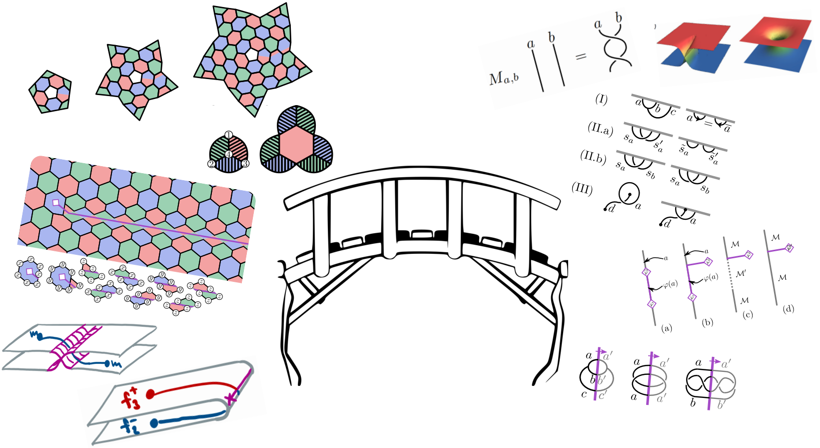
The twists and boundaries of the 2D color code

arXiv:1806.02820, Quantum 2, 101 (2018).

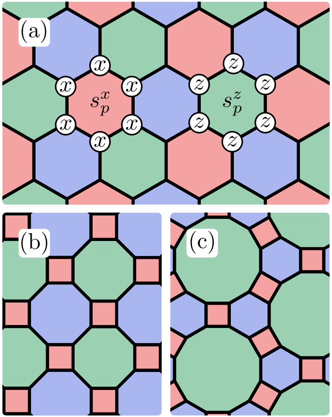
Markus Kesselring, Fernando Pastawski, Jens Eisert, Ben Brown

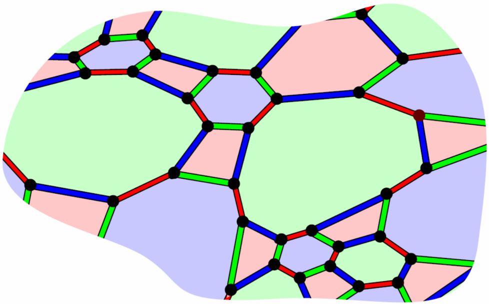
Coogee, Syndey – 5th of February 2019

A bridge between condensed matter and quantum info



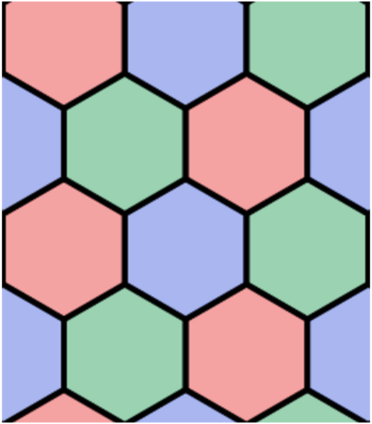
Introduction to 2D color codes: Stabilizers



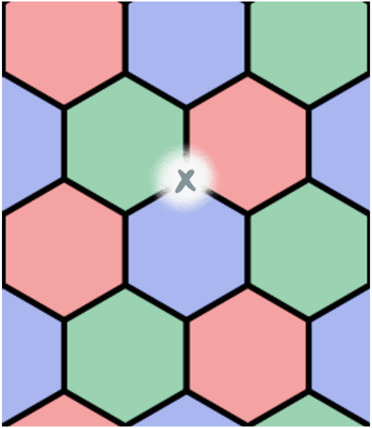


$$H_{CC} = -\sum_p s_p^x - \sum_p s_p^z$$

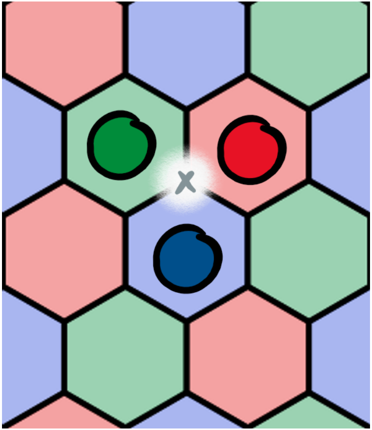
Color code anyons: creation



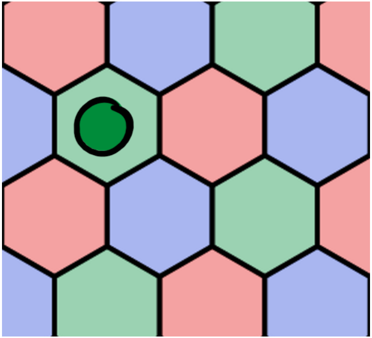
Color code anyons: creation



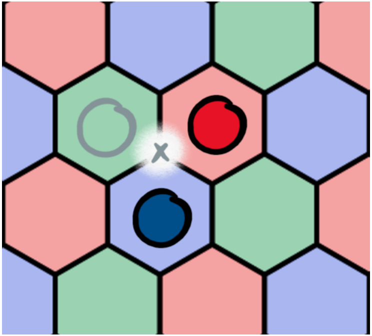
Color code anyons: creation



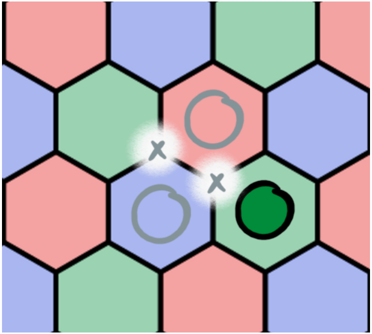
Color code anyons: moving anyons



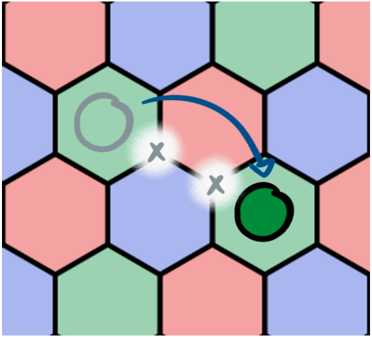
Color code anyons: moving anyons



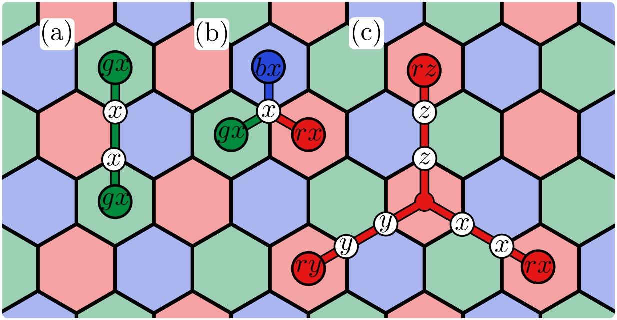
Color code anyons: moving anyons



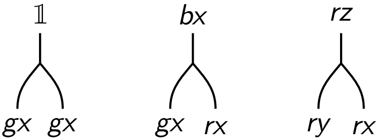
Color code anyons: moving anyons



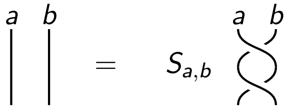
Color code anyons: fusion



rx	gx	bx
ry	gy	by
rz	gz	bz

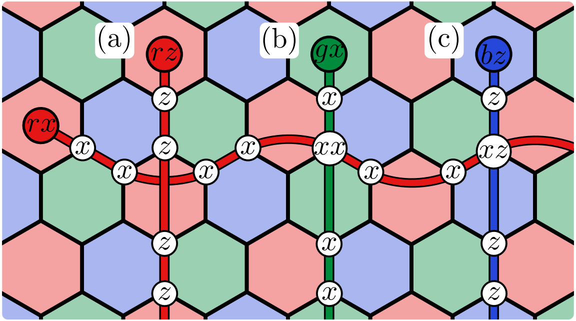


Color code anyons: braiding



For abelian anyons braiding results in a phase: $S_{a,b} = e^{i\theta_{a,b}}$

Color code anyons: braiding



rx	*gx*	*bx*
ry	*gy*	*by*
rz	*gz*	*bz*

Summary anyons

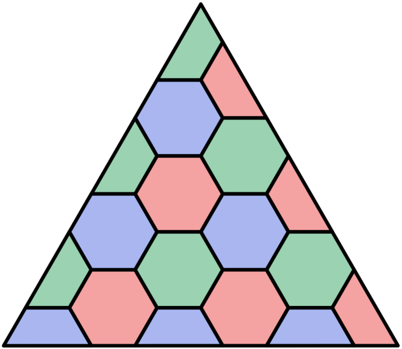
Fusion of two anyons within a row/column results in the third anyon within said row/column.

Braiding of two anyons which are in the same row/column is trivial, otherwise it will result in a phase -1.

rx	gx	bx
ry	gy	by
rz	gz	bz

Boundaries of the color code

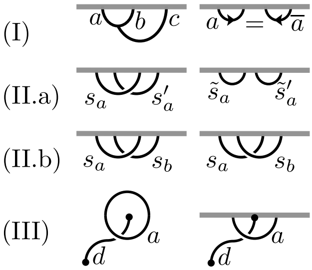
Three known bounadries of the color code:



rx	*gx*	*bx*
ry	*gy*	*by*
rz	*gz*	*bz*

Gapped bounadries

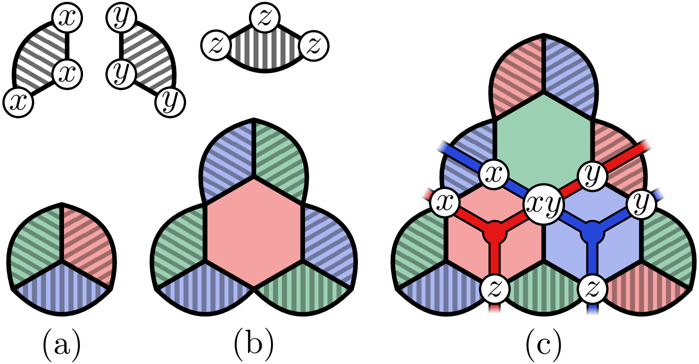
Anyons that condense at a gapped boundary form a *Lagrangian subgroup* $\mathcal{M}$



	color boundaries		
Pauli boundaries	*rx*	*gx*	*bx*
	ry	*gy*	*by*
	rz	*gz*	*bz*

- self bosonic exchange statistics
- trivial braiding with other anyons in $\mathcal{M}$

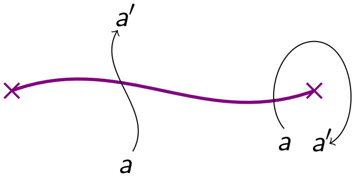
Small codes with Pauli boundaries



A tiny $[[4, 1, 2]]$ color code!

Domain walls and twists

- domain walls correspond to maps $a \mapsto \varphi(a) = a'$
- anyons crossing domain walls get mapped
- anyons circling around twist get mapped

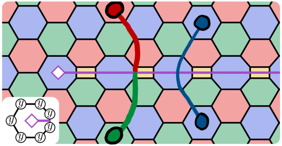


- data of anyon model is left invariant
  - $\varphi(N^c_{a,b}) = N^{c'}_{a',b'} = N^c_{a,b}$
  - $\varphi(S_{a,b}) = S_{a',b'} = S_{a,b}$
  - $\varphi(T_{a,b}) = T_{a',b'} = T_{a,b}$

Color Code Twists

color label permuting domain walls

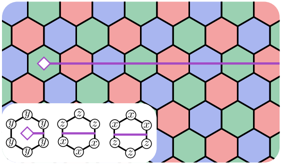
rx	*gx*	*bx*
ry	*gy*	*by*
rz	*gz*	*bz*



$|S_3| = 6$

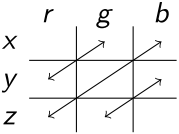
Pauli label permuting domain walls

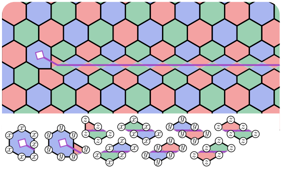
rx	*gx*	*bx*
ry	*gy*	*by*
rz	*gz*	*bz*



$|S_3| = 6$

Duality (or domino) domain walls - exchange color and Pauli label

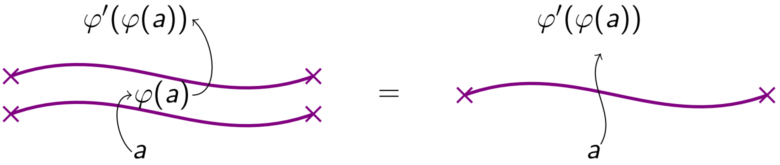




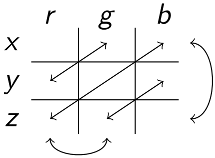
$|Z_2| = 2$

Generating domain walls and twists

Combining domain walls and twists



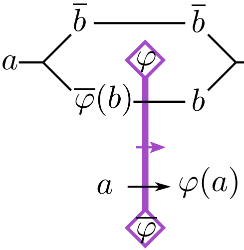
Generating set of twists

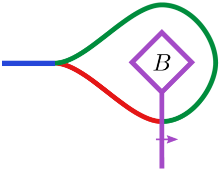


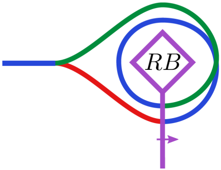
Number of twists

$$(S_3 \times S_3) \rtimes Z_2 = S_3 \wr Z_2 \qquad \text{order } 72$$

Anyon localization at twist defects



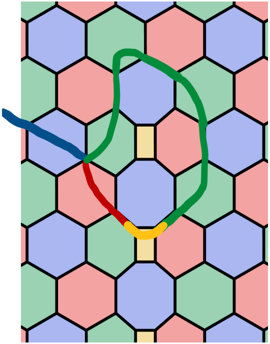




Non-abelian
Ising anyons:

$$\sigma \times \psi = \sigma$$

$$\rightarrow N^{\sigma}_{\sigma,\psi} = 1$$



Quantum dimension of twists

Quantum dimension of a twist φ:

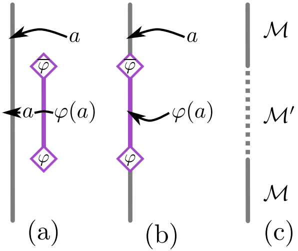
$$d_\varphi^2 = \sum_{a \in \mathcal{C}} d_a N_{\varphi,a}^{\varphi}$$

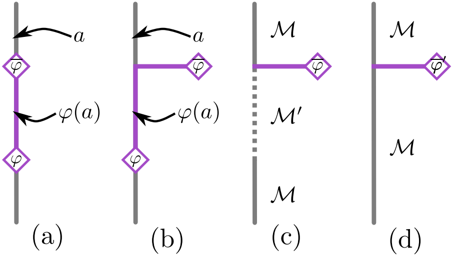
Quantum dimensions of color code twists:

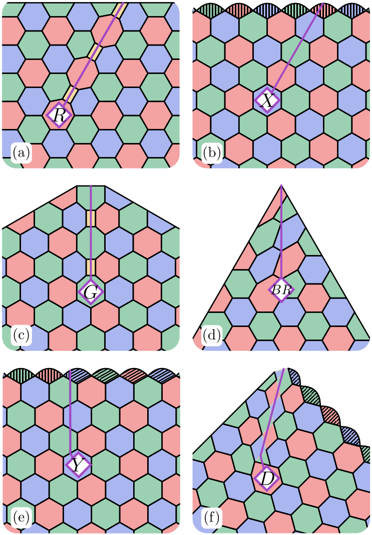
	$\mathbb{1}$	X	Y	Z	XZ	ZX
$\mathbb{1}$	1	2	2	2	4	4
R	2	2	2	2	4	4
G	2	2	2	2	4	4
B	2	2	2	2	4	4
RB	4	4	4	4	2	2
BR	4	4	4	4	2	2

$D \circ ()$	$\mathbb{1}$	X	Y	Z	XZ	ZX
$\mathbb{1}$	$\sqrt{2}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$
R	$\sqrt{8}$	$\sqrt{2}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$
G	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{2}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$
B	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{2}$	$\sqrt{8}$	$\sqrt{8}$
RB	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{2}$
BR	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{8}$	$\sqrt{2}$	$\sqrt{8}$

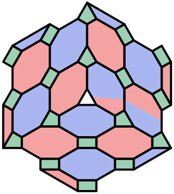
Twist condensation



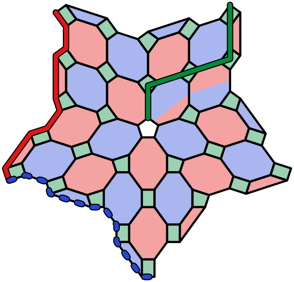




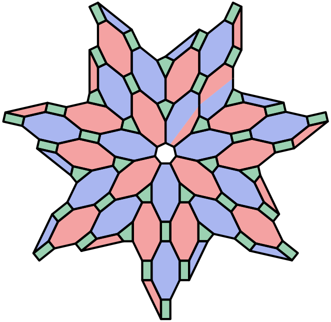
Applications of twists: Stellated codes



$$k\frac{d^2}{n} = 1.33$$



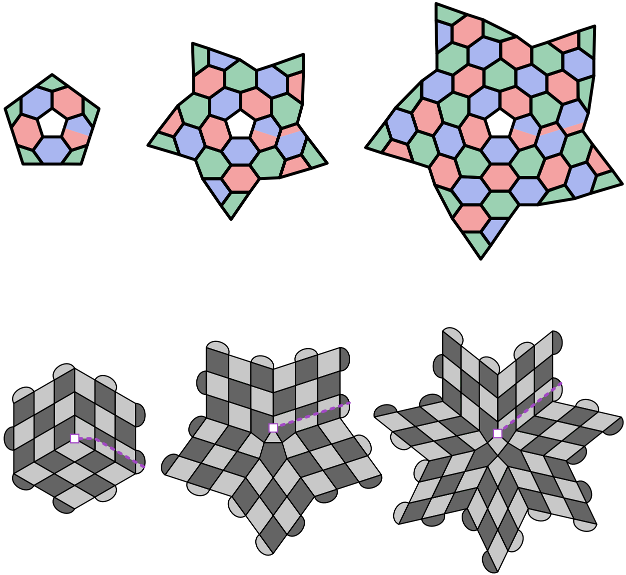
$$k\frac{d^2}{n} = 2.40$$



$$k\frac{d^2}{n} = 2.86$$

General: $k\frac{d^2}{n} = 4 - \frac{8}{s} \to 4$

Applications of twists: Stellated codes



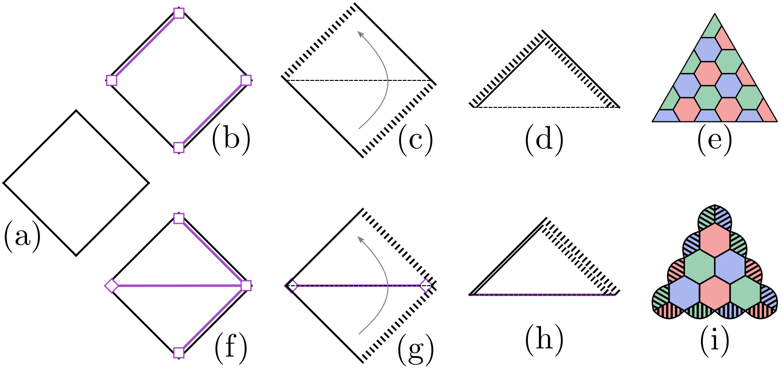
Unfolding the Color Code into 2 Toric Codes

The color code can be "unfolded" into two copies of the toric code:
$CC = TC \otimes TC$

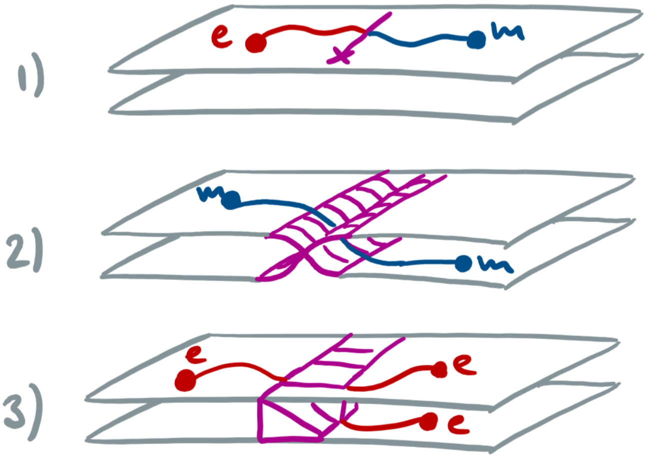
Toric code anyons: $\mathbb{1}$, e, m, $\varepsilon = e \times m$

	r	g	b
x	e^-	$e^- e^+$	e^+
y	$e^- m^+$	$\varepsilon^- \varepsilon^+$	$m^- e^+$
z	m^+	$m^- m^+$	m^-

Unfolding the Color Code into 2 Toric Codes: Boundaries



Unfolding the Color Code into 2 Toric Codes: Twists



Unfolding the Color Code into the Three Fermion Model

Three fermion anyon model:

Like the toric code, but with more fermions!

Anyons and fusion:

$$\mathbb{1}, \qquad f_1, \qquad f_2, \qquad f_3 = f_1 \times f_2$$

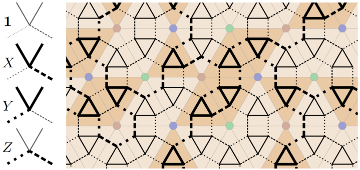
Self exchange:

$$\theta_{f_i} = -1$$

Braiding:

$$S_{f_i,f_j} = \begin{cases} +1, & i = j \\ -1, & i \neq j \end{cases}$$

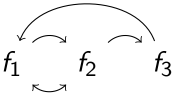
Topological subsystem code:



Unfolding the Color Code into the Three Fermion Model

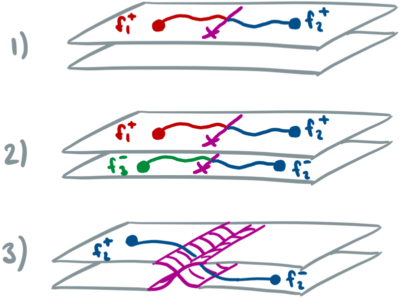
	r	g	b
x	$f_1^- f_1^+$	$f_2^- f_2^+$	$f_3^- f_3^+$
y	$f_2^- f_3^+$	$f_3^- f_1^+$	$f_1^- f_2^+$
z	$f_3^- f_2^+$	$f_1^- f_3^+$	$f_2^- f_1^+$

The Three Fermion Model: Symmetries + CC Twists



$$|S_3| = 6$$

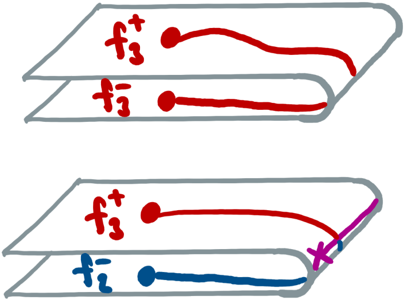
Six permutations of three elements



	r	g	b
x	$f_1^- f_1^+$	$f_2^- f_2^+$	$f_3^- f_3^+$
y	$f_2^- f_3^+$	$f_3^- f_1^+$	$f_1^- f_2^+$
z	$f_3^- f_2^+$	$f_1^- f_3^+$	$f_2^- f_1^+$

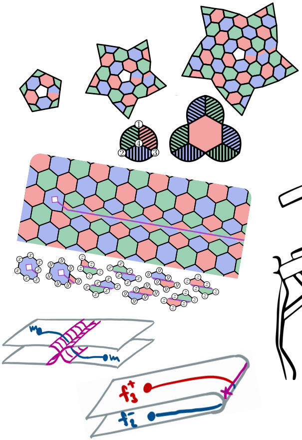
The Three Fermion Model: CC Boundaries Twists

CC boundary = 3F fold + 3F domain wall
6 possible 3F domain walls $\leftrightarrow$ 6 CC boundaries

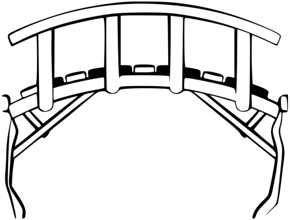


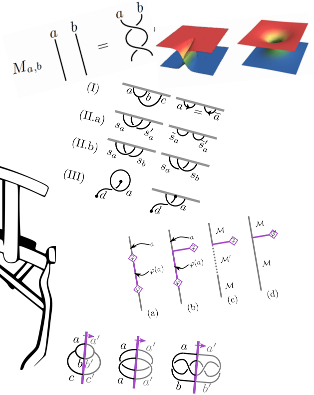
	r	g	b
x	$f_1^- f_1^+$	$f_2^- f_2^+$	$f_3^- f_3^+$
y	$f_2^- f_3^+$	$f_3^- f_1^+$	$f_1^- f_2^+$
z	$f_3^- f_2^+$	$f_1^- f_3^+$	$f_2^- f_1^+$

Summary

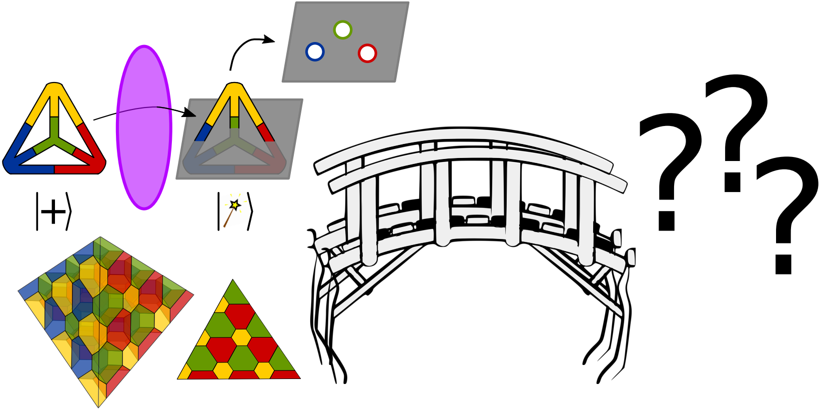


rx	gx	bx
ry	gy	by
rz	gz	bz





And now the same in 3D!



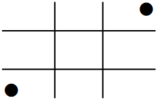
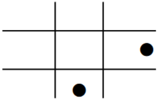
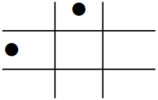
Questions?

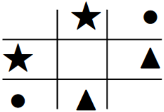
arXiv:1806.02820, Quantum 2, 101 (2018)

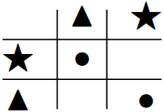
Thanks to my collaborators:
Ben J. Brown, Fernando Pastawski, Jens Eisert

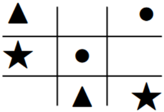
And thank you for your attention!

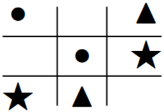
Fermions in the Color Code

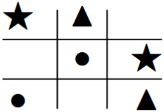
$f_1 =$  $=$  $=$ 

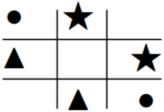
$f_1 =$ 

$f_2 =$ 

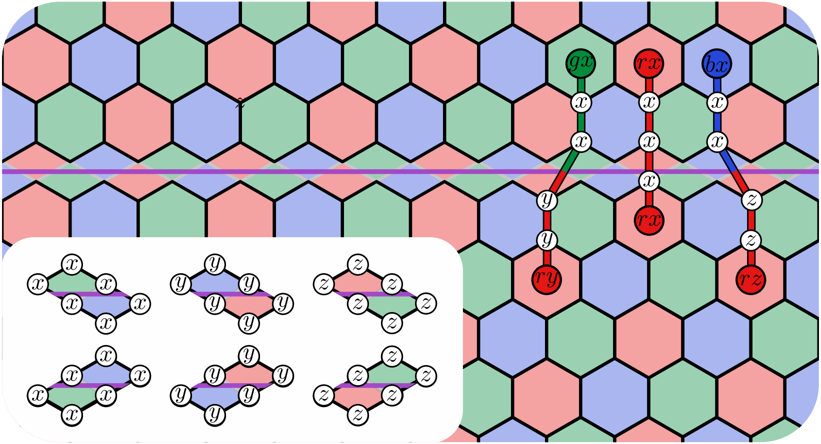
$f_3 =$ 

$f_4 =$ 

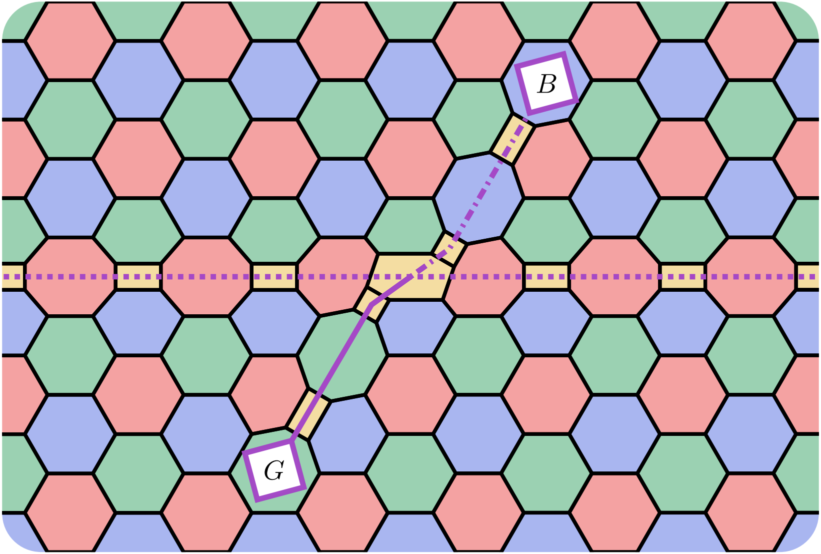
$f_5 =$ 

$f_6 =$ 

Domino Twist: anyons



Twists crossing domain walls



Twist-Corner identification in triangular color codes