Quantum computing with topological wormholes

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Motivation

- Locality — central to quantum error correction.
- Exhibit topological order; simple syndrome extraction.
- Restrictions on storing and processing quantum information.
- Is locality warranted?
    - Deterministic ways to share entanglement have recently been discovered.
    - Some architecture have flying qubits.
    - Some architectures use extended objects such as resonators.

Kitaev, Ann. Phys. 2003
Bravyi, Kitaev, arXiv:quant-ph/9811052
Bombin, Martin-Delgado, PRL 2006

Bravyi, Poulin, Terhal, PRL 2010
Bravyi, Koenig, PRL 2013
Pastawski, Yoshida, PRA 2015

Axline et al., Nat. Phys. 2018
Kurpiers et al. Nature 2018
Campagne-Ibarcq, PRL 2018

Motivation

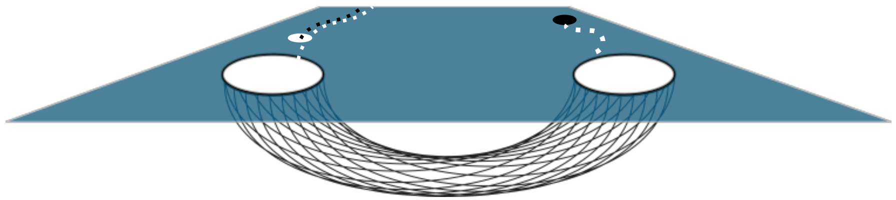
- LDPC codes completely ignore geometry & locality.
  - Achieve constant rate and threshold.
- What is the space in between LDPC and topological?
  - How much non-locality to break no-go theorems?
  - Are the coding-theory gains sufficient to motivate the development of experimental platforms with non-local gates?

Tillich, Zémor, IEEE Trans. Inf. Theory 2014
Leverrier, Tillich, Zémor, FOCS 2015Fawzi et al., STOC 2018

Fawzi et al., FOCS 2018
Grospellier, AK, arXiv:1810.03681v1

Summary

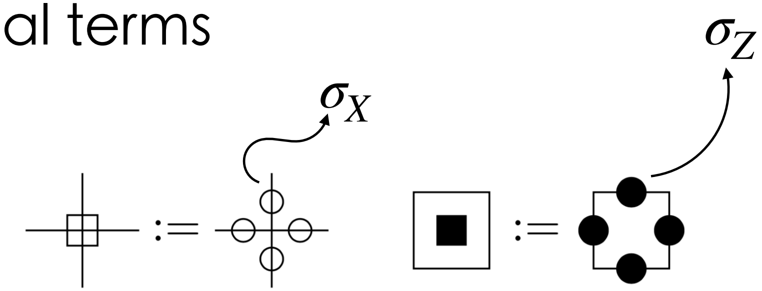
- Introduce new defect called a wormhole using a small amount of non-locality.



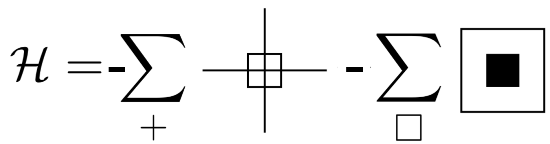
- Entangle two spatially separated sectors of lattice.
- Can encode a logical qubit.
- All Clifford gates with one encoding

Defects on the toric code

- The toric code is defined on a square lattice with periodic conditions.
- Hamiltonian has local terms

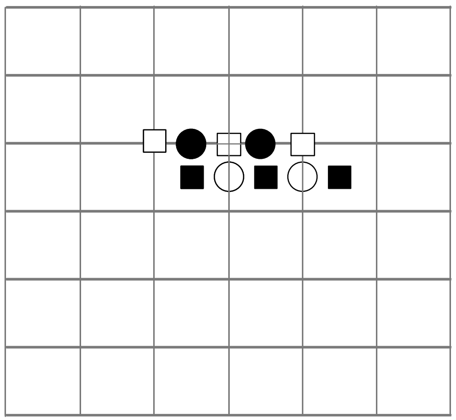


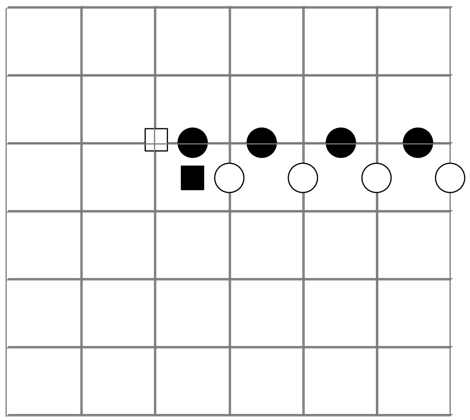
- Can be expressed as the sum of these terms



Defects on the toric code

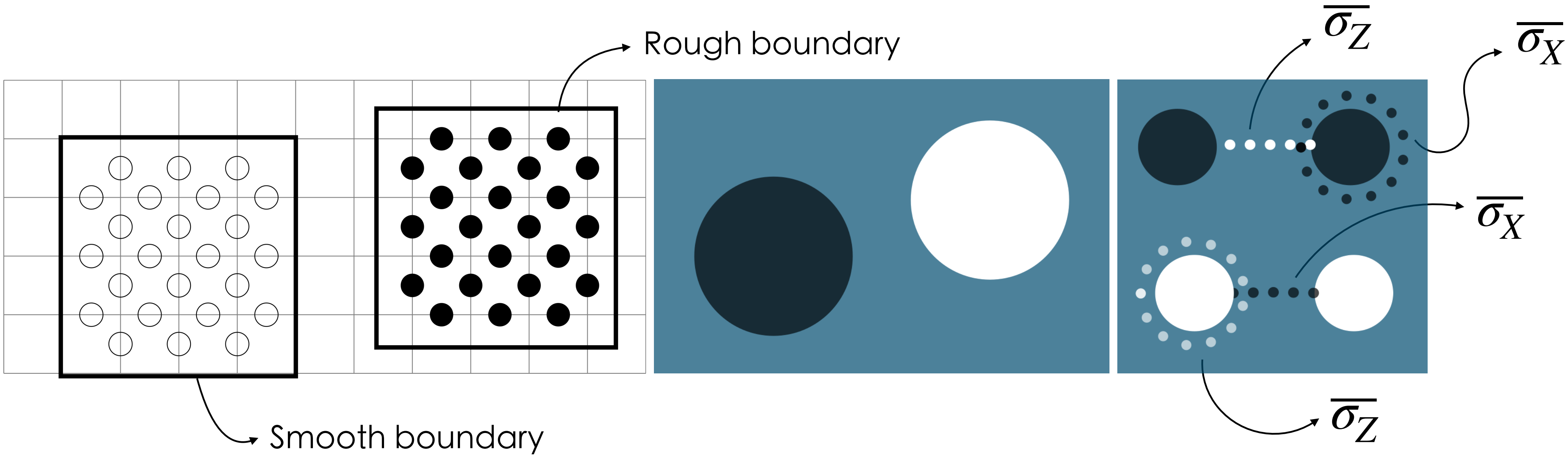
- Defects come in two varieties!
- They can condense on different type of boundaries.





Defects on the toric code

- Punctures come in two varieties!



- Can be used to encode a logical qubit.

Defects on the toric code

- Perform logical CNOT via braiding!
- Example of general transformations called code deformation.

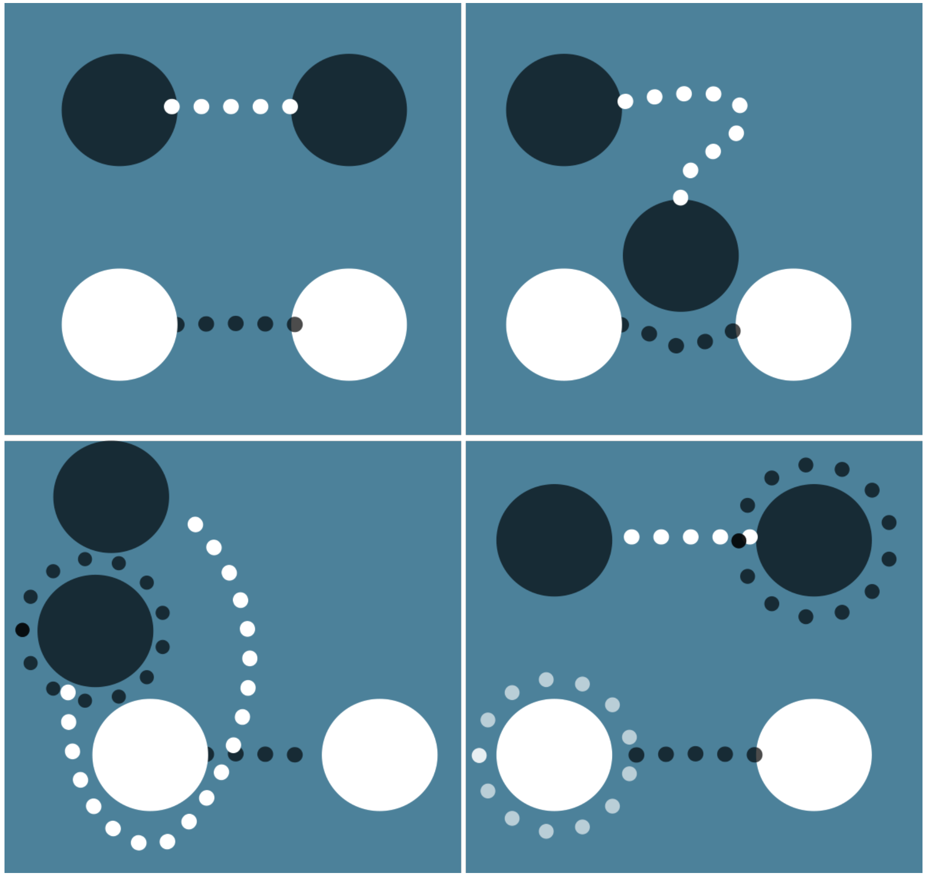
$$\mathscr{C}_1 \to \mathscr{C}_2 \to \ldots \mathscr{C}_N = \mathscr{C}_1$$

- In each step, permitted operations:
  - Measuring stabilizer/ gauge operators.
  - Performing transversal gates.
- Each step modifies the code on a patch smaller than minimal distance $\Rightarrow$ Fault tolerant

Raussendorf, Harrington PRL 2007
Fowler et al., PRA 2012
Vuillot et al., arXiv:1810.10037

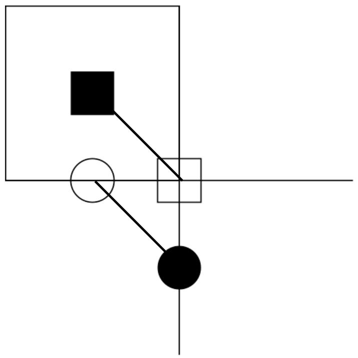
Defects on the toric code

- Perform logical CNOT via braiding!
  - ZI → ZZ
  - IX → XX
- Move punctures by measurement alone.
- Smooth punctures are control & rough punctures are target
- Difficult to perform single-qubit Cliffords.
- CSS code throughout ⇒ CSS-preserving gates only



Defects on the toric code

- Twists: two-body measurements.

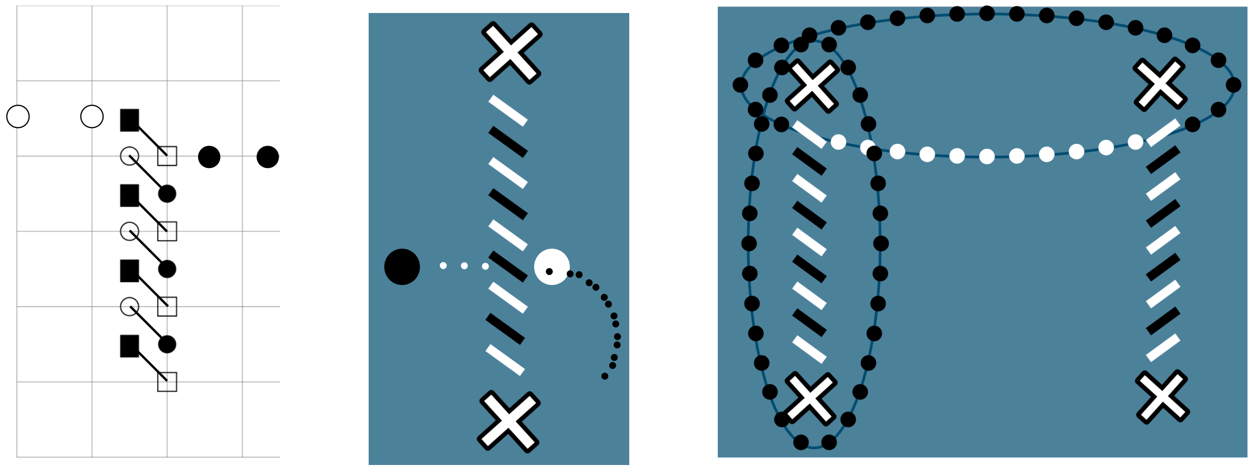



- Break the CSS nature of code.
- Yields hybrid operators of weight 6.

Bombin, PRL 2010
Bombin, NJP 2011
Zheng et al., PRB 2015
Brown et al., PRX 2017

Defects on the toric code

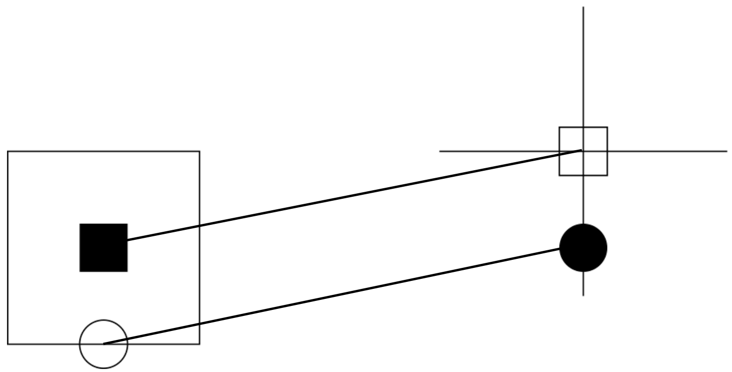
- Measure two-body operators along a ‘defect line’



- Can transform anyon type.
- Can encode qubits using two pairs of twists.
- Permits single-qubit Clifford gates.

Wormholes

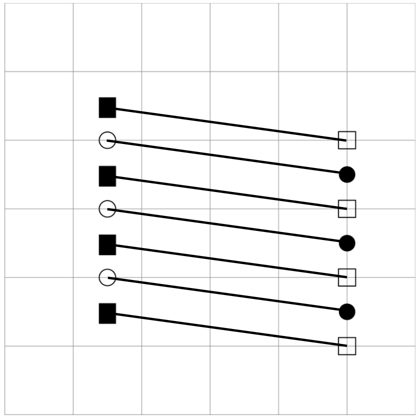
- Separate entangling measurements.

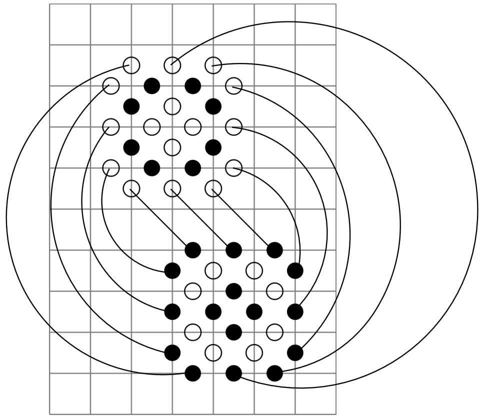


- Yields hybrid nonlocal operators of weight 8.
- (With local Clifford, non-local operators can be of weight 4.)

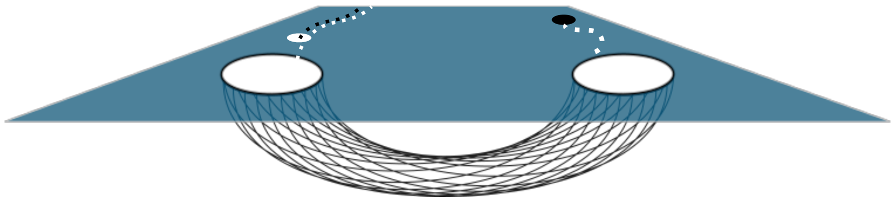
Wormholes

- Measure two-body operators that are spatially separated.



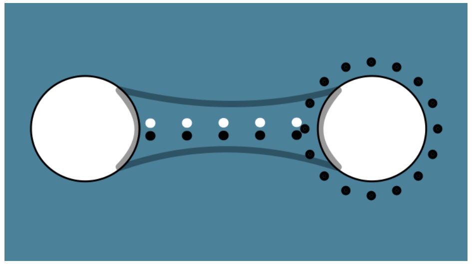


- Can extend to entangling the boundary of two regions.

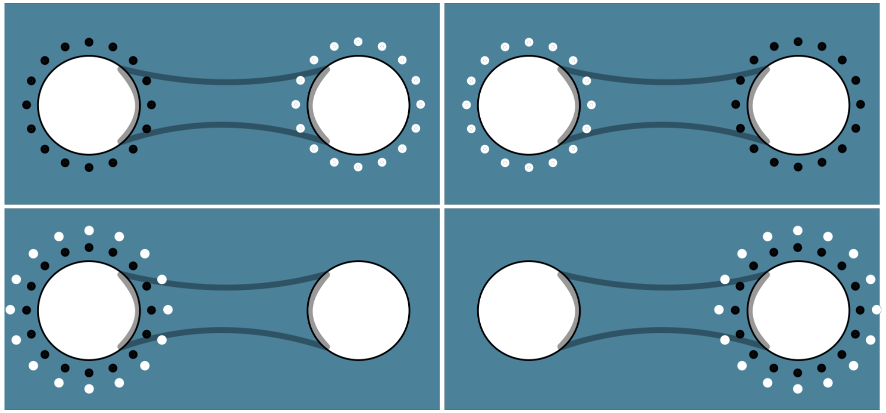


Wormholes

- Can encode a single logical qubit in a wormhole.



- The mouths are topologically indistinguishable.

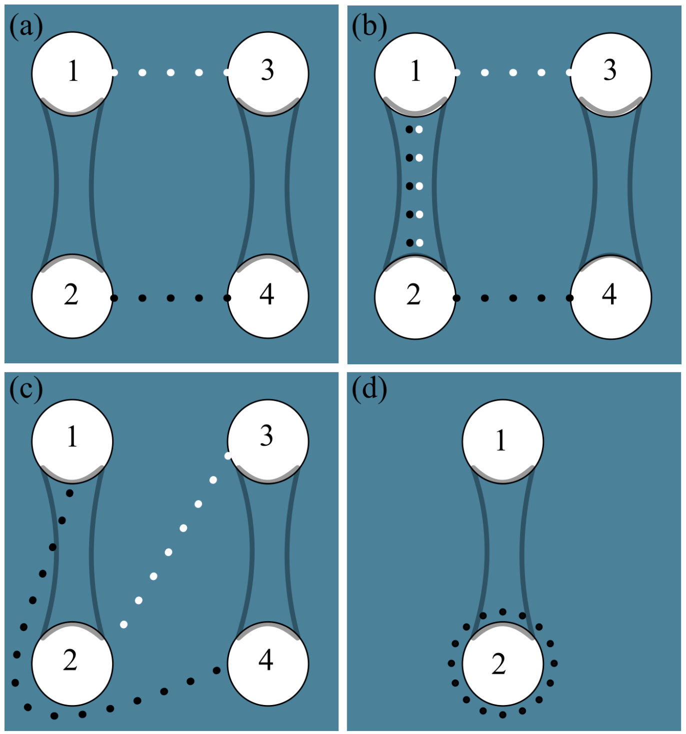


Clifford gates

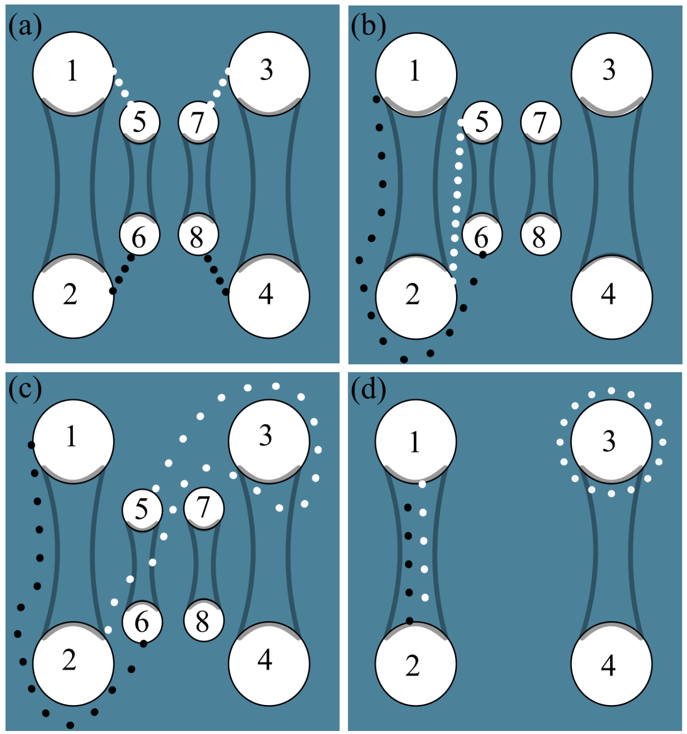
Example : Phase gate

Clifford gates

Hadamard gate



c-z gate



Discussion

- Can use two-body measurements to create non-local defect.
- To use as a code, you need higher-weight non-local measurements.
- Could imagine 2D lattice where non-local connection is defined by fiat.
- On the other hand, entanglement lets us decide where to insert non-local connections.
- Interesting, but does not permit new gates. Still requires state injection + magic state distillation!

Bravyi, Kitaev PRA 2005
Knill, Nature 2005
Bravyi, Haah PRA 2012

Hastings, Haah, PRL 2018
AK, J.-P. Tillich, arXiv: 1811.08461

Discussion

- Codes play an increasingly important role in our understanding of gravity.
- These wormholes behave much like gravitational wormholes.
- Example of ER=EPR?
  - Geometry experienced by excitations is induced by entanglement in underlying qubit substrate.
  - Entropy of wormhole mouth proportional to its boundary.

Susskind, Rep. Prog. Phys. 2016
Pastawski et al., JHP 2015

LDPC codes

- Preserve some properties of toric code but constant rate?
- Code family where
  - qubits participate in O(1) stabilizers; and
  - stabilizers act on O(1) qubits.
- Overhead: ratio of physical qubits to logical qubits.
- Gottesman showed that if certain kinds of LDPC codes exist, then we can achieve constant overhead.
- Codes satisfying these criteria have not been found.
- Hypergraph product codes are good candidate class.

Gottesman, Quantum Inf. Comput. 2014

Hypergraph product codes

- Let H be the parity check matrix of a classical code $\mathscr{C}$
- Hypergraph product code $\mathscr{Q}$

$$H_X = (1 \otimes H \,|\, H^T \otimes 1) \qquad H_Z = (H \otimes 1 \,|\, 1 \otimes H^T)$$

- If $\mathscr{C}$ is LDPC, then so is $\mathscr{Q}$

$$\mathscr{C} = [n, k, d] \implies \mathscr{Q} = [\![O(n^2), O(k^2), O(d)]\!]$$

- Possess efficient decoding algorithm that runs in linear time.
- Threshold estimates provided for some classes.

Tillich, Zémor, IEEE Trans. Inf. Theory 2014
Leverrier, Tillich, Zémor, FOCS 2015

Fawzi et al., STOC 2018
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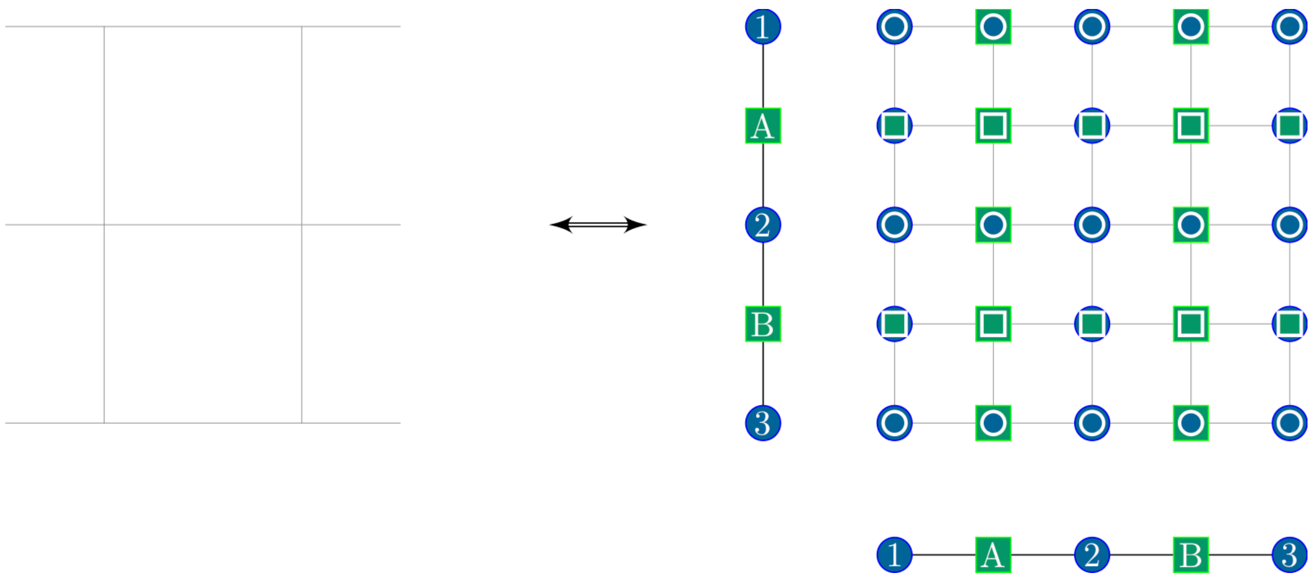
Hypergraph product codes



Example: 3 x 3 surface code

Hypergraph product codes

- We can perform Clifford gates by deforming $\mathcal{Q}$
- Punctures can be described as a hypergraph product of sub-codes.
- Can be done entirely algebraically; process described in terms of constituent graphs.
- Moving a puncture is similar; can use linear algebra to decide when a logical operator has been performed.
- Interesting objects: braiding on an abstract graph.

END