Measurement-Induced Phase Transitions and Nearly Random Stabilizer Codes

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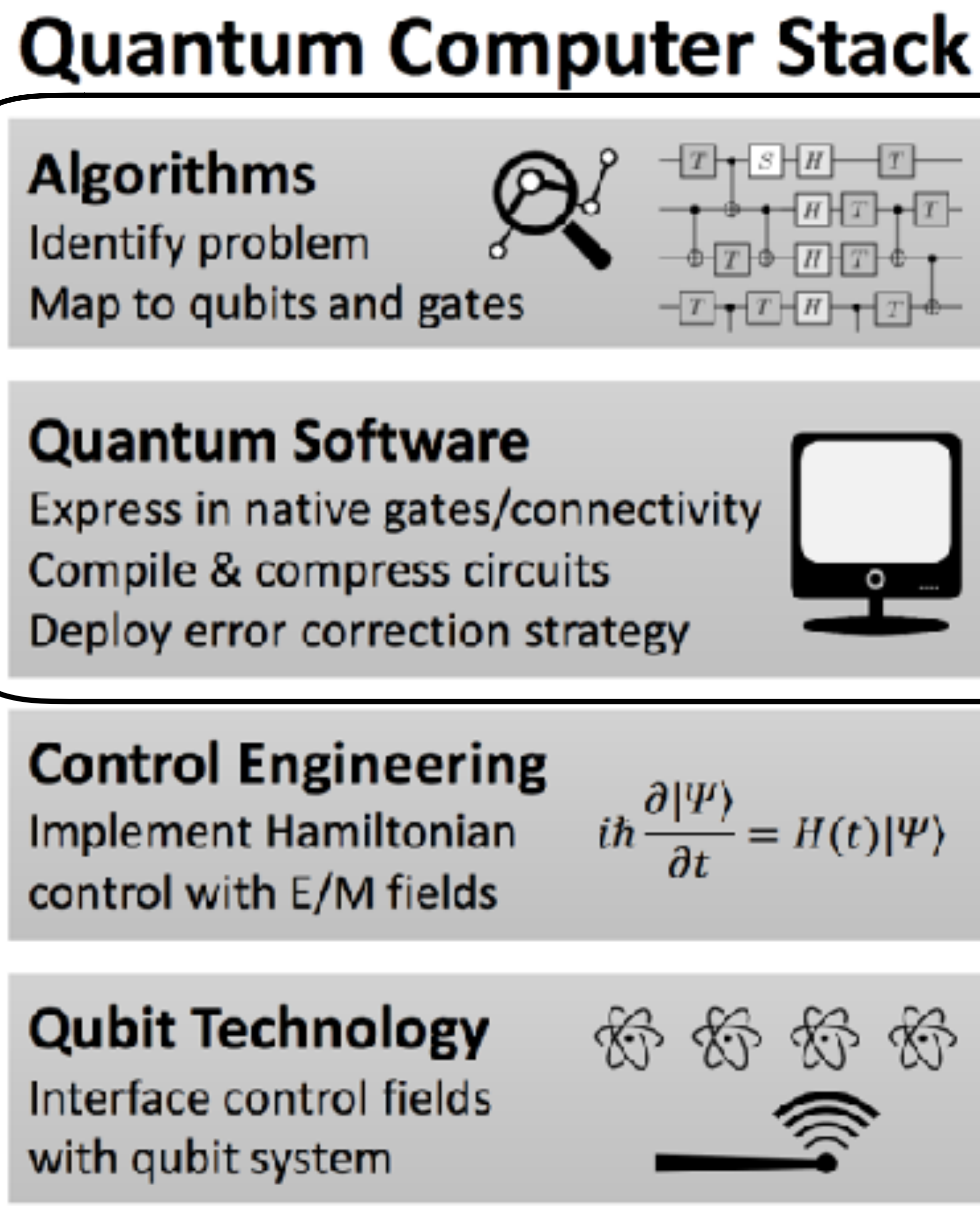
Entanglement Frontier: Quantum Computing

A universal quantum computer can significantly advance many fields of science and engineering

Building a quantum computer requires innovation at every level of the "stack"



Experimental and theoretical quantum error correction

- Quantum error correction thresholds
- Measurement-induced transition
- Quasirandom stabilizer codes

Quantum engineering of qubit platforms

- Silicon-spin qubits

Alexeev *et al.*, arXiv:1912.07577

Overview

Measurement-induced transitions

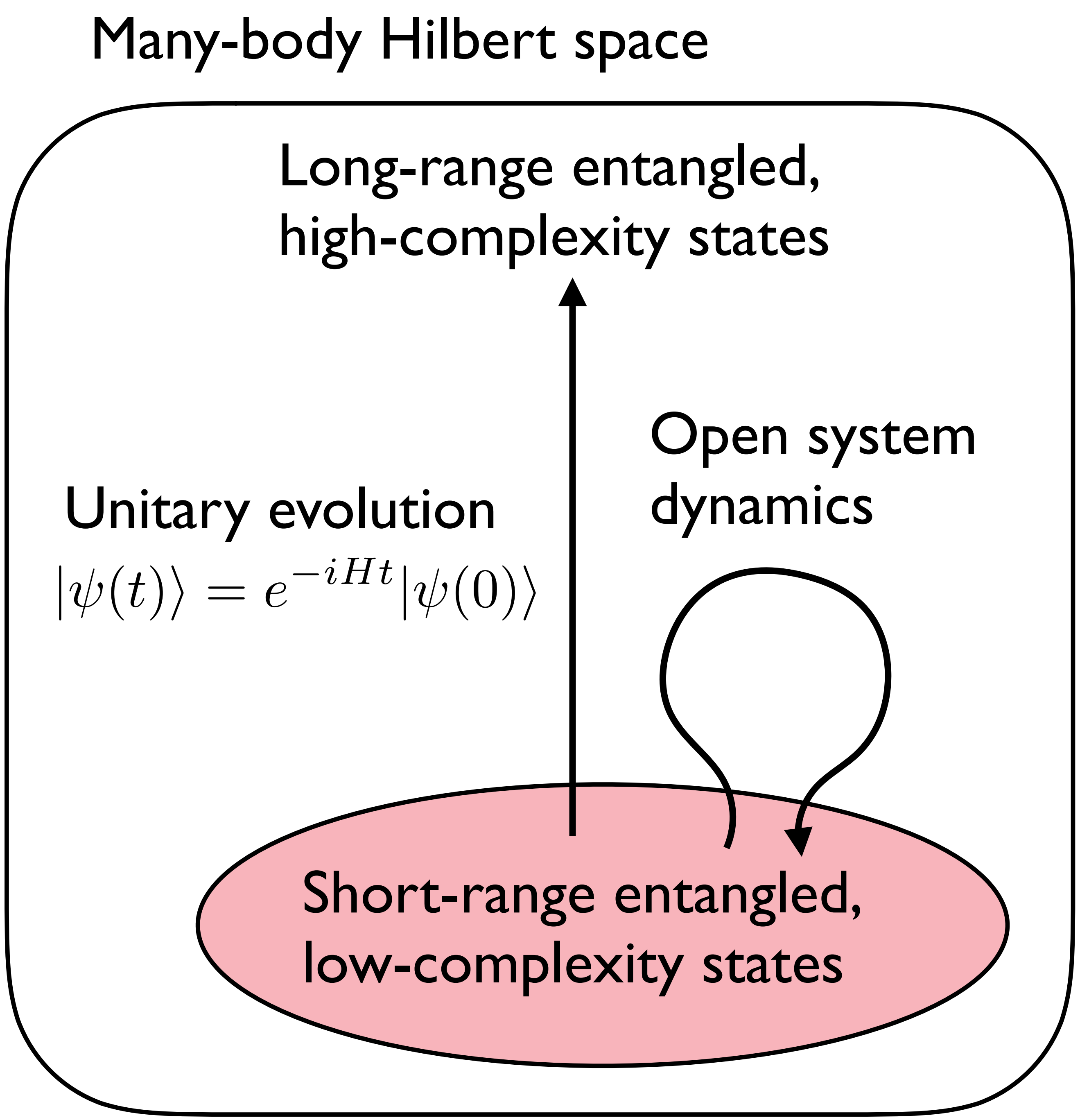
Collaborators:
David Huse - Princeton
Aidan Zabalo - Rutgers
Justin Wilson - Rutgers
Sarang Gopalakrishnan - CUNY
Jed Pixley - Rutgers

Gullans and Huse, arxiv:1905.05195
Gullans and Huse, arxiv:1910.00020

Quasirandom stabilizer codes

Collaborators:
Stefan Krastanov - Yale ⟶ MIT
Steve Flammia - Sydney/Yale
Steve Girvin - Yale
Liang Jiang - Yale ⟶ Chicago
David Huse - Princeton

Entanglement Frontier: Nonequilibrium Quantum Dynamics



Unitary dynamics in closed systems
- Frontier across physics

- CM/AMO physics - quantum simulation, quantum chaos, many-body localization, …

- Quantum information - quantum complexity, speedup …

- High-energy physics - quantum gravity, black holes, cosmology, …

Entanglement Frontier: Error Correction and Fault-Tolerance

Classical computation

- Error correction is irrelevant for transistors

Error rate $\ll 1/10^9$ year

- Data storage

- Crucial application: Wireless data transmission

Quantum computing platforms

Gate fidelities

Single-qubit gates
- Trapped ions < 99.998 %
  Brown *et al.*, PRA (2011).
- Superconducting qubits < 99.9 %
  Barends *et al.*, Nature (2014).

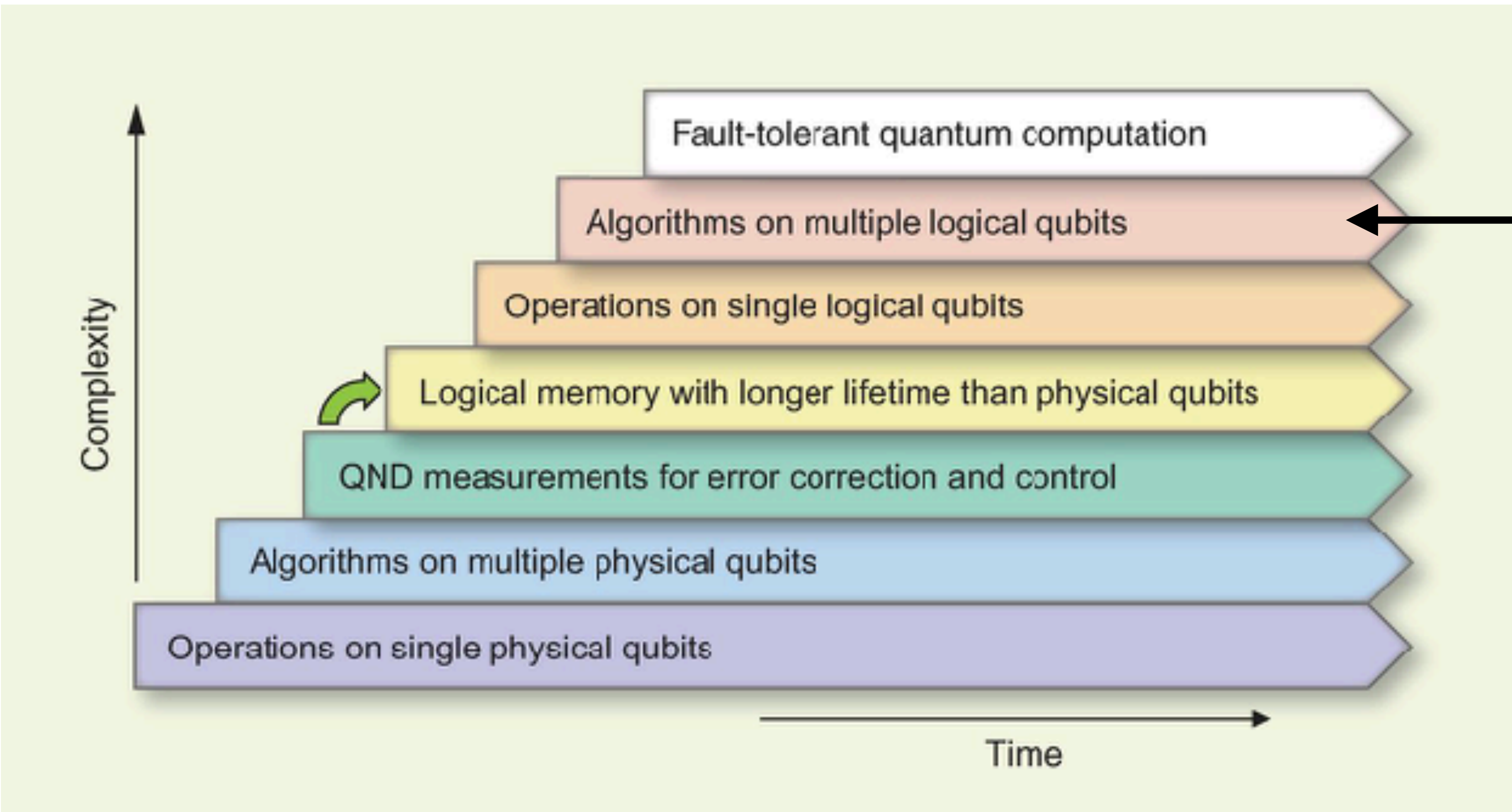
Two-qubit gates
- Trapped ions < 99.92 %
  Gaebler *et al.*, PRL (2016).
- Superconducting qubits < 99.5 %
  Barends *et al.*, Nature (2014).

Similar results for neutral atoms, spin qubits, and photons

Topological qubits - 99.9999 % (???)

Error correction is fundamental to quantum computing

Error Correction and Fault-Tolerance



Devoret and Schoelkopf, Science (2013).

Today's talk:

- Random quantum dynamics on logical qubits
- Statistical physics of fault-tolerant operation

Fault-Tolerant Threshold as a Phase Transition

PHYSICAL REVIEW A, VOLUME 62, 062311

Quantum to classical phase transition in noisy quantum computers

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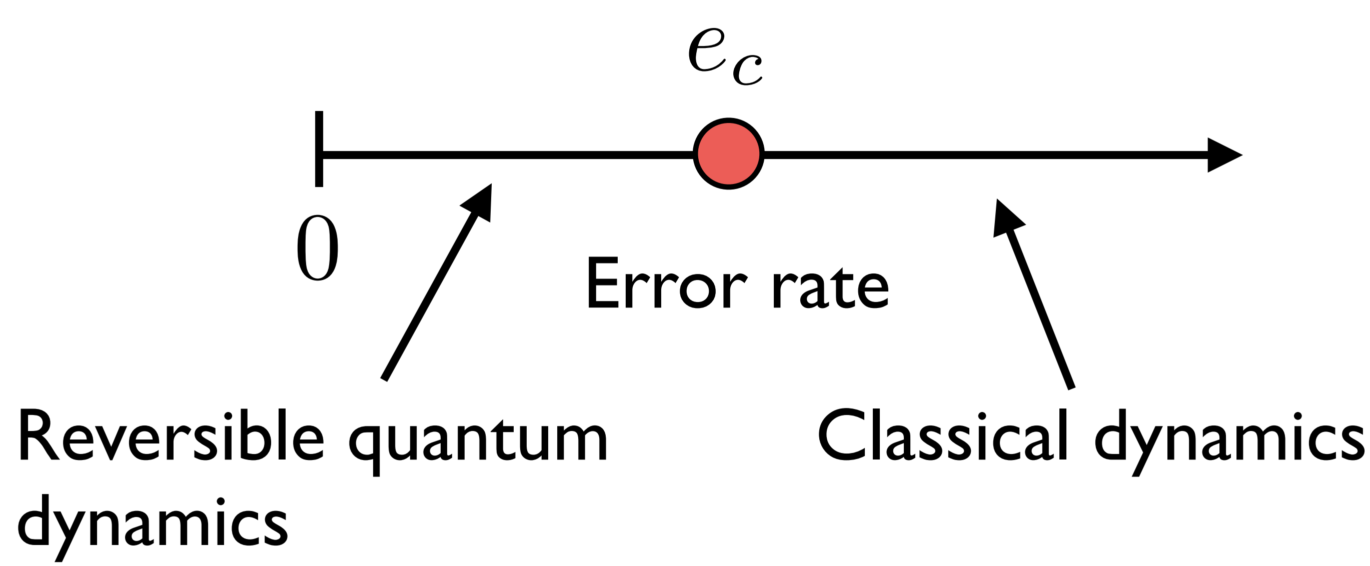
(Received 21 October 1999; published 14 November 2000)



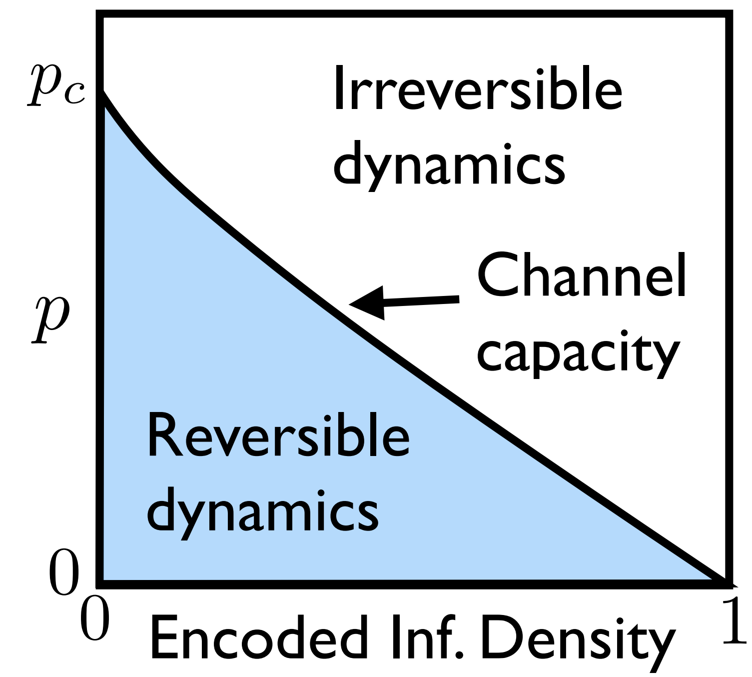
Points to inherent robustness of quantum computing in many-body systems

What is the nature of this phase transition?
How general is this phenomena?

Outline

Unitary-measurement dynamics in open systems

- Superconducting qubit example [1]
- Measurement-induced entanglement transition [2,3]
- Purification or memory transition [4]
- Quantum error correction thresholds [4,5]

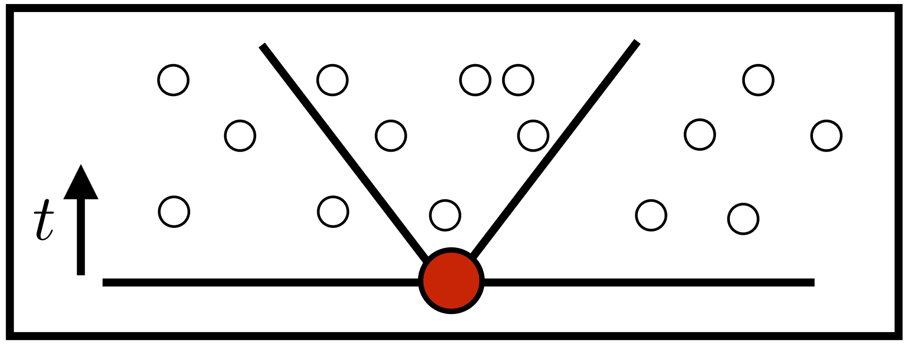


[1] Minev *et al.*, Nature (2019).
[2] Li, Chen, Fisher, PRB (2018/19).
[3] Skinner, Ruhman, Nahum, PRX (2019).
[4] Gullans and Huse, arxiv:1905.05195
[5] Choi, Bao, Qi, Altman, arxiv:1903.05124

Defining a local order parameter

Scalable probes of the "ordered" phase

Gullans and Huse, arxiv:1910.00020

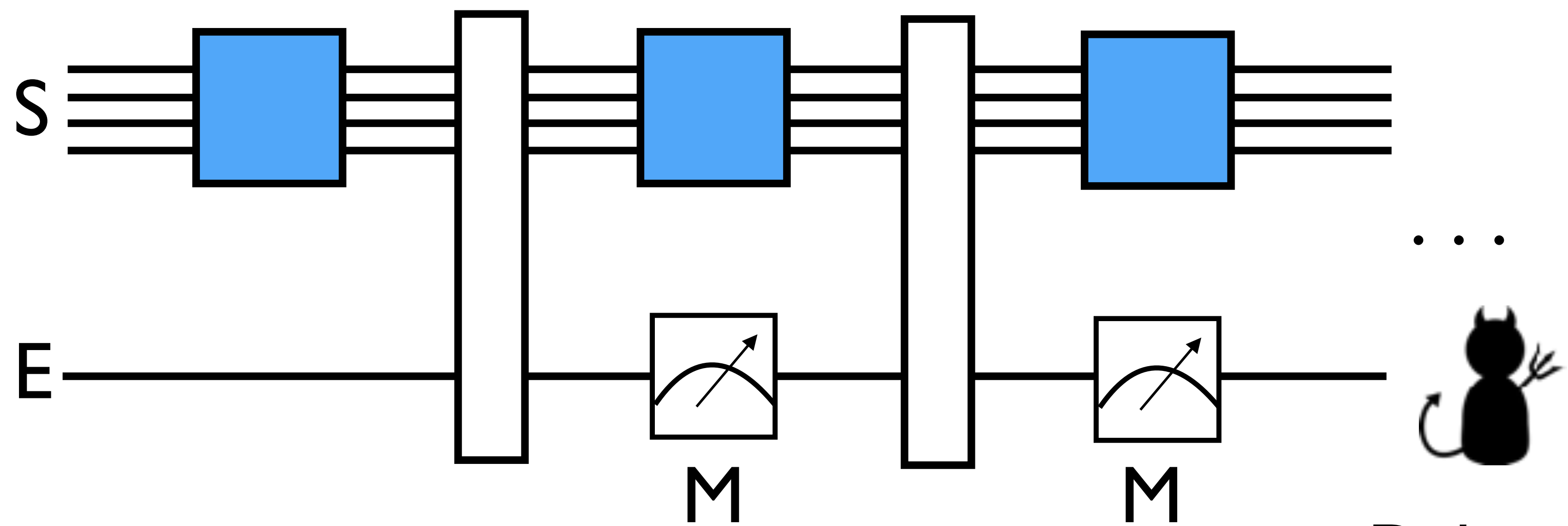


Quasirandom stabilizer codes

Unitary-Measurement Dynamics

Generic open system dynamics with a monitored environment

- Less control required than full error correction
- Discrete time version:



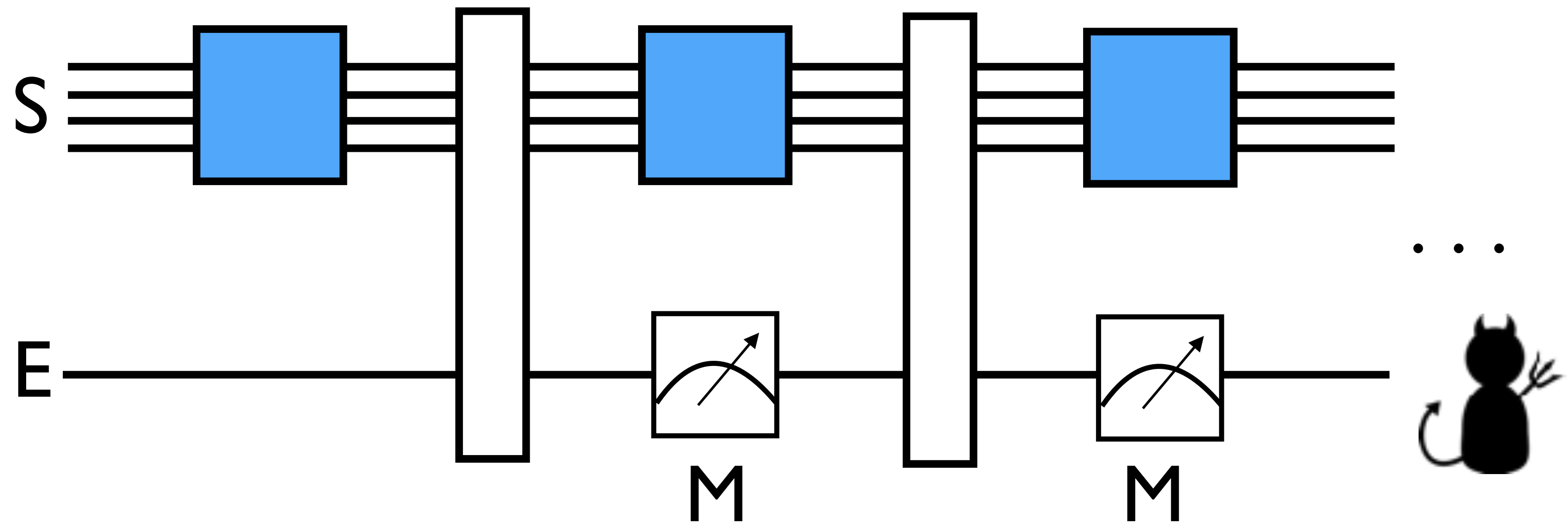
Delayed feedback

"Informationally efficient environment"

- Continuous time version: Stochastic master equation

Carmichael, *An Open Systems Approach to Quantum Optics* (1991).
Gardiner and Zoller, *Quantum Noise* (2004).

Quantum Channel Description of Dynamics



Quantum channel: $\mathcal{N}(\rho) = \sum_{\vec{m}} K_{\vec{m}} \rho K_{\vec{m}}^\dagger$

Kraus operator

Unitary-measurement dynamics:

$$K_{\vec{m}} = P_t^{m_t} U_t \cdots P_1^{m_1} U_1$$

Measurement record:

$$\vec{m} = (0, 1, 1, 0, \ldots)$$

Example: single qubit

$$P_t^{m_t} = |m_t\rangle\langle m_t|$$

Quantum trajectory: Quantum state conditioned on measurements

$$\rho \to K_{\vec{m}} \rho K_{\vec{m}}^\dagger / p_{\vec{m}}$$

Probability: $p_{\vec{m}} = \mathrm{Tr}[K_{\vec{m}}^\dagger K_{\vec{m}} \rho]$

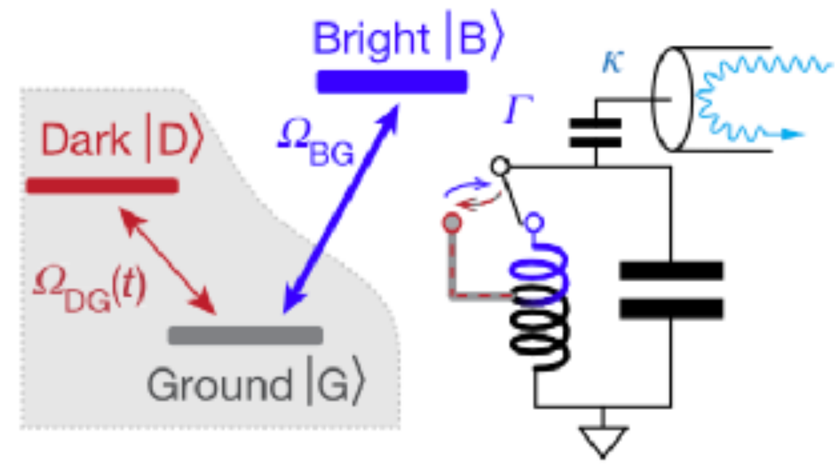
Nonunitary and nonlinear dynamics

Experimental Realization of Unitary-Measurement Dynamics

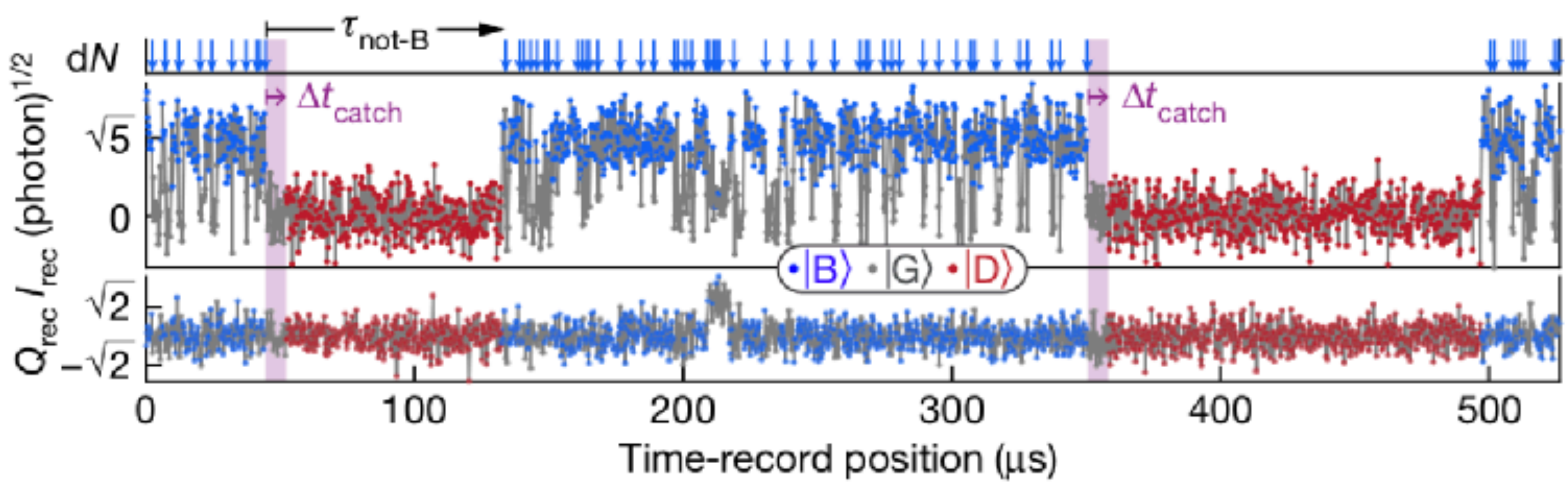
To catch and reverse a quantum jump mid-flight

Z. K. Minev[1,5]*, S. O. Mundhada[1], S. Shankar[1], P. Reinhold[1], R. Gutiérrez-Jáuregui[2], R. J. Schoelkopf[1], M. Mirrahimi[3,4], H. J. Carmichael[2] & M. H. Devoret[1]*

Three-level system formed from two transmon qubits



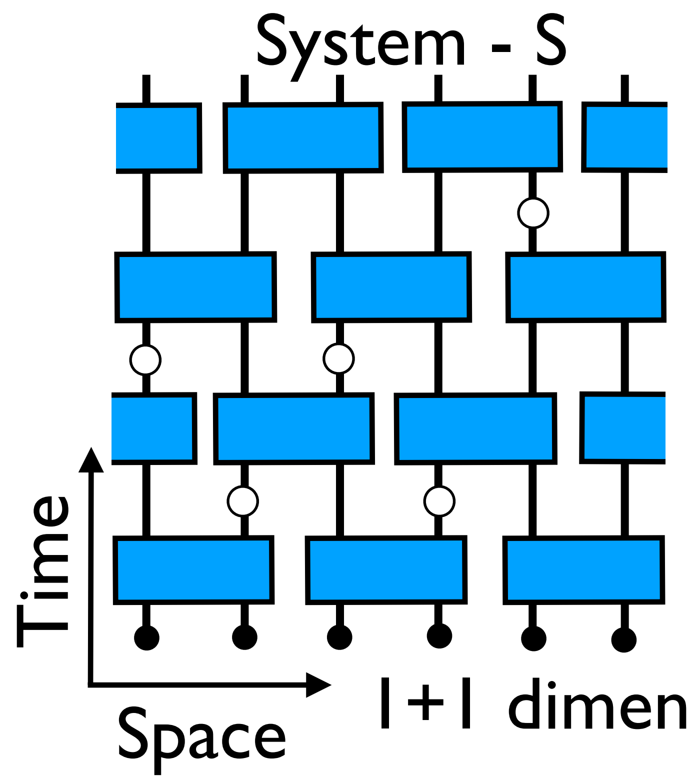
Continuous time version:



0+1 dimensional system

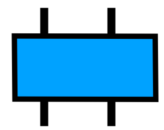
Minev *et al.*, Nature (2019).

The Measurement-Induced Transition



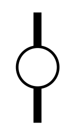
1+1 dimensions

Unitary



Random two-site gate

Measurement



- M/E

Measurement rate p
- Coupling to environment

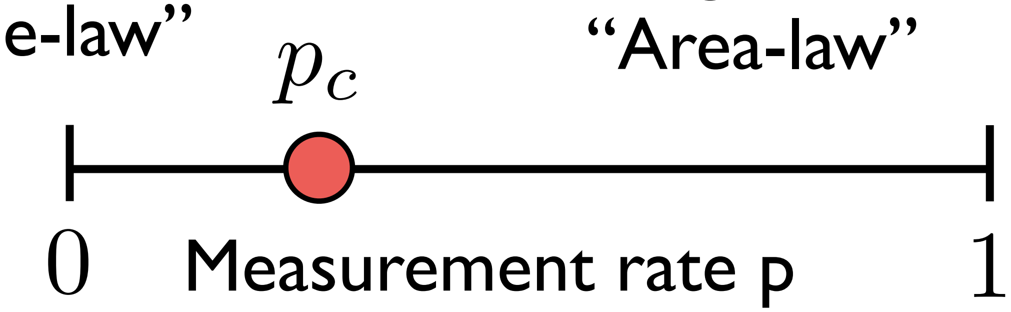
Gullans and Huse, arxiv:1905.05195

Reversible dynamics

Long-range entanglement - "Volume-law"

Irreversible dynamics

Short-range entanglement - "Area-law"



Li, Chen, Fisher, PRB (2018/19).
Skinner, Ruhman, Nahum, PRX (2019).
See also Aharanov, PRA (2000).

Limiting cases: p = 1 - Random product state
p = 0 - Random unitary dynamics

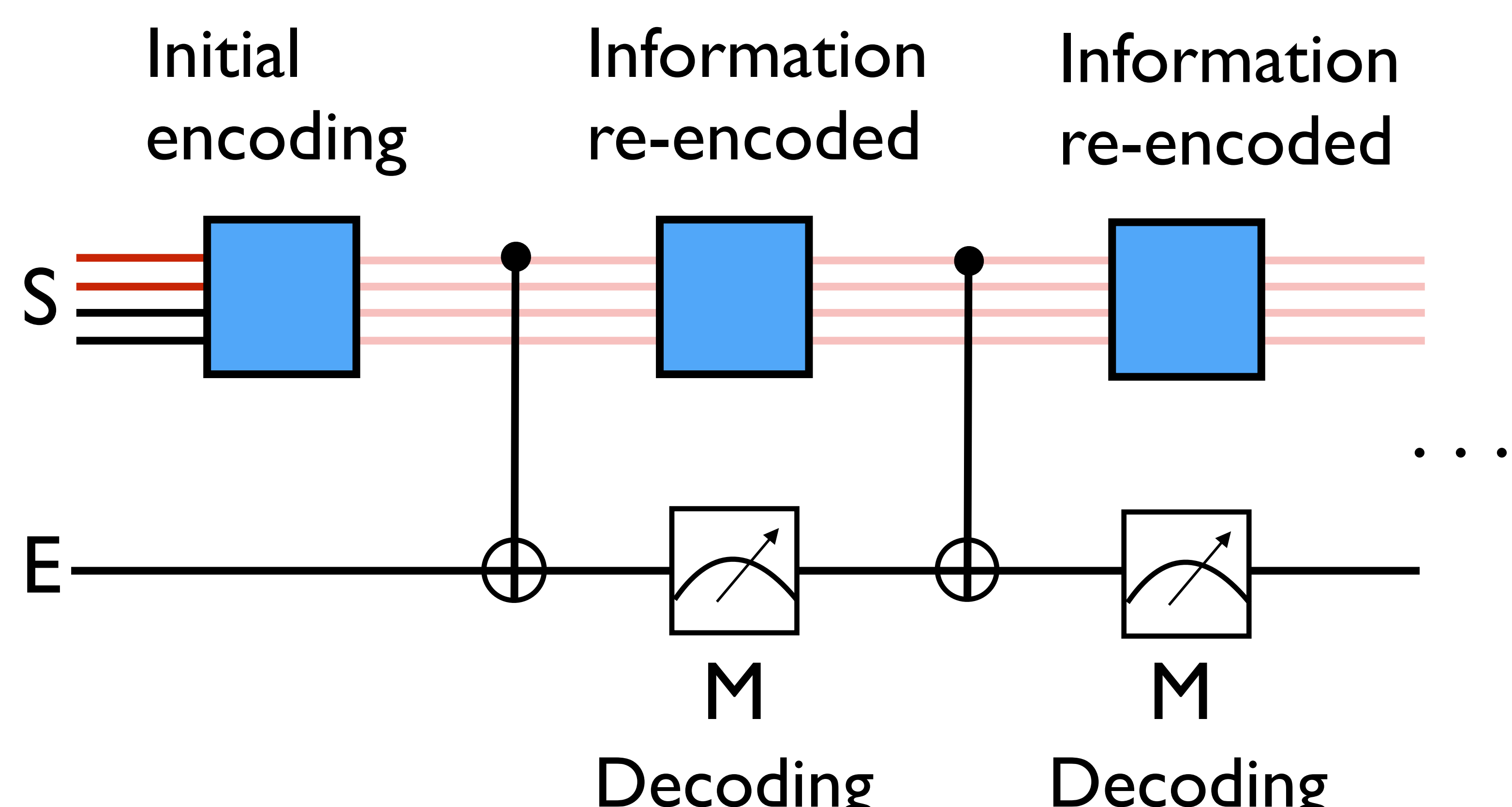
Purification or Memory Transition

Start in a mixed state: $\rho \to K_{\vec{m}} \rho K^{\dagger}_{\vec{m}} / p_{\vec{m}}$

Do the measurements collapse the system to a pure state?
Equivalent: Does the system forget initial conditions?

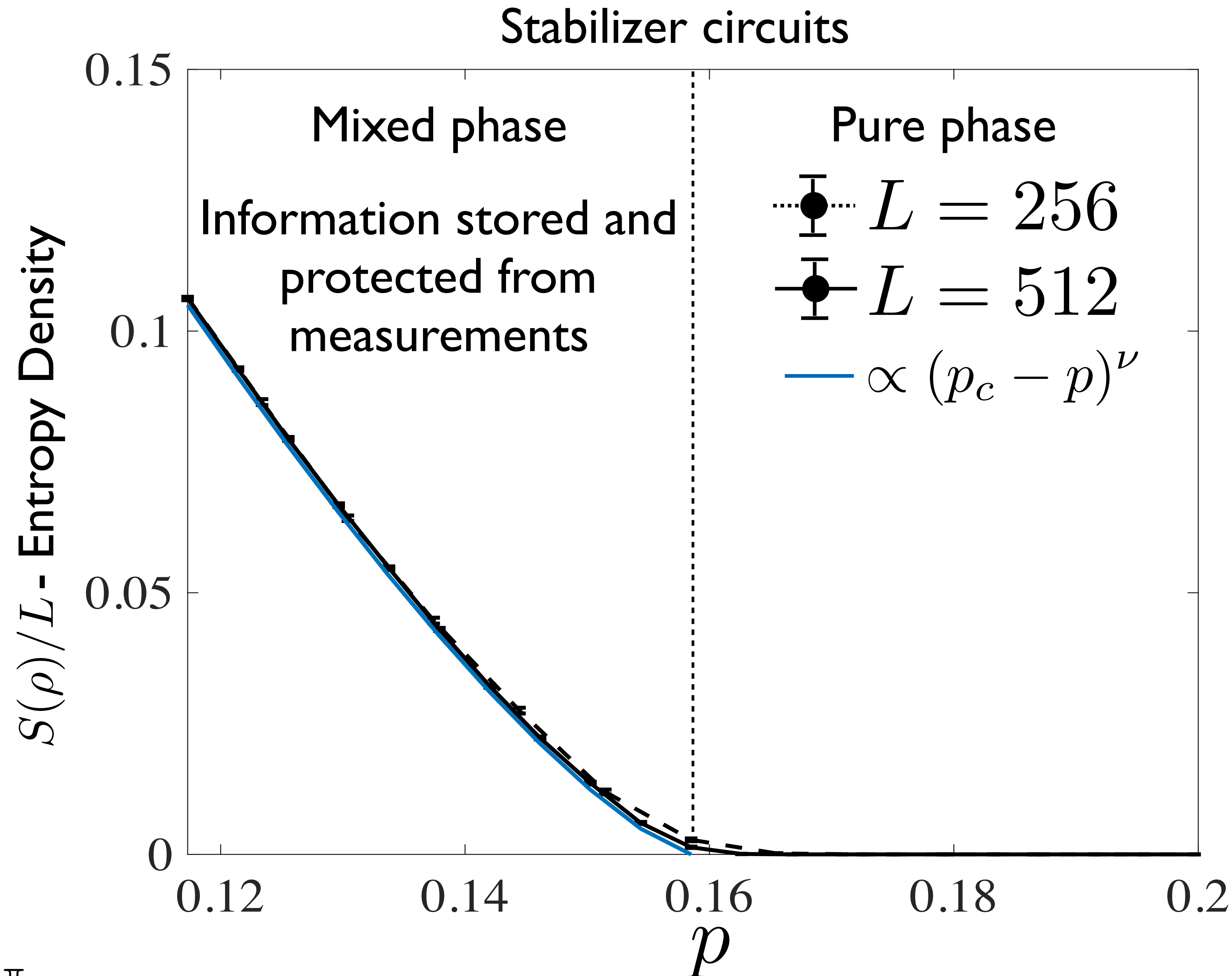
Limiting cases: p = 1 - state evolves to a random pure product state

p infinitesimal compared to system scrambling time ~ L
- not until times exp(L)



Gullans and Huse, arxiv:1905.05195

Mixed and Pure Phase



$\rho = \frac{1}{2^L}\mathbb{I}$ - Initial state

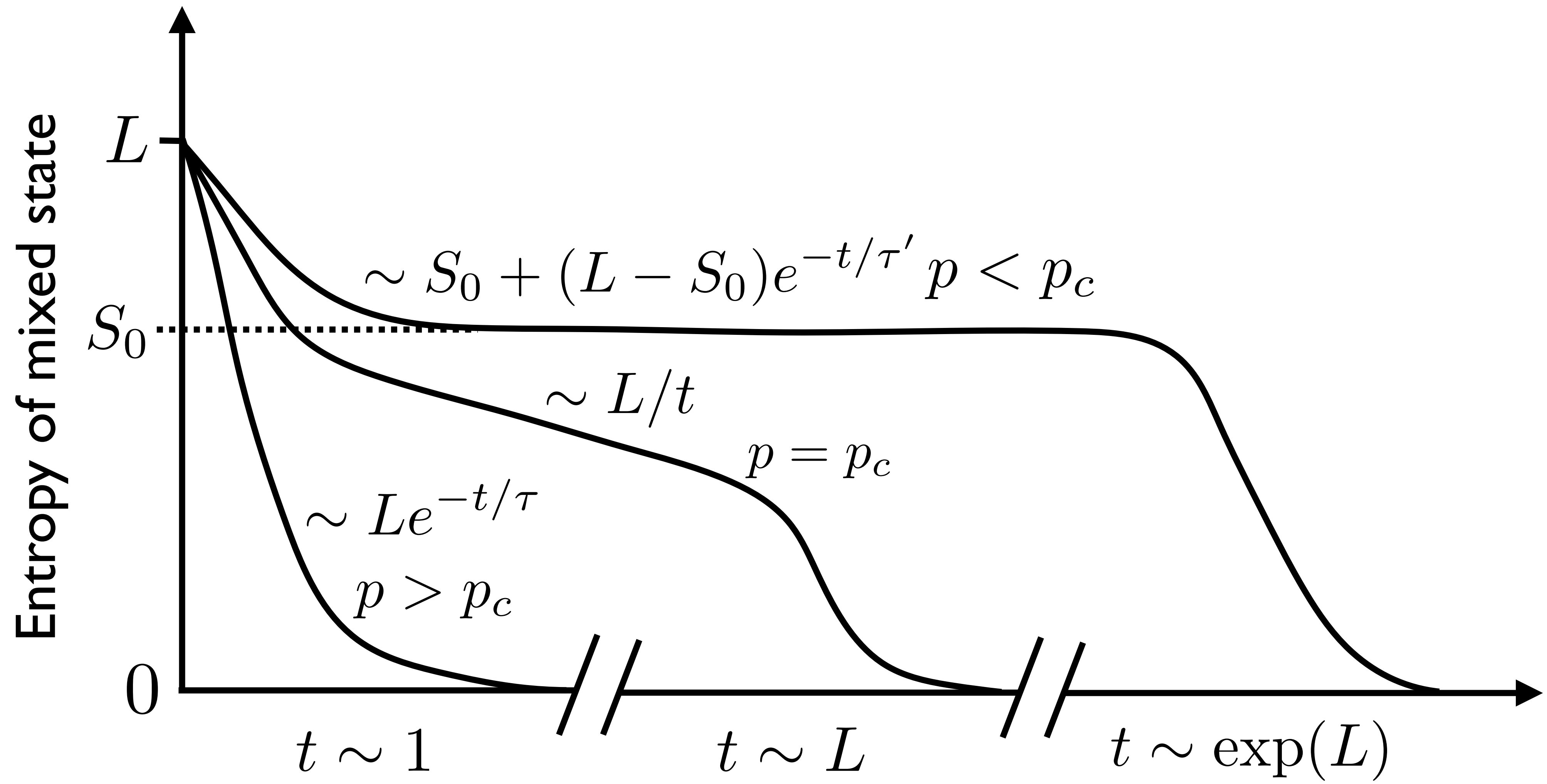
L - Number of qubits in 1D chain

Simulations using Gottesman-Knill theorem

Aaronson, Gottesman, PRA (2004).

Gullans and Huse, arxiv:1905.05195

Purification Dynamics



$\tau \propto \frac{1}{|p - p_c|^{\nu}}$ - Correlation time

S_0/L - Density of logical qubits

Gullans and Huse, arxiv:1905.05195

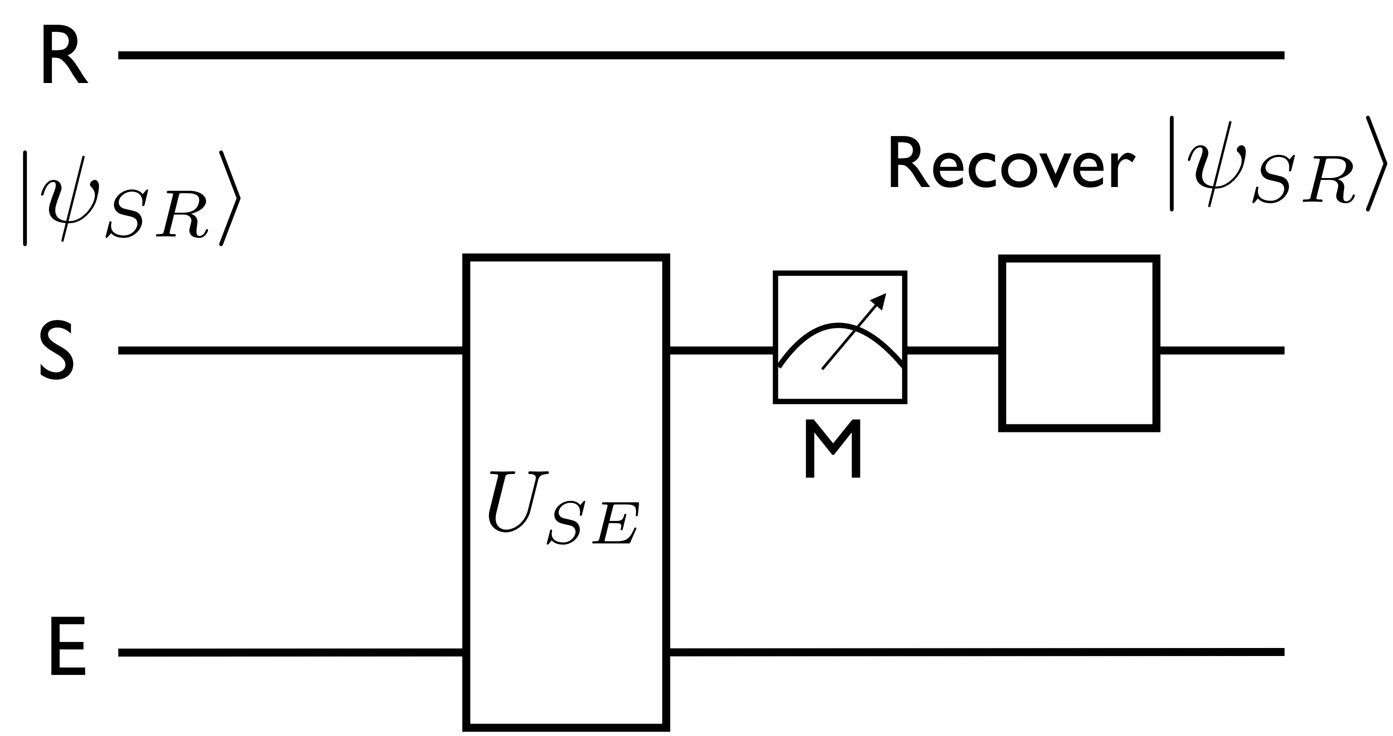
Quantum Channel Capacity

Shannon channel capacity - maximum amount of information that can be sent down a noisy channel

Purify the quantum channel

$$N(\rho_S) = \sum_{\vec{m}} K_{\vec{m}} \rho_S K_{\vec{m}}^{\dagger} \to U_{SE} |\psi_{SR}\rangle \otimes |0_E\rangle$$

$$\rho_S = \mathrm{Tr}_R(|\psi_{SE}\rangle\langle\psi_{SE}|)$$



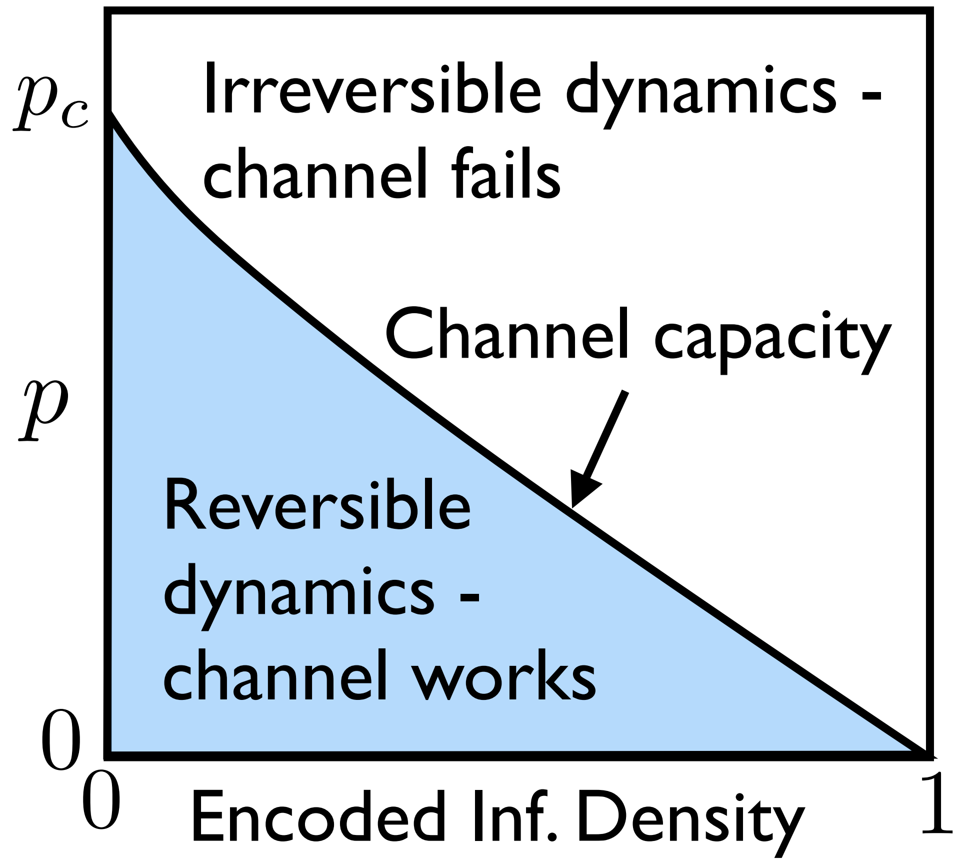
Quantum channel capacity - maximum number of recoverable qubits in R

Schumacher, Nielsen, PRA (1996), Lloyd, PRA (1997).

Purification Transition as a Quantum Error Correction Threshold

Entropy of the mixed state measures the number of "recoverable" qubits in the reference system - lower bound on the channel capacity

Full phase diagram



Corollary: The mixed phase naturally generates finite-rate quantum error correcting codes. Found entirely new class of QEC codes!

Gullans and Huse, arxiv:1905.05195

See also Choi, Bao, Qi, Altman, arxiv:1903.05124

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- Measurement-induced entanglement transition
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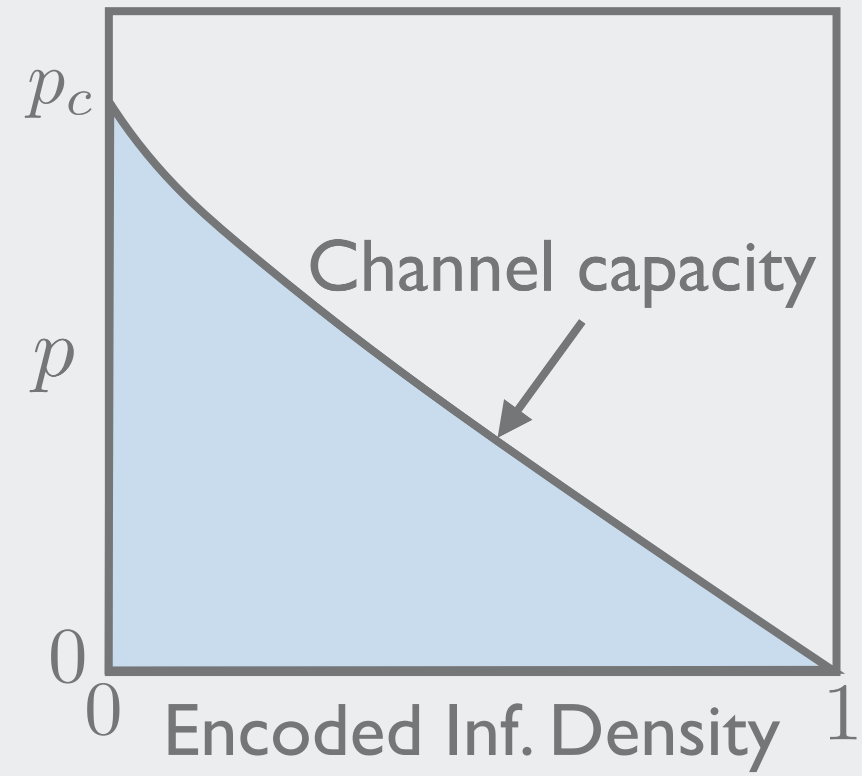
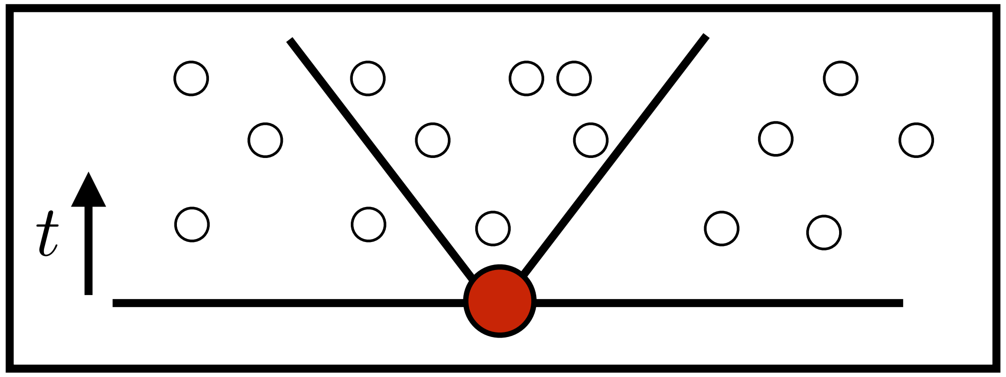
- Quantum error correction thresholds



Defining a local order parameter

Scalable probes of the "ordered" phase



Quasirandom stabilizer codes

Gullans and Huse, arxiv:1910.00020

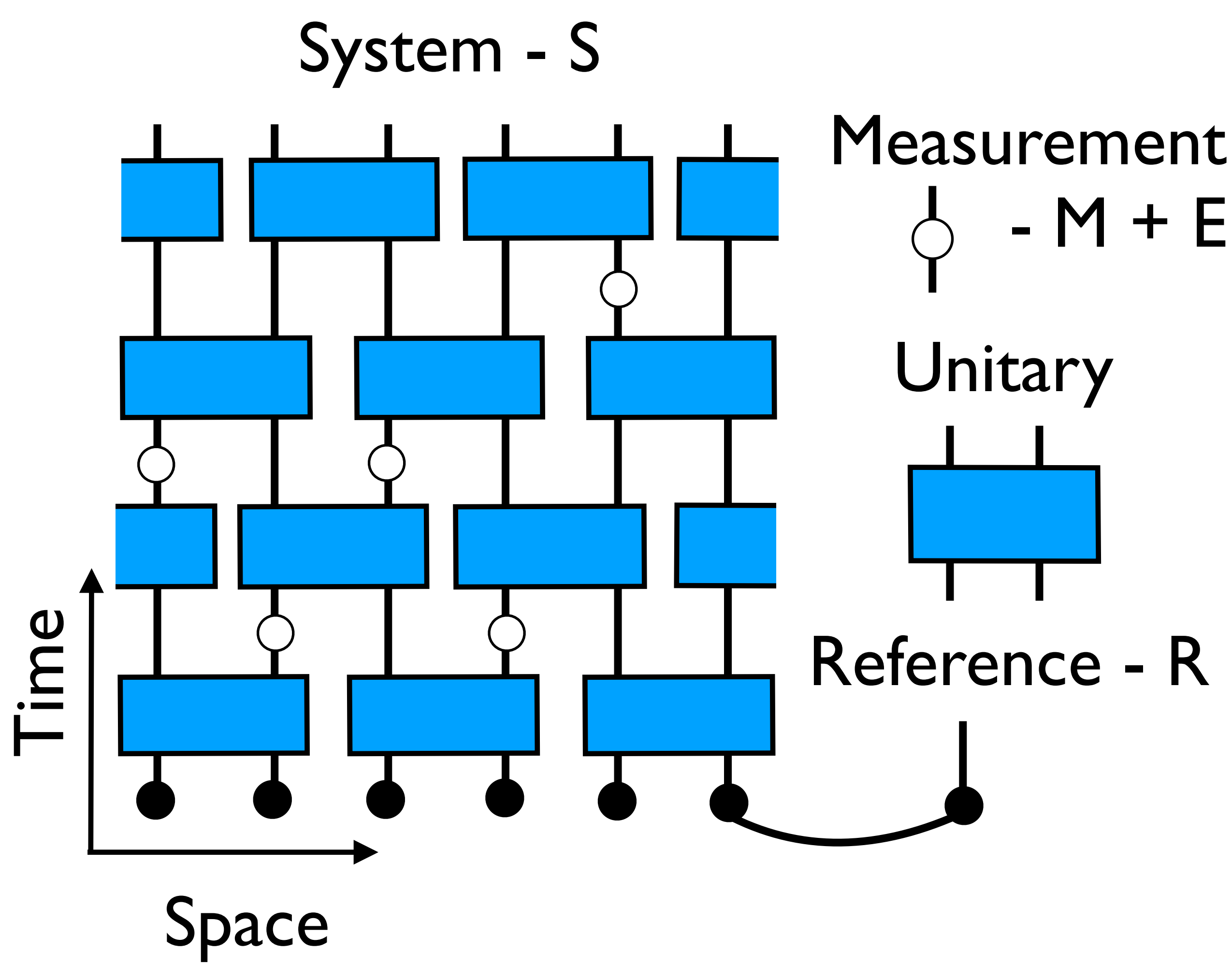
Defining a Local Order Parameter

Entanglement and entropy are often highly non-local observables

- Difficult to measure
- May not be a reliable diagnosis for the ordered phase (e.g., all-to-all models)

Mixed phase is defined by the ability to store and protect local quantum information

- Use this to define an order parameter



Order parameter definition:

$$\langle S_Q \rangle = \sum_{\vec{m}} p_{\vec{m}} S(\rho_{R\vec{m}})$$

Probability of measurement record:

$$p_{\vec{m}}$$

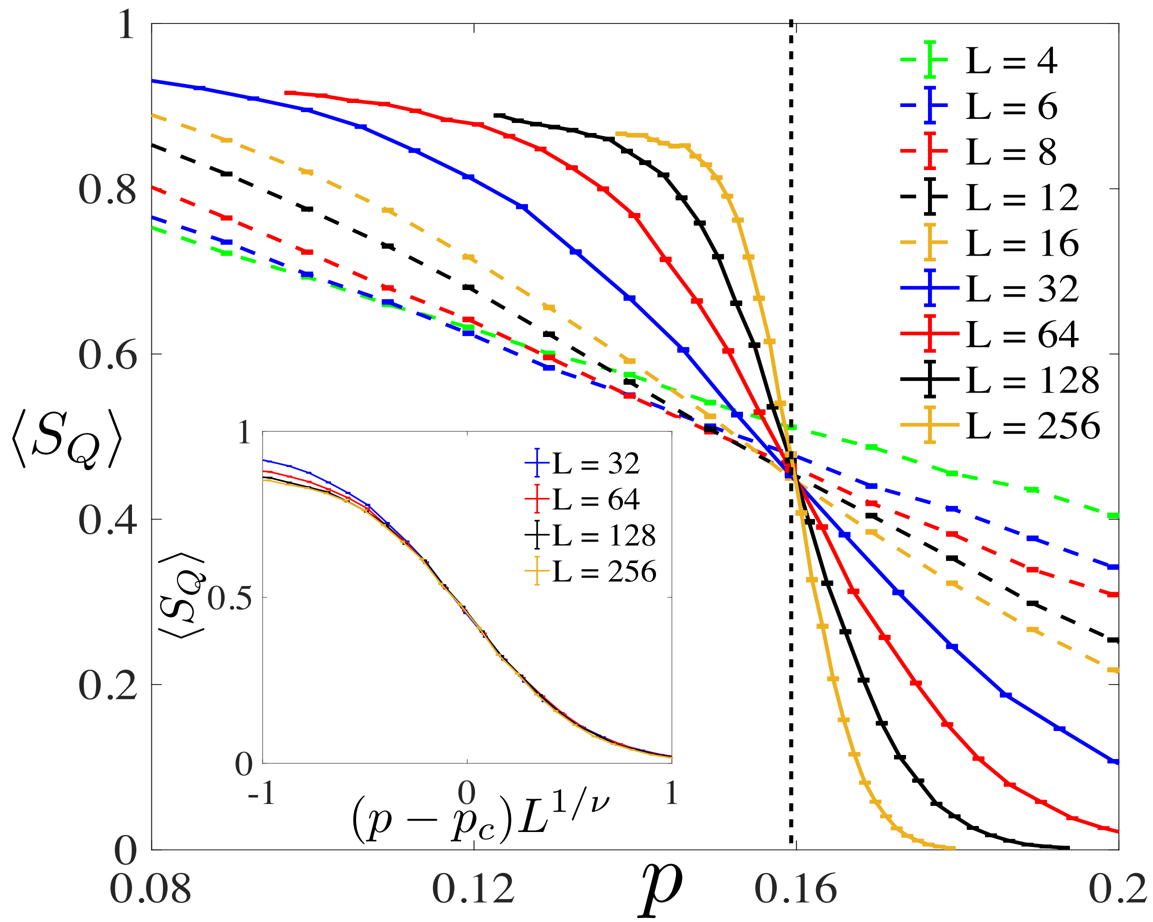
Density matrix of reference qubit:

$$\rho_{R\vec{m}} = \begin{pmatrix} p_{1\vec{m}} & O_{\vec{m}} \\ O^*_{\vec{m}} & p_{0\vec{m}} \end{pmatrix}$$

Gullans and Huse, arxiv:1910.00020

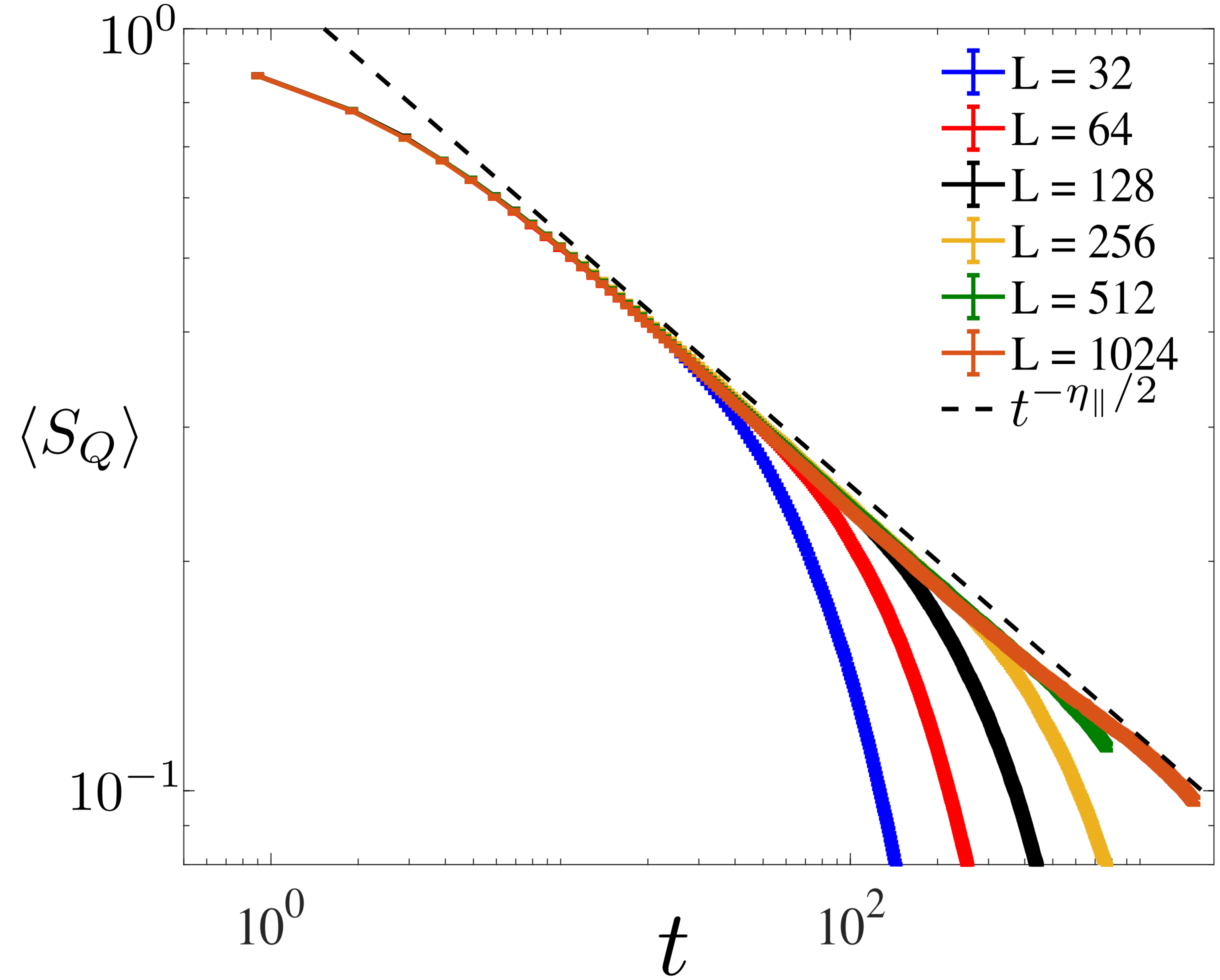
Results for Stabilizer Circuits

Initial state - pseudorandom stabilizer state



Works on small systems

Initial state - product state at p_c



At critical point p_c: Power-law decay of order parameter

Works for generic/nonstabilizer dynamics:
Zabalo *et al.* (MJG), arxiv:1911.00008

Gullans and Huse, arxiv:1910.00020

Experimental Challenges

Main challenge - exponentially large number of trajectories - difficult to prepare many copies of a given trajectory - hidden information

$$\langle S_Q \rangle = \sum_{\vec{m}} p_{\vec{m}} S(\rho_{R\vec{m}}) \quad \rho_{R\vec{m}} = \begin{pmatrix} p_{1\vec{m}} & O_{\vec{m}} \\ O^*_{\vec{m}} & p_{0\vec{m}} \end{pmatrix}$$

Analyzing the data requires finding the "decoding function"

$$\vec{m} \rightarrow (p_{0\vec{m}}, O_{\vec{m}})$$

Measurement and control errors - in progress

- Uncorrected errors eventually wash out the transition
- Intermediate measurements can be used to help detect/correct errors

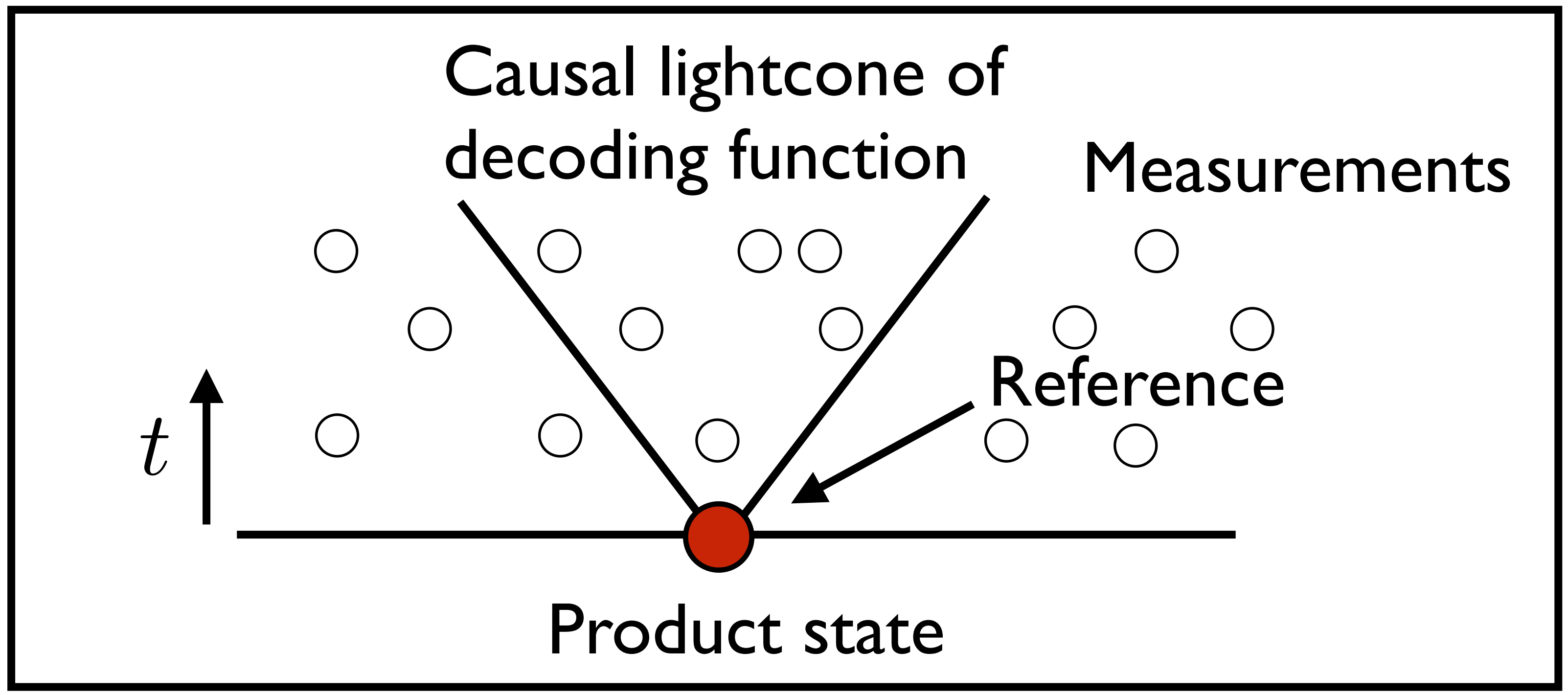
Potential built-in fault-tolerance to the dynamics

Finding the Decoding Function in Experiment

Three basic approaches:

(I) Experimentally sample each quantum trajectory within the lightcone

- Efficient for constant depth circuits - poly(exp(t^{d+1}))



- Works in the ordered phase - decoding function saturates after time $\sim \tau$

(II) Classical models - stabilizer circuits, 1D matrix-product states, effective statistical mechanics description - max-likelihood decoding, …

(III) Hybrid algorithms - use experimental data to learn classical model

Gullans and Huse, arxiv:1910.00020

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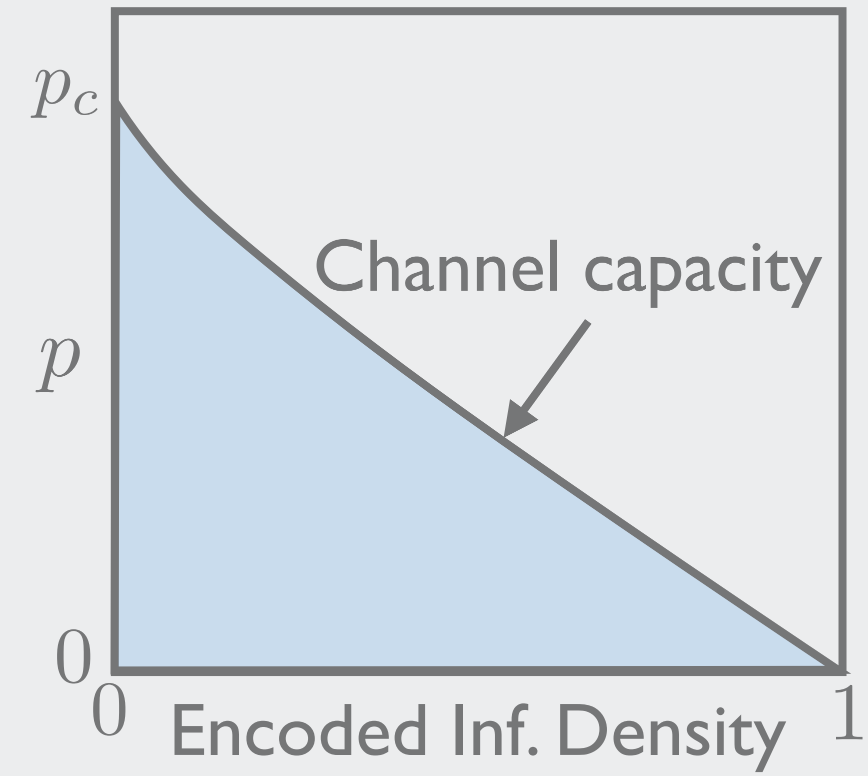

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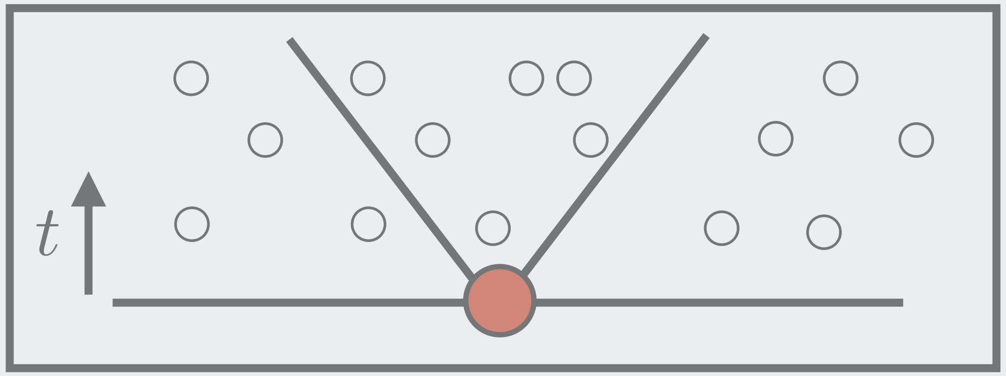


Quasirandom stabilizer codes

Stabilizer States, Mixed States, and Stabilizer Codes

Pauli group element on n-qubits - tensor product of single-site Paulis

$$g = \underbrace{Z \otimes X \otimes \cdots \otimes \mathbb{I} \otimes Z}_{n}$$

S is a commutative subgroup of Pauli group of dimension 2^n

$|S\rangle$ - stabilizer state $\quad g \in S \quad g|S\rangle = |S\rangle$

$$|S\rangle\langle S| = \frac{1}{2^n}\prod_{i=1}^{n}(\mathbb{I} + g_i)$$

Minimal generating set is of size n: $\{g_i\}_{i=1}^{n}$

Mixed stabilizer state - projector onto a subspace of dimension 2^k:

$$\rho = \frac{1}{2^n}\prod_{i=1}^{n-k}(\mathbb{I} + g_i)$$ The density matrix defines an [n,k] stabilizer code

At p = 0 dynamics generates a random code

For 0<p<pc the codes have good QEC properties

- High threshold (perfect QEC), large contiguous distance ~ $L^{1/3}$, and finite rate
- Degenerate and highly structured

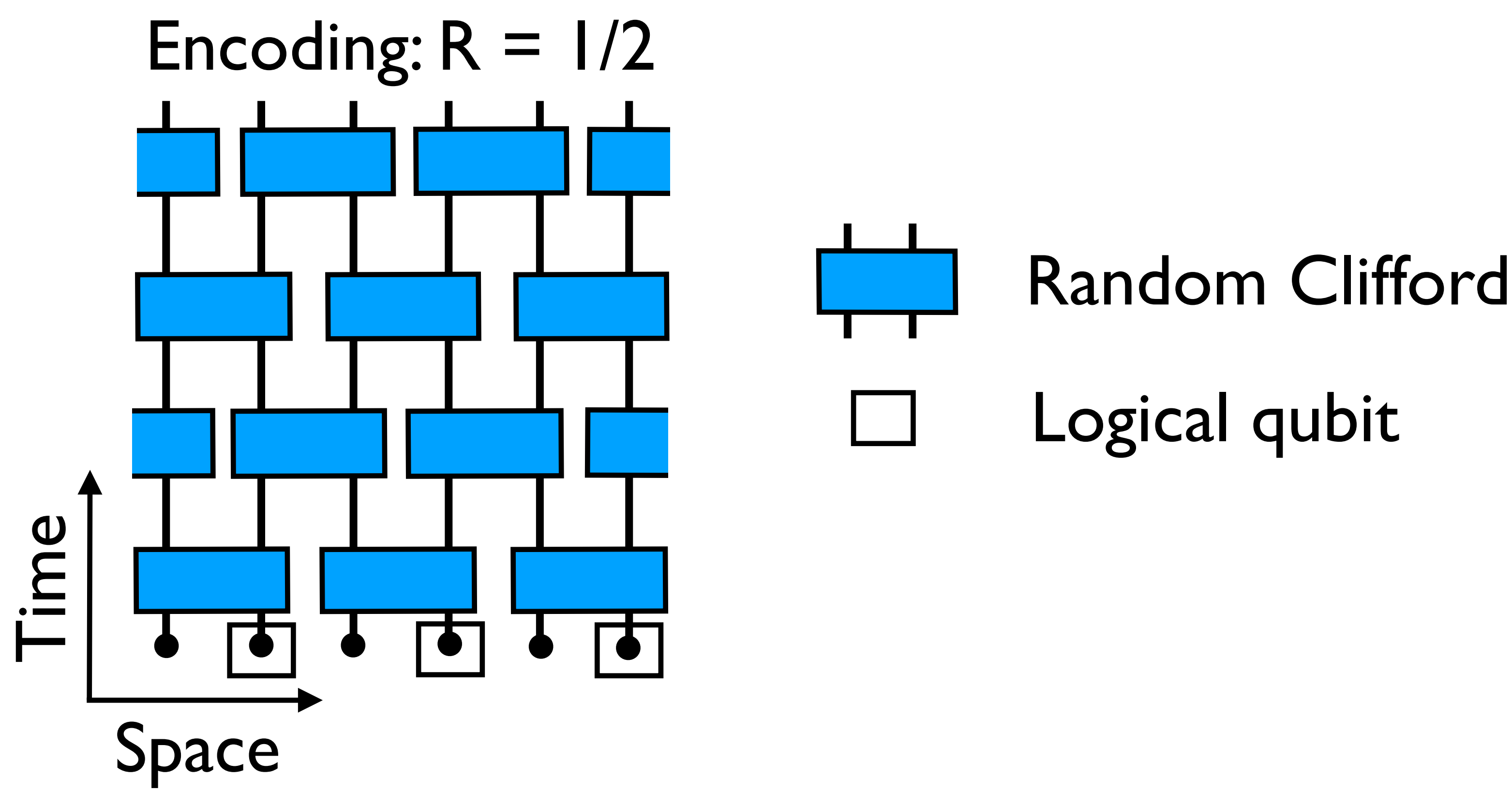
Dynamically Generated Codes at p = 0

Classical error correction: Random code constructions saturate channel capacity bound for the binary symmetric channel (independent bit flip errors)

Quantum error correction: Random stabilizer codes saturate quantum channel capacity bound for erasure channel

Code rate: $R \leq 1 - 2e$

Random code critical erasure rate (a.s.): $e_c = (1-R)/2$



At what depth do we converge to random code behavior?

Stabilizer Codes Generated by Random Circuits

Short random circuits define good quantum error correcting codes

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arXiv:1312.7646 - Proceedings of ISIT 2013

All-to-all model with random 2-qubit Clifford gates

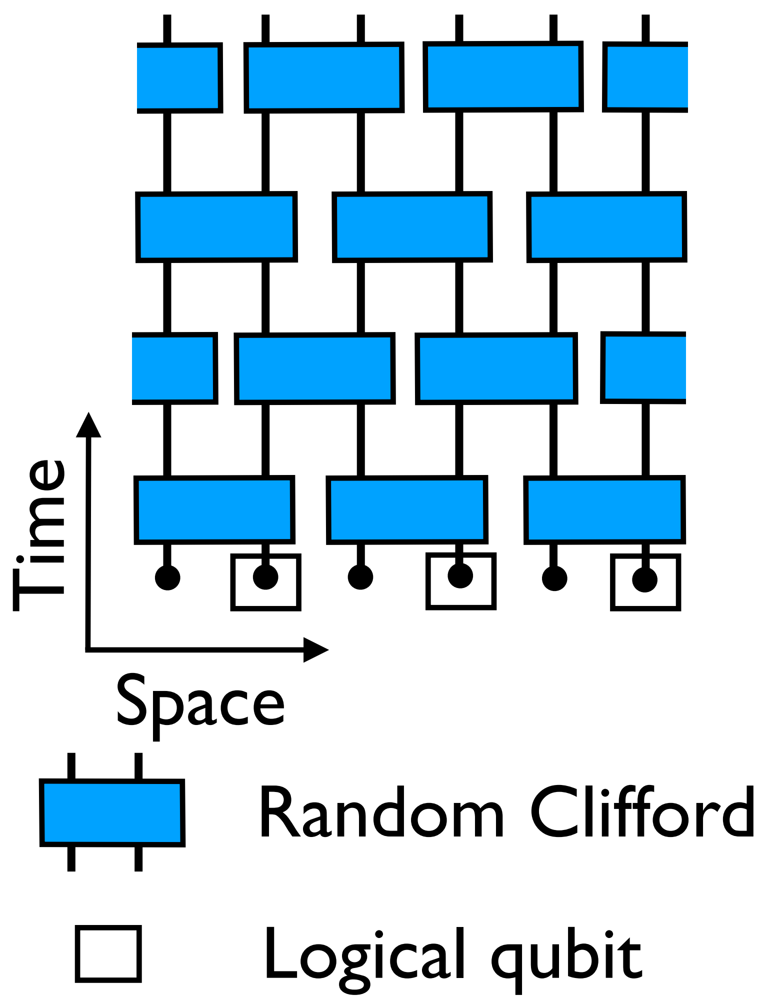
Distance converges to random code behavior in depth ~ $\log^3(N)$

Our result: Critical properties (including e_c) converge much more rapidly than scrambling time

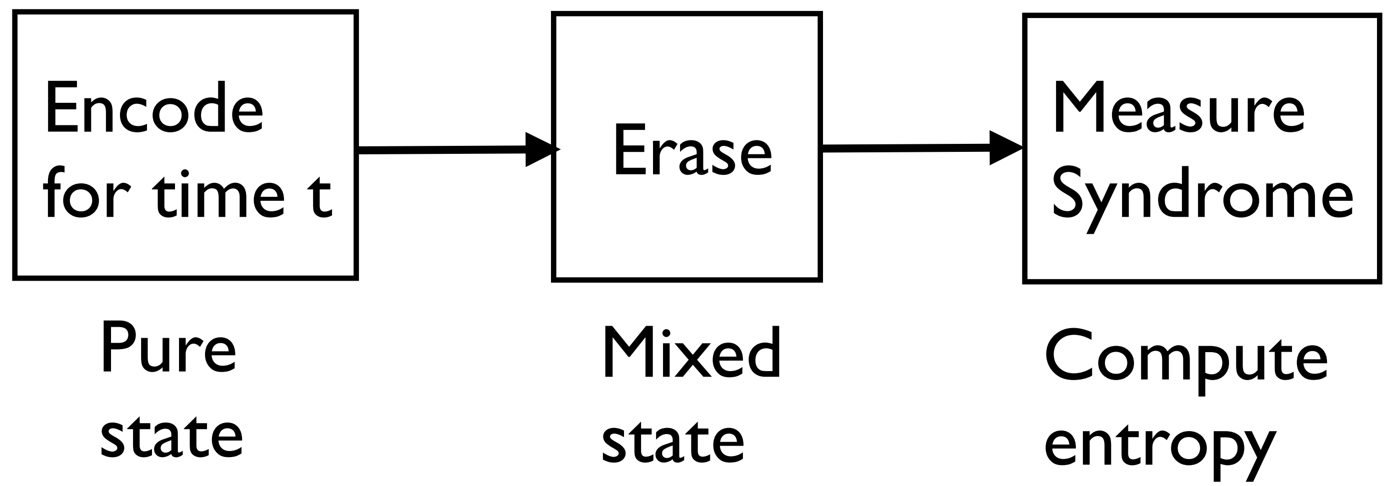
Dynamically Generated Stabilizer Codes at p = 0

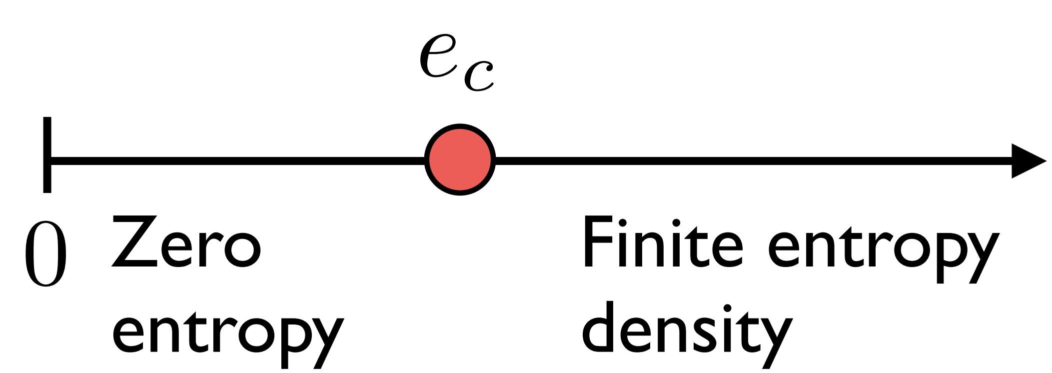
Look at 2-local random Clifford circuits in 1D, 2D and all-to-all

Encoding: R = 1/2



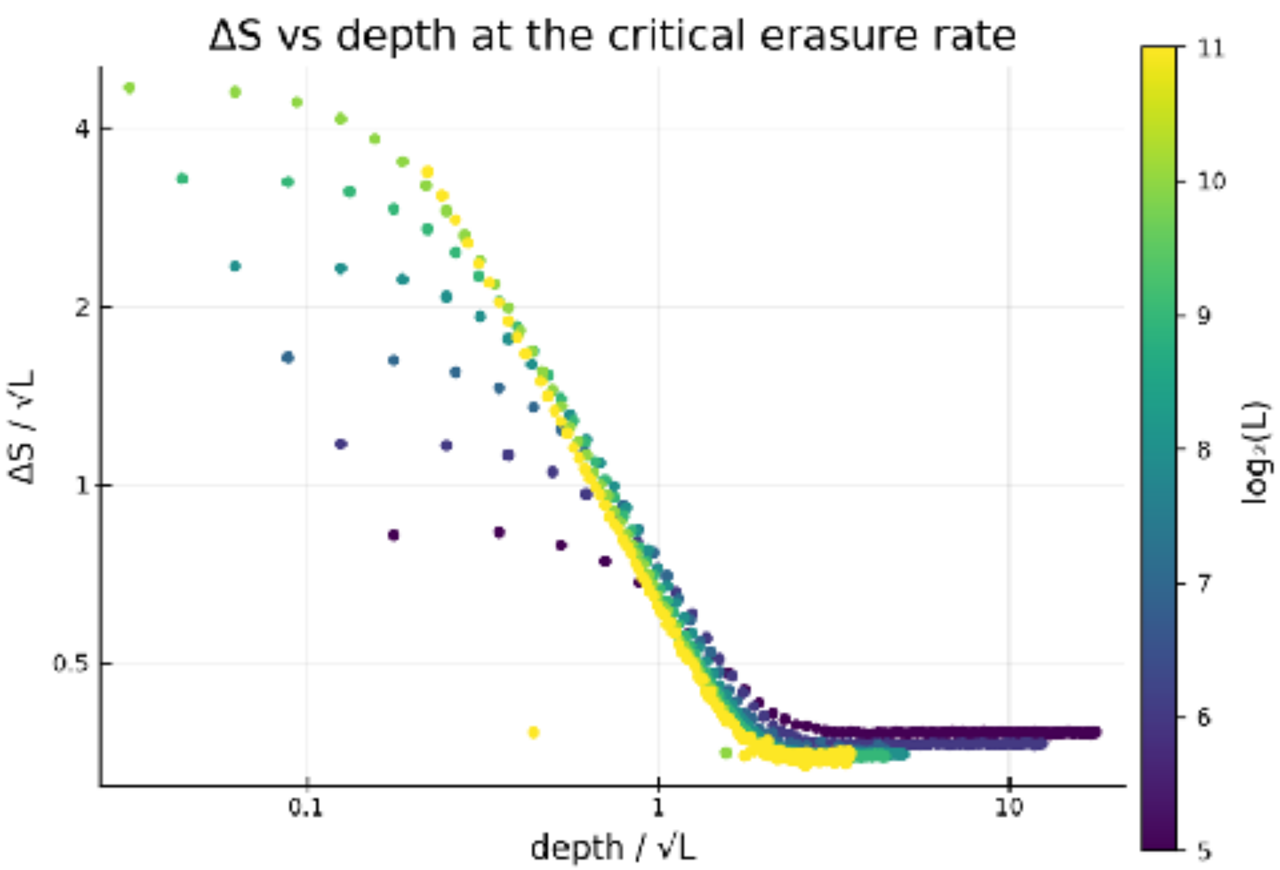
QEC Model:





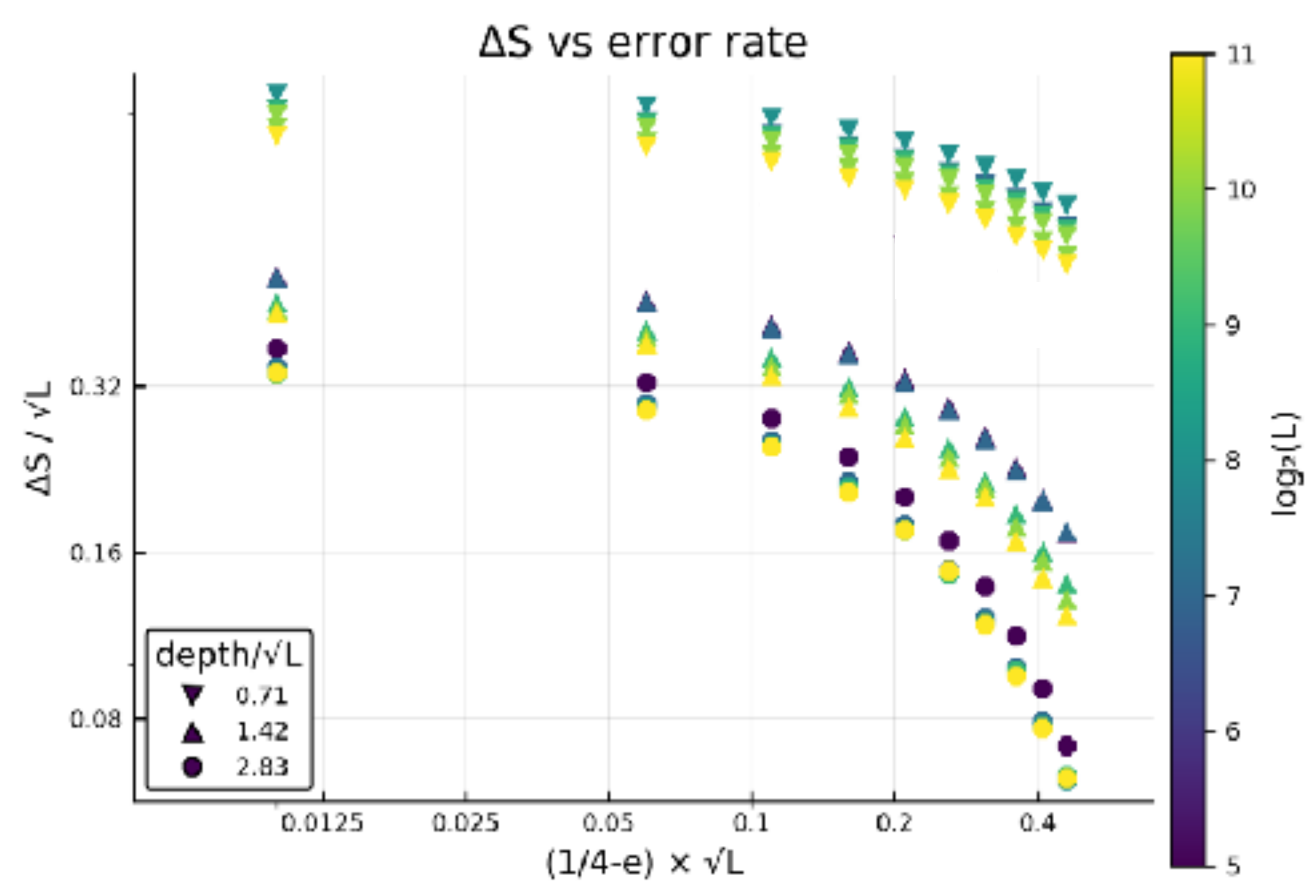
Entropy density - Order parameter for phase above threshold

Convergence to Critical Properties of Random Code in 1D



Converges to random code behavior at e_c at depth $\sim L^{1/2}$

Critical region - continuous phase transition

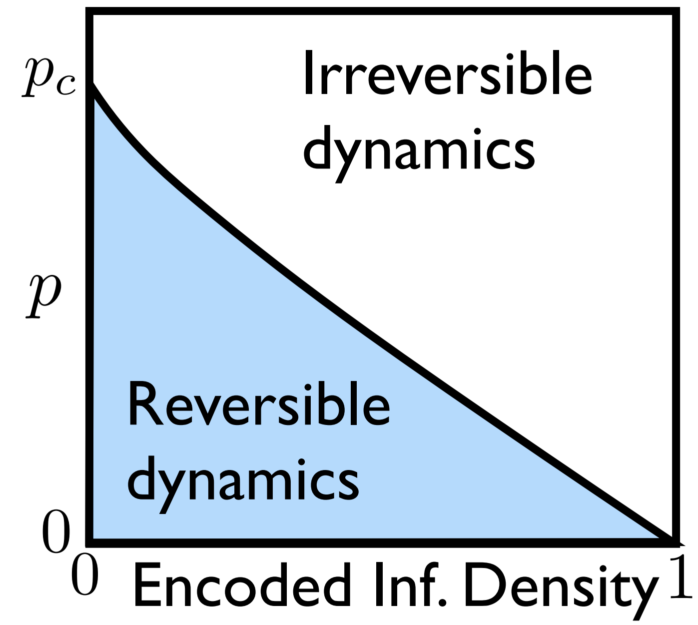


Overview

- Measurement-induced transitions are a generic property of open quantum system dynamics

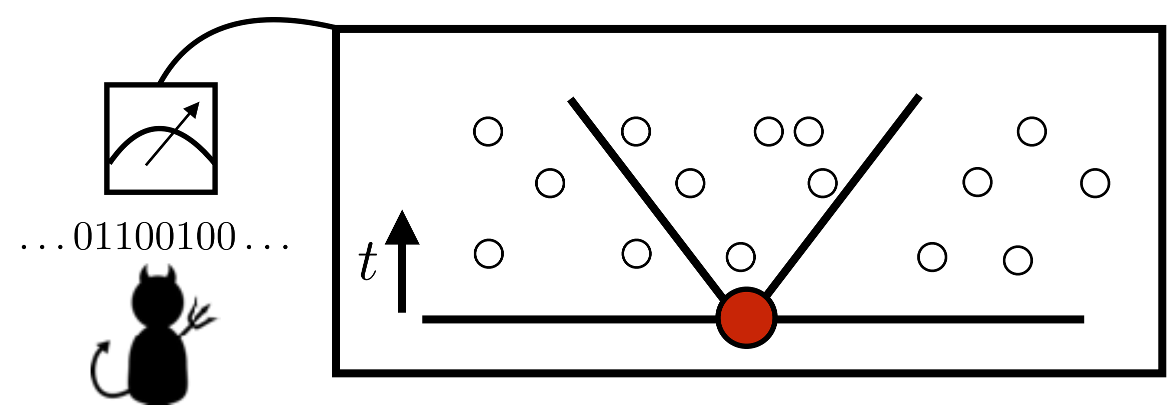
- Established precise connections between these phase transitions and quantum error correction thresholds

Gullans and Huse, arxiv:1905.05195



- Introduced a local order parameter that points towards scalable experimental probes

Gullans and Huse, arxiv:1910.00020



- Random code behavior is achieved much more quickly than the scrambling time - in progress

Nature may be pointing us to new approaches to fault-tolerant quantum computing

David Huse

Thanks for your attention!

Jason Petta

Collaborators: Measurement transition
Aidan Zabalo - Rutgers
Justin Wilson - Rutgers
Sarang Gopalakrishnan - CUNY
Jed Pixley - Rutgers
David Huse - Princeton

Collaborators: Quasirandom codes
Stefan Krastanov - Yale ⟶ MIT
Steve Flammia - Sydney/Yale
Steve Girvin - Yale
Liang Jiang - Yale ⟶ Chicago
David Huse - Princeton