Detecting topological order in the Heisenberg picture

A 1D numerical approach for 2D quantum systems

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Département de Physique
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Sydney Quantum Information Theory Workshop
Coogee, Australia, February 2016

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Detecting topological order

Given access to the ground state:

- Topological entanglement entropy $S(\rho_R) = \alpha|\partial R| - \gamma + \mathcal{O}(|R|^{-1})$
  - $\gamma = \log(\sqrt{\sum_c d_c^2})$.
  - Same γ for different TQFT (Heisenberg antiferromagnet on the Kagome).
  - $\gamma \neq 0$ with no topological order.
- Entanglement spectrum $\rho_R = e^{-H_{\text{eff}}}$.
- PEPS description of ground state.
  - String-like operators that pull through the tensors on the virtual level.

These all require access to the ground state

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Our contribution

- A numerical method to detect features of a TQFT without actually knowing the ground state!
- Can extract all the topological S matrix elements.
- The numerical problem boils down to 1D DMRG (at the operator level).
- The approach is not rigorous... it works better than it should!
- Perhaps it will fail for more challenging models.

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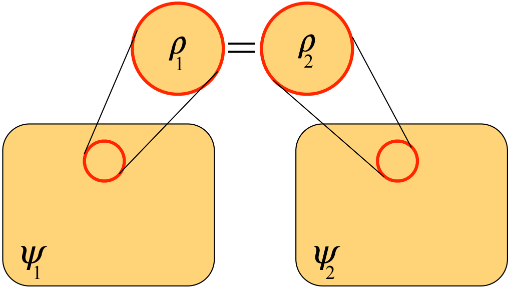
- A 2D system with a degenerate ground state.
- Degeneracy depends on the topology.
- No local order parameter
    - Impossible to discriminate between the different ground sates by looking at a small region.



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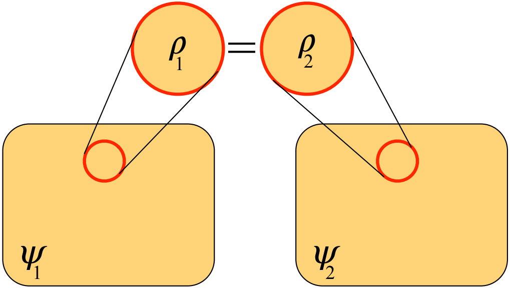

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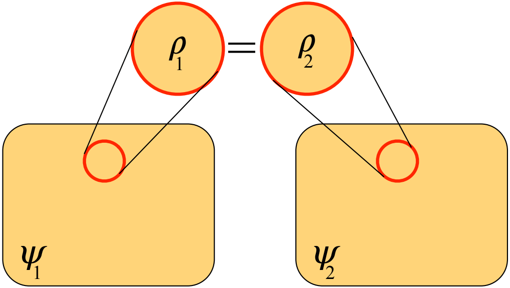

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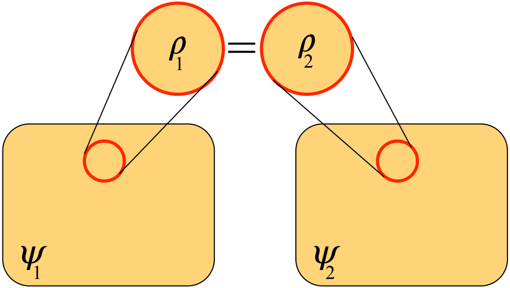

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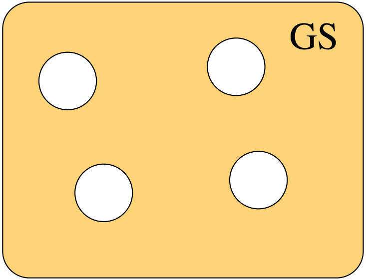
- Point-like excitations that can move freely.
- Topological charge defined by equivalent class of shallow quantum circuits (conservation).
- Non-trivial braiding.



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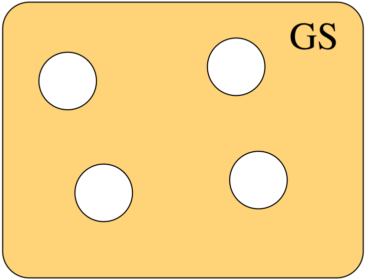

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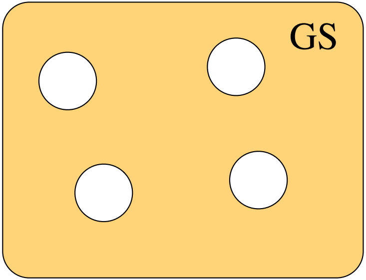

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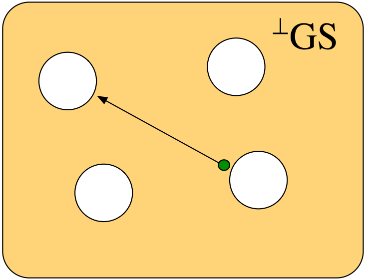

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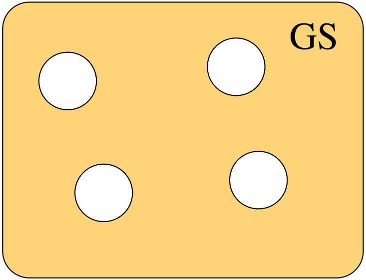

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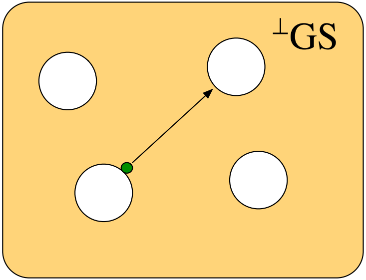

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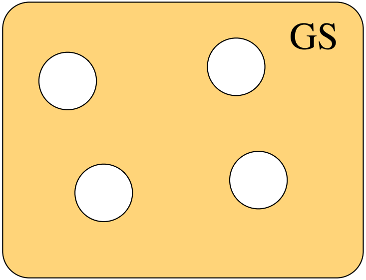

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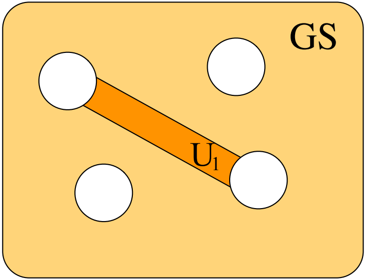

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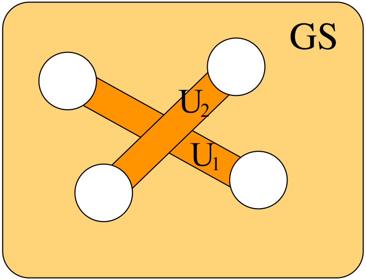

Degeneracy

- Suppose that there is a unique GS, ψ.
- Since U_1 maps GS to GS, we have $U_1|\psi\rangle = e^{i\phi_1}|\psi\rangle$.
- Since U_2 maps GS to GS, we have $U_2|\psi\rangle = e^{i\phi_2}|\psi\rangle$.
- This implies $U_1 U_2 \Pi_{\rm GS} = U_2 U_1 \Pi_{\rm GS}$.

$U_1 U_2 \Pi_{\rm GS} \neq U_2 U_1 \Pi_{\rm GS}$ implies that the ground state is degenerate.

- U_1 and U_2 are logical operators, i.e., operators which act inside this degenerate ground space.

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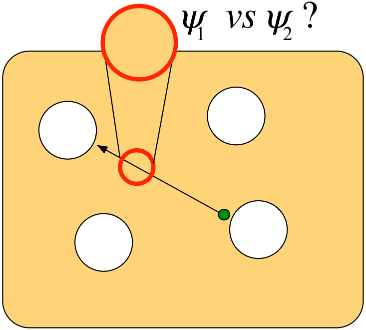
- Can I locally detect that the string operator U_1 has been applied?
  - This would enable me to learn information about which GS the system is in.
- No because the particle can always avoid any topologically trivial region.



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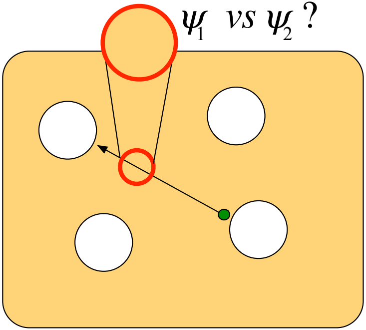

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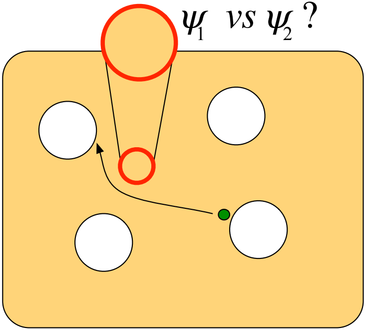

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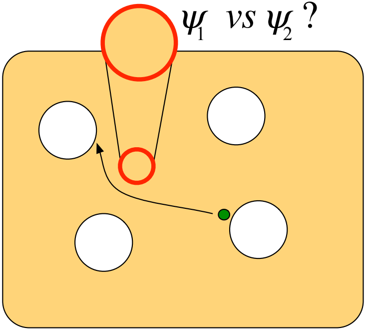

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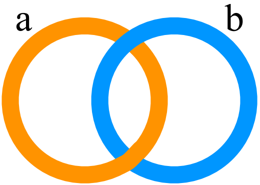
Topological data from string operators



- Ground space expectation of twist product $U_a \infty U_b \Pi_{GS} = \tilde{S}_{ab} \Pi_{GS}$ reveals (close cousin of) topological S-matrix element.
- Can be evaluated efficiently from a shallow circuit representation of $U_{a/b}$ or MPO representation.
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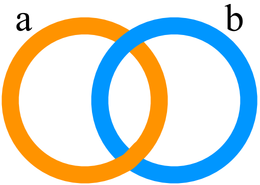

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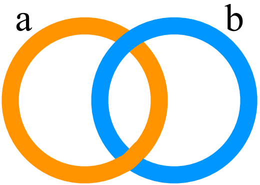

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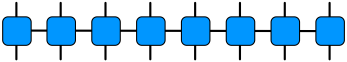
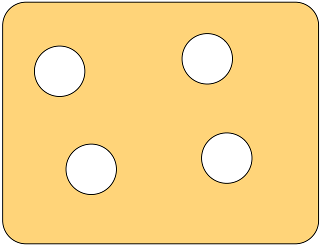
$$M = \sum_{j_1,j_2,\dots j_n} X^{j_1} \otimes X^{j_1,j_2} \otimes \dots \otimes X^{j_{n-1},j_n} \otimes X^{j_n}$$

$=$ 

When string operators arise from anyon propagation, it is an MPO.



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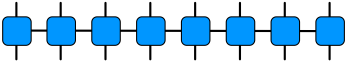
$=$ 

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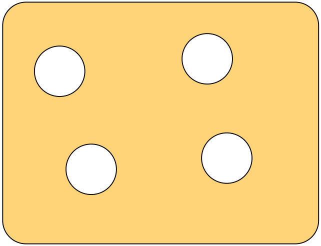

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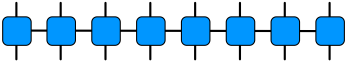
$=$ 

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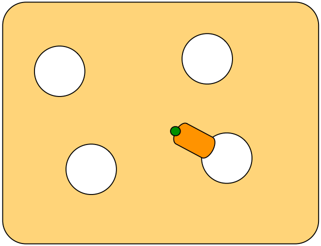

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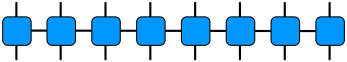

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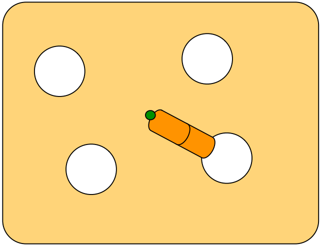

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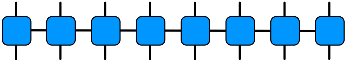

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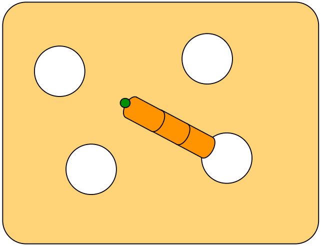

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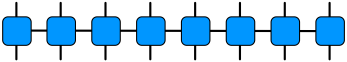

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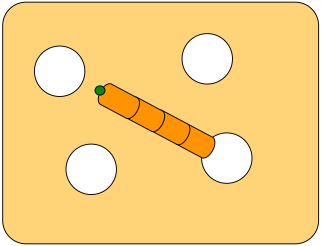

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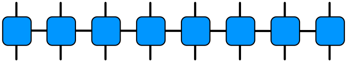
$=$ 

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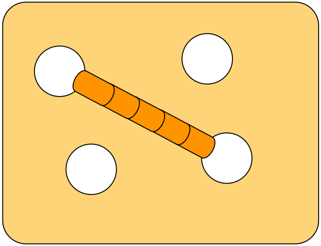

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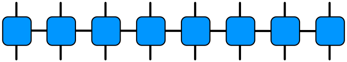

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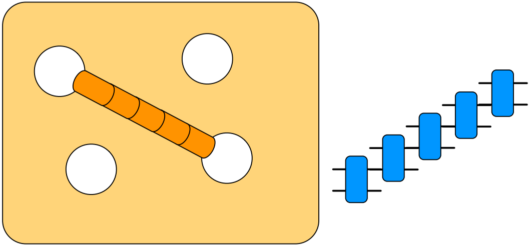

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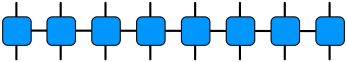
$=$ 

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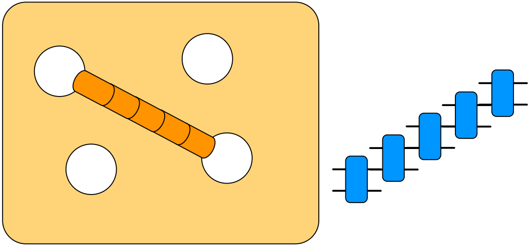

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- These exact MPO representations are special:
    - RG fixed points, commuting cases, etc.
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    - Dressed by perturbation theory.
    - MPO is fatten and bond dimension is increased.
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Outline

Objective function

Logical string-like operators should ...

- Be supported on a finite-width region R.
- Preserve the ground state: $[U_a^R, H]\Pi_{GS} = 0$
- Reveal non-trivial topological data $U_a^R U_b^{R'} \Pi_{GS} = \eta U_b^{R'} U_a^R \Pi_{GS}$.
- Be deformable, i.e. changing the location of R should not affect the above.

Objective function: $C(U_a, U_b, \eta) =$

$$\sum_{R \text{ crosses } R'} \|[H, U_a^R]\Pi_{GS}\|^2 + \|[H, U_b^{R'}]\Pi_{GS}\|^2 + \lambda \|U_a^R U_b^{R'} \Pi_{GS} - \eta U_b^{R'} U_a^R \Pi_{GS}\|^2$$

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- Vectorize matrices:
  - $M = |\phi\rangle\langle\psi| \rightarrow |M\rangle = |\phi\rangle \otimes |\psi\rangle$.
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Outline

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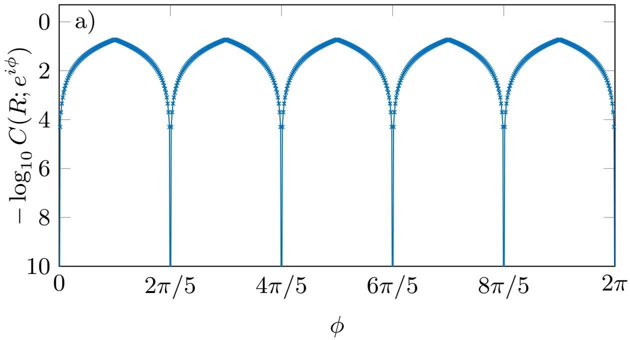
$$H = J \cdot Toric - \frac{h}{2}\sum j(X_j + X_j^\dagger) - \frac{\lambda}{4}\sum_{\langle j,k\rangle}(Z_j + Z_j^\dagger)(Z_k + Z_k^\dagger)$$



$\mathbb{Z}_5$, $\{J, h, \lambda\} = \{1, 0, 0\}$, $\chi = 5$, $w = 1$

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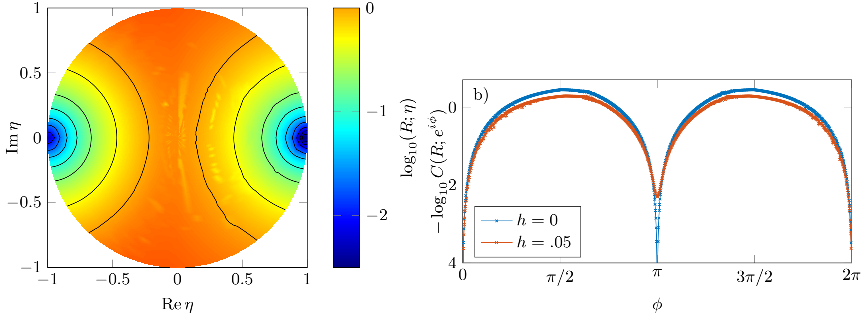

$\mathbb{Z}_2$, $\{J, h, \lambda\} = \{1, 0.05, 0\}$, $\chi = 5$, $w = 2$

Toric-Ising

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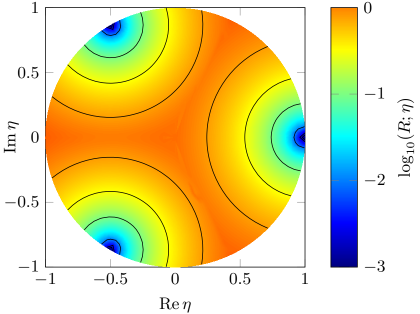
Alternating minimization



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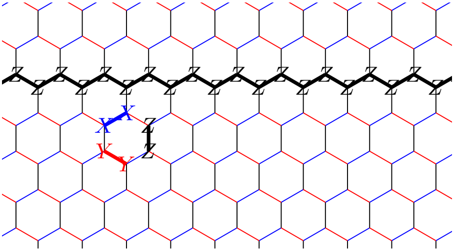
Honeycomb

$$H = -J_x \sum_{j,k \in \mathrm{x-link}} X_j X_k - J_y \sum_{j,k \in \mathrm{y-link}} Y_j Y_k - J_z \sum_{j,k \in \mathrm{z-link}} Z_j Z_k$$

$\mathbb{Z}_2$ phase for $0 < |J_x| + |J_y| < J_z$.



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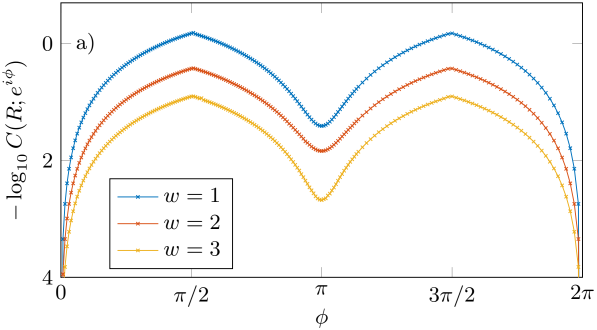

$$\{J_x, J_y, J_z\} = \{0.1, 0.1, 1\}, \chi = 5$$

Honeycomb

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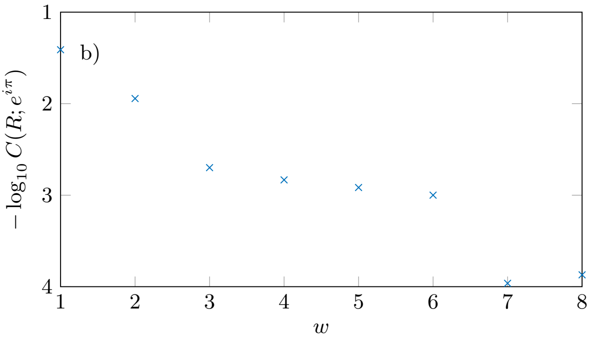

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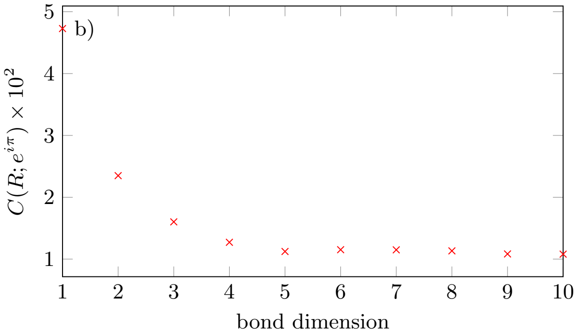

$$\{J_x, J_y, J_z\} = \{0.1, 0.1, 1\},\ w = 2$$

Honeycomb

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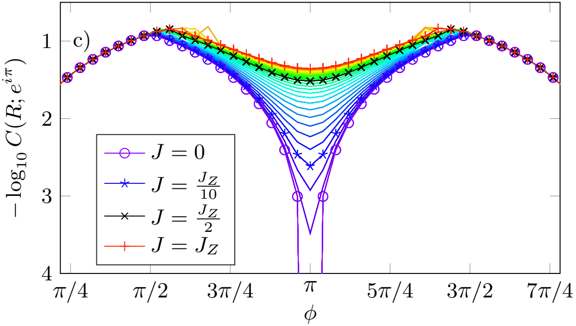
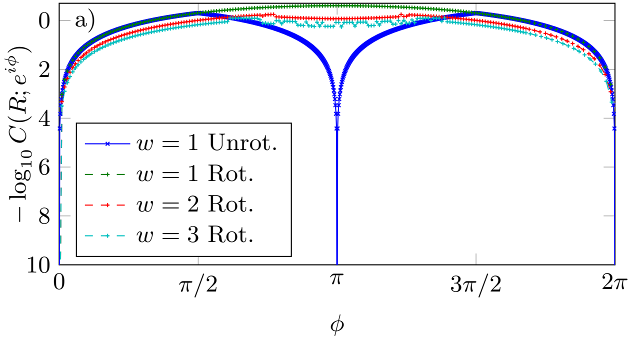


$$\{J_x, J_y, J_z\} = \{J, J, 1\}, \chi = 4, w = 3$$

Compass model – Not topologically ordered

$$H = -J_x \sum_{j,k \in \text{x}-\text{link}} X_j X_k - J_z \sum_{j,k \in \text{z}-\text{link}} Z_j Z_k$$

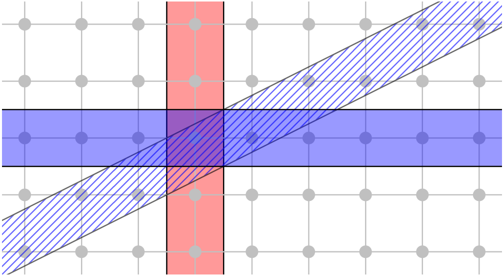


$J_x = J_z$, $\chi = 5$

Compass model – Not topologically ordered

$$H = -J_x \sum_{j,k \in \mathrm{x-link}} X_j X_k - J_z \sum_{j,k \in \mathrm{z-link}} Z_j Z_k$$

Supports vertical and horizontal logical operators,



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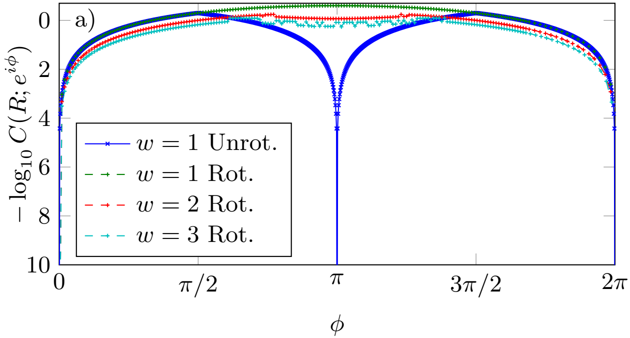
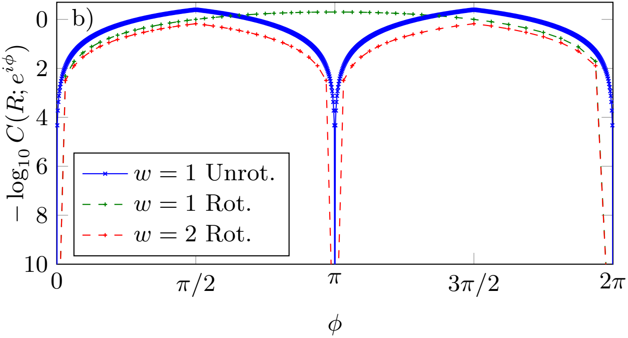


$J_x = J_z$, $\chi = 5$

Toric-Ising

$$H = J \cdot Toric - \frac{h}{2}\sum j(X_j + X_j^\dagger) - \frac{\lambda}{4}\sum_{\langle j,k\rangle}(Z_j + Z_j^\dagger)(Z_k + Z_k^\dagger)$$



$\mathbb{Z}_2$, $\{J, h, \lambda\} = \{1, 0, 0\}$, $\chi = 5$

Outline

Why does it work?

- $\Pi_{GS} U_a^R \Pi_{GS} \Rightarrow U_a^R$ enforce commutation relation on entire spectrum.
- $\Pi_{GS} U_a^R \Pi_{GS} \approx \exp\{-H/\Delta\} U_a^R \exp\{-H/\Delta\}$.
- For a local Hamiltonian, $\exp\{-H/\Delta\}$ maps a ribbon MPO to a (fatter and heavier) ribbon MPO.

If the commutations relations can only be achieved on the low energy sector, given enough width and bond dimension, the minimization problem should output the projected ribbon operator $\Pi_{GS} U_a^R \Pi_{GS}$.

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- Numerically equivalent to 1D DMRG.
- Why does it work at all?
  - Why can we replace ground-state expectations by operator equalities?
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  - Can substitute the twist product by group commutator (simpler).
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