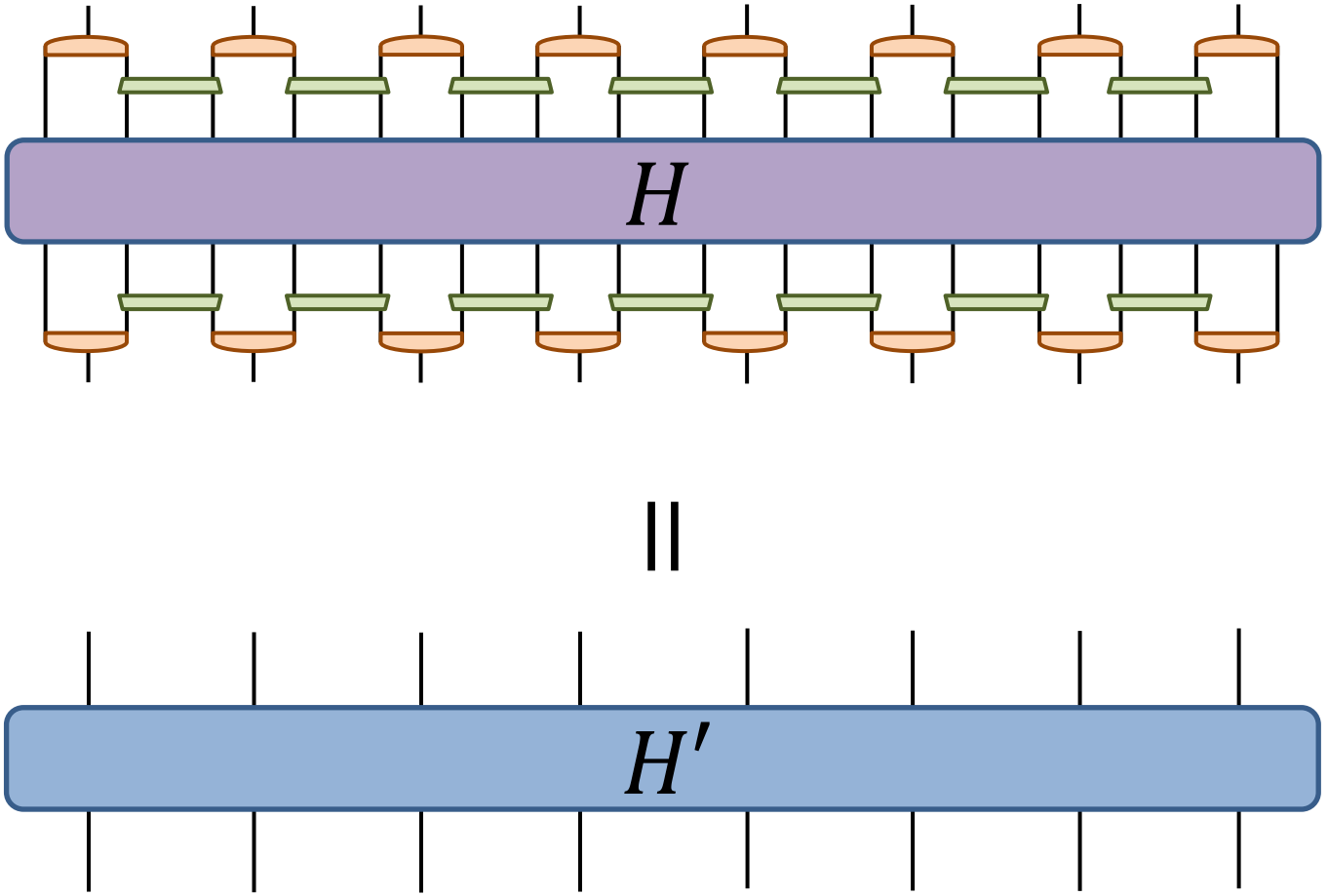
as parameterized by tensors u, w





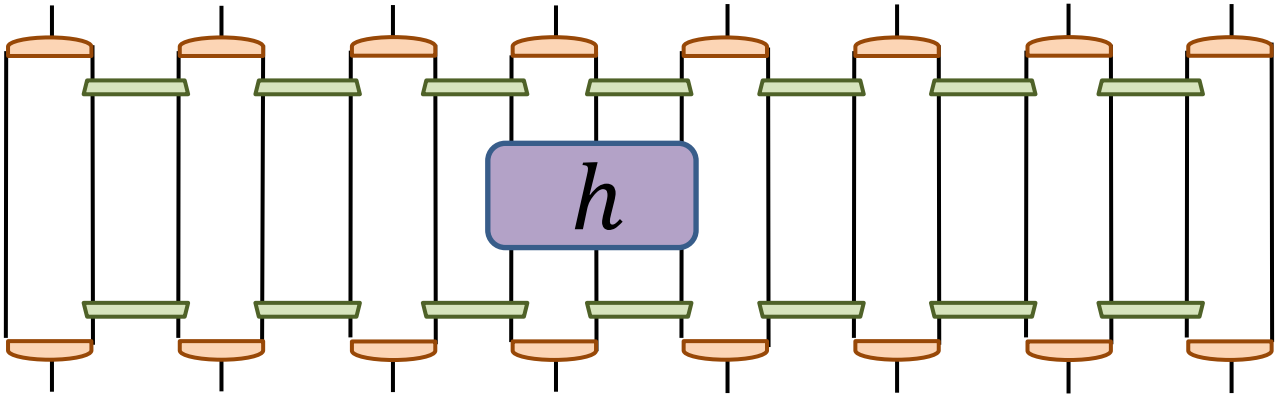


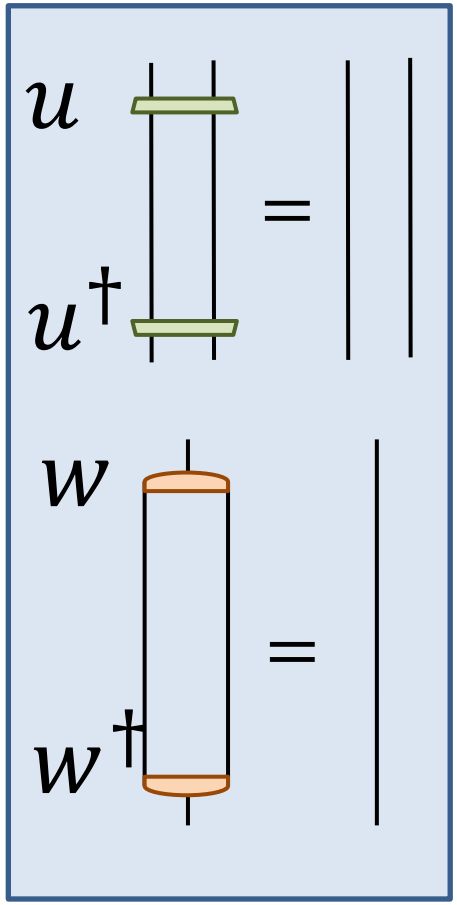
RG transformation on Hamiltonians



RG transformation on Hamiltonians

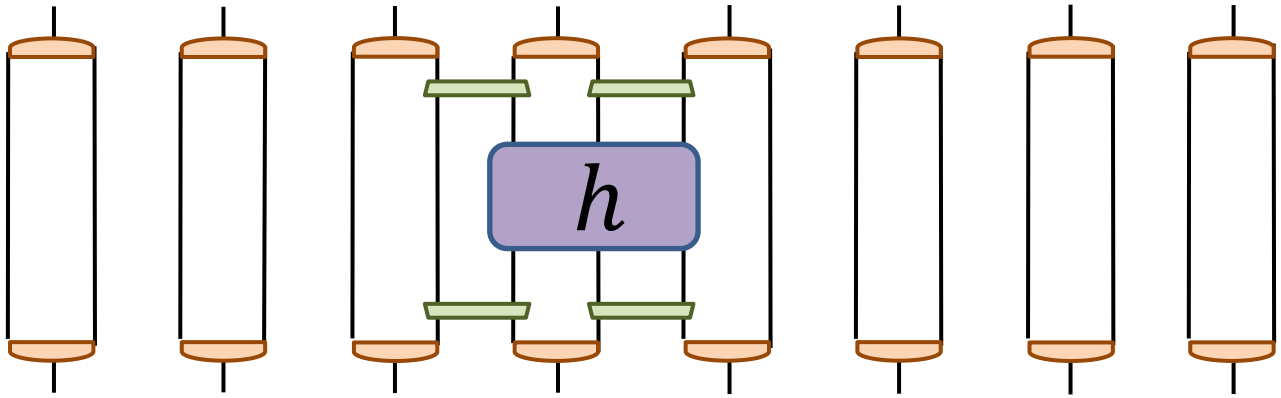
Local Hamiltonian $H = \sum_i h_i$

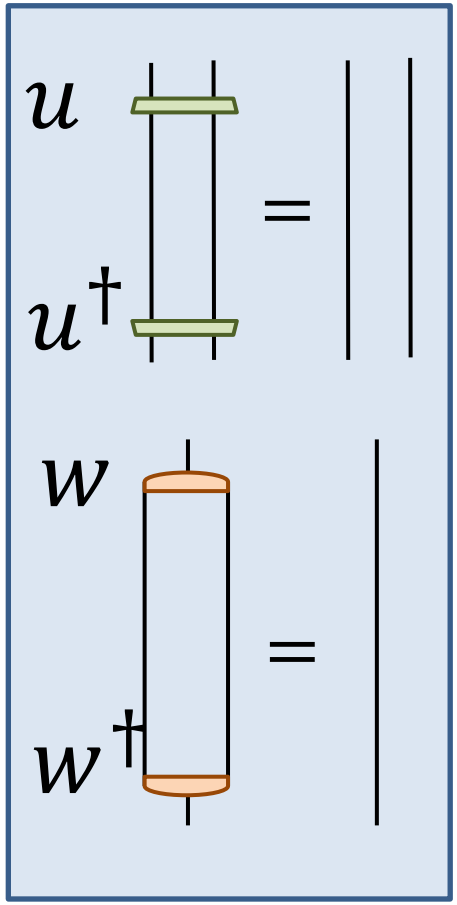




RG transformation on Hamiltonians

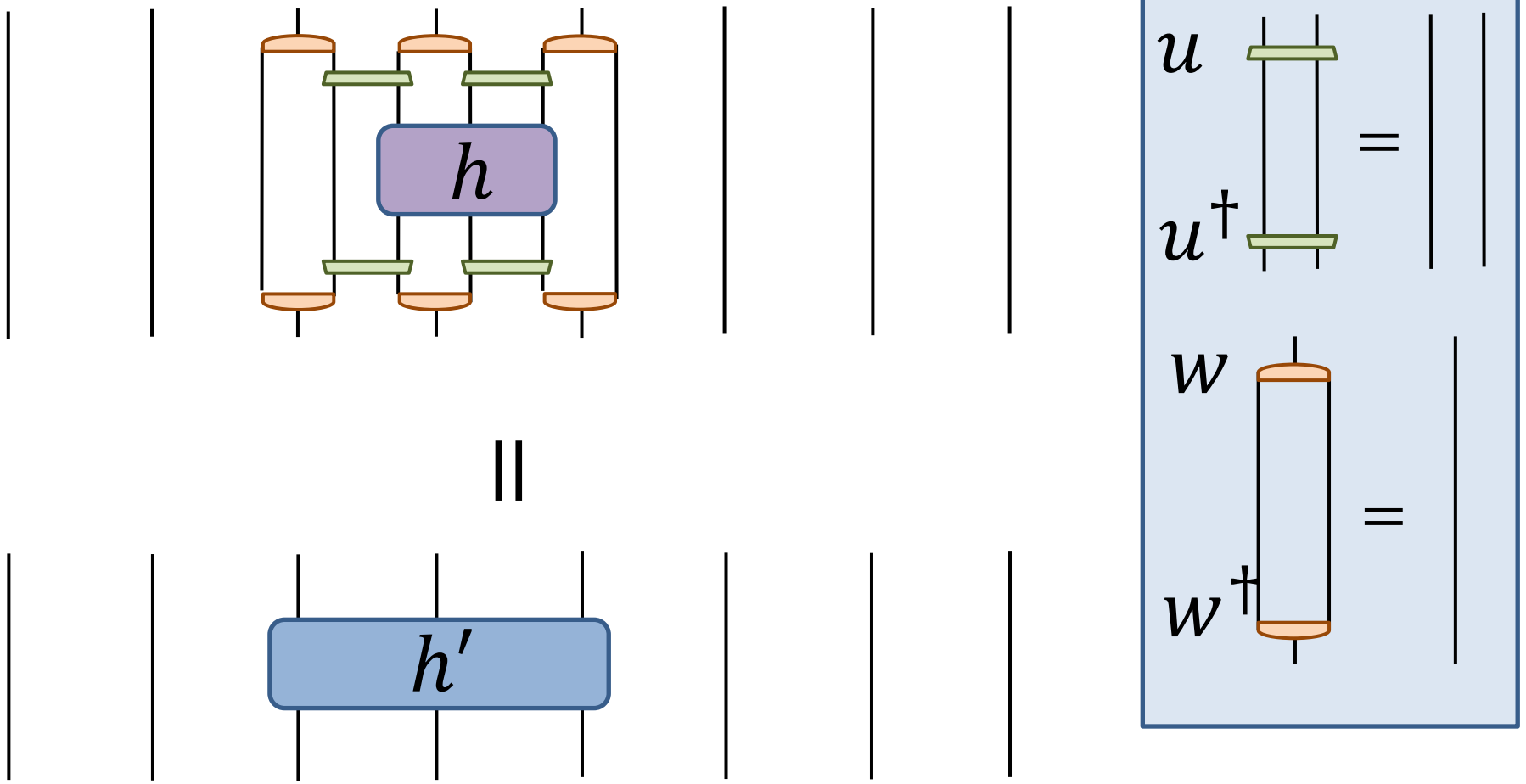
Local Hamiltonian $H = \sum_i h_i$





RG transformation on Hamiltonians

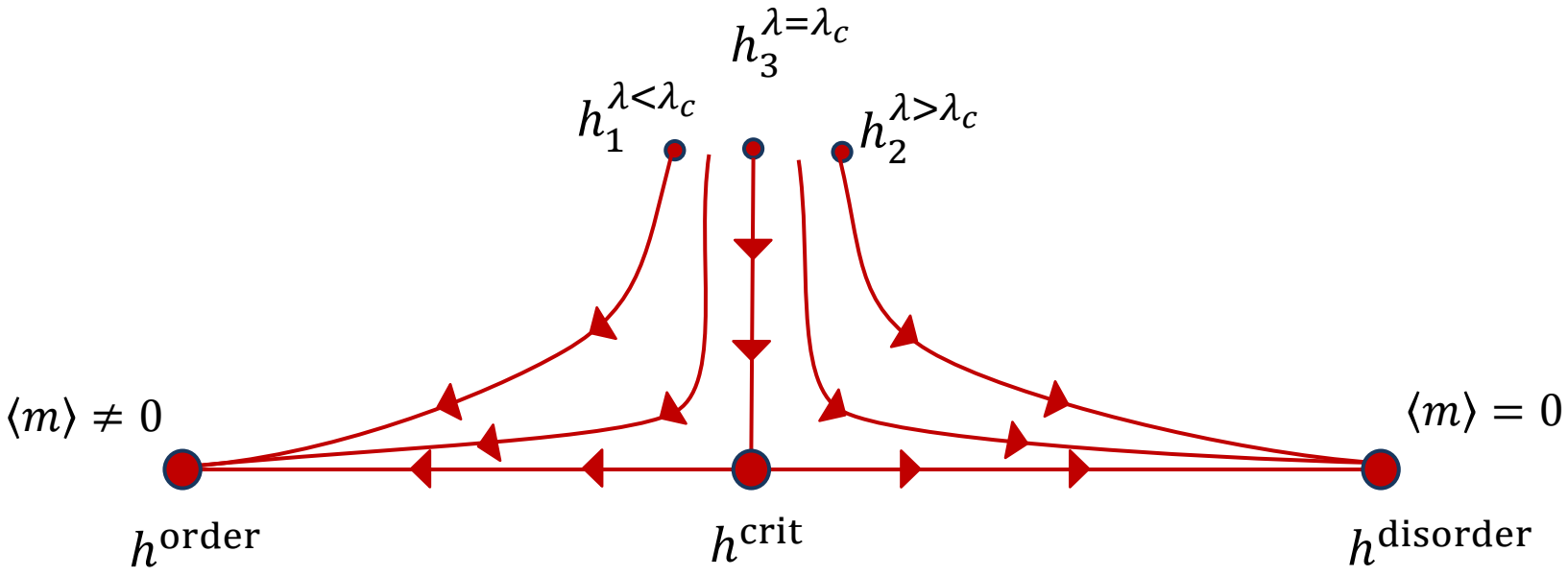
Local Hamiltonian $H = \sum_i h_i$



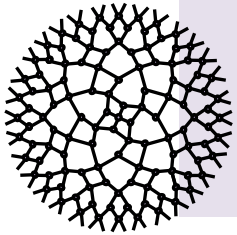
RG flow in the space of local Hamiltonias,

$$H = \sum_i h_i$$

as parameterized by a strictly local Hamiltonian term h



- entanglement renormalization (old stuff)



Real space RG transformation for
→ ground state wave-functions
→ Hamiltonians

MANY OPEN QUESTIONS

RG on Euclidean path integral
(Hamiltonian → Lagrangian)

MERA for thermal states?
Classical partition functions?

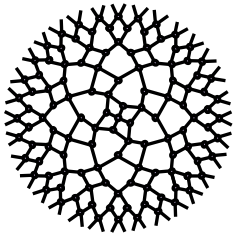
When is MERA
a good approximation?

Theory of minimal updates
(impurities, boundaries, etc)

Better algorithms?
(→ branching, 2D)

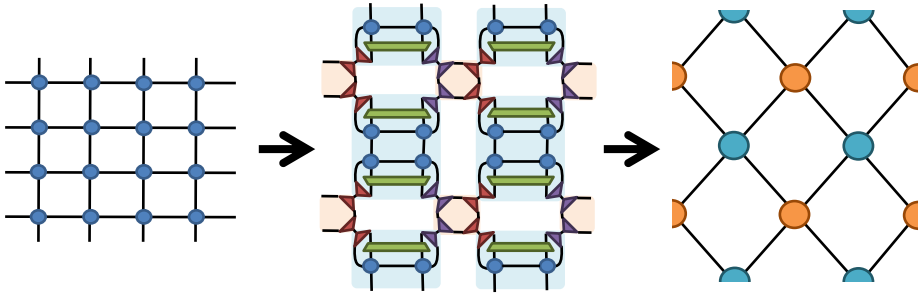
Outline

- entanglement renormalization (old stuff)



Real space RG transformation for
→ ground state wave-functions
→ Hamiltonians

- tensor network renormalization (new stuff)
  - Real space RG of Euclidean path integral — arXiv:1412.0732
  - TNR → MERA (+ thermal states!) — arXiv:1501.xxx
  - Theory of minimal updates — arXiv:1501.yyy
  - Conformal transformations — arXiv:1501.zzz



Tensor network renormalization (new stuff)

local Hamiltonian

ground state

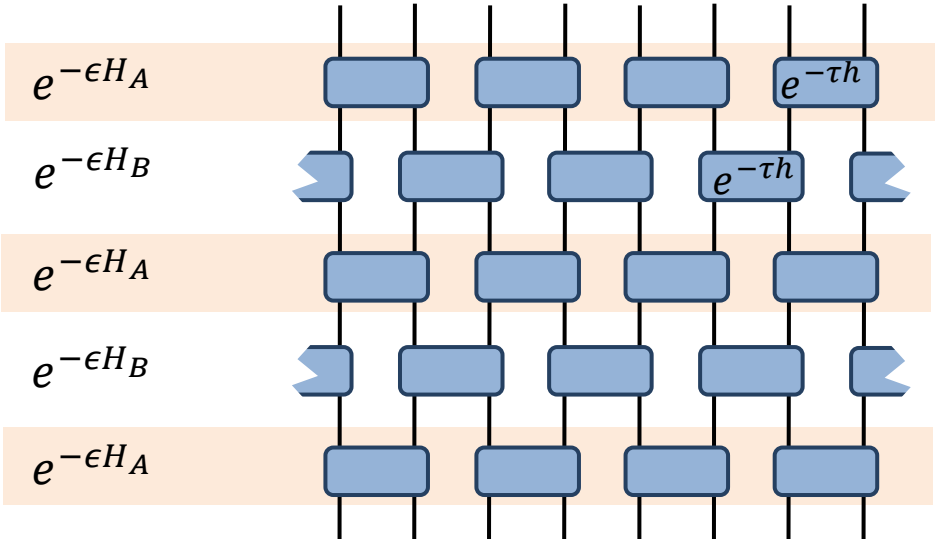
$$H \longrightarrow |\Psi\rangle \qquad |\Psi\rangle = \lim_{\beta\to\infty} \frac{e^{-\beta H}|\phi_0\rangle}{||e^{-\beta H}|\phi_0\rangle||}$$

Suzuki-Trotter decomposition

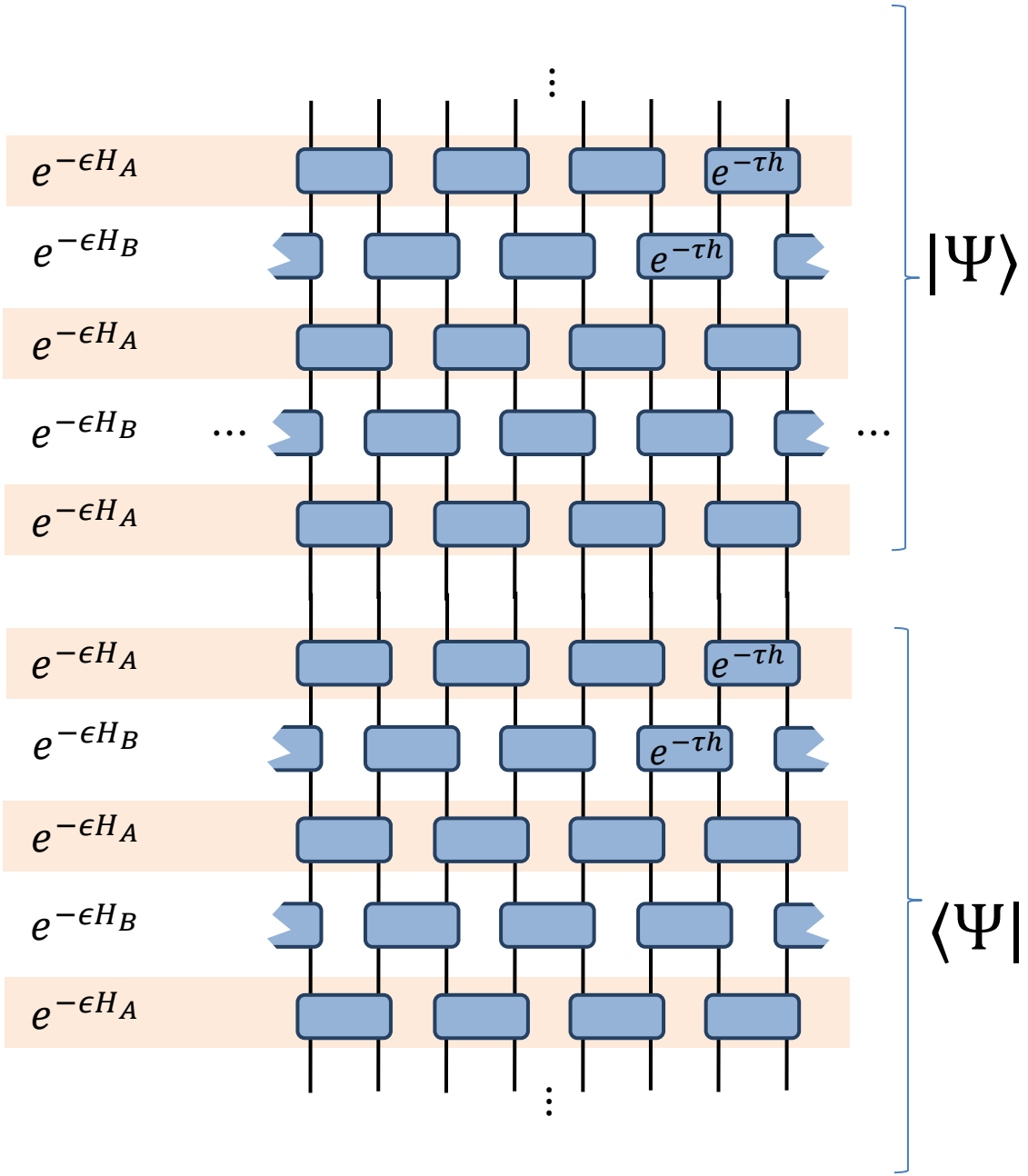
$$e^{-\beta H} \approx e^{-\epsilon H_A} e^{-\epsilon H_B} e^{-\epsilon H_A} e^{-\epsilon H_B} \ldots$$

$$H_A = \sum_{i\ odd} h_{i,i+1}$$

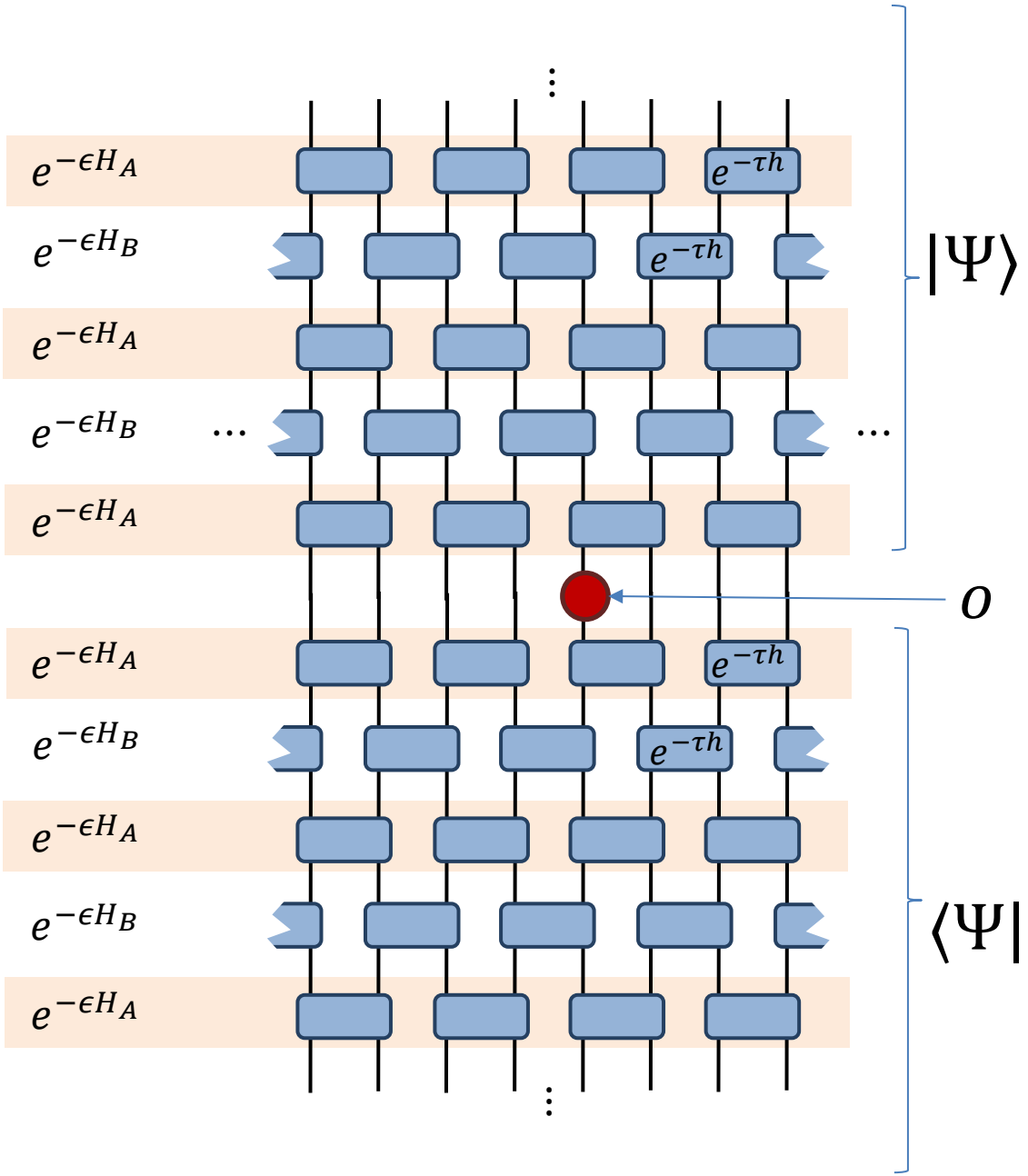
$$H_B = \sum_{i\ even} h_{i,i+1}$$

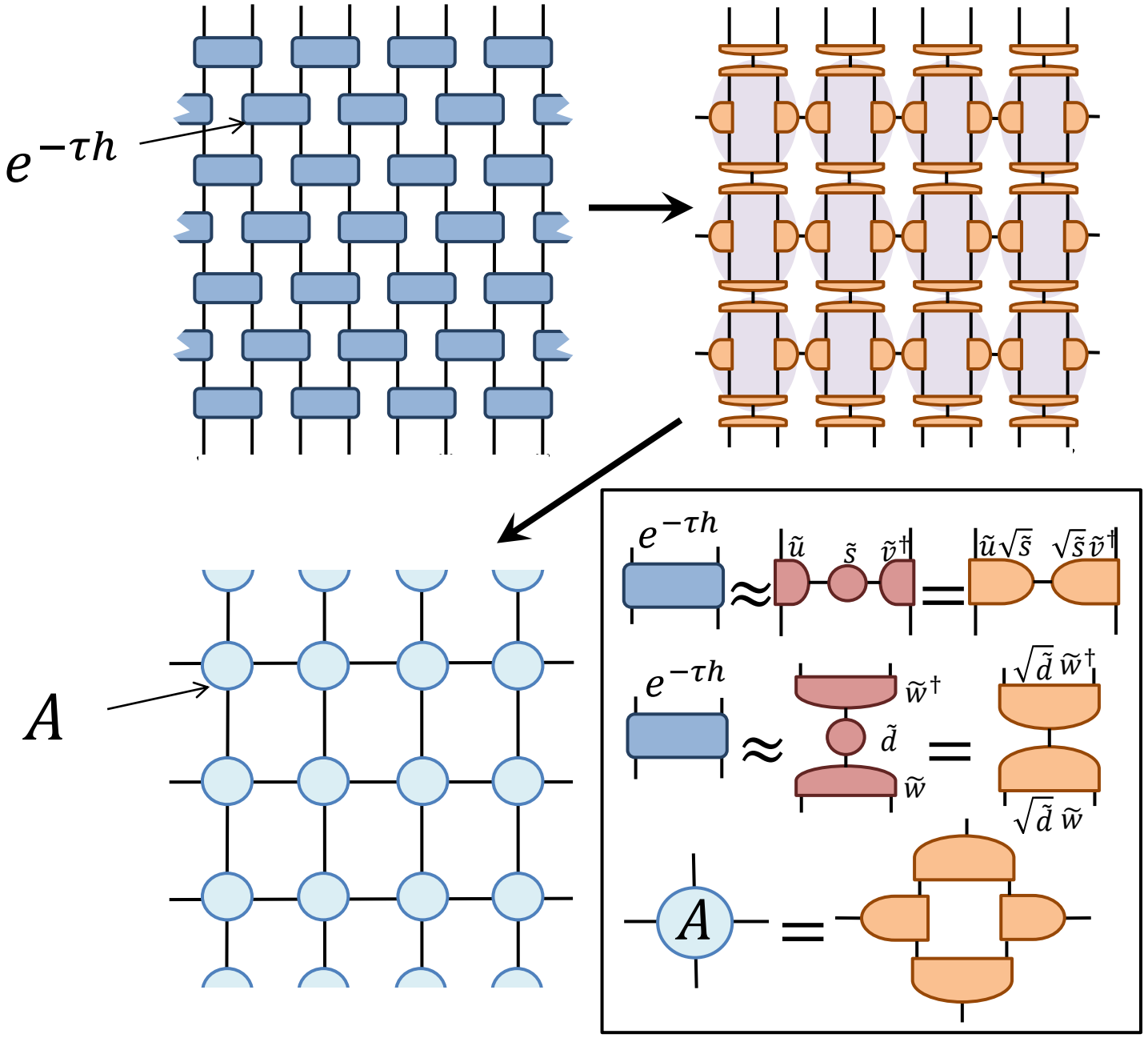


$\langle\Psi|\Psi\rangle \propto$

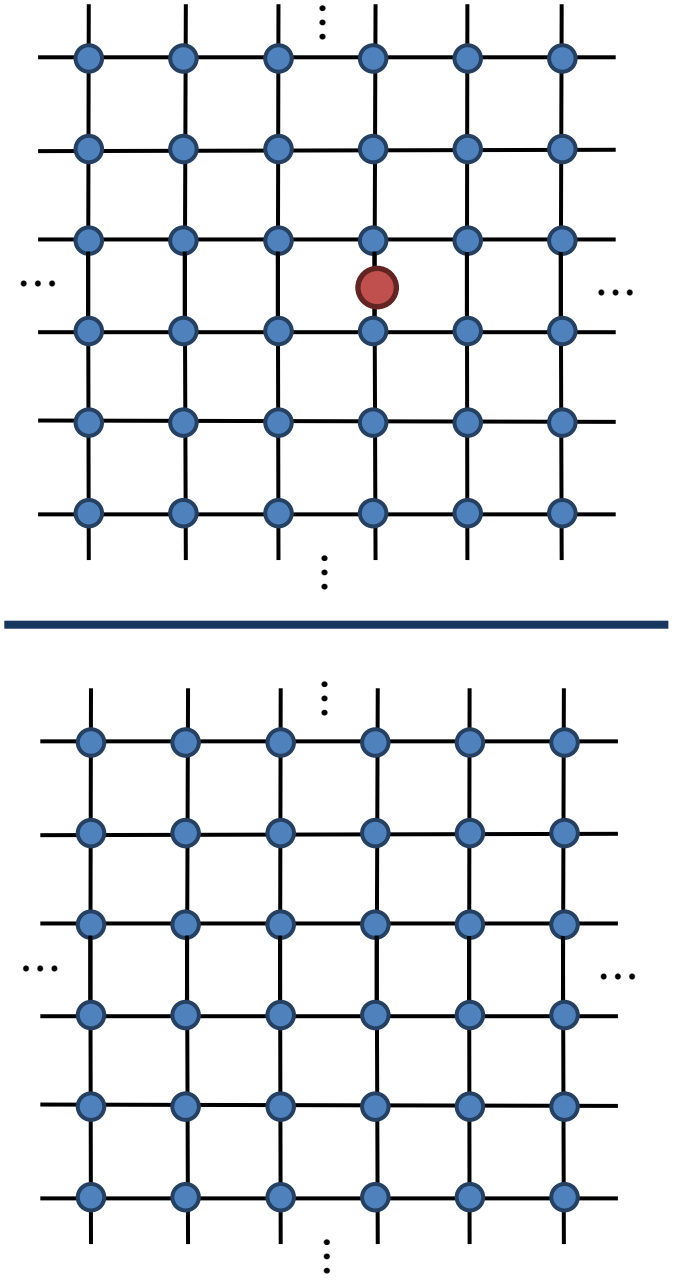


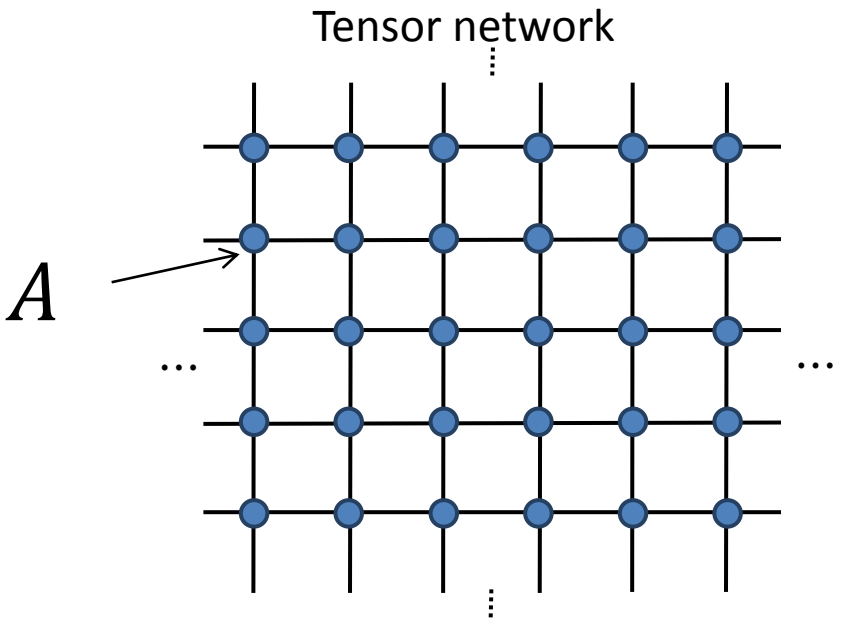
$$\langle \Psi | o | \Psi \rangle \propto$$




$e^{-\tau h}$
A
$e^{-\tau h}$
$\approx$
$\tilde{u}$
$\tilde{s}$
$\tilde{v}^\dagger$
$=$
$\tilde{u}\sqrt{\tilde{s}}$
$\sqrt{\tilde{s}}\tilde{v}^\dagger$
$e^{-\tau h}$
$\approx$
$\tilde{w}^\dagger$
$\tilde{d}$
$\tilde{w}$
$=$
$\sqrt{\tilde{d}}\,\tilde{w}^\dagger$
$\sqrt{\tilde{d}}\,\tilde{w}$
A
$=$

$$\frac{\langle \Psi | o | \Psi \rangle}{\langle \Psi | \Psi \rangle} =$$



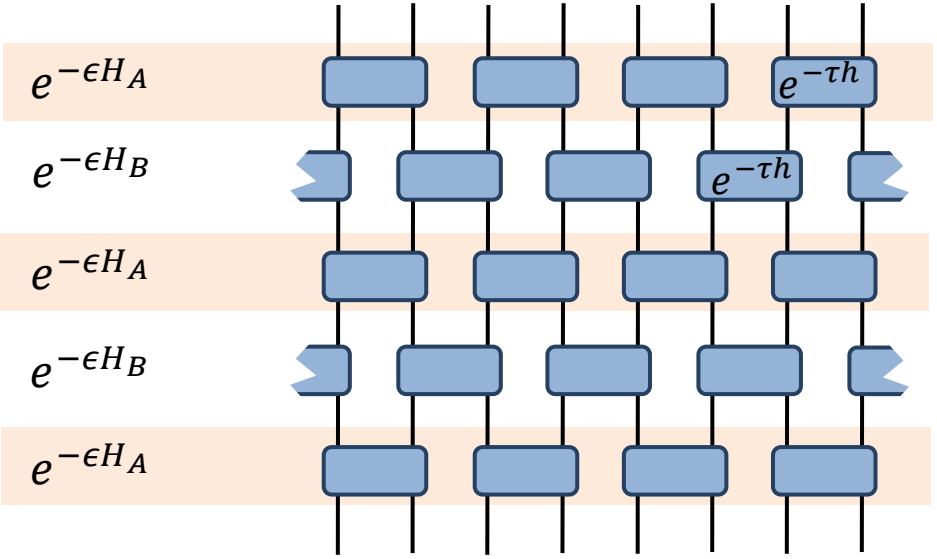


Euclidean path integral /
Euclidean time evolution operator

Classical partition function

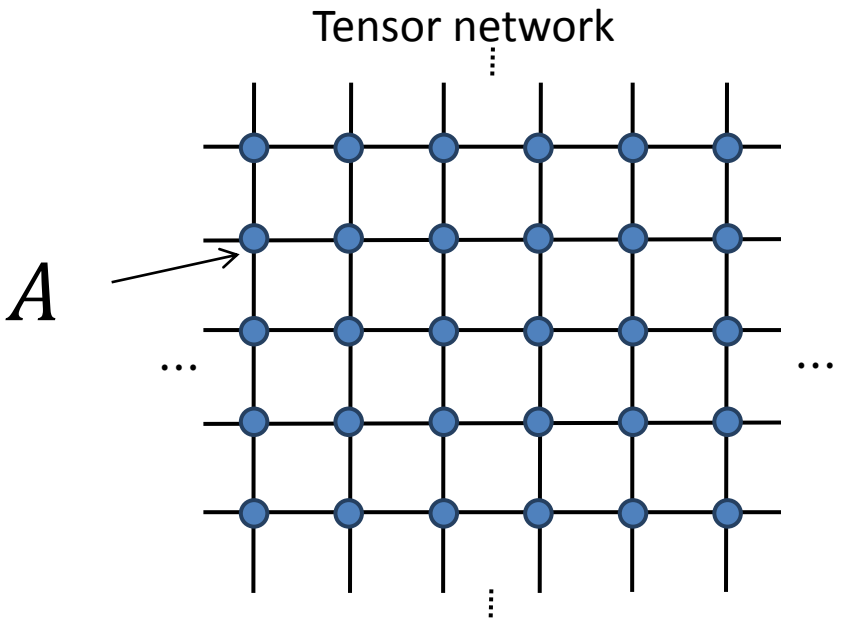
Overlap between two PEPS

Monte Carlo sampling



Plan: sample important amplitudes!

- sampling error
- (sign problem!)



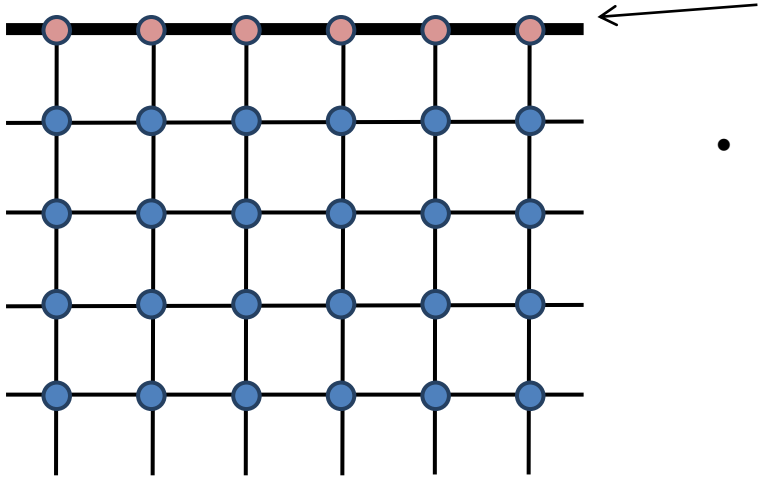
Euclidean path integral /
Euclidean time evolution operator

Classical partition function

Overlap between two PEPS

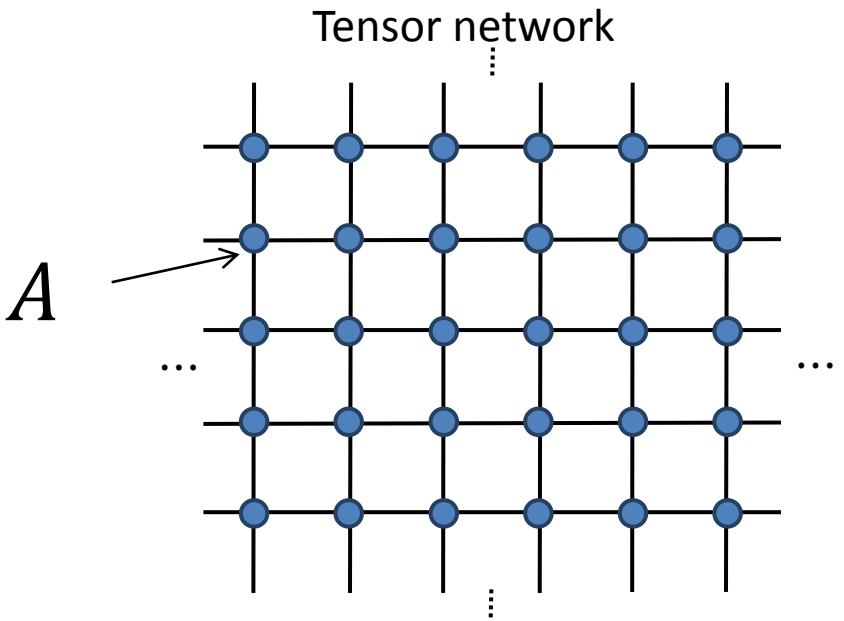
Tensor network techniques (I)

Plan: sum over all amplitudes



boundary MPS (or CTM)

- poor approximation at/near criticality



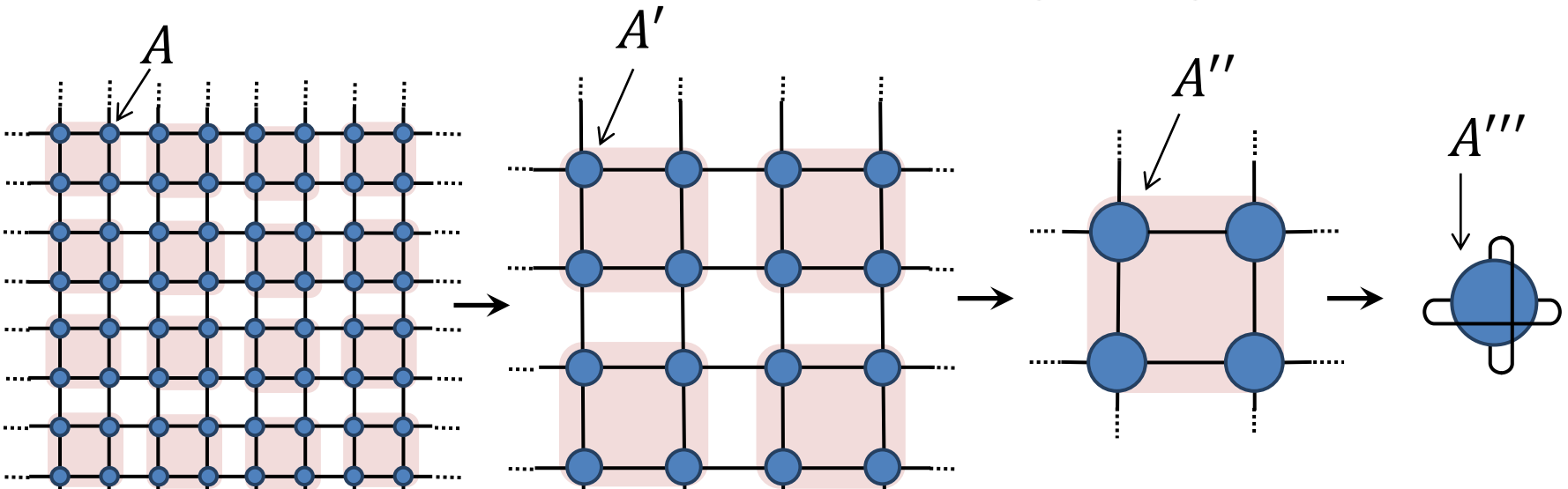
Euclidean path integral /
Euclidean time evolution operator

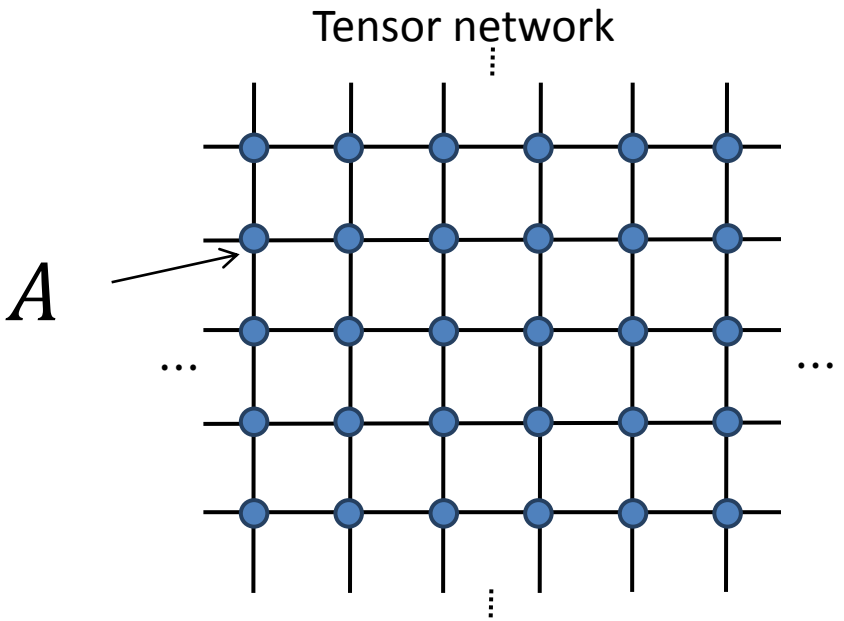
Classical partition function

Overlap between two PEPS

Tensor network techniques (II)

Plan: sum over all amplitudes by coarse-grainning the tensor network





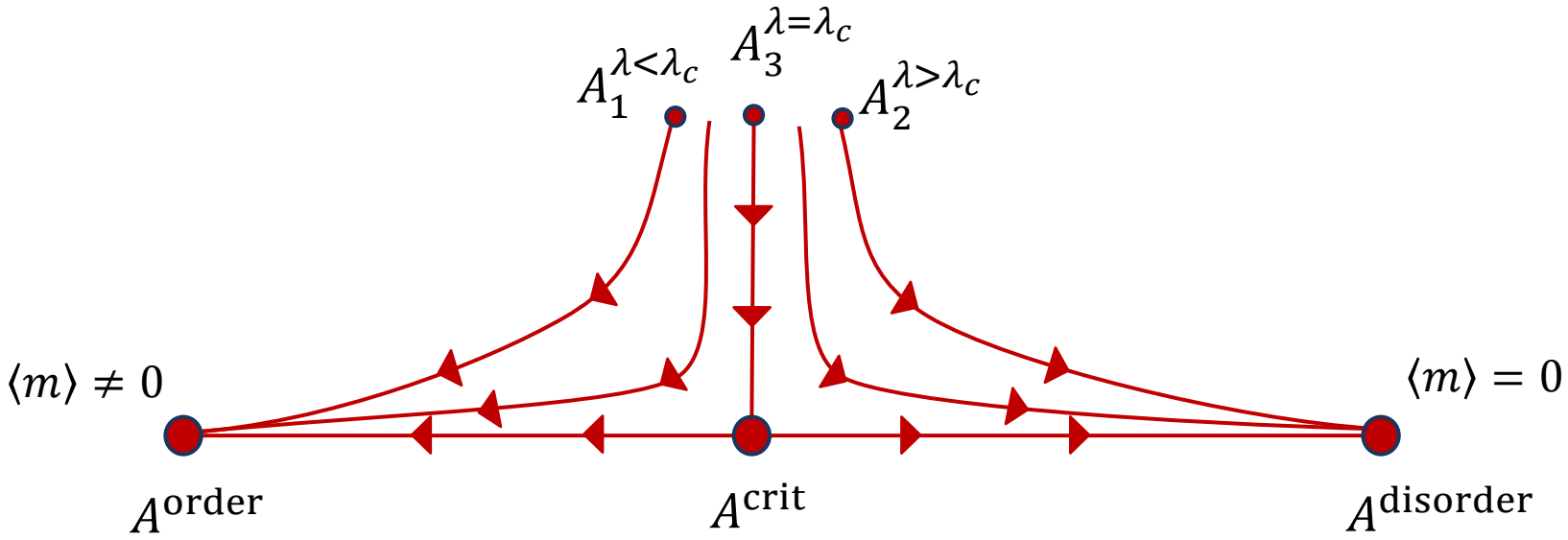
Euclidean path integral /
Euclidean time evolution operator

Classical partition function

Overlap between two PEPS

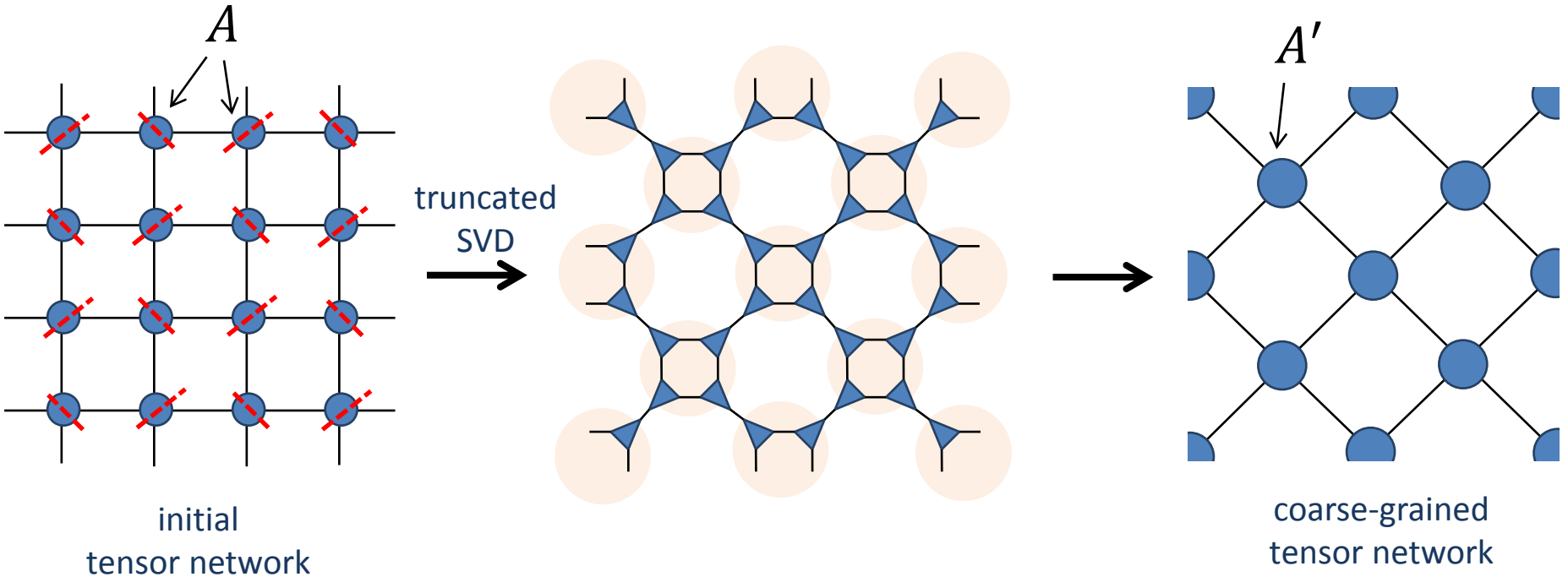
Can we define an RG flow in the space of tensors A ?

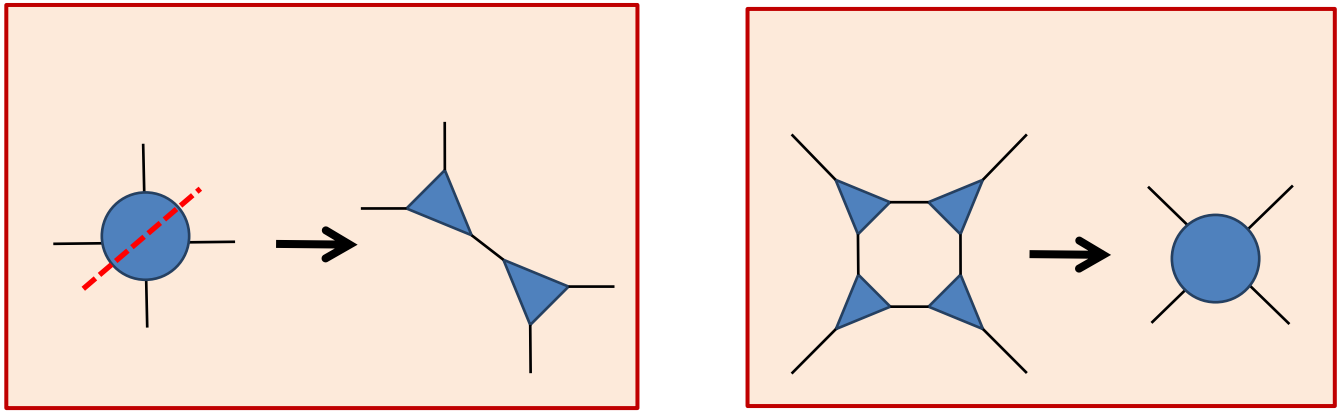
$$A \to A' \to A'' \to \cdots$$



Tensor Renormalization Group (TRG)

(Levin, Nave, 2006)





Tensor Renormalization Group (TRG)

(Levin, Nave, 2006)

Accurate results for gapped 1D systems

Many improvements and generalizations:

- Second Renormalization Group (SRG) (Xie, Jiang, Weng, Xiang, 2008)
- Tensor Entanglement Filtering Renormalization (TEFR) (Gu, Wen, 2009)
- Higher Order Tensor Renormalization Group (HOTRG) (Xie, Chen, Qin, Zhu, Yang, Xiang, 2012)

+ many more…

Improved accuracy, extensions to 2D systems (!?), etc

However, TRG fails to remove some of the short-range correlations.

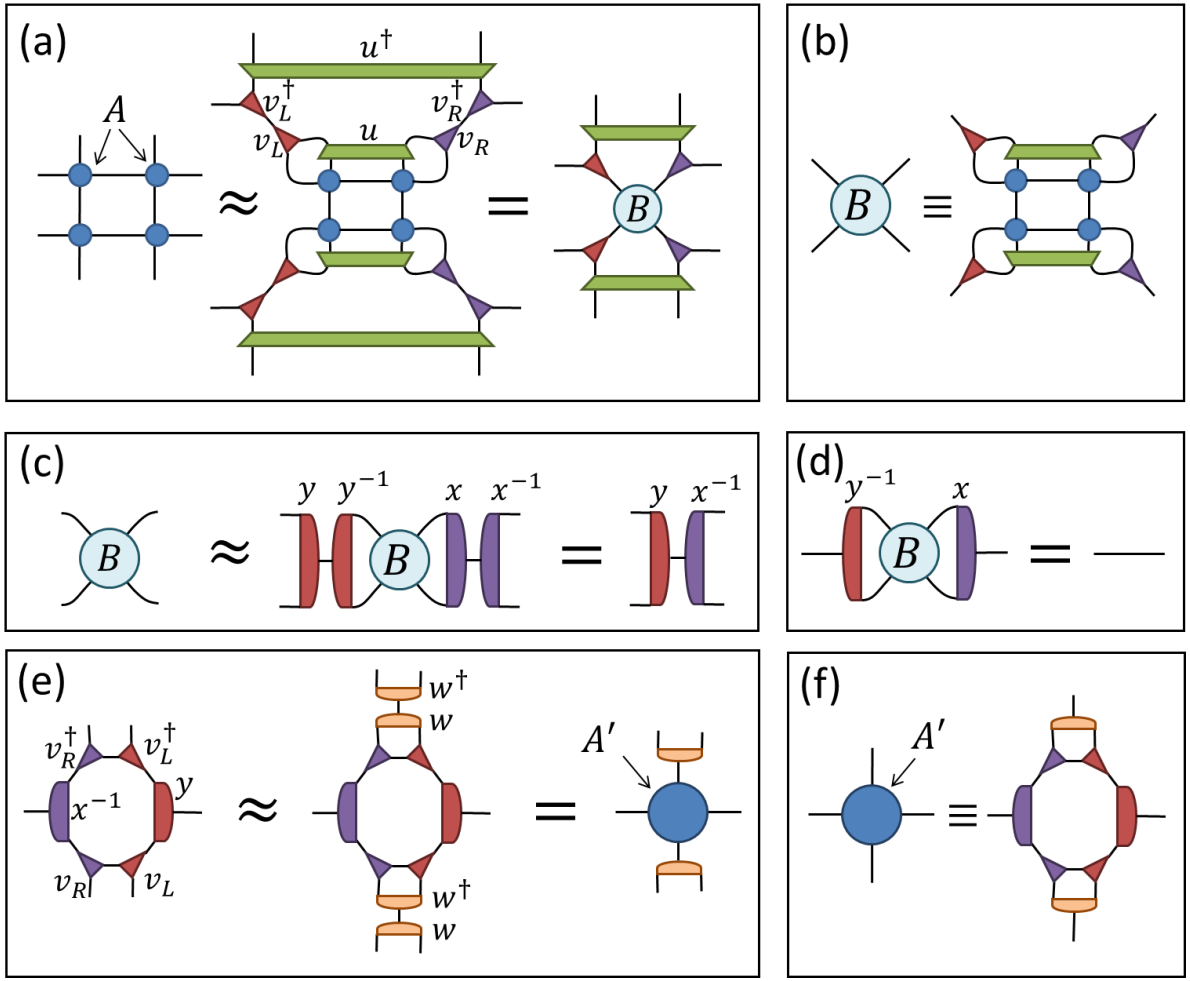
- Not a proper RG flow (wrong structure of fixed points)
- Computationally not sustainable at quantum criticality

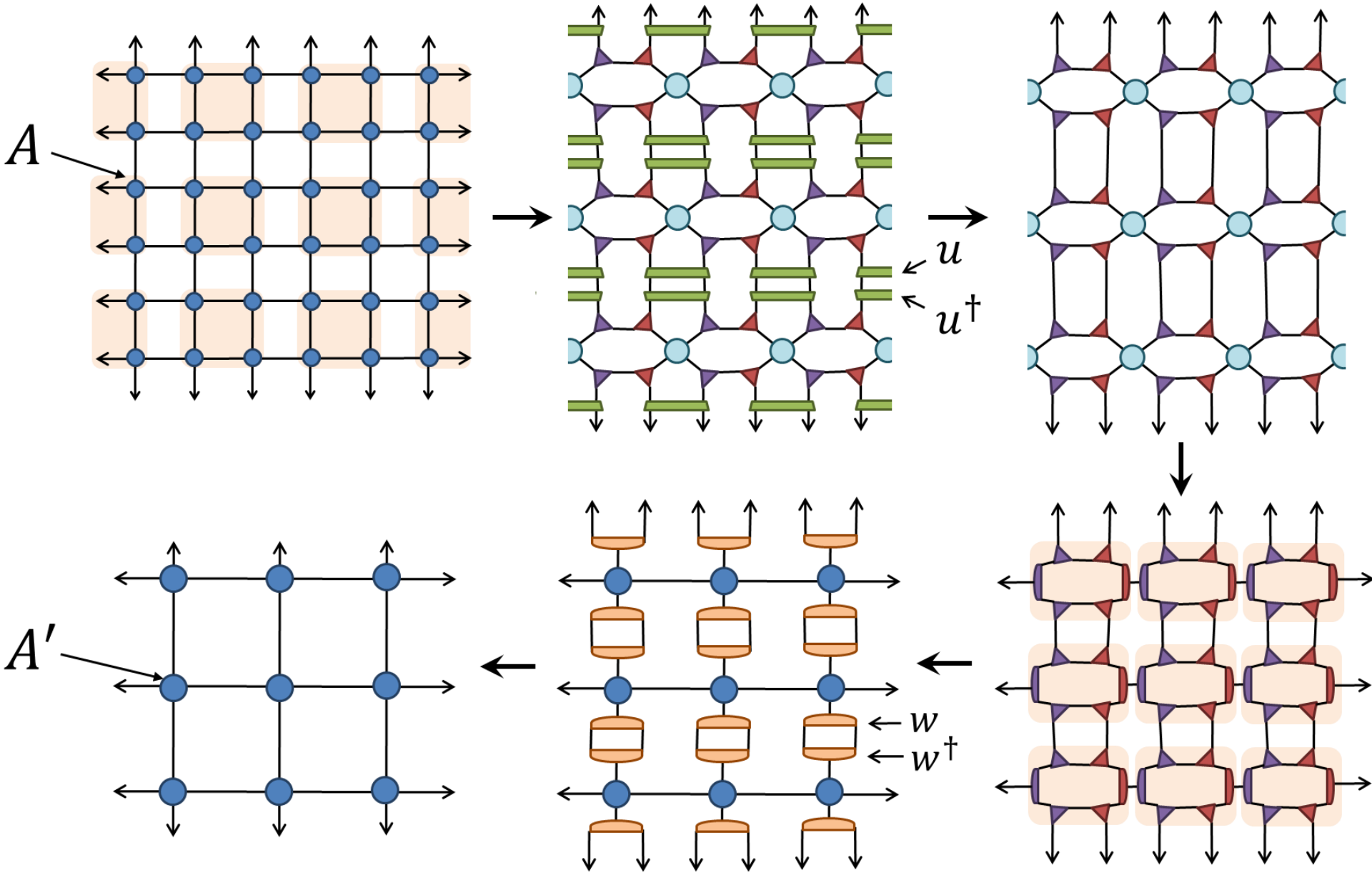
Tensor Network Renormalization

(Evenbly, Vidal, 2014)

Introduce disentanglers to remove all short-range correlations.

- Proper RG flow (correct structure of fixed points)
- Works near/at quantum criticality
- Can be generalized to $D > 1$ dimensions

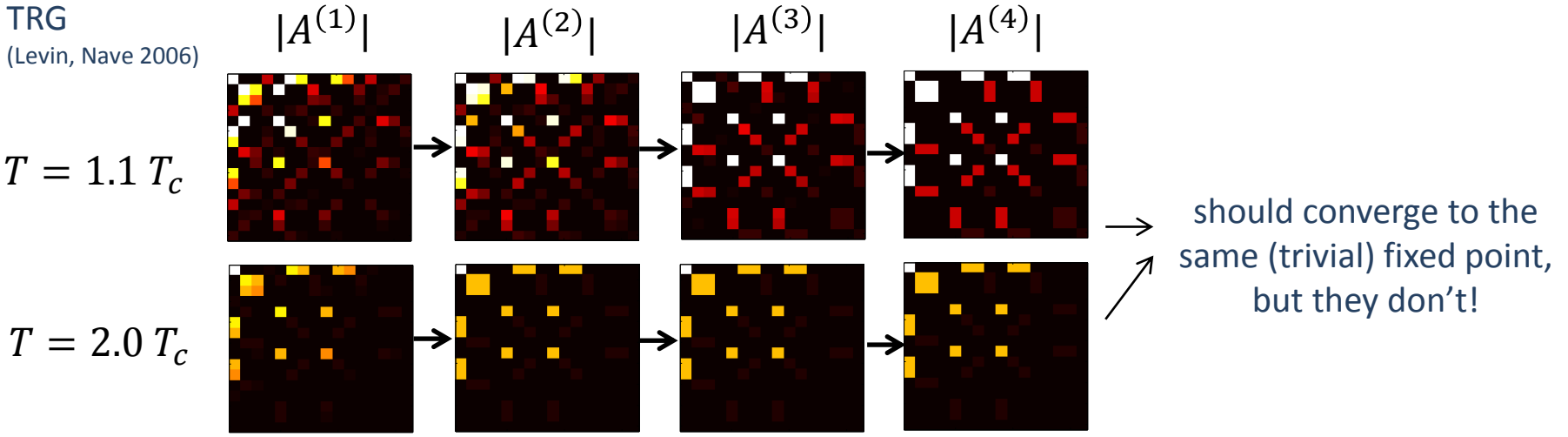



A
u
u†
w
w†
A′

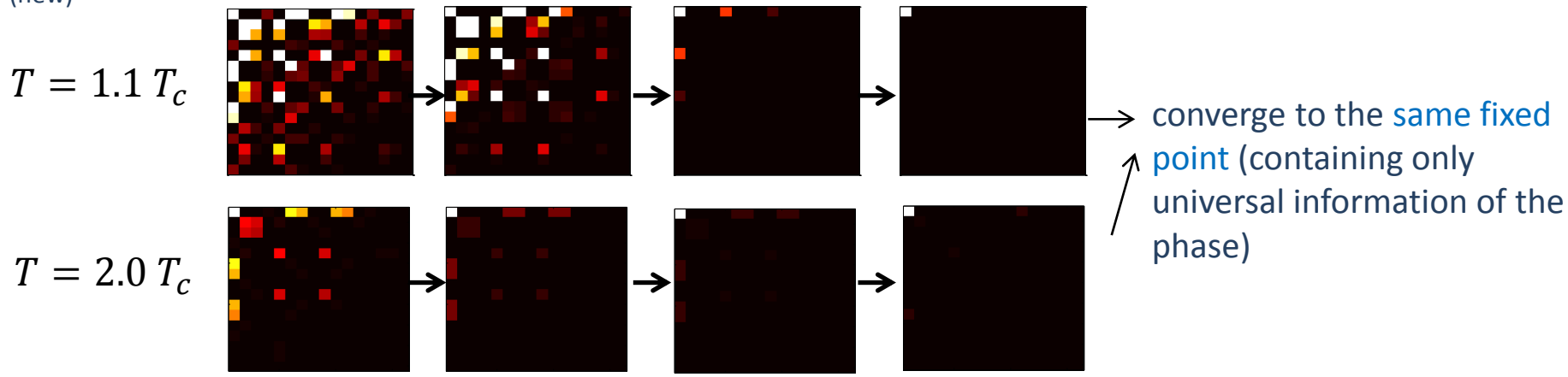
Proper RG flow: 2D classical Ising

e.g. disordered phase $T > T_c$

TRG
(Levin, Nave 2006)



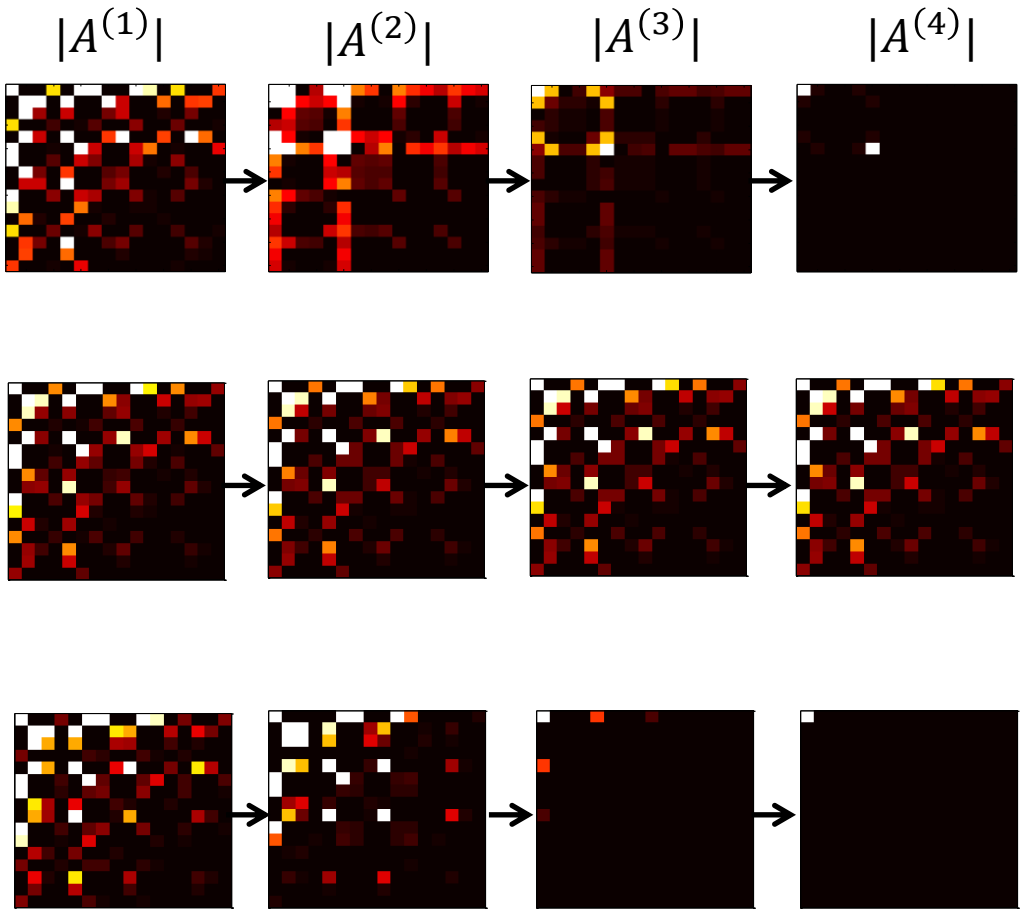
TNR
(new)



Proper RG flow: 2D classical Ising

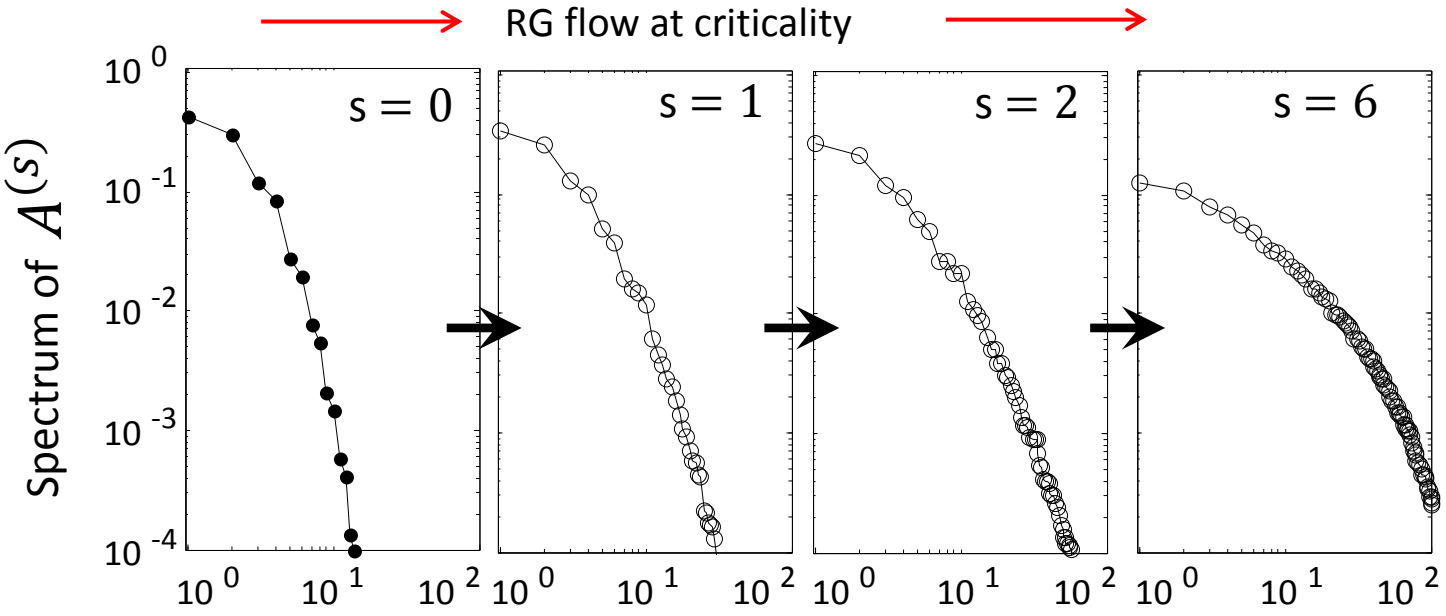
Tensor Network Renormalization (TNR):

- Converges to one of three RG fixed points, consistent with a proper RG flow



Sustainable RG flow: 2D classical Ising

Does TRG give a sustainable RG flow?



Bond dimension χ required to maintain fixed truncation error ($\sim 10^{-3}$):

TRG: $\sim 10 \longrightarrow \sim 20 \longrightarrow \sim 40 \longrightarrow > 100$

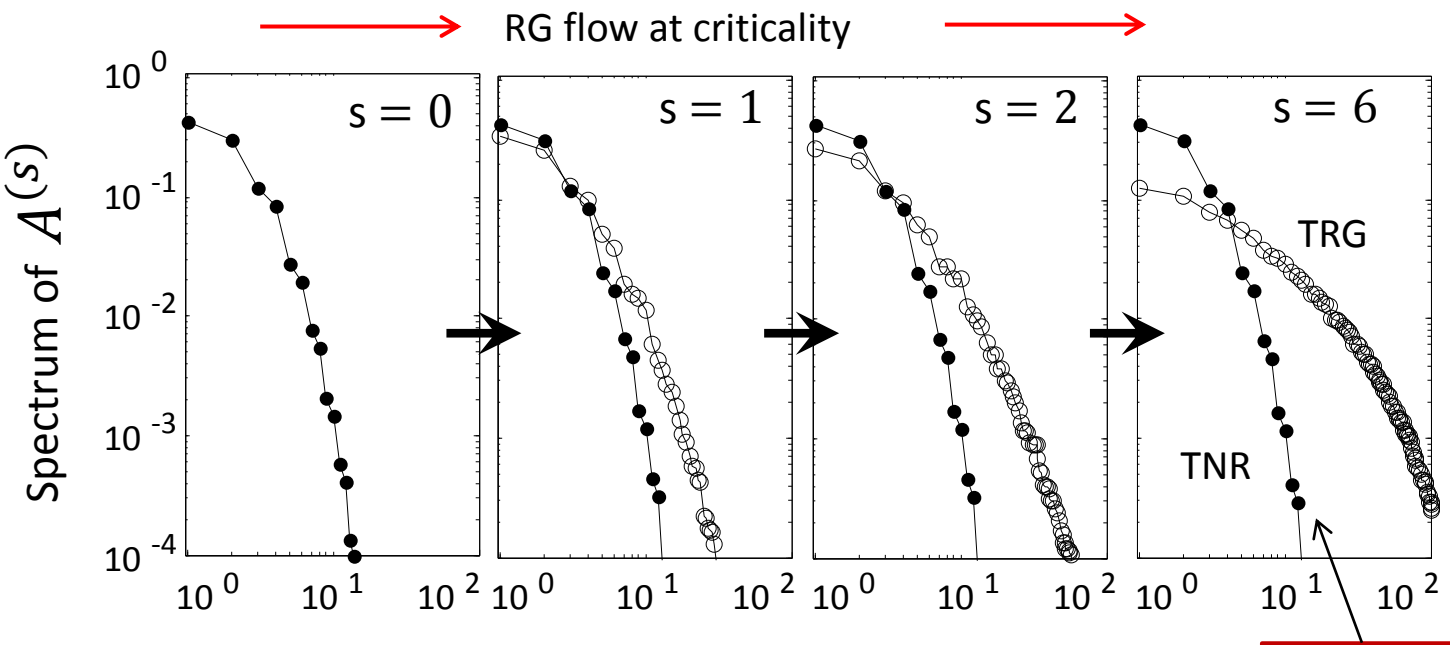
Computational cost:

TRG $O(\chi^6)$: $1 \times 10^6 \rightarrow 6 \times 10^7 \rightarrow 4 \times 10^9 \rightarrow > 10^{12}$

Cost of TRG scales exponentially with RG iteration! ✗

Sustainable RG flow: 2D classical Ising

Does TRG give a sustainable RG flow?



Bond dimension χ required to maintain fixed truncation error ($\sim 10^{-3}$):

TRG: $\sim 10 \longrightarrow \sim 20 \longrightarrow \sim 40 \longrightarrow > 100$

TNR: $\sim 10 \longrightarrow \sim 10 \longrightarrow \sim 10 \longrightarrow \sim 10$

Computational costs:

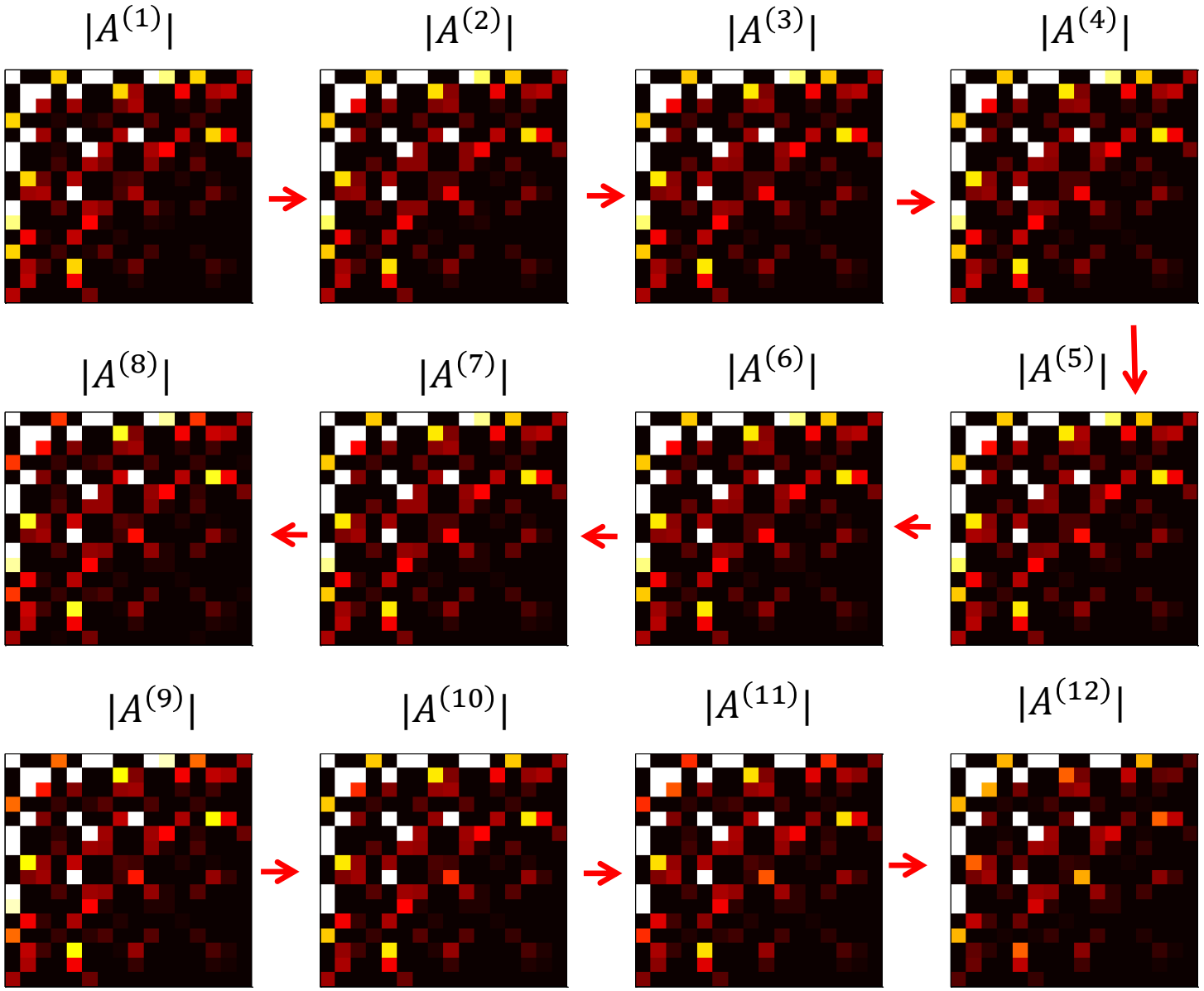
TRG $O(\chi^6)$: $1 \times 10^6 \rightarrow 6 \times 10^7 \rightarrow 4 \times 10^9 \rightarrow > 10^{12}$

TNR $O(k\chi^6)$: $5 \times 10^7 \rightarrow 5 \times 10^7 \rightarrow 5 \times 10^7 \rightarrow 5 \times 10^7$

Proper RG flow: 2D classical Ising

critical point:
$T = T_c$

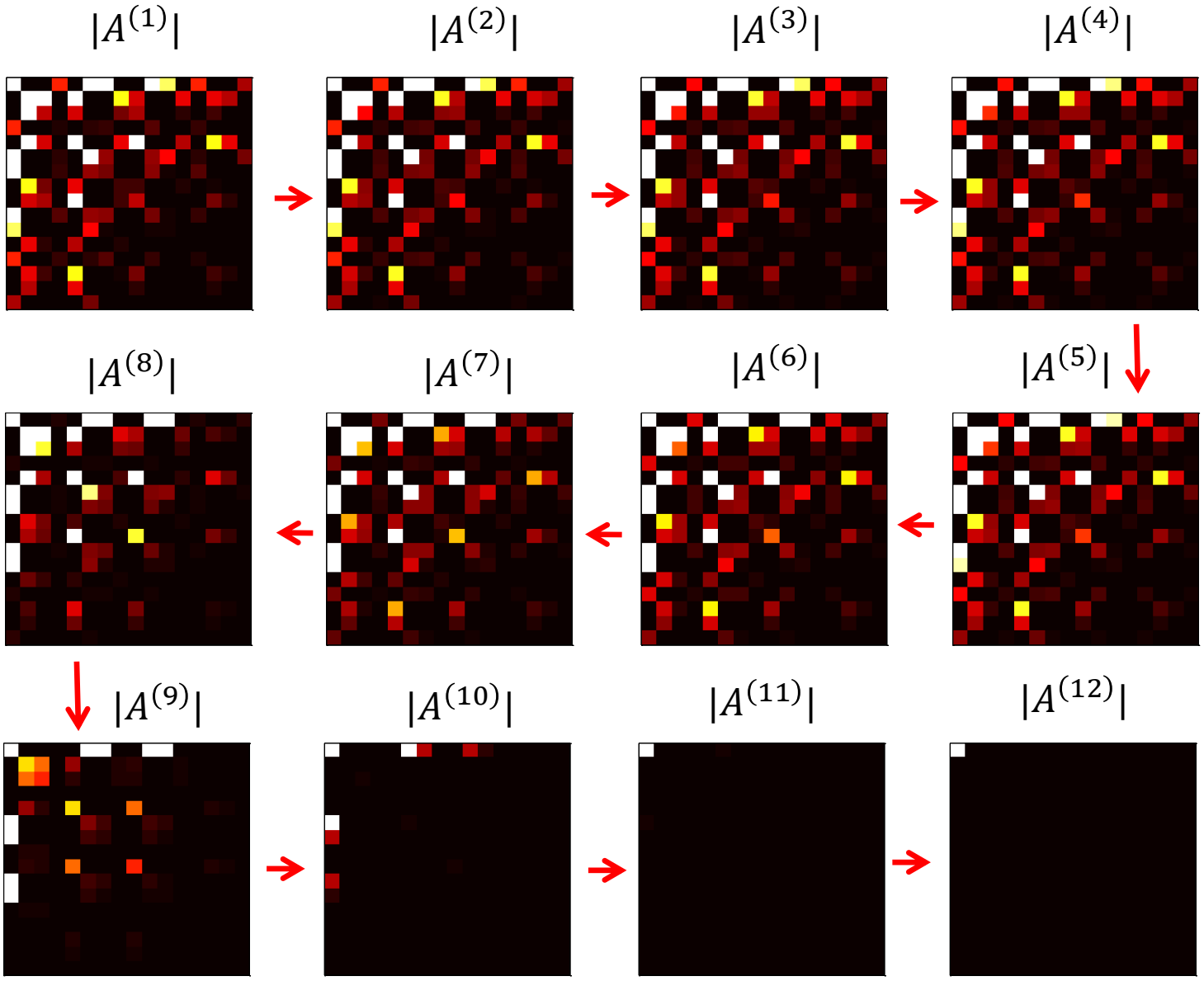
TNR bond dimension:
$\chi = 4$



Proper RG flow: 2D classical Ising

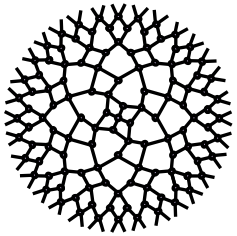
more difficult!
$T = 1.002\ T_c$

TNR bond dimension:
$\chi = 4$



Outline

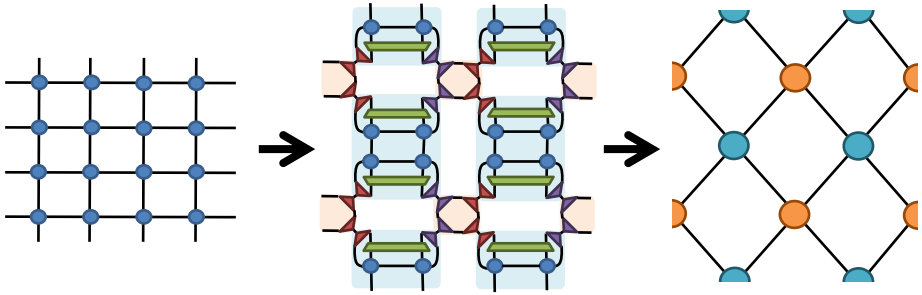
- entanglement renormalization (old stuff)



Real space RG transformation for
→ ground state wave-functions
→ Hamiltonians

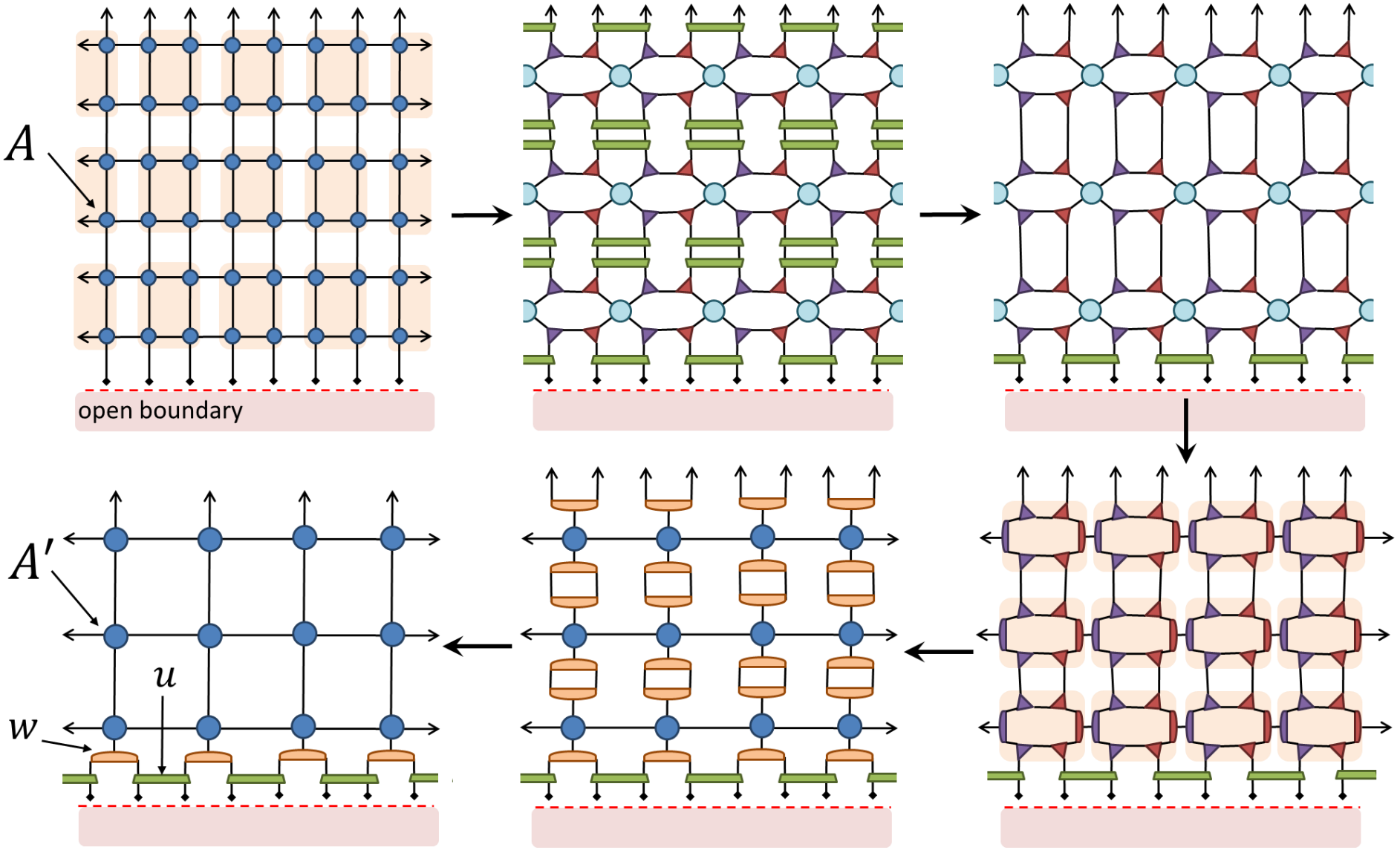
- tensor network renormalization (new stuff)

- Real space RG of Euclidean path integral arXiv:1412.0732
- TNR → MERA (+ thermal states!) arXiv:1501.xxx
- Theory of minimal updates arXiv:1501.yyy
- Conformal transformations arXiv:1501.zzz



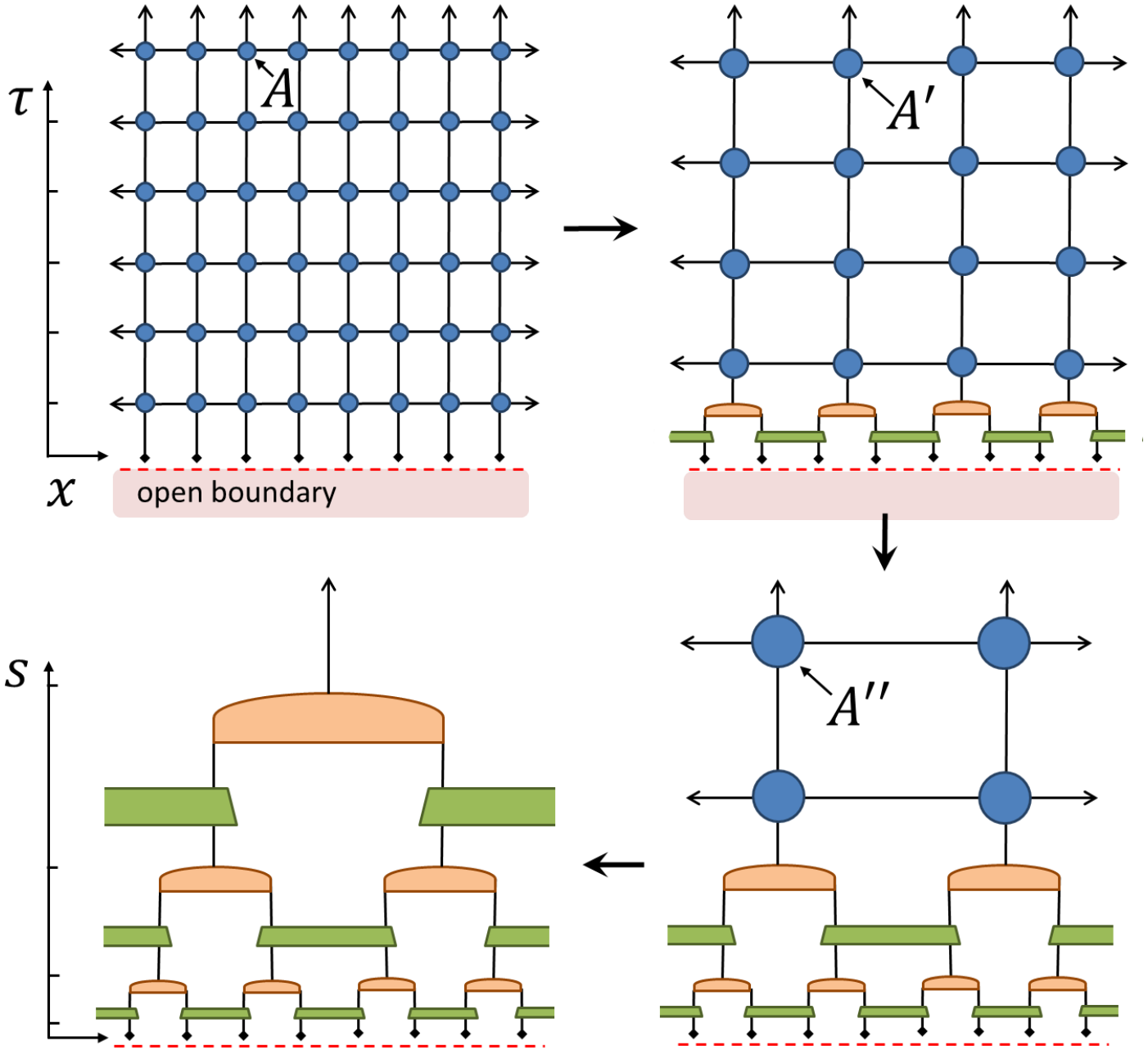
TNR yields MERA

(Evenbly, Vidal, 2015, in prep)



TNR yields MERA

(Evenbly, Vidal, 2015, in prep)

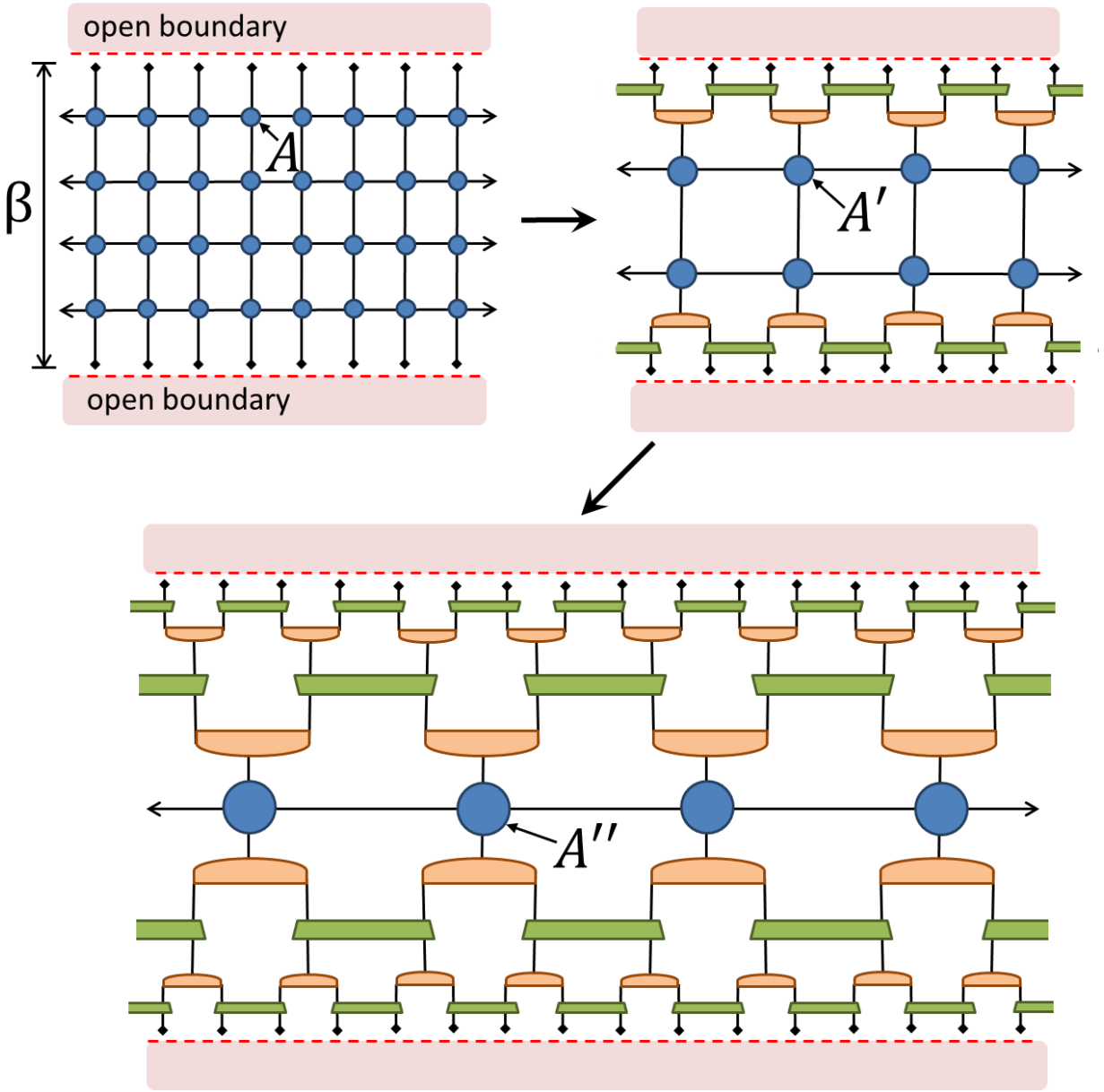


Traditional perspective		New perspective
MERA as a variational class of many-body states	→	MERA as a byproduct of tensor network renormalization (TNR)
		• reshape an existing representation of the ground state • TNR truncation errors: accuracy certificate
energy minimization	→	trace optimization
• 1000's of iterations on scale • non-local in scale		• 1 iteration on scale • local in scale

Extra bonus: thermal MERA

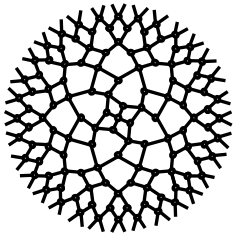
(Evenbly, Vidal, 2015, in prep)

$e^{-\beta H}$



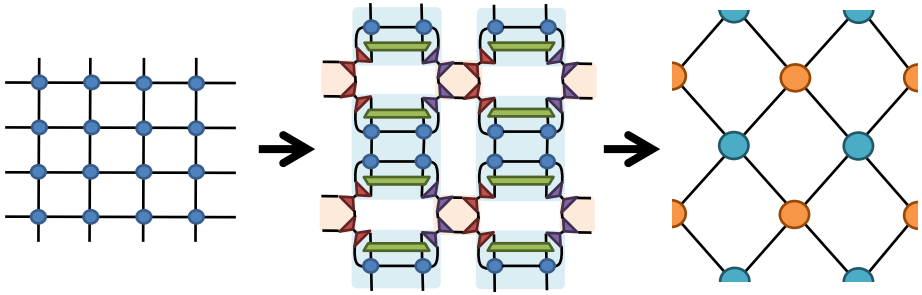
Outline

- entanglement renormalization (old stuff)



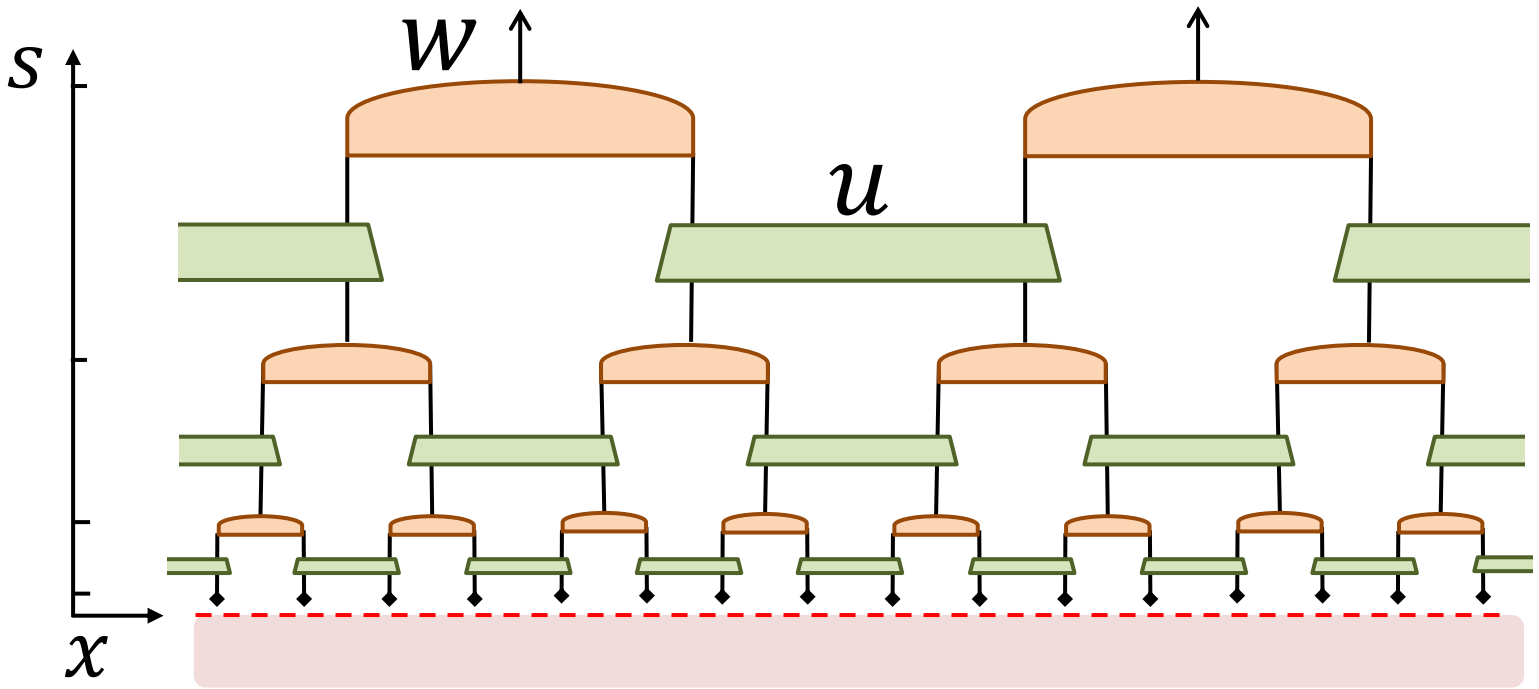
Real space RG transformation for
→ ground state wave-functions
→ Hamiltonians

- tensor network renormalization (new stuff)
  - Real space RG of Euclidean path integral — arXiv:1412.0732
  - TNR → MERA (+ thermal states!) — arXiv:1501.xxx
  - Theory of minimal updates — arXiv:1501.yyy
  - Conformal transformations — arXiv:1501.zzz



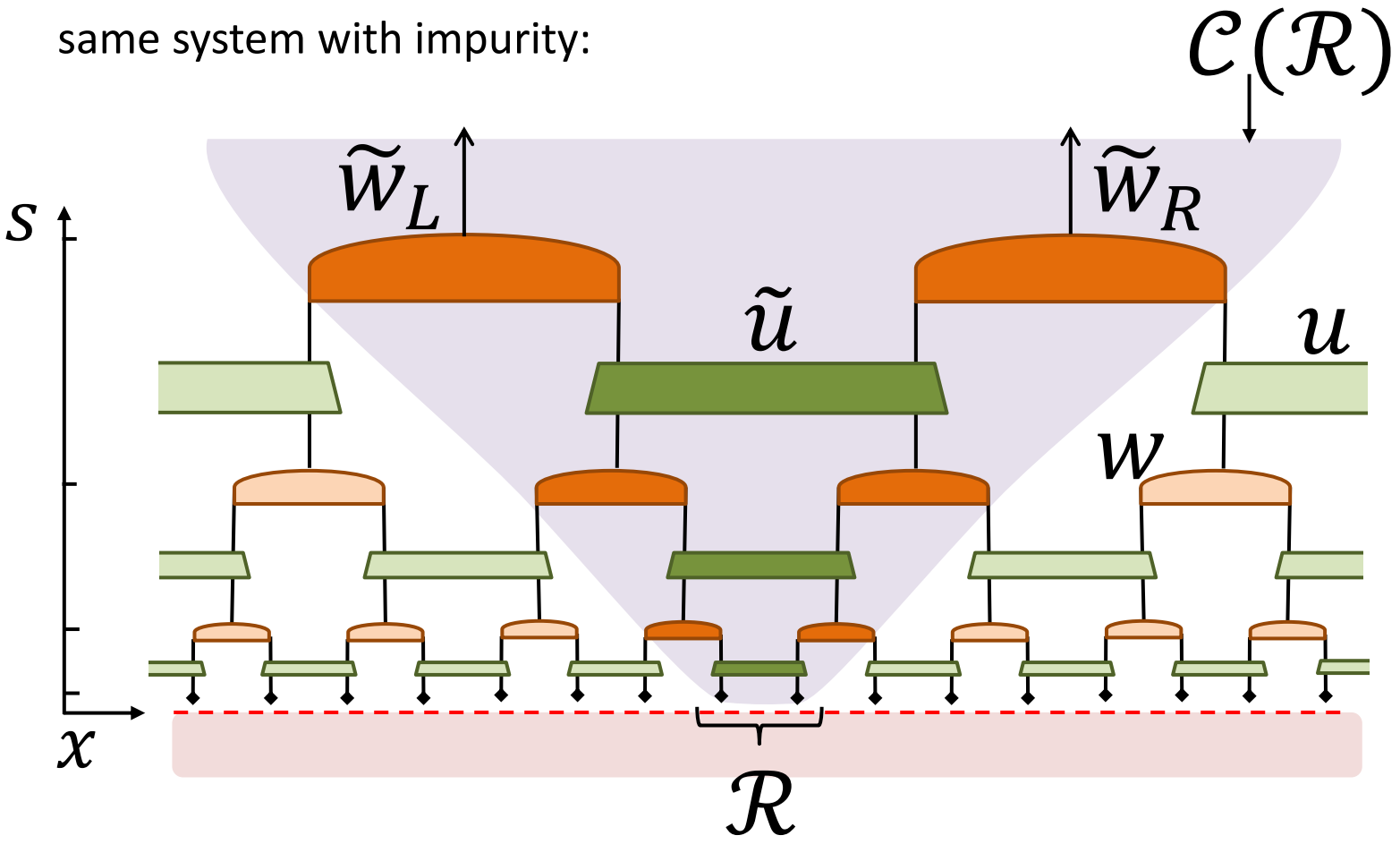
Extra bonus: theory of minimal updates (Evenbly, Vidal, 2015, in prep)

homogeneous system:



Extra bonus: theory of minimal updates (Evenbly, Vidal, 2015, in prep)

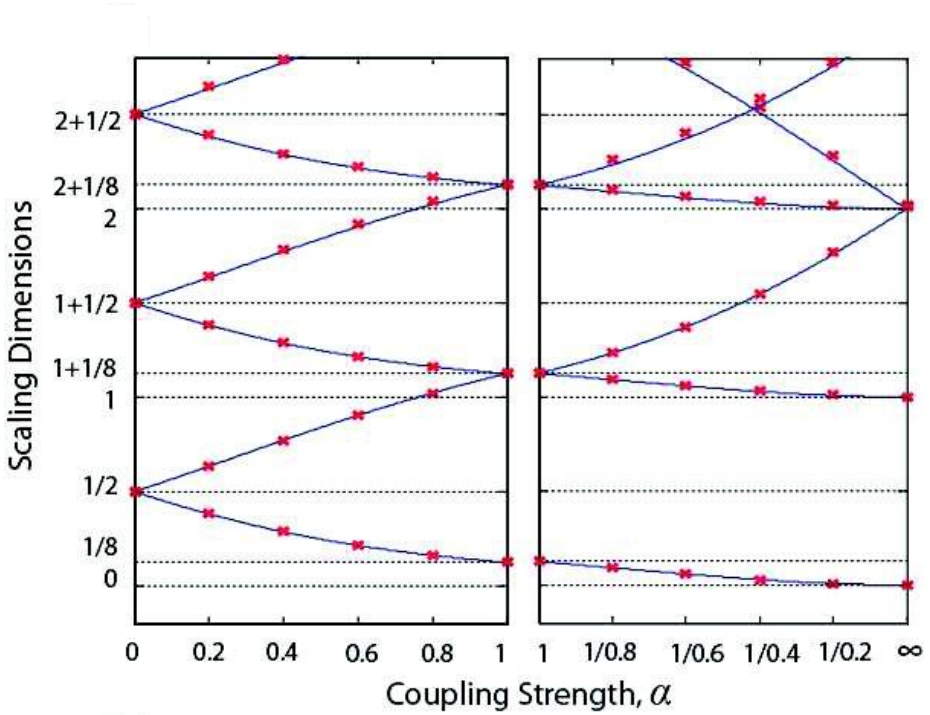
same system with impurity:

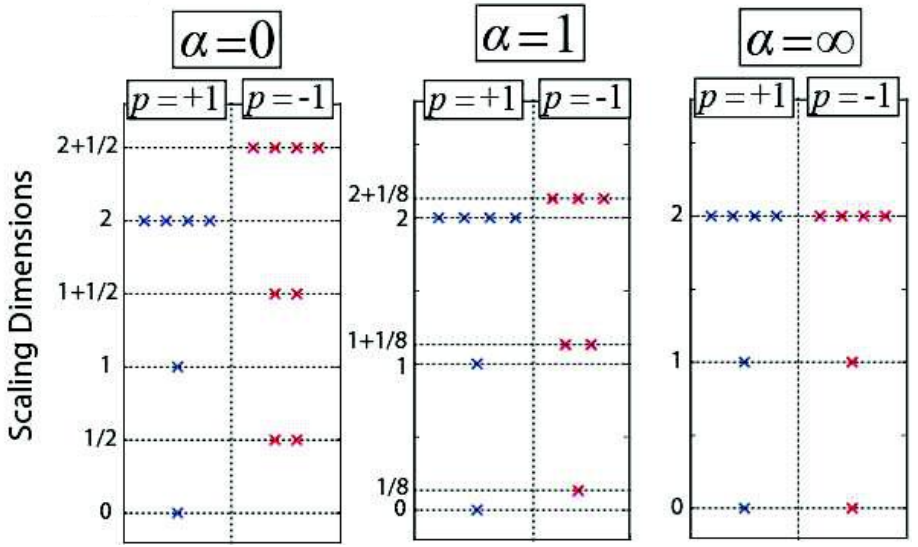


Example: critical Ising model with one modified bond

$$H = \sum_i (-X_r X_{r+1} + Z_r)$$

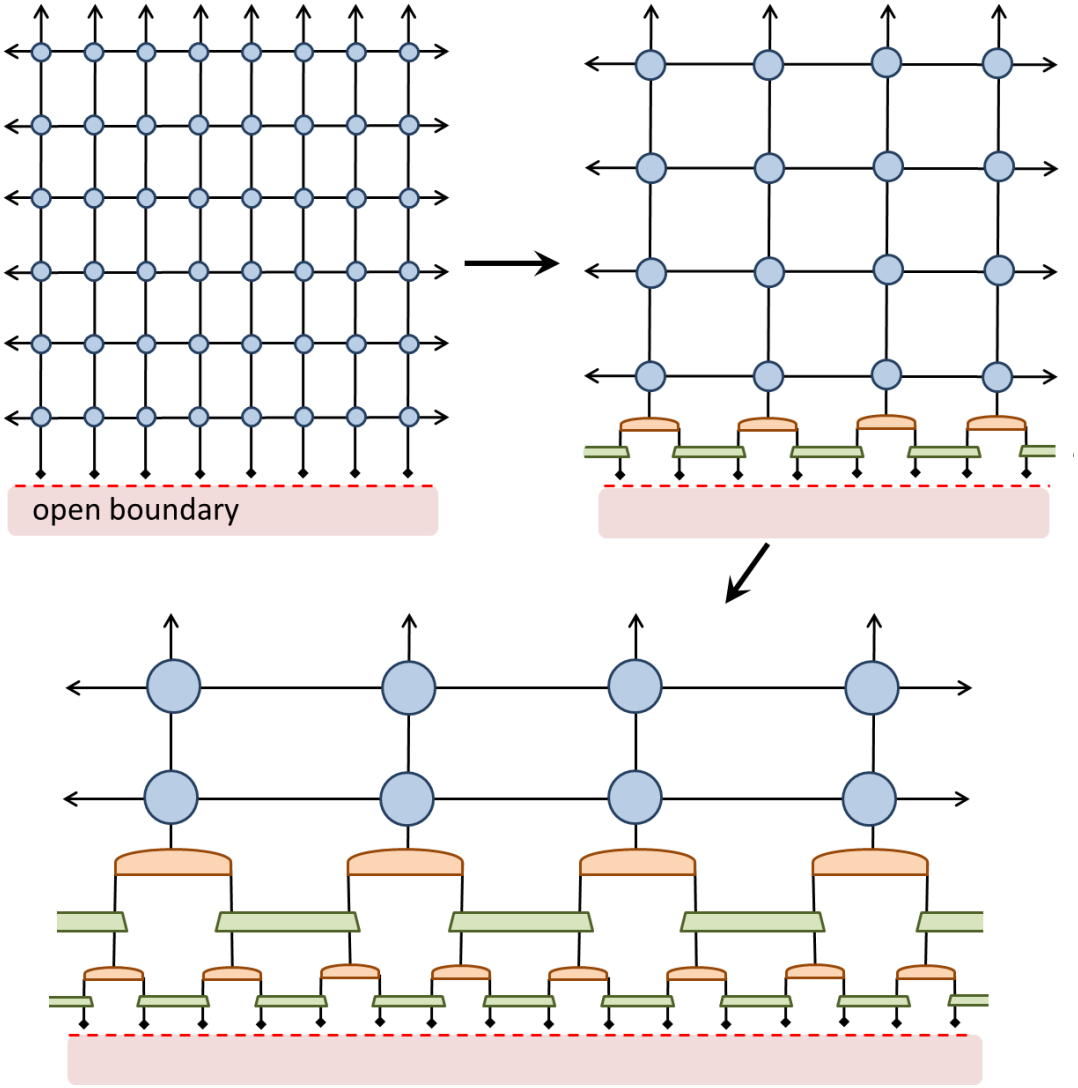
$$H^{imp} = (1-\alpha) X_0 X_1$$





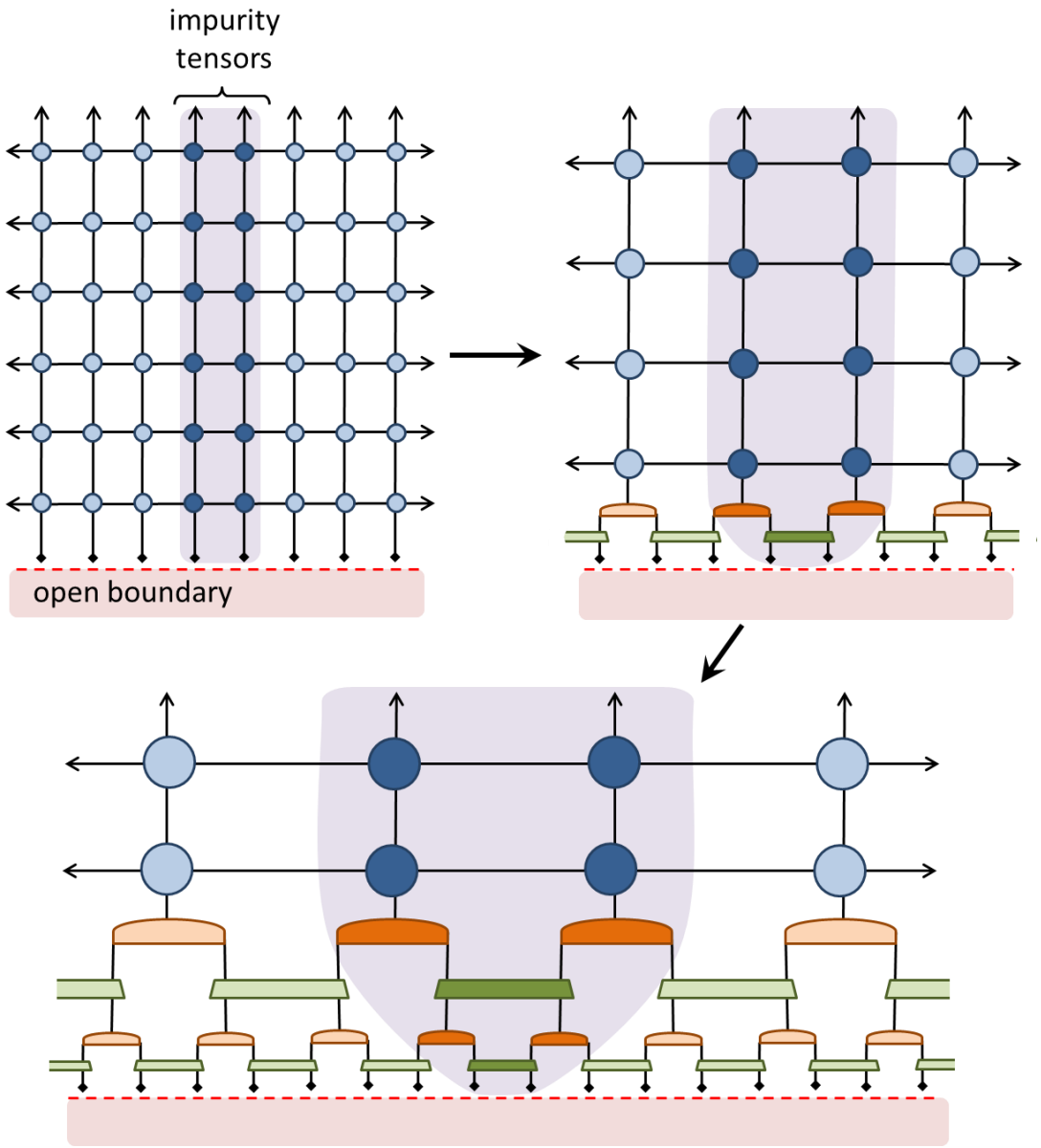
Extra bonus: theory of minimal updates

homogeneous system:



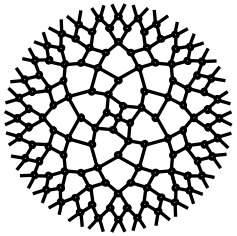
Extra bonus: theory of minimal updates

same system with impurity:



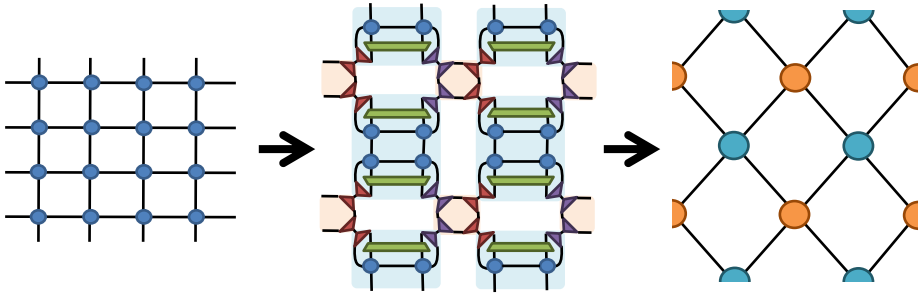
Outline

- entanglement renormalization (old stuff)

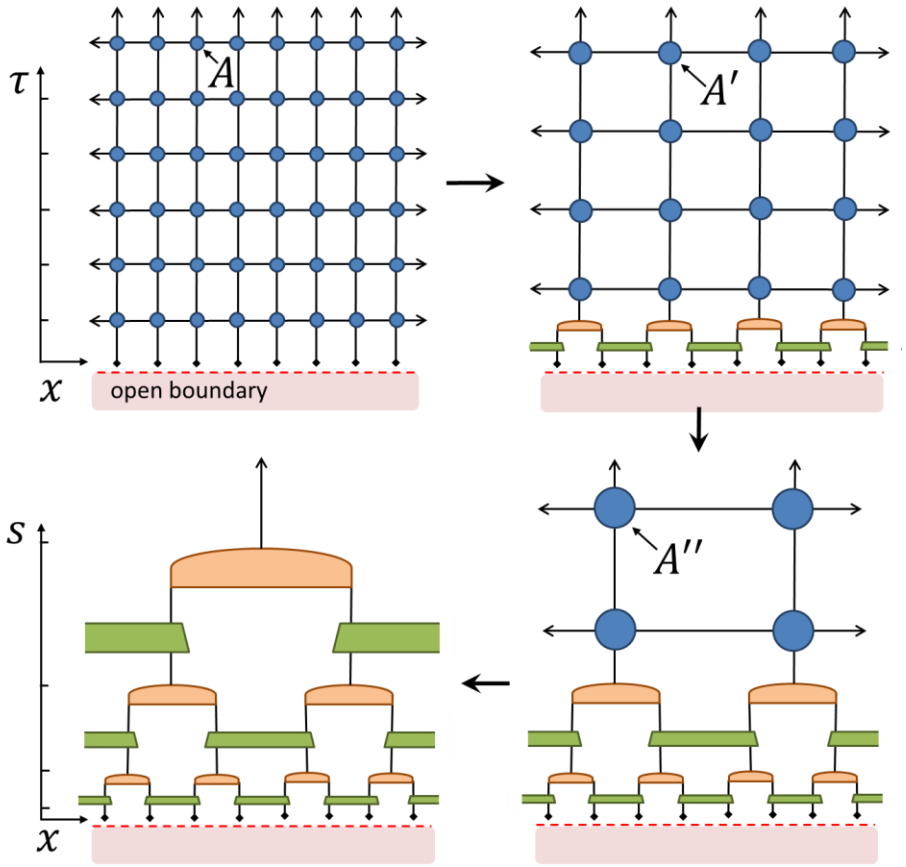


Real space RG transformation for
→ ground state wave-functions
→ Hamiltonians

- tensor network renormalization (new stuff)
  - Real space RG of Euclidean path integral — arXiv:1412.0732
  - TNR → MERA (+ thermal states!) — arXiv:1501.xxx
  - Theory of minimal updates — arXiv:1501.yyy
  - Conformal transformations — arXiv:1501.zzz



Extra bonus: conformal transformations (Evenbly, Vidal, 2015, in prep)



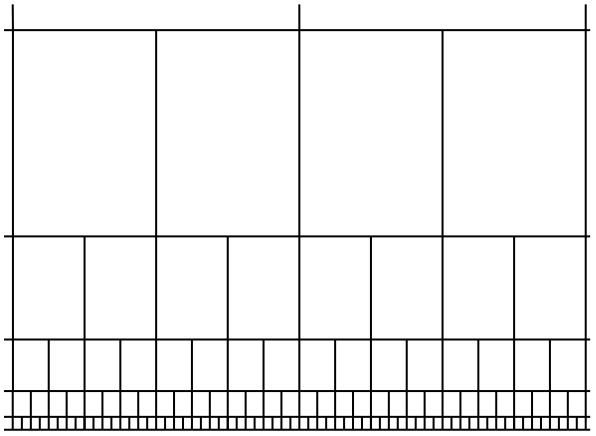
Conformal transformation 1:

Upper half-plane to AdS_2

$$dx^2 + d\tau^2 = \tau^2\left(\frac{dx^2}{\tau^2} + \frac{d\tau^2}{\tau^2}\right)$$

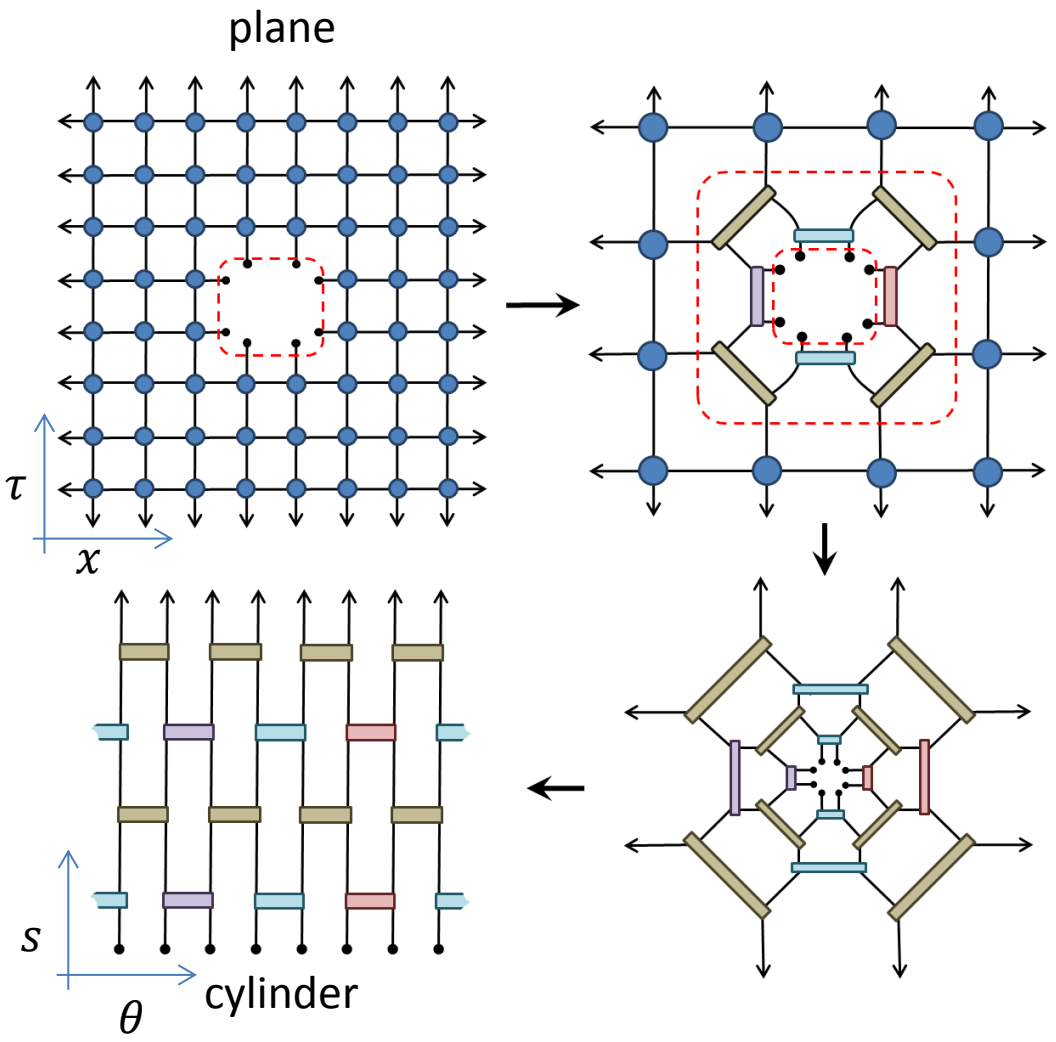
$$\rightarrow \left(\frac{dx^2}{\tau^2} + \frac{d\tau^2}{\tau^2}\right) = \frac{dx^2}{2^{2s}} + ds^2$$

$$s = \log_2(\tau)$$



Holographic description

Extra bonus: conformal transformations (Evenbly, Vidal, 2015, in prep)



Conformal transformation 2:

Plane to cylinder

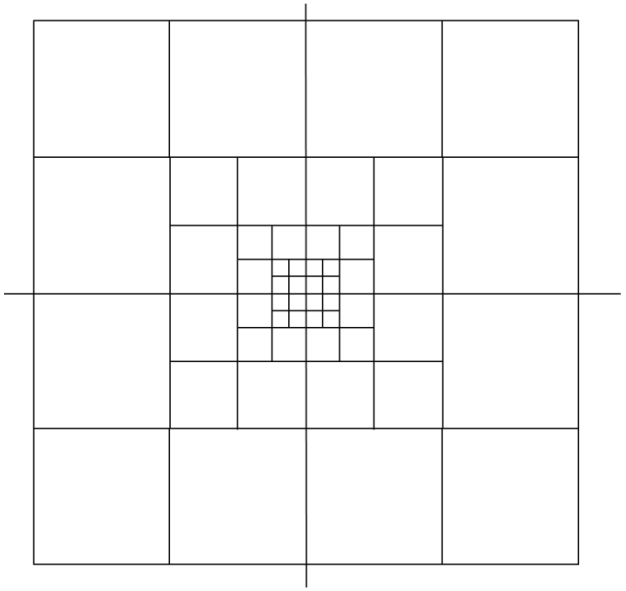
(radial quantization in CFT)

$z \equiv x + i\tau$

$$z = 2^w$$

$w \equiv s + i\theta$

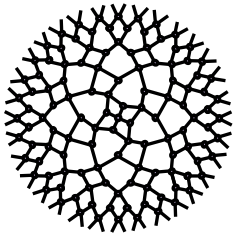
$s \equiv \log_2(x^2 + \tau^2)$



- Extraction of scaling dimensions

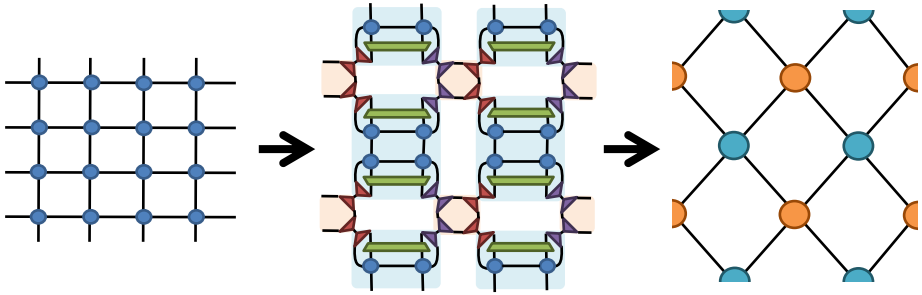
Summary

- entanglement renormalization (old stuff)



Real space RG transformation for
→ ground state wave-functions
→ Hamiltonians

- tensor network renormalization (new stuff)
  - Real space RG of Euclidean path integral — arXiv:1412.0732
  - TNR → MERA (+ thermal states!) — arXiv:1501.xxx
  - Theory of minimal updates — arXiv:1501.yyy
  - Conformal transformations — arXiv:1501.zzz



THANKS !