Symmetry Enriched Topological Phases

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Sydney Quantum Information Theory Workshop, Coogee

February 3, 2016

work done in collaboration with:

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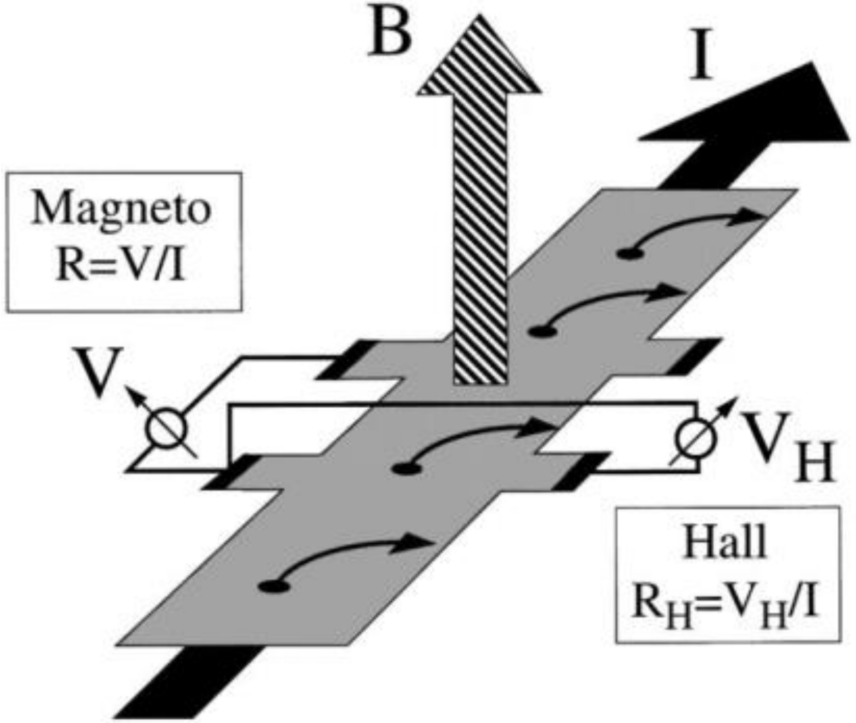
arXiv:1410.4540

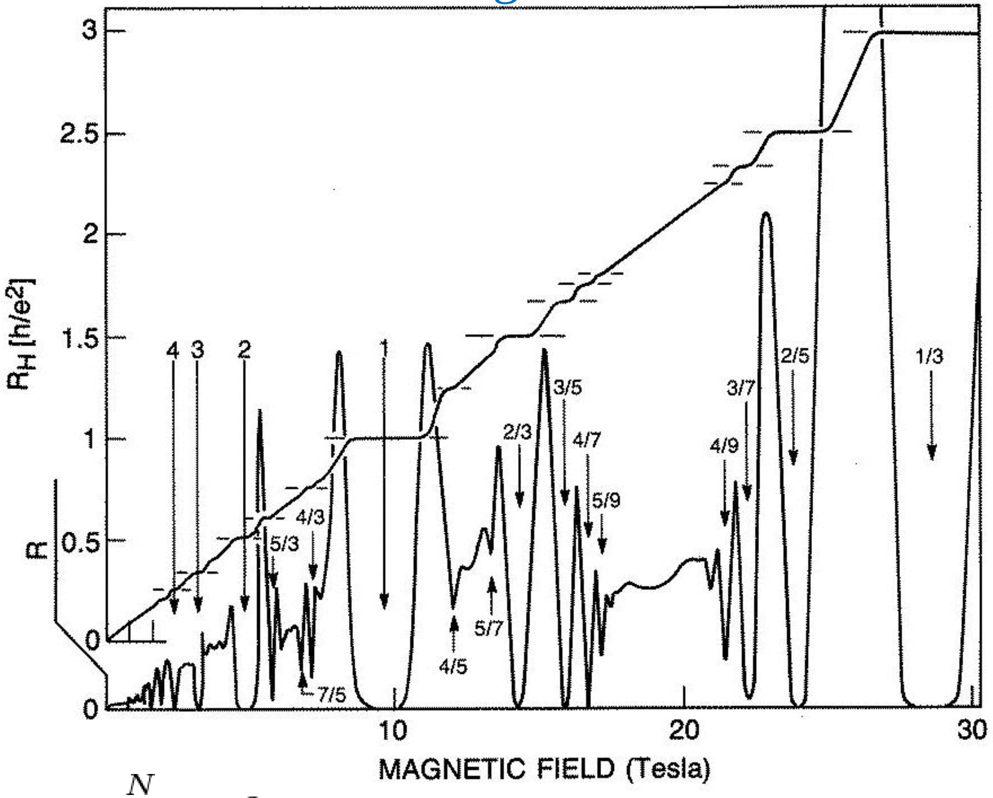
Topological Phases

- Gapped (bulk) local Hamiltonian
- Finite correlation length
- Localized, finite energy excitations (quasiparticles) carry emergent topological quantum numbers
- Low energy effective theory described by a TQFT

Fractional Quantum Hall States

Tsui, Stormer, Gossard 1982
Laughlin 1983



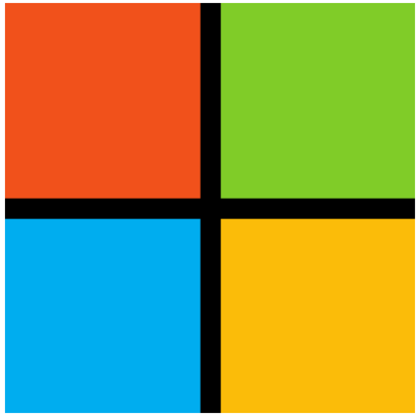


$$\Psi_{\frac{1}{M}}(z_1, \ldots, z_N) = \prod_{i<j}^{N} (z_i - z_j)^M e^{-\frac{1}{4}\sum_{i=1}^{N} |z_i|^2}$$

Topological Phases

Without symmetry imposed, gapped phases are fully characterized by “topological order”

e.g. for (2+1)D = the UMTC $\mathcal{C}$ and chiral central charge c_-

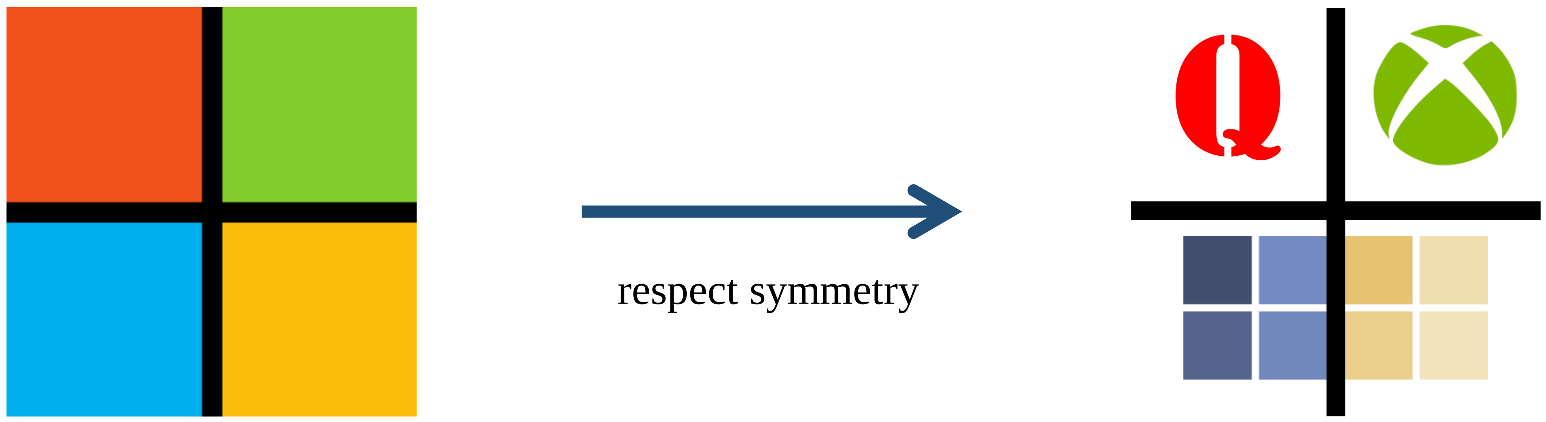


no symmetry

Two Hamiltonians (points in parameter space) realize the same phase if they can be continuously connected without closing the gap

Symmetry Enriched Topological Phases

With symmetry imposed, there is a finer scale of classification characterized by the “SET order”



no symmetry

symmetry enriched

Two Hamiltonians (points in parameter space) realize the same SET phase if they can be continuously connected while **respecting the symmetry** without closing the gap

Symmetry Enriched Topological Phases

With symmetry imposed, there is a finer scale of classification characterized by the “SET order”

- Topological Insulators
- Quantum Spin Liquids
- Fractional Quantum Hall States
- Majorana and Parafermion Systems

symmetry enriched

Symmetry Enriched Topological Phases

General framework to characterize and classify SET order?

Partial and limited approaches:

- Projective symmetry group — Wen 2002
- $H^{d+1}(G, \mathrm{U}(1))$ cohomology for SPT, trivial TO — Chen, Gu, Liu, Wen 2011
- Some exactly solved models, no perm — Mesaros, Ran 2012
- Chern-Simon for Abelian (2+1)D — Lu, Vishwanath 2012
- Symmetry fractionalization for (2+1)D Abelian, no perm — Essin, Hermele 2013

Symmetry Enriched Topological Phases

General framework to characterize and classify SET order?

Our approach, complete for (2+1)D:

Barkeshli, Bonderson, Meng, Wang 2014

1. Symmetry Fractionalization:

   Classify different ways that quasiparticles may carry fractionalized quantum numbers

2. Symmetry Defects:

   Develop and classify algebraic theory of defects ("fluxes")

G-Crossed UMTC

Turaev 2000

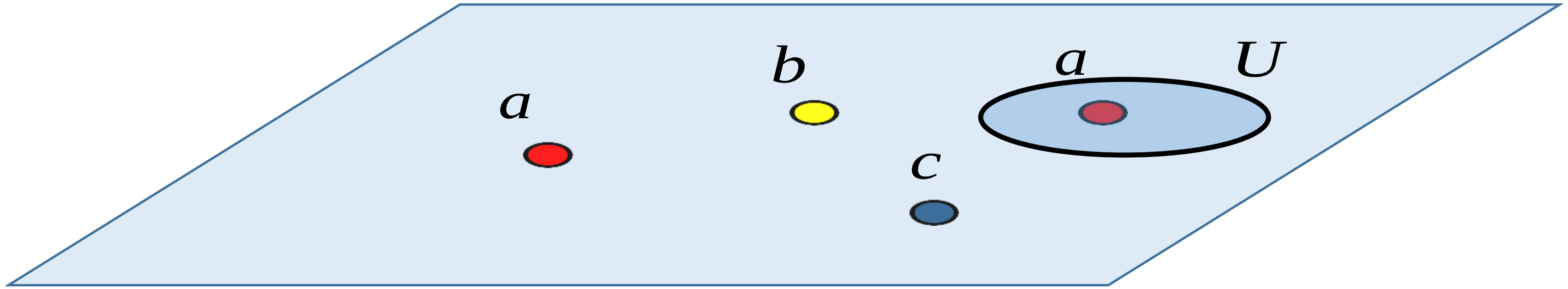
Etingof, Nikshych, Ostrik 2010

Topological Phases (2+1)D

Effective theory described by UMTC $\mathcal{C}$

Topological charges: $a, b, c \ldots \in \mathcal{C}$

(anyon types)

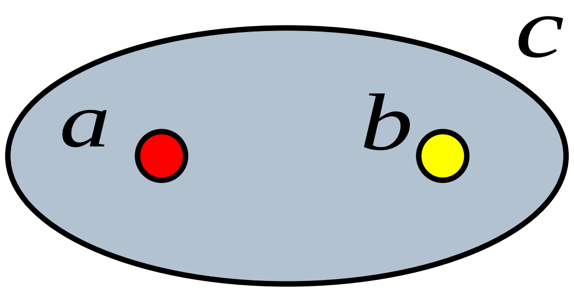


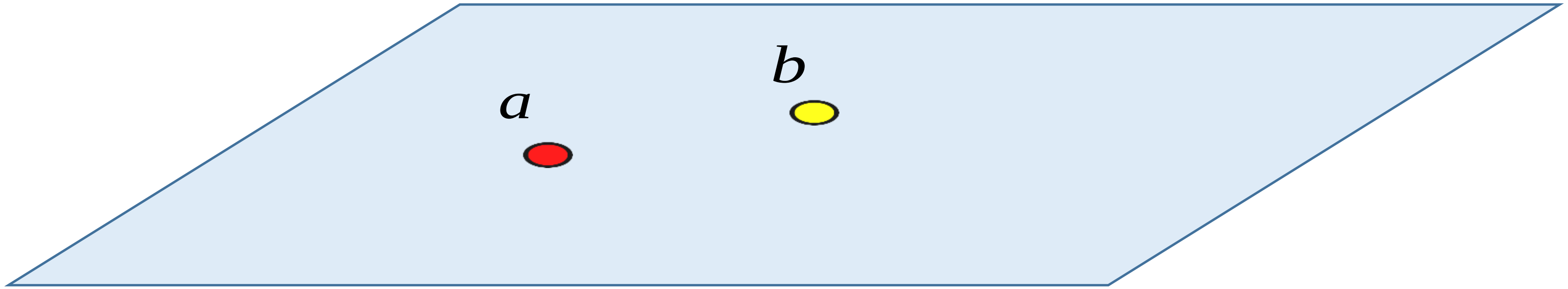
Equivalence classes of states: local operations do not change topological charge

Topological Phases (2+1)D

Effective theory described by UMTC $\mathcal{C}$

Fusion: $a \times b = \sum_{c \in \mathcal{C}} N_{ab}^c c$





No internal DOFs, but different fusion outcomes are possible

ang. mom. analog: $\frac{1}{2} \times \frac{1}{2} = 0 + 1$

Topological Phases (2+1)D

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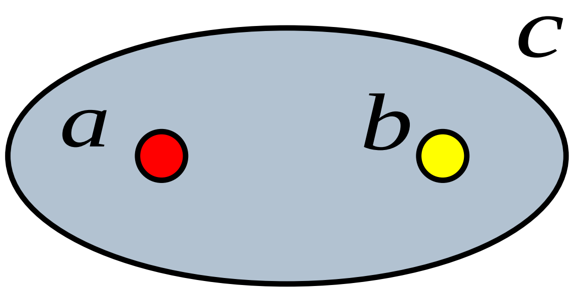

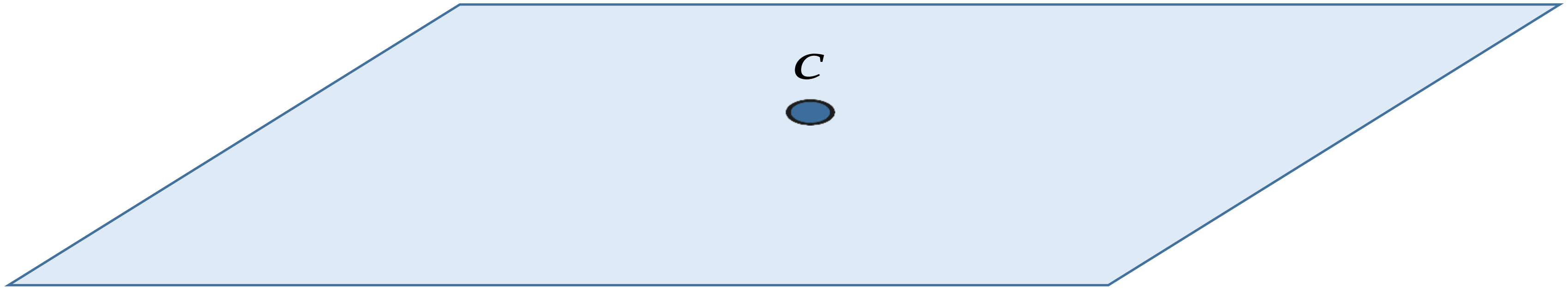

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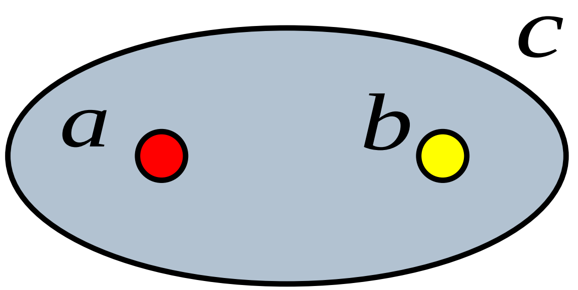
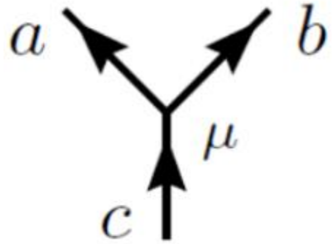
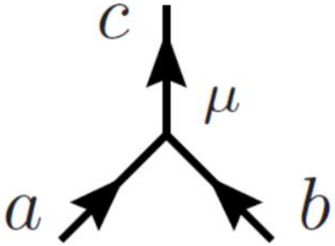


Topological (nonlocal) state space:



$$= |a, b; c, \mu\rangle \in V_c^{ab}$$



$$= \langle a, b; c, \mu| \in V_{ab}^{c}$$

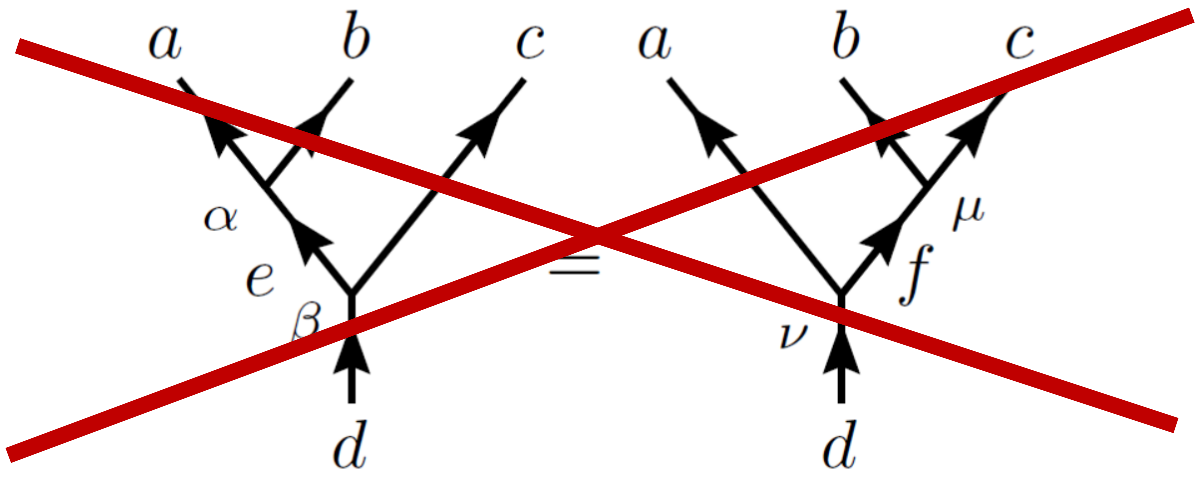
$$\mu = 1, \ldots, N_{ab}^{c}$$

Topological Phases (2+1)D

Effective theory described by UMTC $\mathcal{C}$

Associativity: $(a \times b) \times c = a \times (b \times c)$

$$\sum_e N_{ab}^e N_{ec}^d = \sum_f N_{af}^d N_{bc}^f$$

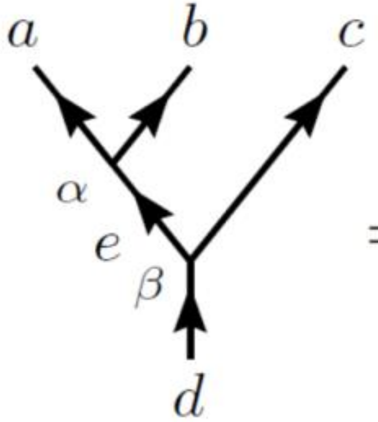


Topological Phases (2+1)D

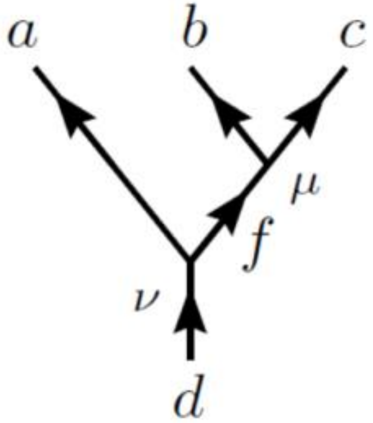
Effective theory described by UMTC $\mathcal{C}$

Associativity: $(a \times b) \times c = a \times (b \times c)$

$$\sum_e N_{ab}^{e} N_{ec}^{d} = \sum_f N_{af}^{d} N_{bc}^{f}$$



$$= \sum_{f,\mu,\nu} \left[F_d^{abc}\right]_{(e,\alpha,\beta)(f,\mu,\nu)}$$



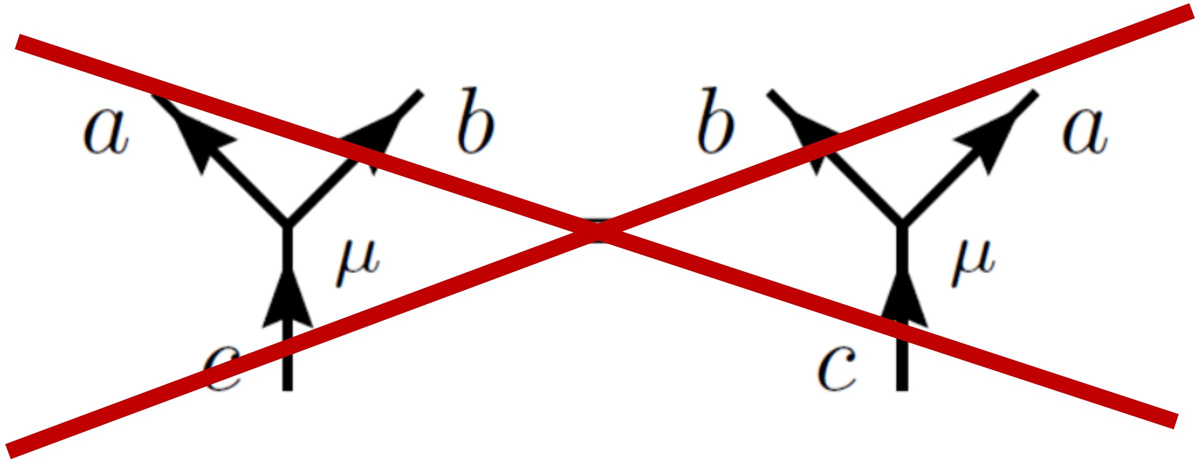
ang. mom. analog: 6j-symbols

Topological Phases (2+1)D

Effective theory described by UMTC $\mathcal{C}$

Commutativity: $a \times b = b \times a$

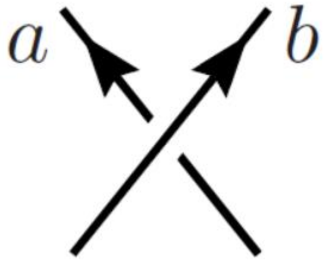
$$N_{ab}^{c} = N_{ba}^{c}$$

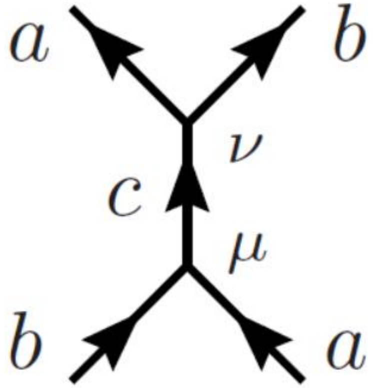


Topological Phases (2+1)D

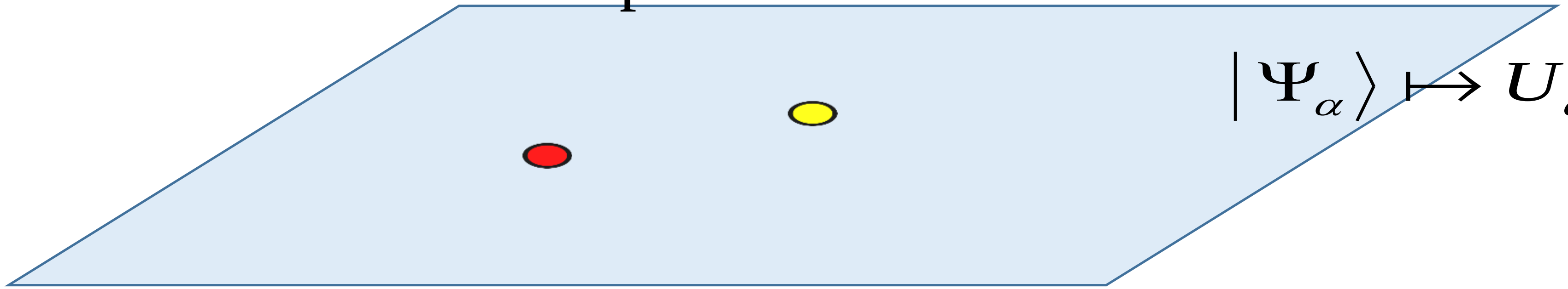
Effective theory described by UMTC $\mathcal{C}$

Braiding:

$R^{ab} =$ 

$= \sum_{c,\mu,\nu} \left[R^{ab}_c\right]_{\mu\nu}$ 

non-Abelian if there are multiple fusion channels c

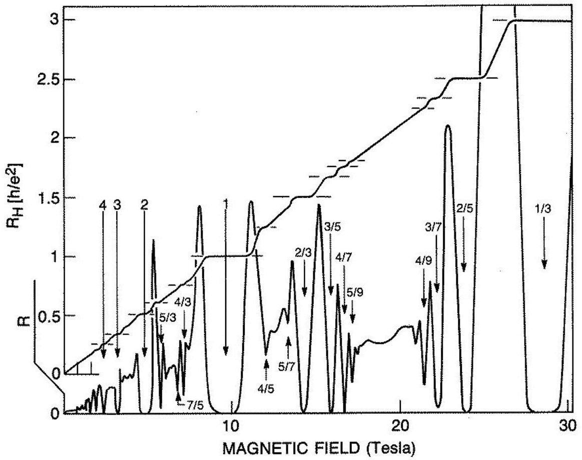


$$|\Psi_\alpha\rangle \mapsto U_{\alpha\beta}[R]|\Psi_\beta\rangle$$

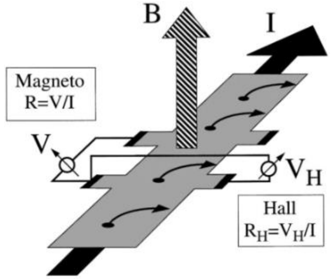
modular = non-degenerate braiding = unitary S-matrix

Fractional Quantum Hall States

Laughlin 1983



$$\nu = \frac{1}{m}$$ Laughlin FQH states



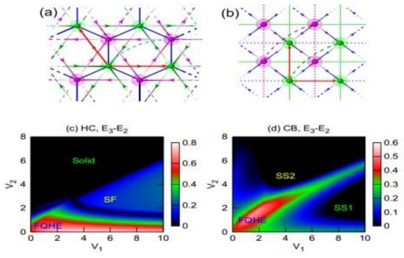
<u>m even (bosonic)</u> $\mathbb{Z}_m^{(1/2)}$

topo charges: $0, 1, 2, \ldots, m-1$

fusion: $a \times b = [a+b] \mathrm{mod}\ m$

associativity: $F^{abc} = e^{i\frac{\pi}{m} a(b+c-[b+c]\mathrm{mod} m)}$

braiding: $R^{ab} = e^{i\frac{\pi}{m} ab}$



<u>m odd (fermionic)</u> $\mathbb{Z}_{2m}^{(1)} = \mathbb{Z}_m^{(\frac{m+1}{2})} \times \mathbb{Z}_2^{(1)}$

Topological Symmetry

Autoequivalence maps $\varphi : \mathcal{C} \to \mathcal{C}$

invertible maps that leave topological properties invariant

Topological Symmetry

Autoequivalence maps $\varphi : \mathcal{C} \to \mathcal{C}$

$$\varphi(a) = a'$$

$$\begin{aligned}\varphi\left(|a,b;c,\mu\rangle\right) &= \widetilde{|a',b';c',\mu\rangle} \\ &= \sum_{\mu'} \left[u_{c'}^{a'b'}\right]_{\mu\mu'} |a',b';c',\mu'\rangle\end{aligned}$$

$$\begin{aligned}\varphi\left(N_{ab}^{c}\right) &= N_{a'b'}^{c'} = N_{ab}^{c} \\ \varphi\left(\left[F_d^{abc}\right]_{(e,\alpha,\beta)(f,\mu,\nu)}\right) &= \left[\widetilde{F}_{d'}^{a'b'c'}\right]_{(e',\alpha,\beta)(f',\mu,\nu)} = \left[F_d^{abc}\right]_{(e,\alpha,\beta)(f,\mu,\nu)} \\ \varphi\left(\left[R_c^{ab}\right]_{\mu\nu}\right) &= \left[\widetilde{R}_{c'}^{a'b'}\right]_{\mu\nu} = \left[R_c^{ab}\right]_{\mu\nu}\end{aligned}$$

Topological Symmetry

Autoequivalence maps $\varphi : \mathcal{C} \to \mathcal{C}$

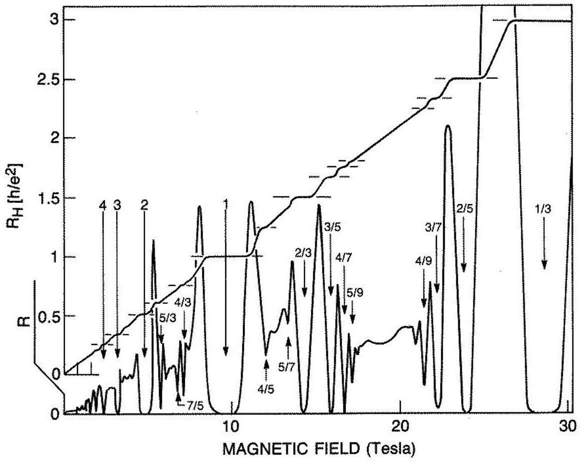
Mod out natural isomorphisms (trivial autoequivalences)

$$\Upsilon(a) = a \qquad \Upsilon(|a,b;c,\mu\rangle) = \frac{\gamma_a \gamma_b}{\gamma_c} |a,b;c,\mu\rangle$$

$\longrightarrow$ $\mathrm{Aut}(\mathcal{C})$ "topological symmetry group"

Fractional Quantum Hall States

Laughlin 1983



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$$\mathrm{Aut}(\mathcal{C}) = \mathbb{Z}_2^n$$

e.g. “quasiparticle-quasihole” conjugation: $a \to [-a]$

Global Symmetry

Symmetries of microscopic Hamiltonian $\mathbf{g} \in G$

$$[R_{\mathbf{g}}, H] = 0$$

Action on physical Hilbert space

$$|\Psi\rangle \mapsto R_{\mathbf{g}} |\Psi\rangle$$

How do the symmetries act on the emergent topological properties?

Global Symmetry

Action on emergent topological theory

$$\rho : G \to \mathrm{Aut}(\mathcal{C})$$

$$\rho_{\mathbf{g}}(a) = {}^{\mathbf{g}}a$$

$$\rho_{\mathbf{g}}\left(|a,b;c,\mu\rangle\right) = \sum_{\mu'} \left[U_{\mathbf{g}}({}^{\mathbf{g}}a, {}^{\mathbf{g}}b; {}^{\mathbf{g}}c)\right]_{\mu\mu'} |{}^{\mathbf{g}}a, {}^{\mathbf{g}}b; {}^{\mathbf{g}}c, \mu'\rangle$$

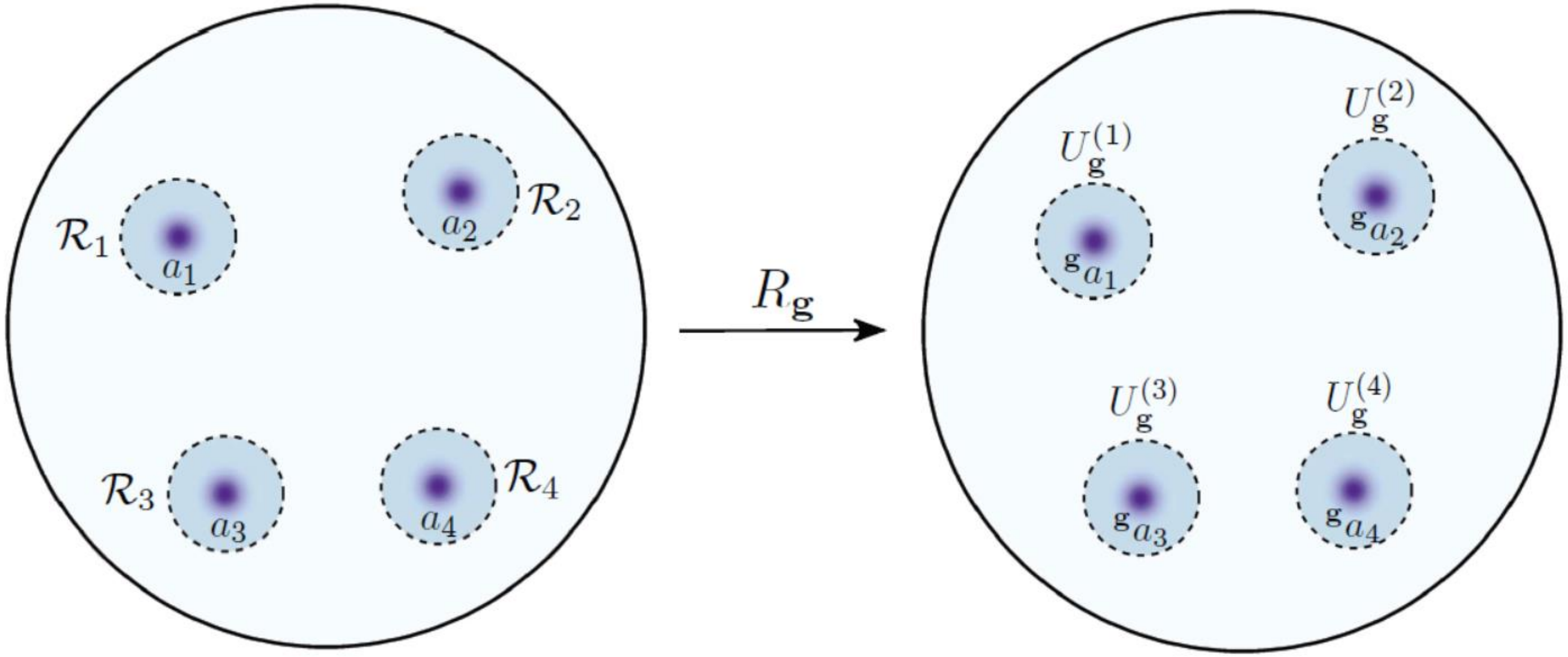
$$\kappa_{\mathbf{g},\mathbf{h}} \circ \rho_{\mathbf{g}} \circ \rho_{\mathbf{h}} = \rho_{\mathbf{gh}}$$

natural isomorphism $\longrightarrow$ defines an invariant $[\mathfrak{O}] \in H^3_{[\rho]}(G, \mathcal{A})$

Symmetry Fractionalization

“Localization” of symmetry action

$$R_{\mathbf{g}}|\Psi_{\{a;c,\mu\}}\rangle = U_{\mathbf{g}}^{(1)}\dots U_{\mathbf{g}}^{(n)}\rho_{\mathbf{g}}|\Psi_{\{a;c,\mu\}}\rangle$$



e.g. for on-site symmetry $R_{\mathbf{g}} = \prod_{k\in I} R_{\mathbf{g}}^{(k)}$

Projective Representations

Recall: projective representations are classified by $H^2(G, \mathrm{U}(1))$

Multiplication is projective:

$$U_{\mathbf{g}} U_{\mathbf{h}} = \eta(\mathbf{g}, \mathbf{h}) U_{\mathbf{gh}}$$

Associativity gives cocycle condition:

$$\eta(\mathbf{g}, \mathbf{h}) \eta(\mathbf{gh}, \mathbf{k}) = \eta(\mathbf{g}, \mathbf{hk}) \eta(\mathbf{h}, \mathbf{k})$$

Coboundaries are "trivial" (mod out):

$$\eta(\mathbf{g}, \mathbf{h}) \sim \eta(\mathbf{g}, \mathbf{h}) \frac{\zeta(\mathbf{g}) \zeta(\mathbf{h})}{\zeta(\mathbf{gh})}$$

Symmetry Fractionalization

Local actions have projective form

$$U_{\mathbf{g}}^{(j)} \rho_{\mathbf{g}} U_{\mathbf{h}}^{(j)} \rho_{\mathbf{g}}^{-1} |\Psi_{a;c,\mu}\rangle = \eta_{a_j}(\mathbf{g},\mathbf{h}) U_{\mathbf{gh}}^{(j)} |\Psi_{a;c,\mu}\rangle$$

Similar to projective representations,
but with additional properties:

1. Symmetry action
2. Must be consistent with fusion $\prod_{j=1}^{n} \eta_{a_j}(\mathbf{g},\mathbf{h}) = 1$

Symmetry Fractionalization

Local actions have projective form

$$U_{\mathbf{g}}^{(j)} \rho_{\mathbf{g}} U_{\mathbf{h}}^{(j)} \rho_{\mathbf{g}}^{-1} |\Psi_{a;c,\mu}\rangle = \eta_{a_j}(\mathbf{g}, \mathbf{h}) U_{\mathbf{gh}}^{(j)} |\Psi_{a;c,\mu}\rangle$$

Results:

1. Possibly obstructed $[\mathfrak{O}] \in H^3_{[\rho]}(G, \mathcal{A})$

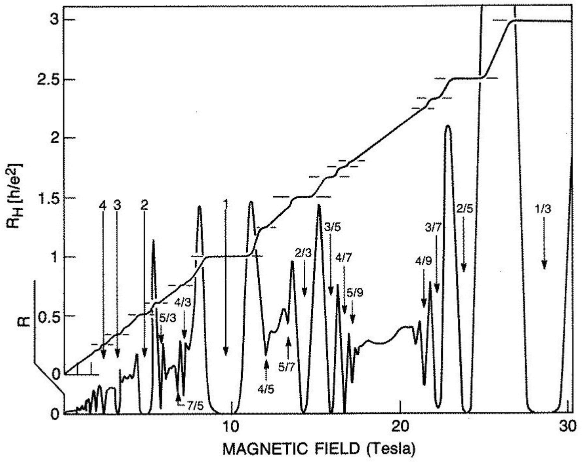
2. Classified (when unobstructed) by cohomology

$$H^2_{[\rho]}(G, \mathcal{A})$$

$\mathcal{A} =$ group formed by the Abelian anyons

Fractional Quantum Hall States

Laughlin 1983



$$\nu = \frac{1}{m}$$ Laughlin FQH states

<u>m even (bosonic)</u> $\quad \mathbb{Z}_m^{(1/2)}$

<u>m odd (fermionic)</u> $\quad \mathbb{Z}_{2m}^{(1)} = \mathbb{Z}_m^{(\frac{m+1}{2})} \times \cancel{\mathbb{Z}_2^{(1)}}$

$$\mathcal{A} = \mathbb{Z}_m$$

Consider $G = \mathrm{U}(1)$ charge conservation:

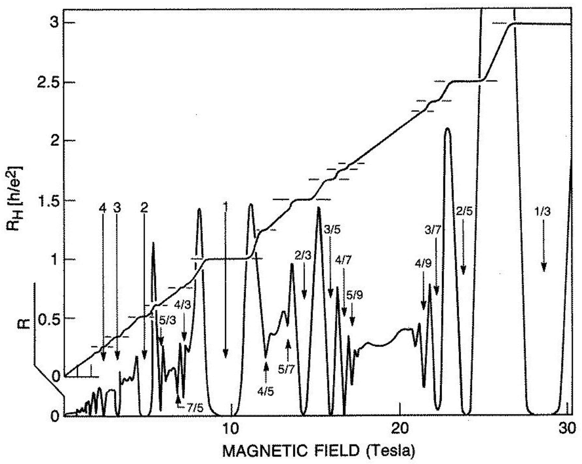
$$\rho_{\mathbf{g}}(a) = a$$

$$H^2_{[\rho]}(G, \mathcal{A}) = \mathbb{Z}_m$$

quasiparticle with topo charge *a* carries electric charge $\frac{pa}{m}e$
where $p \in \mathbb{Z}_m$

Fractional Quantum Hall States

Laughlin 1983



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$\mathcal{A} = \mathbb{Z}_m$

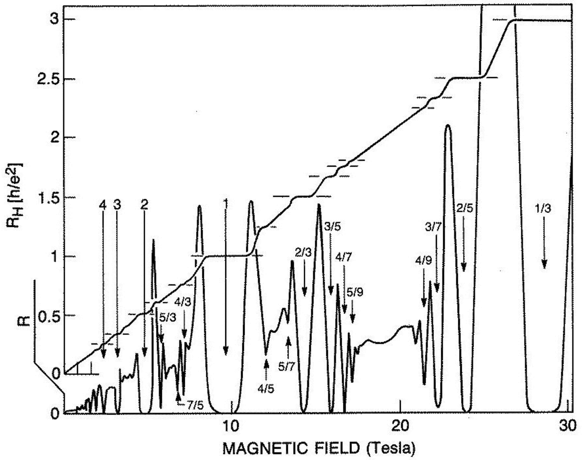
Consider $G = \mathbb{Z}_2$ qp-qh symmetry:

$\rho_{\mathbf{g}}(a) = [-a]$

$$H^2_{[\rho]}(G, \mathcal{A}) = 0$$

Fractional Quantum Hall States

Laughlin 1983



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$$\mathcal{A} = \mathbb{Z}_m$$

Consider $G = \mathbb{Z}_2$ qp-qh conj symmetry:

$$\rho_{\mathbf{g}}(a) = [-a]$$

$$H^2_{[\rho]}(G, \mathcal{A}) = \mathbb{Z}_2$$

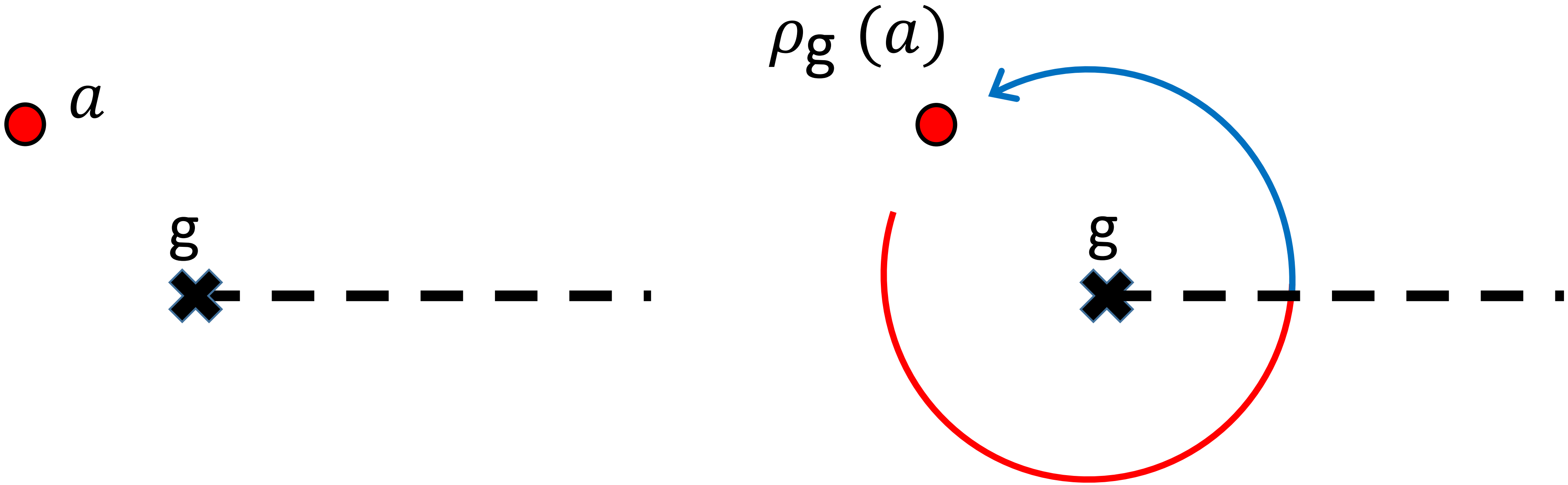
nontrivial fractionalization class assigns "half charges" of *G* symmetry to the odd integer topo charges (quasiparticles)

Symmetry Defects

Extrinsic (confined) objects that locally enact **g**-action

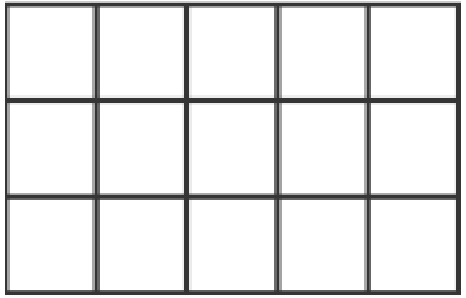
"symmetry fluxes"

$\mathbf{g} \in G$



Symmetry Defects

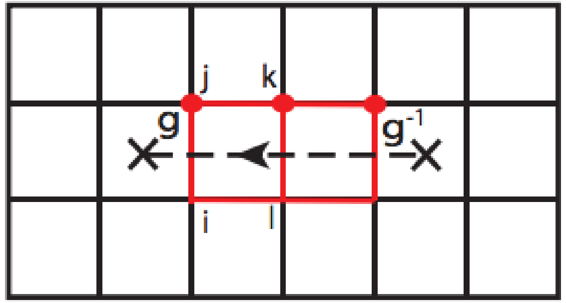
e.g. Lattice model with on-site symmetry: $R_{\mathbf{g}} = \prod_{k \in I} R_{\mathbf{g}}^{(k)}$

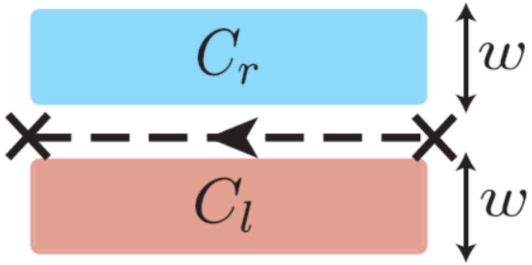


$$H_0 = \sum_i h_i + \sum_{\langle ij \rangle} h_{ij} + \sum_{[ijkl]} h_{ijkl}$$

Symmetry Defects

e.g. Lattice model with on-site symmetry: $R_{\mathbf{g}} = \prod_{k \in I} R_{\mathbf{g}}^{(k)}$





$$H_{\mathbf{g},\mathbf{g}^{-1}} = H_0 + \sum_{\substack{\langle ij\rangle: \\ i\in C_l; j\in C_r}} [R_{\mathbf{g}}^{(j)} h_{ij} R_{\mathbf{g}}^{(j)-1} - h_{ij}]$$

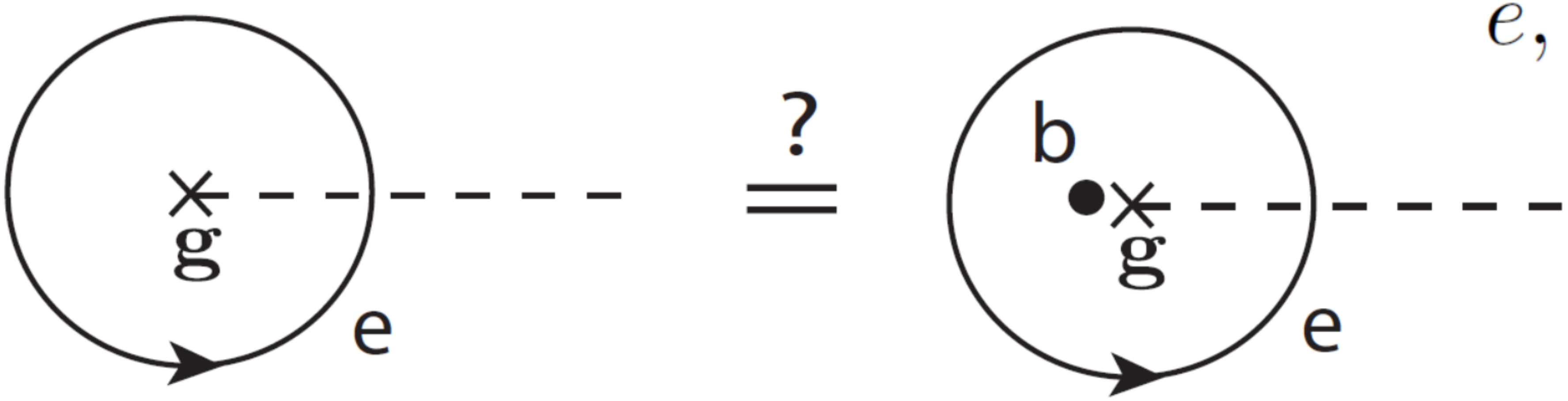
$$+ \sum_{\substack{[ijkl]: \\ i,l\in C_l; j,k\in C_r}} [R_{\mathbf{g}}^{(j)} R_{\mathbf{g}}^{(k)} h_{ijkl} R_{\mathbf{g}}^{(j)-1} R_{\mathbf{g}}^{(k)-1} - h_{ijkl}] \, .$$

Symmetry Defects

Effective theory described by G-crossed UMTC $\mathcal{C}_G^{\times}$

Distinct types of **g**-defects?

$e, b \in \mathcal{C}$

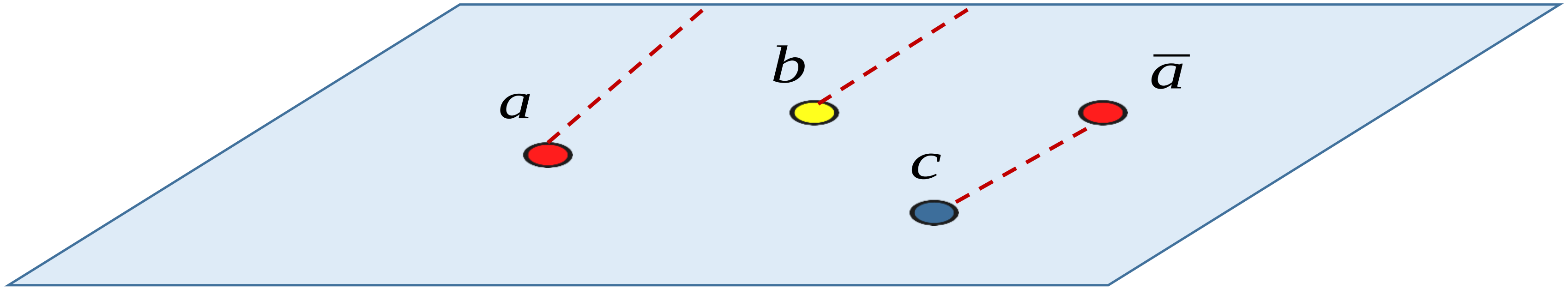


Symmetry Defects

Effective theory described by G-crossed UMTC $\mathcal{C}_G^{\times}$

Topological charges: $a, b, c, \ldots \in \mathcal{C}_{\mathbf{g}}$

(distinct types of **g**-flux)



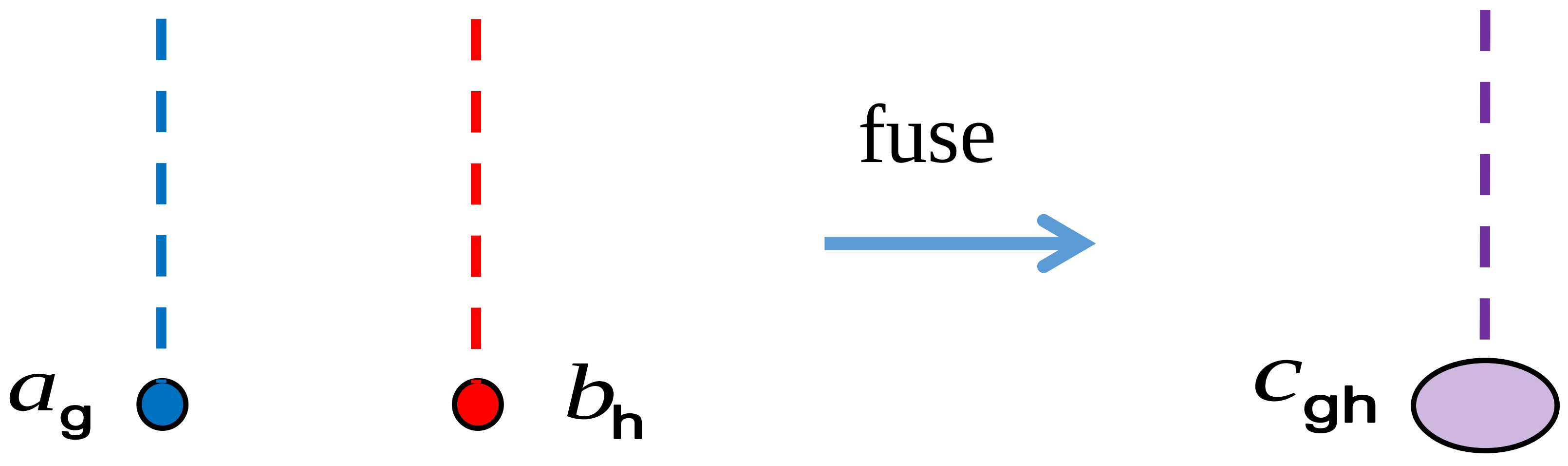
$|\mathcal{C}_{\mathbf{g}}| =$ # of **g**-invariant quasiparticle types in $\mathcal{C}_0 = \mathcal{C}$
(anyon topological charges)

Symmetry Defects

Effective theory described by G-crossed UMTC $\mathcal{C}_G^{\times}$

G-graded fusion: $$a_{\mathbf{g}} \times b_{\mathbf{h}} = \sum_c N_{ab}^c c_{\mathbf{gh}}$$

$a_{\mathbf{g}} \in \mathcal{C}_{\mathbf{g}},\ b_{\mathbf{h}} \in \mathcal{C}_{\mathbf{h}},\ \ldots$



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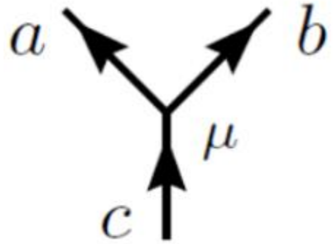
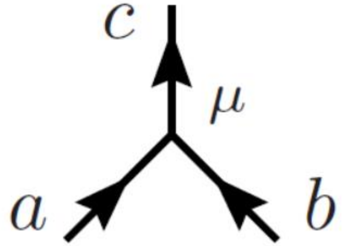
Topological state space (same as before, but G-graded):



$= |a, b; c, \mu\rangle \in V_c^{ab}$



$= \langle a, b; c, \mu| \in V_{ab}^c$

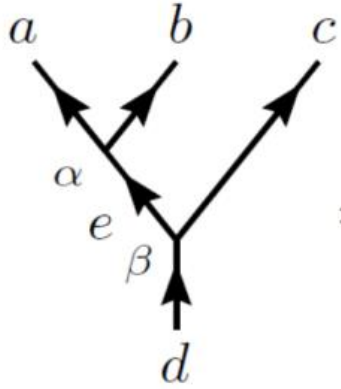
$\mu = 1, \ldots, N_{ab}^c$

Symmetry Defects

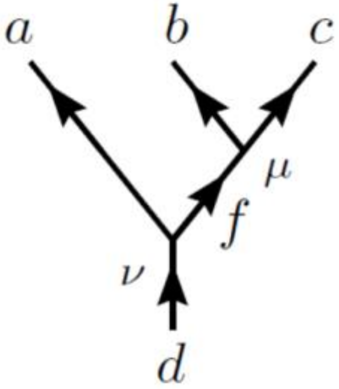
Effective theory described by G-crossed UMTC $\mathcal{C}_G^{\times}$

Associativity (same as before, but G-graded):

$$(a \times b) \times c = a \times (b \times c) \qquad \sum_e N_{ab}^{e} N_{ec}^{d} = \sum_f N_{af}^{d} N_{bc}^{f}$$



$$= \sum_{f,\mu,\nu} \left[F_d^{abc}\right]_{(e,\alpha,\beta)(f,\mu,\nu)}$$



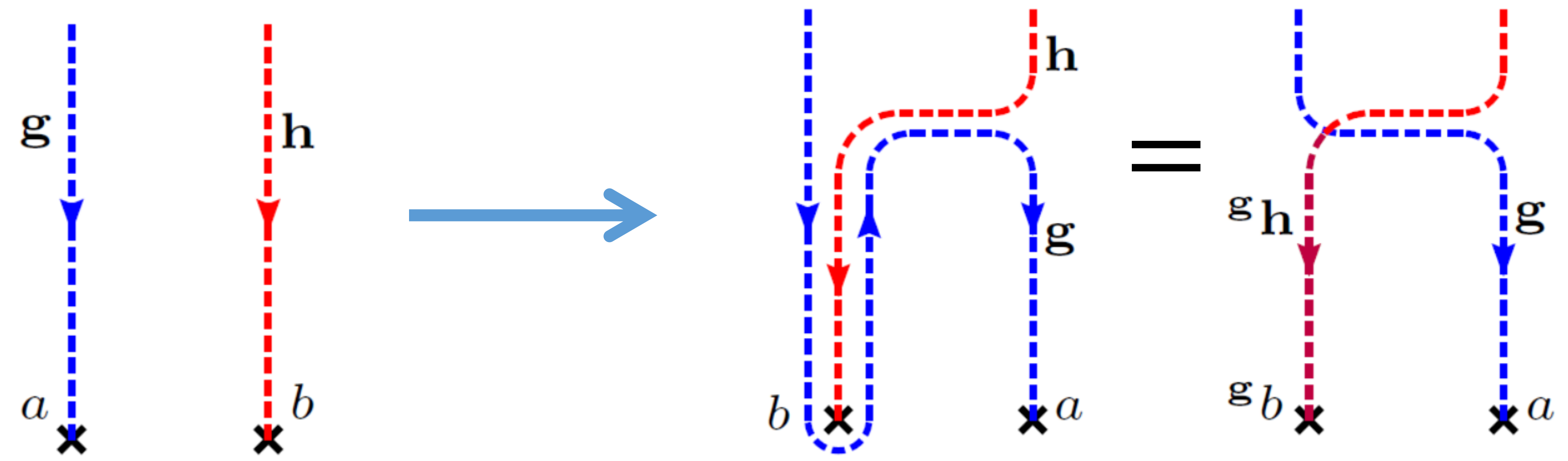
$$\mathcal{C}_G = \bigoplus_{\mathbf{g} \in G} \mathcal{C}_{\mathbf{g}}$$

$$\mathcal{C}_{\mathbf{0}} = \mathcal{C}$$

Symmetry Defects

Effective theory described by G-crossed UMTC $\mathcal{C}_G^\times$

Braiding:



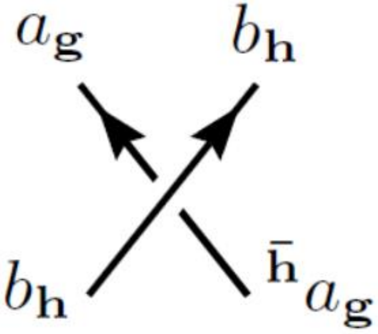
$${}^{\mathbf{g}}\mathbf{h} = \mathbf{g}\mathbf{h}\bar{\mathbf{g}} \qquad {}^{\mathbf{g}}b_{\mathbf{h}} \in \mathcal{C}_{\mathbf{g}\mathbf{h}\bar{\mathbf{g}}}$$

Symmetry Defects

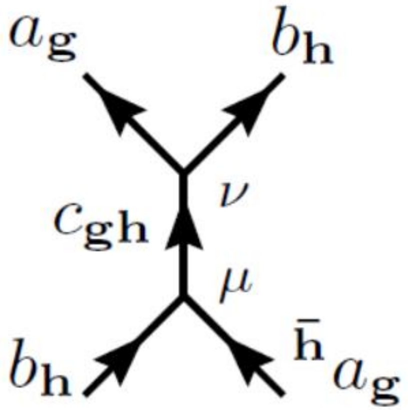
Effective theory described by G-crossed UMTC $\mathcal{C}_G^{\times}$

G-crossed braiding:

$$R^{a_{\mathbf{g}} b_{\mathbf{h}}} =$$



$$= \sum_{c,\mu,\nu} \sqrt{\frac{d_c}{d_a d_b}} \left[R^{a_{\mathbf{g}} b_{\mathbf{h}}}_{c_{\mathbf{gh}}} \right]_{\mu\nu}$$



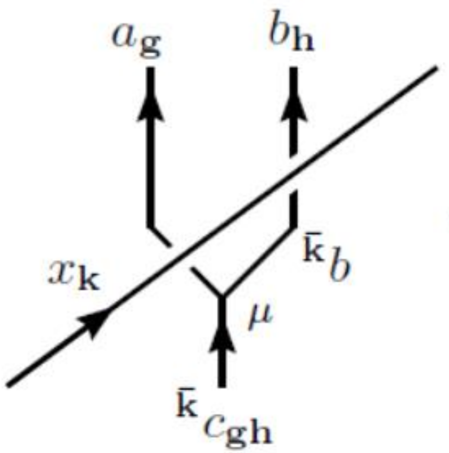
includes symmetry action on topological charges

(defect branch sheets implicitly into screen)

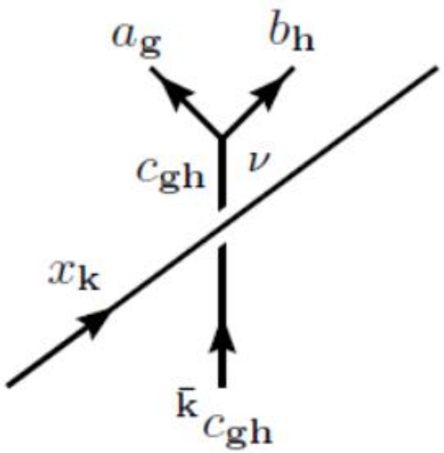
Symmetry Defects

Effective theory described by G-crossed UMTC $\mathcal{C}_G^{\times}$

Sliding lines over vertices:



$$= \sum_{\nu} \left[U_{\mathbf{k}}\left(a,b;c\right)\right]_{\mu\nu}$$

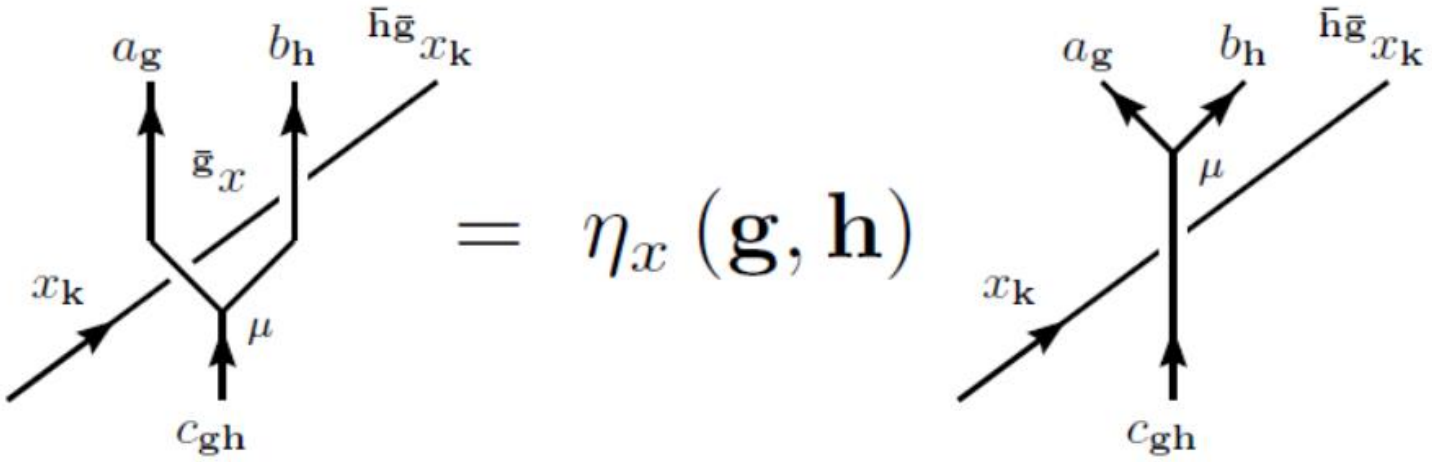


extension of symmetry action on topological theory

Symmetry Defects

Effective theory described by G-crossed UMTC $\mathcal{C}_G^\times$

Sliding lines under vertices:



extension of local projective form factors

Symmetry Defects

Effective theory described by G-crossed UMTC $\mathcal{C}_G^\times$

Solutions to consistency equations gives G-crossed extensions (symmetry enrichment) of the original topological phase $\mathcal{C}_0 = \mathcal{C}$

Classified by cohomology

(torsors)

$$\begin{cases} H^2_{[\rho]}(G, \mathcal{A}) & \text{(symm frac class)} \\ H^3(G, \mathrm{U}(1)) & \text{(defect/SPT)} \end{cases}$$

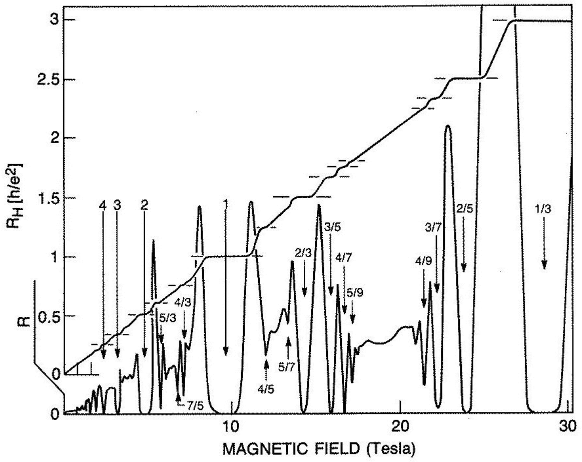
Possibly obstructed

$$\begin{cases} [\mathfrak{O}] \in H^3_{[\rho]}(G, \mathcal{A}) & \text{(symm frac class)} \\ [\Omega] \in H^4(G, \mathrm{U}(1)) & \text{(defect)} \end{cases}$$

Examples of Symmetry Defects

- Fluxes in symmetry protected topological (SPT) phases
- Majorana and Parafermion zero-modes
  (topological phase - superconductor heterostructures)
- "Genons" in multi-layer topological phases with layer interchange
- Defects in topological phases in lattice models with on-site symmetry
- Dislocations in topological phases with translation symmetry

Fractional Quantum Hall States



$$\nu = \frac{1}{m}$$ Laughlin FQH states

<u>m odd (fermionic)</u> $$\mathbb{Z}_{2m}^{(1)} = \mathbb{Z}_m^{(\frac{m+1}{2})} \times \cancel{\mathbb{Z}_2^{(1)}}$$

$$\mathcal{A} = \mathbb{Z}_m$$

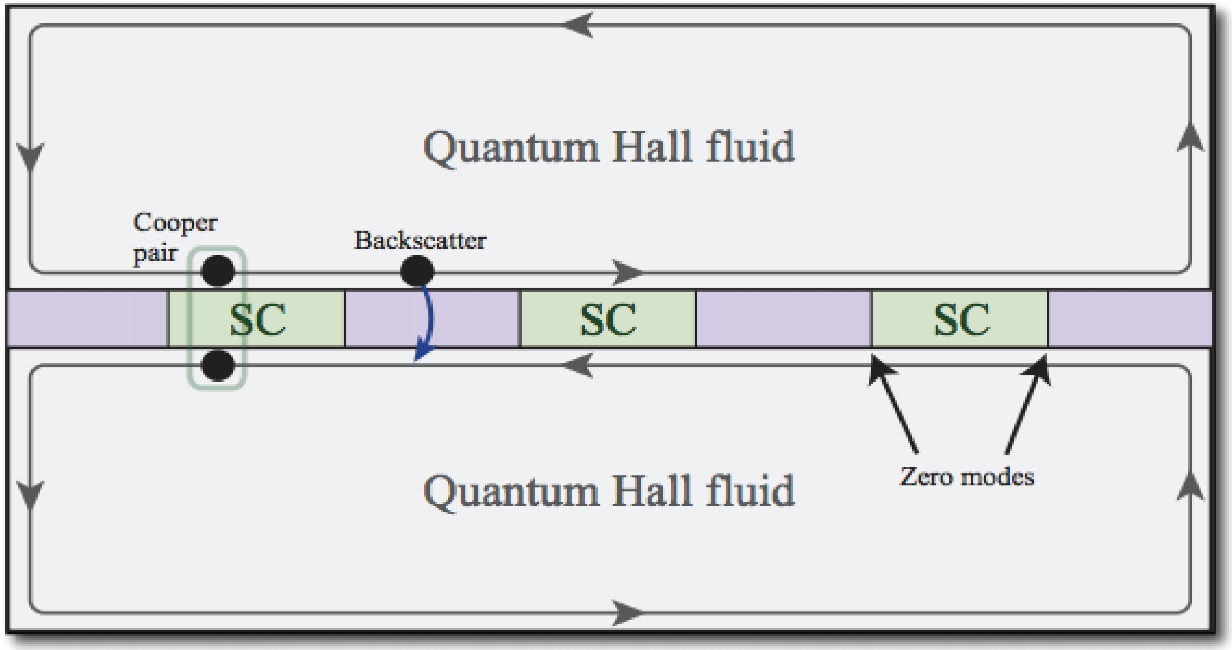
Consider $G = \mathbb{Z}_2$ qp-qh symmetry: $\rho_{\mathbf{g}}(a) = [-a]$

$$H^2_{[\rho]}(G, \mathcal{A}) = 0$$

$$H^3(G, \mathrm{U}(1)) = \mathbb{Z}_2$$

Fractional Quantum Hall States

qp-qh symmetry defects in FQH states may be obtained by interfacing FQH systems with superconductors



Clarke et al. 2012
Lindner et al. 2012
Cheng 2012

Parafermion zero modes

Fractional Quantum Hall States

$$\nu = \frac{1}{m}$$ Laughlin FQH states

<u>m odd (fermionic)</u>

$$\mathcal{C}_{\mathbf{0}} = \mathcal{A} = \mathbb{Z}_m$$

$$G = \mathbb{Z}_2$$

$$\rho_{\mathbf{g}}(a) = [-a]$$

$$\mathcal{C}_{\mathbf{g}} = \{\sigma\}$$

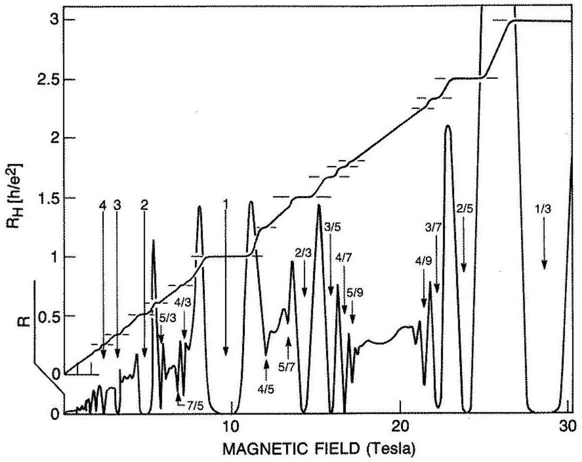
$$\sigma \times a \;=\; a \times \sigma = \sigma$$

$$\sigma \times \sigma \;=\; \sum_{a \in \mathcal{C}_0} a$$

$$[F^{\sigma\sigma\sigma}_{\sigma}]_{ab} = \pm \frac{1}{\sqrt{N}} e^{-i\frac{\pi(m+1)}{m}ab}$$

$$R^{\sigma\sigma}_{a} / R^{\sigma\sigma}_{0} = e^{i\frac{\pi(m+1)^2}{2m}a^2}$$

Fractional Quantum Hall States



$$\nu = \frac{1}{m}$$ Laughlin FQH states

<u>m even (bosonic)</u> $\mathbb{Z}_m^{(1/2)}$ $\mathcal{A} = \mathbb{Z}_m$

Consider $G = \mathbb{Z}_2$ qp-qh symmetry: $\rho_{\mathbf{g}}(a) = [-a]$

$$H^2_{[\rho]}(G, \mathcal{A}) = \mathbb{Z}_2$$

$$H^3(G, \mathrm{U}(1)) = \mathbb{Z}_2$$

Fractional Quantum Hall States

$$\nu = \frac{1}{m}$$ Laughlin FQH states

m even (bosonic)

$$\mathcal{C}_{\mathbf{0}} = \mathcal{A} = \mathbb{Z}_m$$

$$G = \mathbb{Z}_2$$

$$\rho_{\mathbf{g}}(a) = [-a]$$

$$\mathcal{C}_{\mathbf{g}} = \{\sigma_+, \sigma_-\}$$

$$\sigma_+ \times \sigma_+ = \sigma_- \times \sigma_- = \sum_{a \text{ even}} a \qquad \sigma_+ \times \sigma_- = \sum_{a \text{ even}} a$$

OR

$$\sigma_+ \times \sigma_- = \sum_{a \text{ odd}} a \qquad \sigma_+ \times \sigma_+ = \sigma_- \times \sigma_- = \sum_{a \text{ odd}} a$$

Gauging Symmetry

Promote symmetry to local gauge invariance

defects become deconfined quasiparticles
(anyons) described by a new UMTC $(\mathcal{C}_G^\times)^G$
fully determined by the defect theory

$$\mathcal{C} \underset{\text{Confinement}}{\overset{\text{Defectification}}{\rightleftarrows}} \mathcal{C}_G^\times \underset{\text{Condensation}}{\overset{\text{Gauging}}{\rightleftarrows}} (\mathcal{C}_G^\times)^G$$

Symmetry, Defects, and Gauging of Topological Phases

arXiv:1410.4540

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