Entanglement, 2-dimensional gravity and tensor networks

Nele Callebaut

Ghent University, Verstraete group & Princeton University

Based on 1808.10431, with Herman Verlinde
and 1808.05583

February 8, 2019

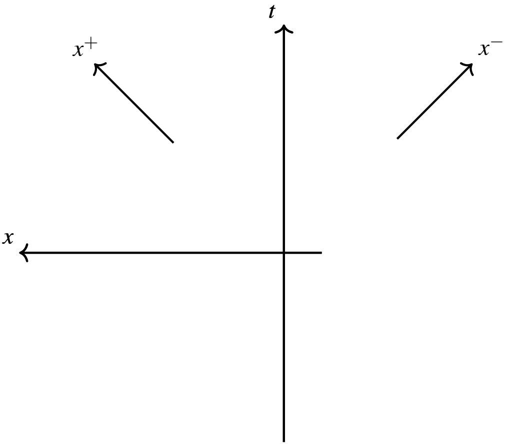
Research Foundation Flanders
Opening new horizons

Outline

- Set-up: bCFT_2 on $ds^2 = -dt^2 + dx^2$, $x \geq 0$
- Goal: develop theory of entanglement dynamics
- Result: entanglement dynamics of bCFT_2 is described by JT gravity
- Entanglement renormalization in bCFT described by Schwarzian QM – connection to cMERA
- Example of 'geometry from entanglement'
- Future directions

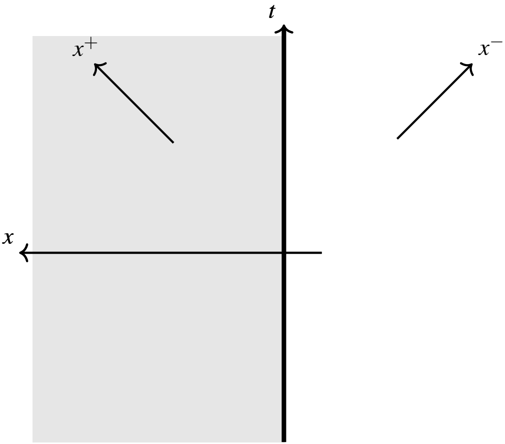
Set-up 1

CFT$_2$ on $ds^2 = -dt^2 + dx^2 = -dx^+ dx^-$ in vacuum state $|0\rangle$



Set-up 2

bCFT_2 on $ds^2 = -dt^2 + dx^2 = -dx^+ dx^-$, $x \geq 0$ in state $|0\rangle$



Set-up 3

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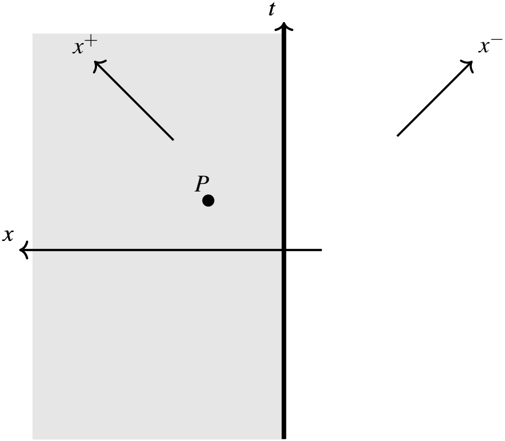
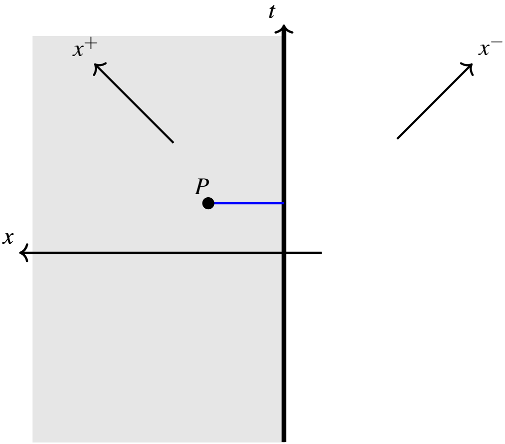


point P at (x_P^+, x_P^-)

Set-up 4

bCFT$_2$ on $ds^2 = -dt^2 + dx^2 = -dx^+ dx^-$, $x \geq 0$ in state $|0\rangle$



point P at (x_P^+, x_P^-)
$\sim$ interval

Set-up 5

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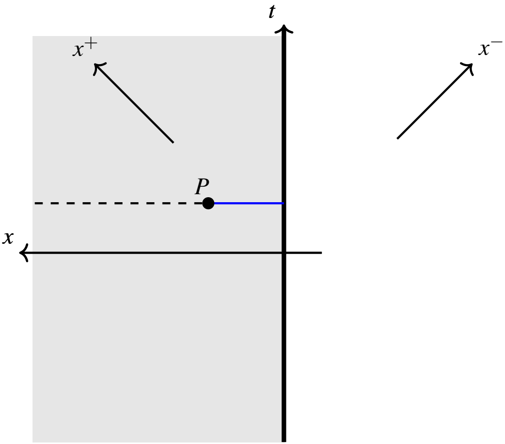

point P at (x_P^+, x_P^-)
$\sim$ interval

$$S_P = -\mathrm{tr}(\rho_A \log \rho_A)$$
$$= \frac{c}{6} \log \frac{x_P}{\delta} = \frac{c}{6} \log \frac{x_P^+ - x_P^-}{2\delta}$$
$$= S_P(x_P^+, x_P^-)$$

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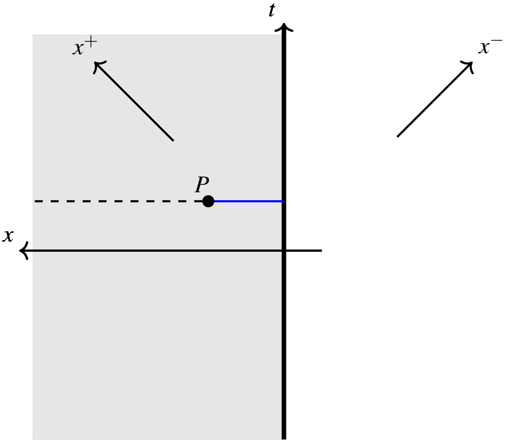

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local field $S_P(x_P^+, x_P^-)$ $\Rightarrow$ dynamics of S_P?

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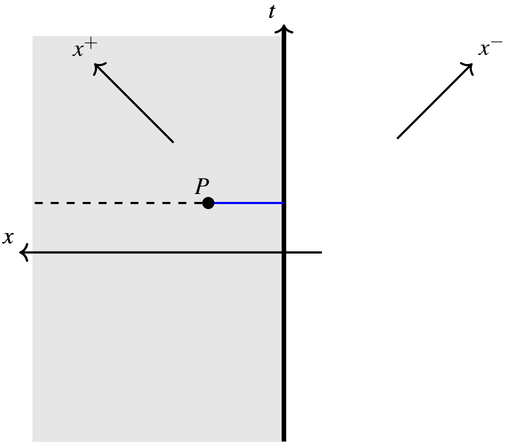

point P at (x^+, x^-)
$\sim$ interval

$$S = -\mathrm{tr}\rho_A \log \rho_A$$
$$= \frac{c}{6} \log \frac{x}{\delta} = \frac{c}{6} \log \frac{x^+ - x^-}{2\delta}$$
$$= S(x^+, x^-)$$

local field $S(x^+, x^-)$ $\Rightarrow$ dynamics of S?

Entanglement

$$S(x^+, x^-) = \frac{c}{6} \log \frac{x^+ - x^-}{2\delta}$$

obeys

$$\partial_+ \partial_- \left(\frac{12}{c} S \right) = \frac{1}{2\delta^2} e^{-\frac{12}{c} S}$$

Goal: develop a field theory for the entanglement ("theory of entanglement dynamics") that reproduces this equation as an EOM

Inspiration from tensor networks

Entanglement

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obeys

$$\partial_+ \partial_- \left(\frac{12}{c} S \right) = \frac{1}{2\delta^2} e^{-\frac{12}{c} S}$$

Goal: develop a field theory for the entanglement ("theory of entanglement dynamics") that reproduces this equation as an EOM

Inspiration from tensor networks

$$\boxed{S \quad \rightsquigarrow \quad \text{(2D) metric } g}$$

$$\Rightarrow \text{ dynamics of entanglement} = \text{dynamics of (2D) geometry}$$
$$= \text{(2D) gravity}$$

Dilaton gravity

Einstein gravity

$$I[g,\phi_m] = \int dxdt\,\sqrt{g}(R+\Lambda) + I_m[g,\phi_m]$$

is trivial in 2D: EOM doesn't allow non-zero cosmological constant Λ or conformal matter stress tensor because $G_{\mu\nu} = 0$.

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Dilaton gravity

$$I[g, \Phi, \phi_m] = \int dxdt \sqrt{g} \left\{\Phi R + V(\Phi)\right\} + I_m[g, \Phi, \phi_m]$$

Dilaton gravity

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Dilaton gravity

$$I[g,\Phi,\phi_m] = \int dxdt\,\sqrt{g}\,\{\Phi R + V(\Phi)\} + I_m[g,\Phi,\phi_m]$$

Jackiw-Teitelboim (JT) dilaton gravity

$$I[g,\Phi,\phi_m] = \int dxdt\,\sqrt{g}\,\Phi(R+\Lambda) + I_m[g,\phi_m]$$

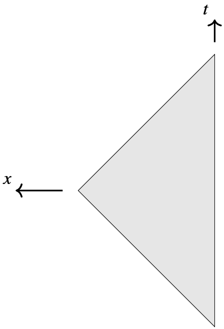
JT gravity and entanglement

$$I_{JT}[g, \Phi, \phi_m] = \int dxdt \sqrt{g}\, \Phi(R + \Lambda) + I_m[g, \phi_m]$$

The EOM following from variation wrt Φ is

$$R = -\Lambda.$$

JT metric solution is always AdS_2!



$$ds^2 = \ell^2 \frac{(-dt^2 + dx^2)}{x^2}$$

JT gravity and entanglement

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JT metric solution is always AdS_2!

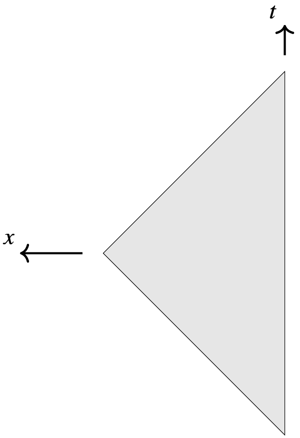
In conformal gauge $g_{\mu\nu} = e^{\omega(x^+,x^-)}\eta_{\mu\nu}$ (or $ds^2 = -e^{\omega}dx^+dx^-$) and $\Lambda = \frac{2}{\ell^2}$

$$\partial_+\partial_-\omega + \frac{1}{2\ell^2}e^{\omega} = 0.$$

$$\partial_+\partial_-\left(\frac{12}{c}S\right) = \frac{1}{2\delta^2}e^{-\frac{12}{c}S}$$

$$\boxed{\omega = -\frac{12}{c}S + 2\log\frac{\ell}{\delta}} \qquad \boxed{ds^2 = -\left(\frac{\ell}{\delta}\right)^2 e^{-\frac{12}{c}S(x^+,x^-)}dx^+dx^-}$$

JT entanglement dynamics of bCFT: metric



Part 1 of our JT entanglement dynamics theory: metric is AdS_2 determined by entanglement of the bCFT.

JT gravity and modular Hamiltonian

$$I_{JT}[g,\Phi,\phi_m] = \int dxdt\,\sqrt{g}\,\Phi(R+\Lambda) + I_m[g,\phi_m]$$

The EOM following from variation wrt $g_{\mu\nu}$ are

$$\begin{aligned}
&\partial_+\Phi\partial_+\omega - \partial_+^2\Phi = 8\pi G T^m_{++} && (-\nabla_+^2\Phi = 8\pi G T^m_{++})\\
&\partial_-\Phi\partial_-\omega - \partial_-^2\Phi = 8\pi G T^m_{--} && (-\nabla_-^2\Phi = 8\pi G T^m_{--})\\
&\partial_+\partial_-\Phi + \frac{\Lambda}{4}e^{\omega}\Phi = 0 && (\Box\Phi - \Lambda\Phi = 0).
\end{aligned}$$

What object in bCFT_2 obeys these EOM?

JT gravity and modular Hamiltonian

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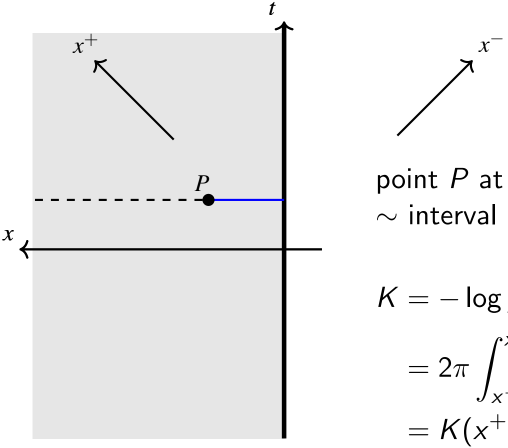
&\partial_+ \partial_- \Phi + \frac{\Lambda}{4} e^{\omega} \Phi = 0 && (\Box \Phi - \Lambda \Phi = 0).
\end{aligned}$$

What object in bCFT$_2$ obeys these EOM?
Answer: modular Hamiltonian K

$$\boxed{K \quad \rightsquigarrow \quad \text{dilaton } \Phi}$$

Modular Hamiltonian

bCFT$_2$ on $ds^2 = -dt^2 + dx^2 = -dx^+ dx^-$, $x \geq 0$ in state $|0\rangle$



point P at (x^+, x^-)
$\sim$ interval

$$K = -\log \rho_A \qquad (\text{or } \rho_A = e^{-K}/\text{tr}(K))$$

$$= 2\pi \int_{x^-}^{x^+} ds \frac{(s - x^-)(x^+ - s)}{x^+ - x^-} T_{++}(s)$$

$$= K(x^+, x^-)$$

Modular Hamiltonian

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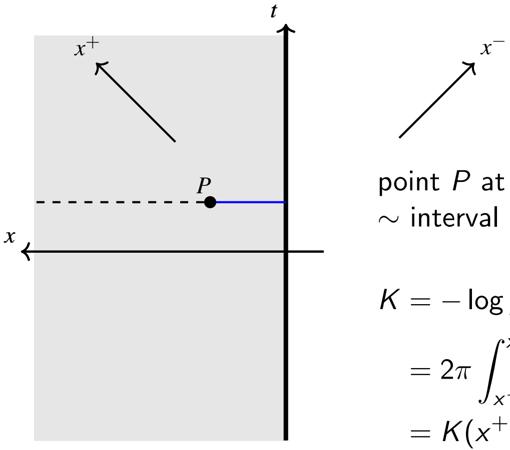

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local field $K(x^+, x^-) \quad \Rightarrow \quad$ dynamics of K?

Modular Hamiltonian

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obeys

$$\begin{aligned} \nabla_+ \partial_+ K &= 2\pi\, T_{++} \\ \nabla_- \partial_- K &= 2\pi\, T_{--} \\ (\Box - \Lambda) K &= 0. \end{aligned}$$

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$$\begin{aligned} -\nabla_+ \partial_+ \Phi &= 8\pi G \, T^m_{++} \\ -\nabla_- \partial_- \Phi &= 8\pi G \, T^m_{--} \\ (\Box - \Lambda) \Phi &= 0 \end{aligned}$$

$$\boxed{\Phi = -4G \, K + \Phi_0} \quad \text{and} \quad \boxed{T^m_{\pm\pm} = T_{\pm\pm}}$$

Modular Hamiltonian

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Part 2 and 3 of our JT entanglement dynamics theory: JT dilaton is determined by modular Hamiltonian of the bCFT, and JT matter sector by the bCFT matter.

JT entanglement dynamics of bCFT

$$I_{JT}[g, \Phi, \phi_m] = \int dxdt \sqrt{g}\, \Phi(R + \Lambda) + I_m[g, \phi_m]$$

$$\boxed{I_{\text{ent dyn of bCFT}}[S, K, \phi_m] = \int dxdt \sqrt{g}\, \Phi(R + \Lambda) + \int dxdt \sqrt{g}\mathcal{L}_{bCFT}}$$

with

$$\boxed{S \rightsquigarrow g, \quad K \rightsquigarrow \Phi, \quad \text{bCFT fields } \rightsquigarrow \phi_m.}$$

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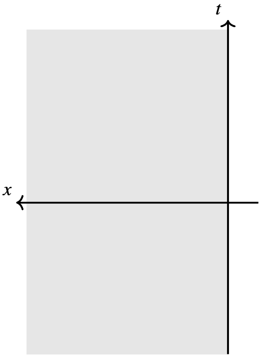
Theory of entanglement dynamics obtained by coupling bCFT to AdS_2 JT gravity.

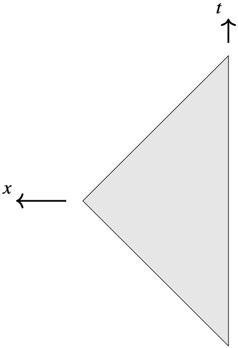
JT entanglement dynamics of bCFT

Given bCFT_2



$$ds^2 = -dt^2 + dx^2$$

Entanglement dynamics of bCFT_2



$$ds^2 = \frac{-dt^2 + dx^2}{x^2}$$

Recap

- bCFT$_2$ on $ds^2 = -dt^2 + dx^2 = -dx^+ dx^-$, $x \geq 0$ in state $|0\rangle$

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- bCFT$_2$ on $ds^2 = -dt^2 + dx^2 = -dx^+ dx^-$, $x \geq 0$ in state $|0\rangle$
- point $P(x^+, x^-) \sim$ interval $\sim S(x^+, x^-)$ and $K(x^+, x^-)$ that obey EOM of JT gravity with

$$\omega = -\frac{12}{c} S + 2 \log \frac{\ell}{\delta}$$

$$\Phi = -4G\, K + \Phi_0$$

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- Theory of entanglement dynamics obtained by coupling bCFT_2 to AdS_2 JT gravity

$$I_{\text{ent dyn of bCFT}} = I_{JT,grav} + I_{bCFT}$$

Recap

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- Theory of entanglement dynamics obtained by coupling bCFT_2 to AdS_2 JT gravity

$$I_{\text{ent dyn of bCFT}} = I_{JT,grav} + I_{bCFT}$$

How general? For class of excited states in bCFT obtained by conformal transformations of the lightcone coordinates $X^{\pm}(x^{\pm})$.

Connection to tensor networks

- Construction coincides with definition of boundary kinematic space [Karch et al 1703.02990]
  Kinematic space / MERA [Czech et al]
- Construction allows description of entanglement renormalization in bCFT
  cfr MERA, cMERA [Vidal, Haegeman et al]

Entanglement renormalization 1

Entanglement interpretation of Φ_0?

$$\nabla_\pm \partial_\pm \Phi_0 = 0$$
$$-e^{\omega} \partial_\pm (e^{-\omega} \partial_\pm \Phi_0) = 0$$

$$\Phi_0(x) \sim -\int^x dx' e^{\omega(x')} \sim -\partial_x \omega \sim \partial_x S = \frac{S(x) - S(x-\epsilon)}{-\epsilon}$$

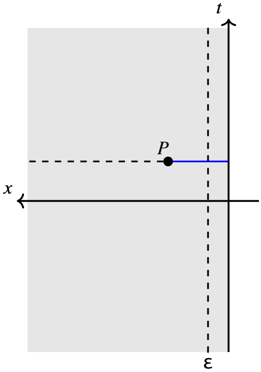
$$-\Phi_0 = \frac{\Phi_b}{\epsilon} \frac{3}{c} (S(x) - S(x-\epsilon))$$

$$\boxed{-\frac{\Phi_0}{4G} = S(x) - S(x-\epsilon)}$$

Entanglement renormalization 2

$$\frac{\Phi_0}{4G} = \frac{\Phi_b}{4G\,\epsilon}\frac{3}{c}(-\delta S_b) \qquad \text{or} \qquad \frac{\Phi_0}{4G} = -\delta S_b$$

with $\delta S_b = S(x) - S(x-\epsilon)$ the amount of entanglement between region left of $P(x,t)$ and boundary layer of width ϵ

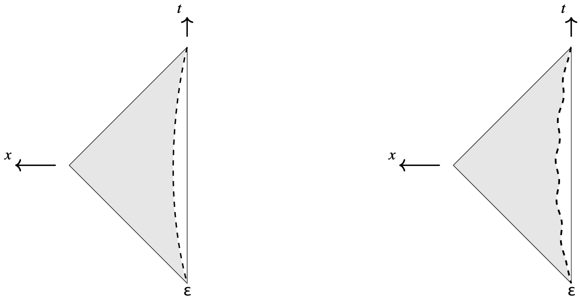


In the JT theory, the boundary condition

$$\Phi_0 = \frac{\Phi_b}{\epsilon}$$

defines location of boundary $\{t(u), x(u)\}$ in $ds^2 = \frac{-dt^2+dx^2}{x^2}$ with boundary time u such that

$$1)\ ds^2|_b = -\frac{du^2}{\epsilon^2}$$
$$2)\ \Phi|_b = \frac{\Phi_b}{\epsilon}$$



In the JT theory, the boundary condition

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$$\begin{aligned} &1)\ ds^2|_b = -\frac{du^2}{\epsilon^2} \\ &2)\ \Phi|_b = \frac{\Phi_b}{\epsilon} \end{aligned}$$

1) Family of trajectories $\{t(u), \epsilon t'(u)\}$ $\Longleftarrow \frac{-dt^2+dx^2}{x^2} = -\frac{du^2}{\epsilon^2}$

2) $t(u)$ as a function of matter content of JT theory determined by Schwarzian QM $\Longleftarrow \Phi(x, t; T) = \frac{\Phi_b}{\epsilon}$

$$\Phi(\epsilon t'(u), t(u); T) = \frac{\Phi_b}{\epsilon}$$

$$\Phi_b \int \{t, u\} du + I_{CFT}$$

$\{t, u\} = \frac{t'''}{t'} - \frac{3}{2}\left(\frac{t''}{t'}\right)^2$

Entanglement renormalization 3

$$\frac{\Phi_0}{4G} = \frac{\Phi_b}{4G\,\epsilon}\,\frac{3}{c}(-\delta S_b) \qquad \text{and} \qquad \frac{\Phi_0}{4G} = -\delta S_b$$

$$\Phi_0 = \frac{\Phi_b}{\epsilon} \quad \text{corresponds to} \quad \delta S_b = -\frac{c}{3}$$

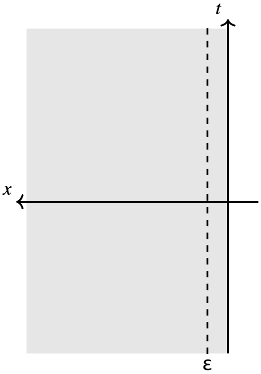
and

$$\Phi_b = \frac{c\epsilon}{3} \quad \Rightarrow \quad \frac{c\epsilon}{3}\int \{t, u\} du + I_{bCFT}$$

$$\boxed{I^{\epsilon}_{bCFT} = I^{\epsilon \to 0}_{bCFT} + \frac{c\epsilon}{3}\int \{t, u\} du}$$

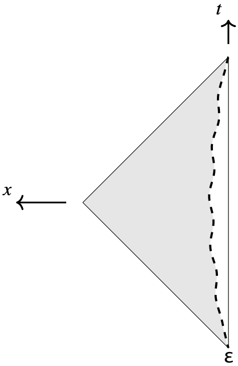
Entanglement renormalization 4

Entanglement renormalization in given $bCFT_2$
similar to cMERA



$$ds^2 = -dt^2 + dx^2$$

Boundary dynamics in entanglement dynamics of $bCFT_2$
described by Schwarzian QM



$$ds^2 = \frac{-dt^2 + dx^2}{x^2}$$

Geometry from entanglement

Holographic argument

$$\delta M = T\delta S \qquad \text{gravitational first law in AdS}_3 \text{ gravity}$$
$$\delta\langle K\rangle_{CFT} = \delta S_{CFT} \qquad \text{entanglement first law in CFT}_2$$

Jacobson argument (non-holographic)

$$\delta M = T\delta S + \delta E \qquad \text{gravitational first law in AdS}_3 \text{ gravity}$$
$$0 = \delta \mathbf{S}_{CFT}|_V \qquad \text{condition on entanglement in CFT}_3$$

Current argument (non-holographic)

$$0 = T\delta S + \delta E \qquad \text{gravitational first law in AdS}_2 \text{ JT gravity}$$
$$0 = \frac{\delta\Phi}{4G} + \delta\langle K\rangle_{CFT} \qquad \text{condition on entanglement in CFT}_2$$

Summary

- bCFT$_2$ on $ds^2 = -dt^2 + dx^2 = -dx^+ dx^-$, $x \geq 0$ in state $|0\rangle_X$
- Theory of entanglement dynamics obtained by coupling bCFT$_2$ to AdS$_2$ JT gravity

$$I_{\text{ent dyn of bCFT}} = I_{JT,grav} + I_{bCFT}$$

- Consequence of construction: Entanglement renormalization in bCFT$_2$ described by Schwarzian QM

$$I^{\epsilon}_{bCFT} = I^{\epsilon\to 0}_{bCFT} + \frac{c\epsilon}{3} \int \{t, u\} du$$

Other directions

- cMERA - Schwarzian
- $T\bar{T}$ deformation of CFT
- bCFT / SPT
- de Sitter version
- $\ldots$

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Thank you!