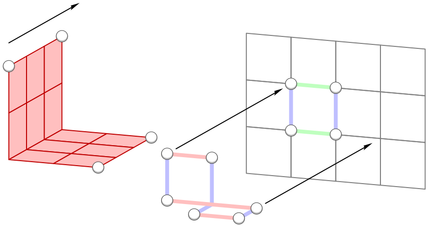
Parallelized quantum error correction with fracton topological codes

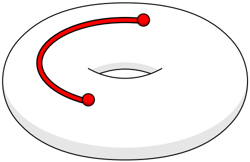
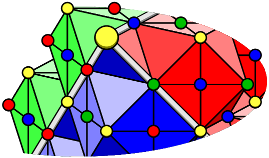
Ben Brown



joint work with Dom Williamson

Decoding

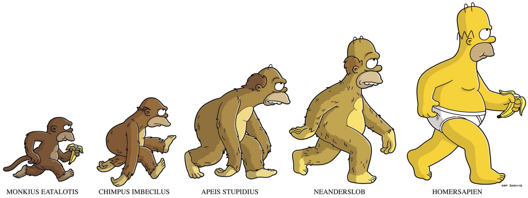
- Decoders correct errors
- they use the locations of excitations to estimate the error
- This may be slow as we grow the code
- We explore **parallelizable** codes to speed up decoders


Parallelized quantum error correction

Parallelized quantum error correction is a *missing link* in topological decoders

- Less evolved than self-correcting codes with cellular automata decoders
- Fills the divide between common 2D topological codes and self correction
- Cousin to fault-tolerant single-shot error correction
- **Completing the picture of evolution gives new insights into decoding**



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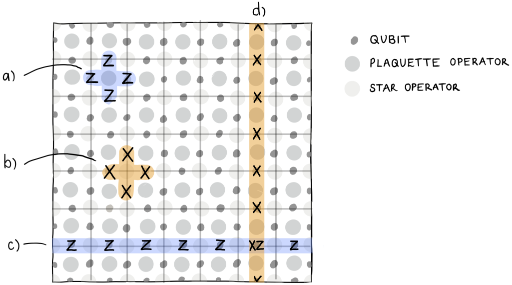
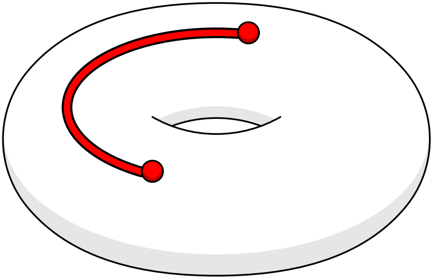
Toric code

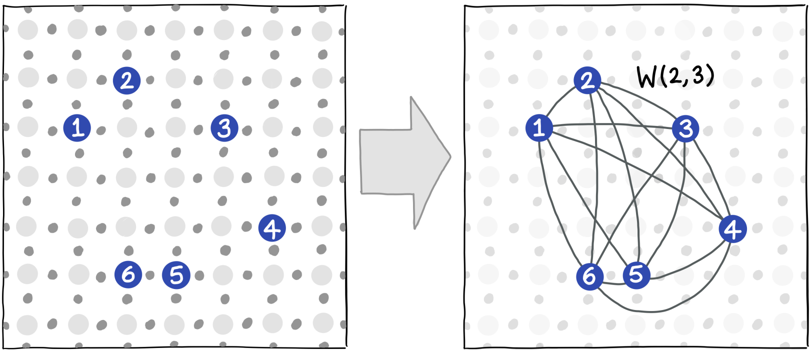
The toric code has point defects at the end points of string errors





Decoding the toric code

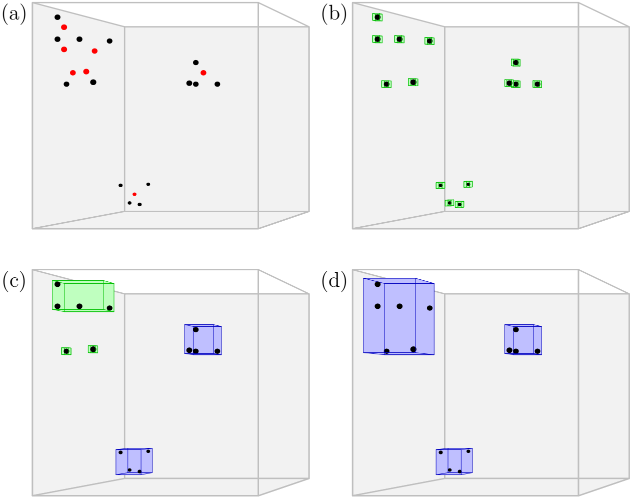
The toric code is decoded using matching



Matching pairs up defects that can be pairwise corrected

Decoding general topological codes

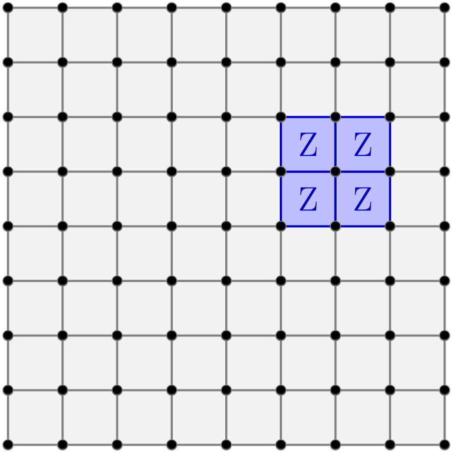
Generically, we can use clustering to decode topological codes



Clustering scales like the code volume - slow

Parallelized decoding

A simple classical example – the eight-vertex model

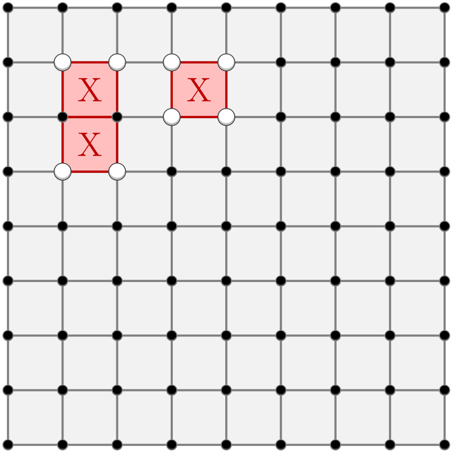


Features of the code:

- Spins on the faces
- Four-body stabilizers on the faces that touch every vertex

Parallelized decoding

A simple classical example – the eight-vertex model

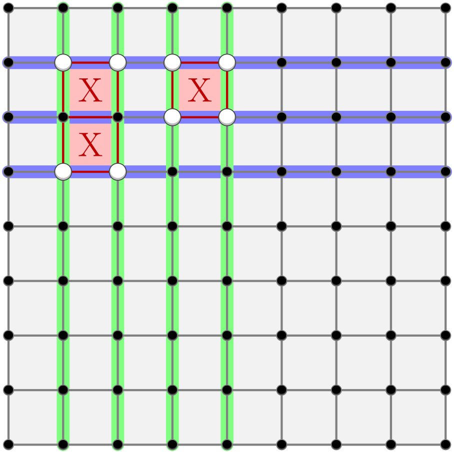


Features of errors and its defects:

- Stabilizer defects light up at the corners of the face errors
- Defects occur in pairs along horizontal and vertical lines

Parallelized decoding

A simple classical example – the eight-vertex model

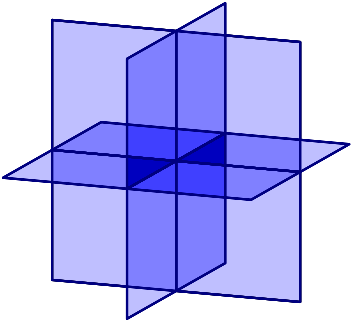


Decoding:

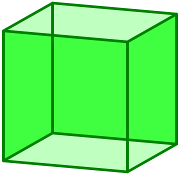
- With this structure we can perform one-dimensional matching
- Edges from the matching form the boundaries of the error

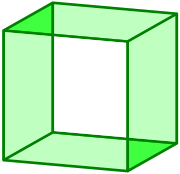
X-cube model

The X-cube model is three-dimensional with qubits on the faces of a cubic lattice



Pauli-X stabilizer

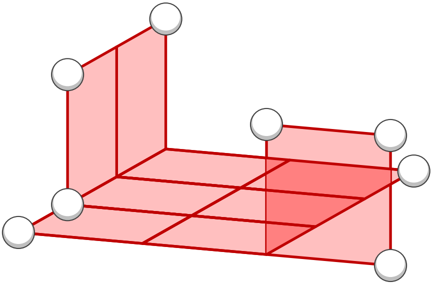


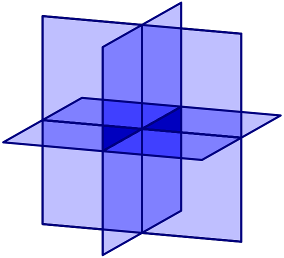


Pauli-Z stabilizer

Planeons

Pauli-Z operators create planeons



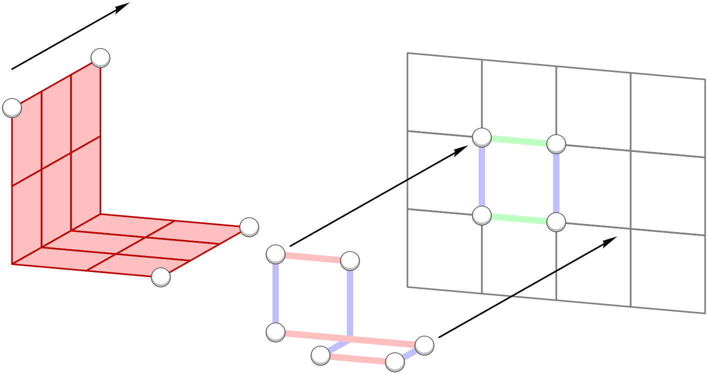


Decoding planeons

Planeons are corrected in clusters that respect their planar symmetries

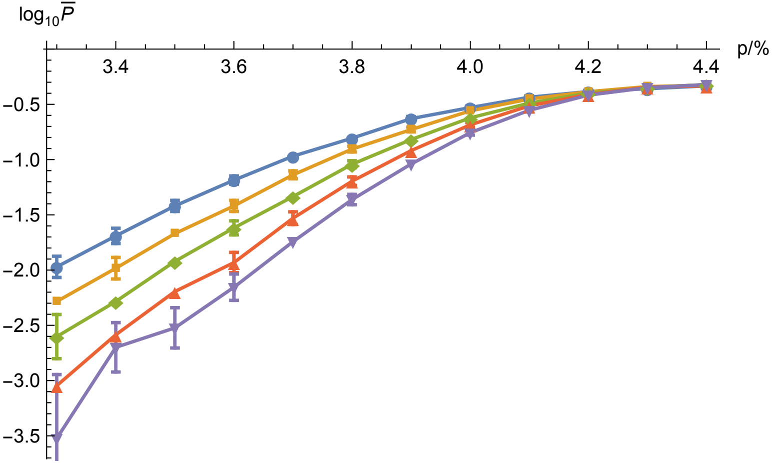
We can find a Pauli operator to correct a cluster of defects if

- Every defect is paired with another on an *xy* plane
- Every defect is paired with another on an *yz* plane
- Every defect is paired with another on an *zx* plane



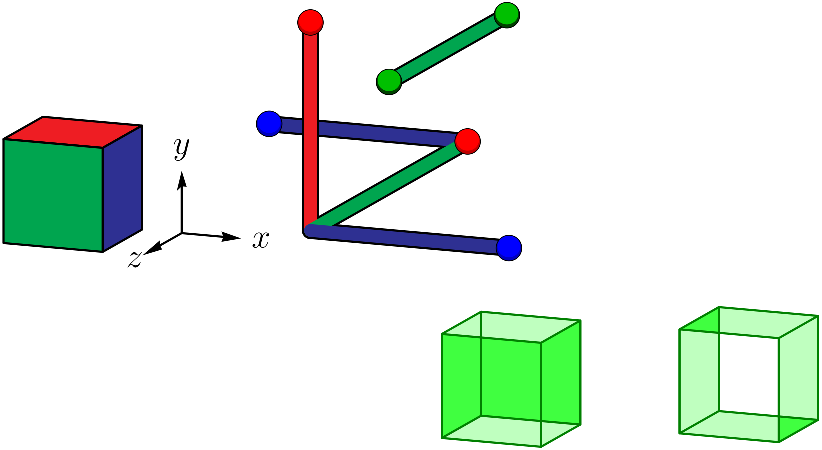
Threshold for planeons

The threshold for planeons is $\sim 4.3\%$



Lineons

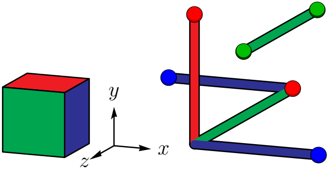
Pauli-X operators create lineons

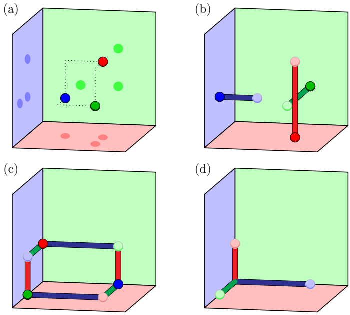


Decoding lineons

We can decode lineons with groups of defects if

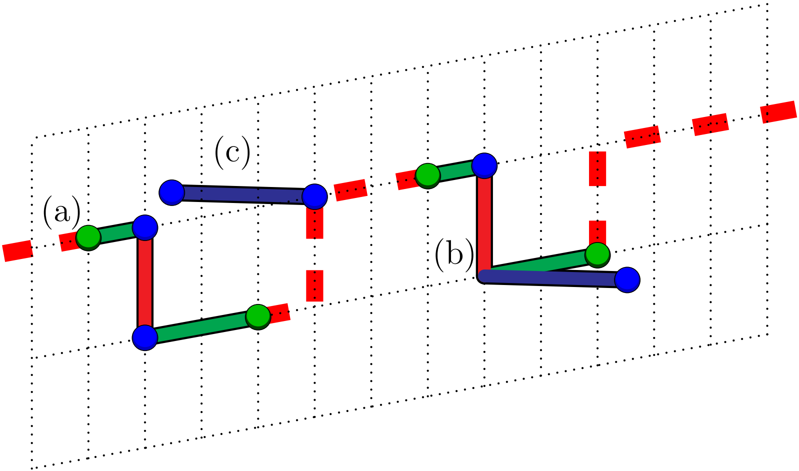
- Every red defect is matched with a red or a blue vertex on its plane of constant z
- Every red defect is matched with a red or a green vertex on its plane of constant x
- Every green defect is matched with a blue or a green vertex on its plane of constant y
- ... and so on





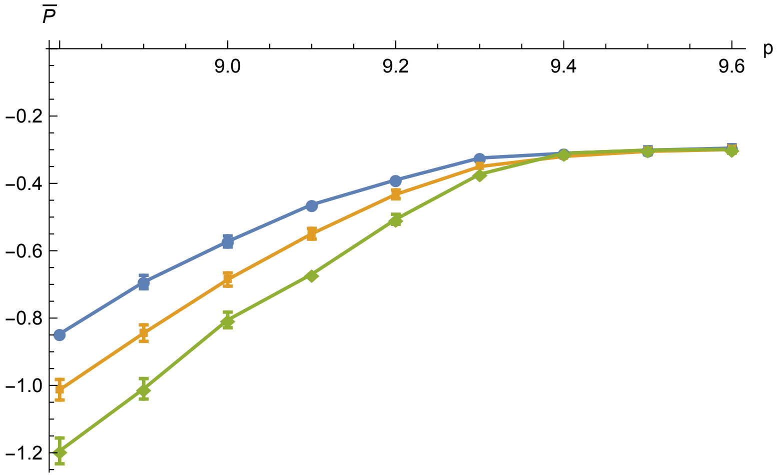
Decoding lineons

We match over planes, not the volume of the lattice, to find groups of defects as above



Threshold with parallelized error correction

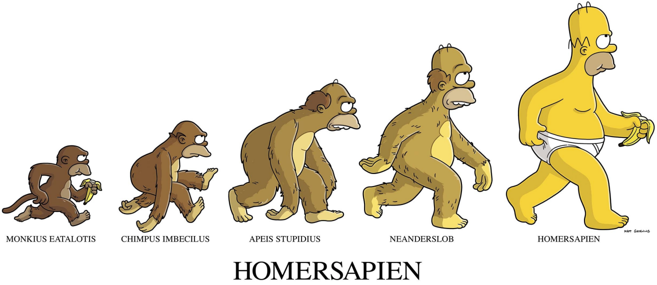
The threshold is close to that of the toric code, as we may expect



Emergent symmetries

We decoded the X-cube model using its emergent symmetries

We decode a lot of code[1] using emergent symmetries

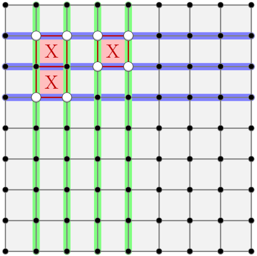


[1] a **LOT** of codes

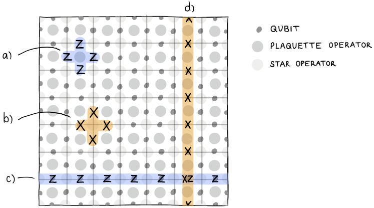
Emergent symmetries

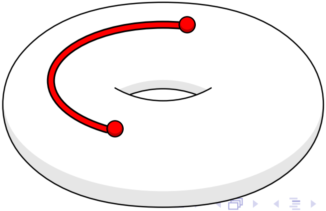
Emergent symmetries promise charge conservation

Eight-vertex model – 1D charge conservation symmetries



Toric code – global charge conservation symmetry

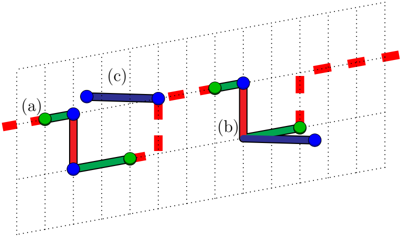


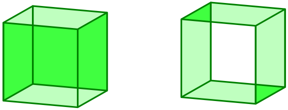


Emergent symmetries

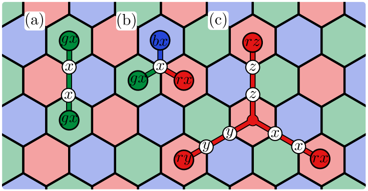
Emergent symmetries promise charge conservation on planes

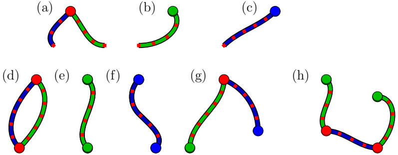
X-cube model planar charge conservation symmetries





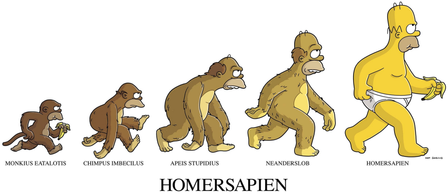
Color code – emergent symmetry among pairs of colored defects





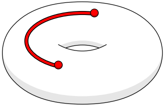
Parallelized quantum error correction

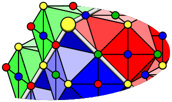
Parallelized quantum error correction is a *missing link* in topological decoders

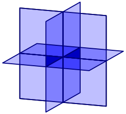


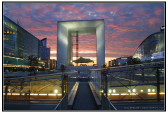
2D toric code
(global symmetries)

X-cube model
(planar symmetries)







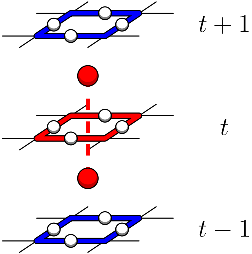


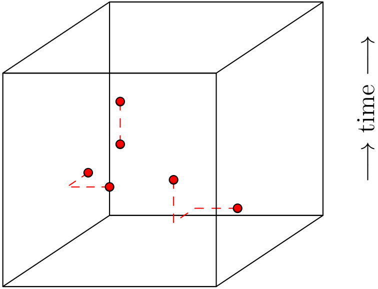
gauge color code
(temporal planar symmetries – single-shot)

4D toric code
(local symmetries)

Single-shot error correction

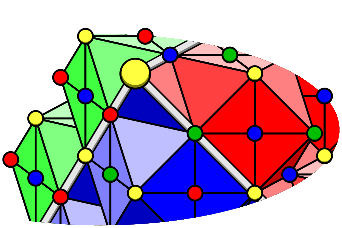
Single-shot error correction is temporally parallelized

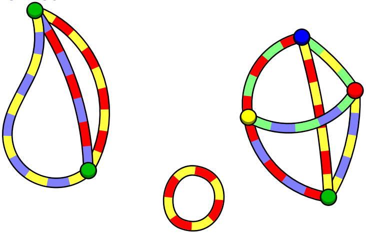


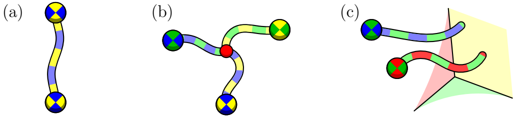


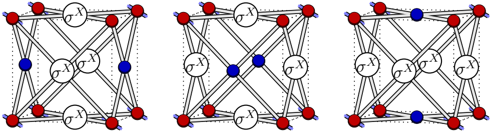
Single-shot error correction

Single-shot error correction is temporally parallelized



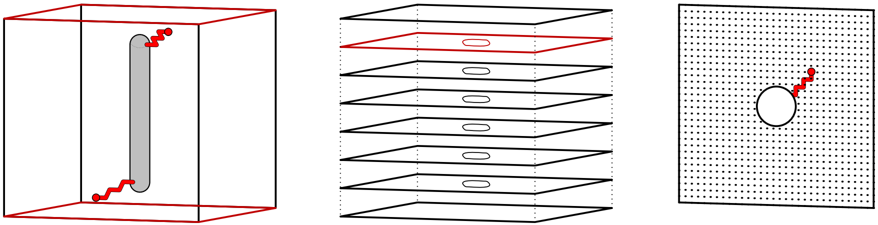






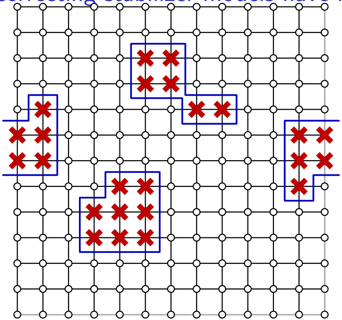
Correlated errors with parallelized quantum error correction

Resilience of single-shot codes to time-correlated errors is understood by parallelization



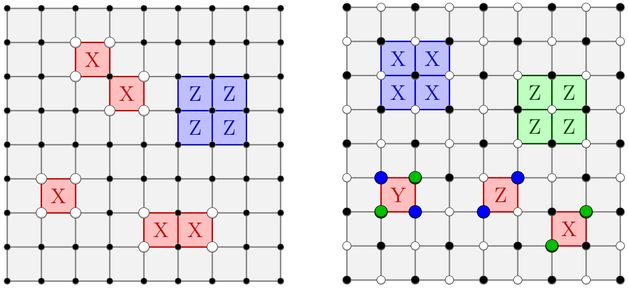
Local decoders with self-correcting memories

Self-correcting stabilizer models have local emergent symmetries



Biased noise on the surface code

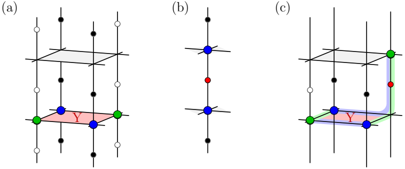
Decoding Pauli-Y errors on the surface code is like decoding the eight-vertex model

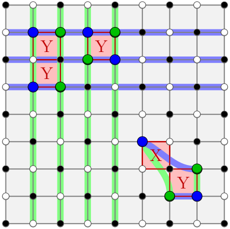


Biased noise on the surface code

The parallelized decoder is generalizable to noisy measurements and finite bias

We deal with measurement errors in the standard way
The model has 2D symmetries in 2+1D spacetime





We generalize to a **weakly symmetry respecting decoder** to decode finite bias

The cubic code model

The cubic code may be more parallelizable than the X-cube model or the toric code