Discretely rotation-invariant tensors

When does a tensor give rise to holographic states?

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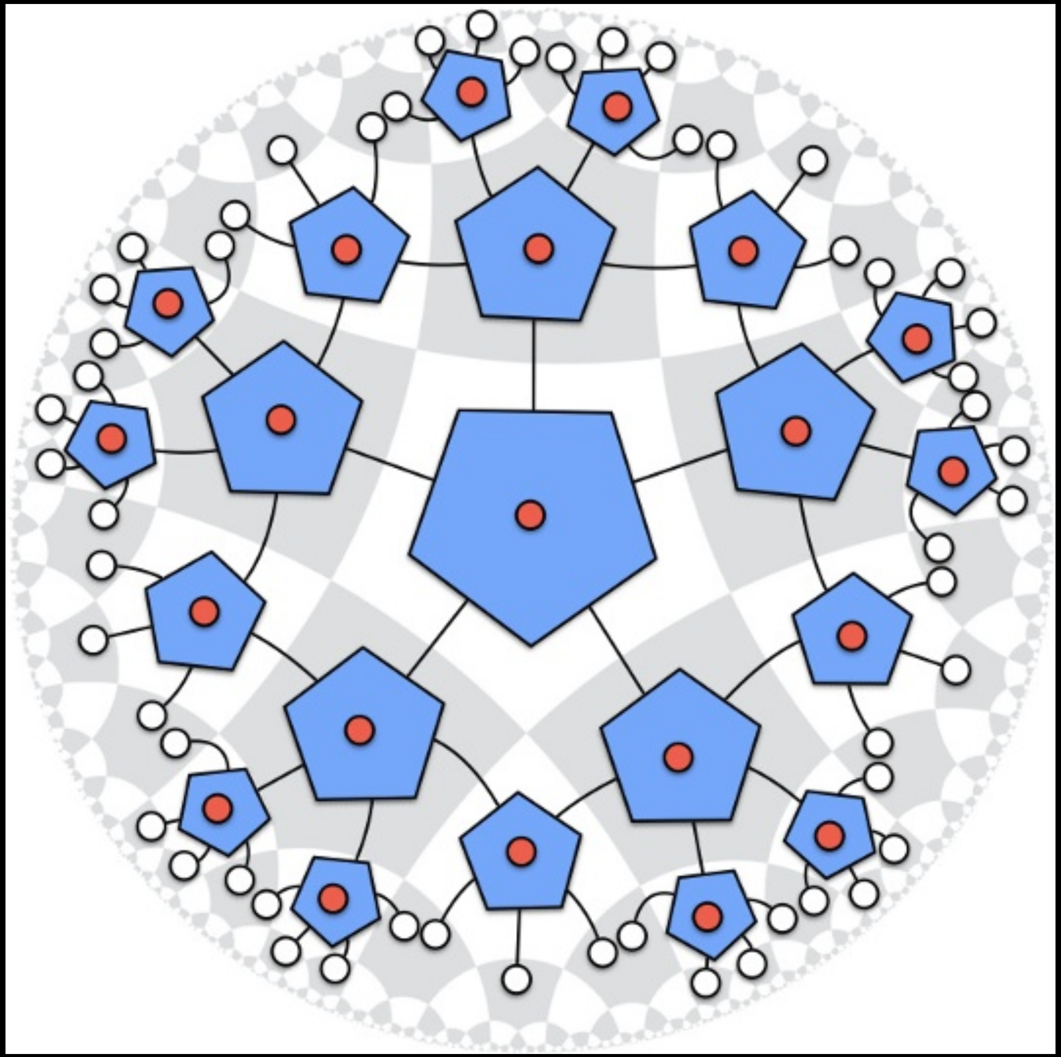
from the original paper:



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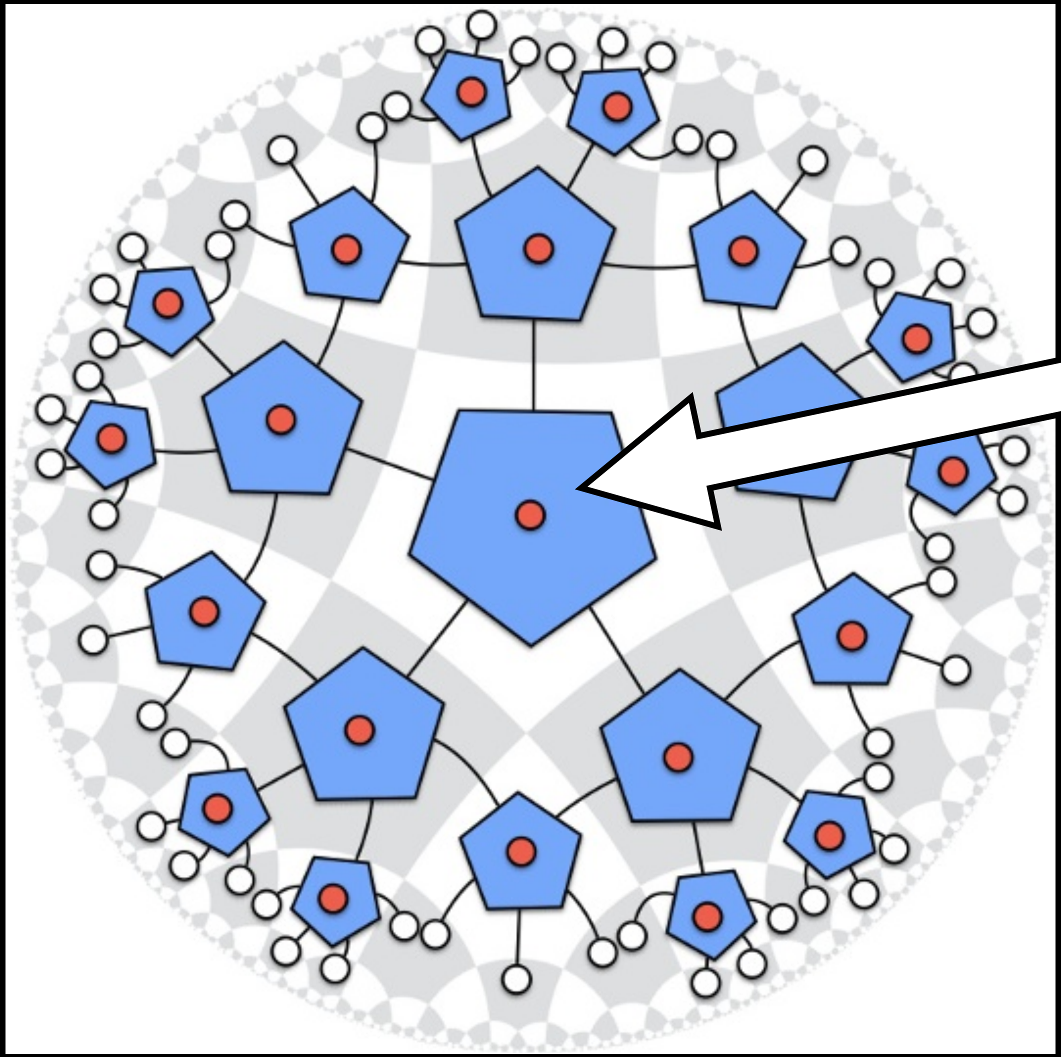
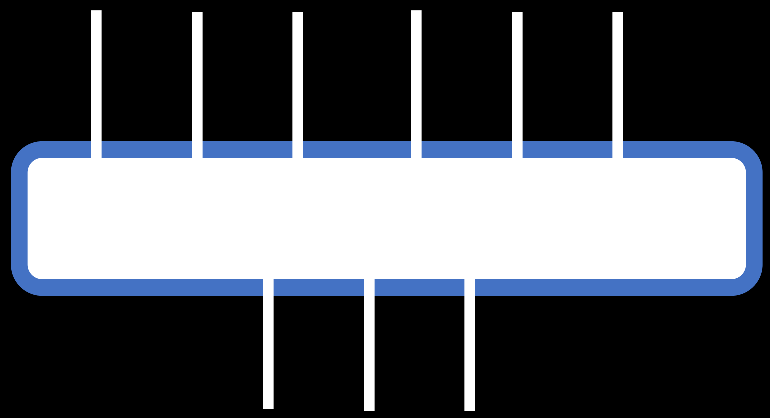


special kind of tensor*

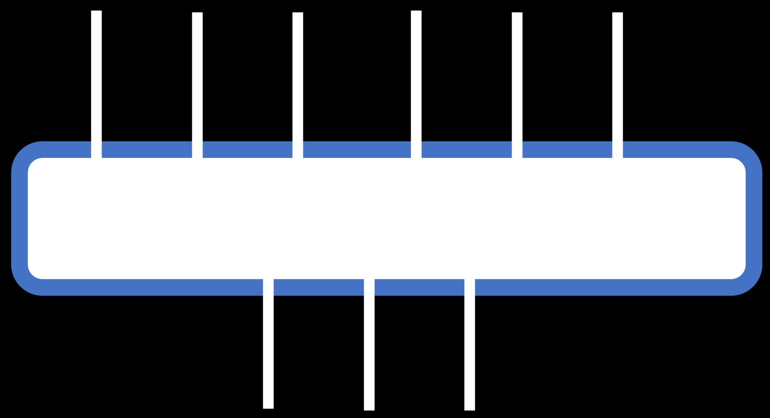
*no bulk indices

What kind of tensor?



What kind of tensor?

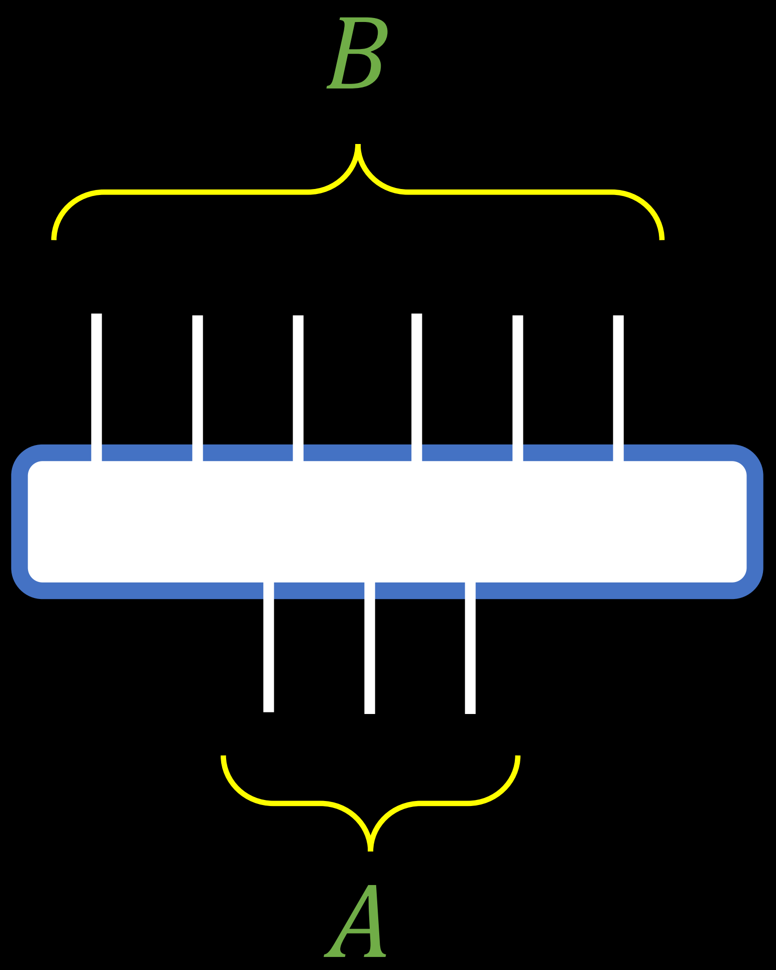
e.g. **perfect**



What kind of tensor?

e.g. **perfect**

$\forall\, A, B$

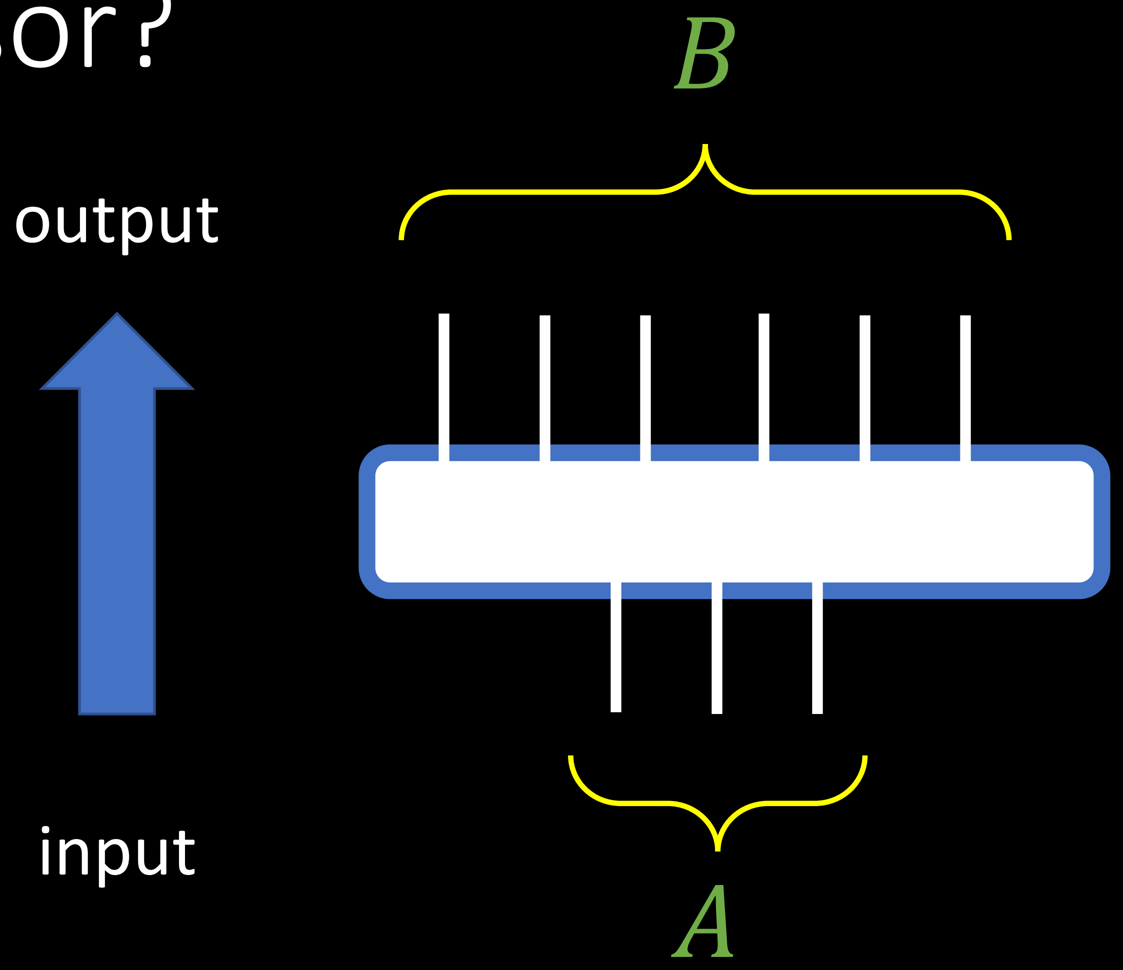


What kind of tensor?

e.g. **perfect**

$\forall\, A, B$:

isometry!

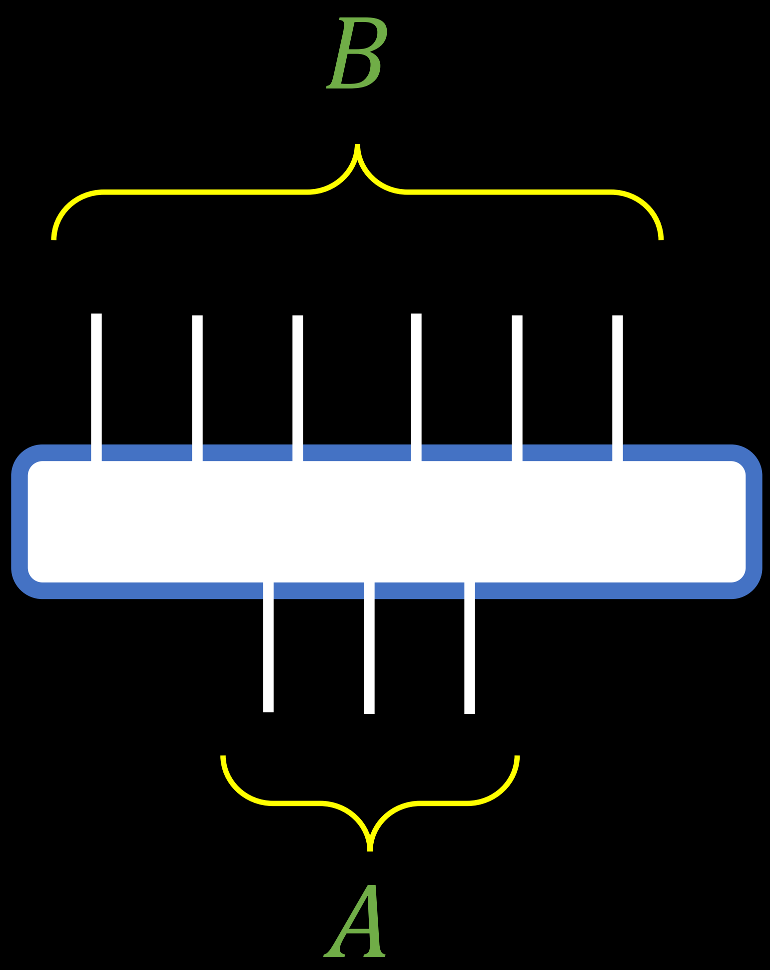


What kind of tensor?

e.g. *block***perfect**

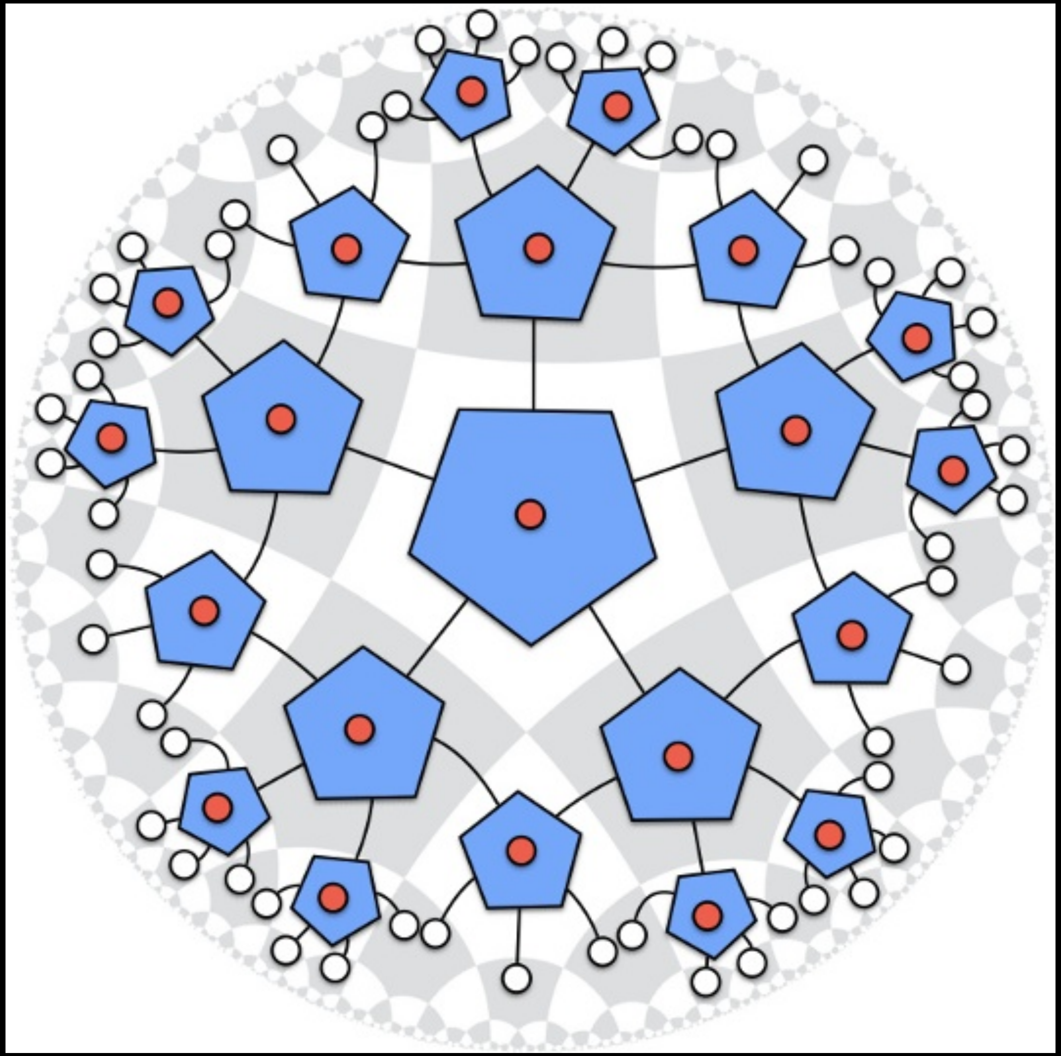
$\forall\, A, B$:

A, B contiguous



These do the job…

…but are they the right notions?



Ingredients:

1. a **tiling** of the disk
2. a finite area **region** made up of tiles
3. a **tensor** to put in the tiles

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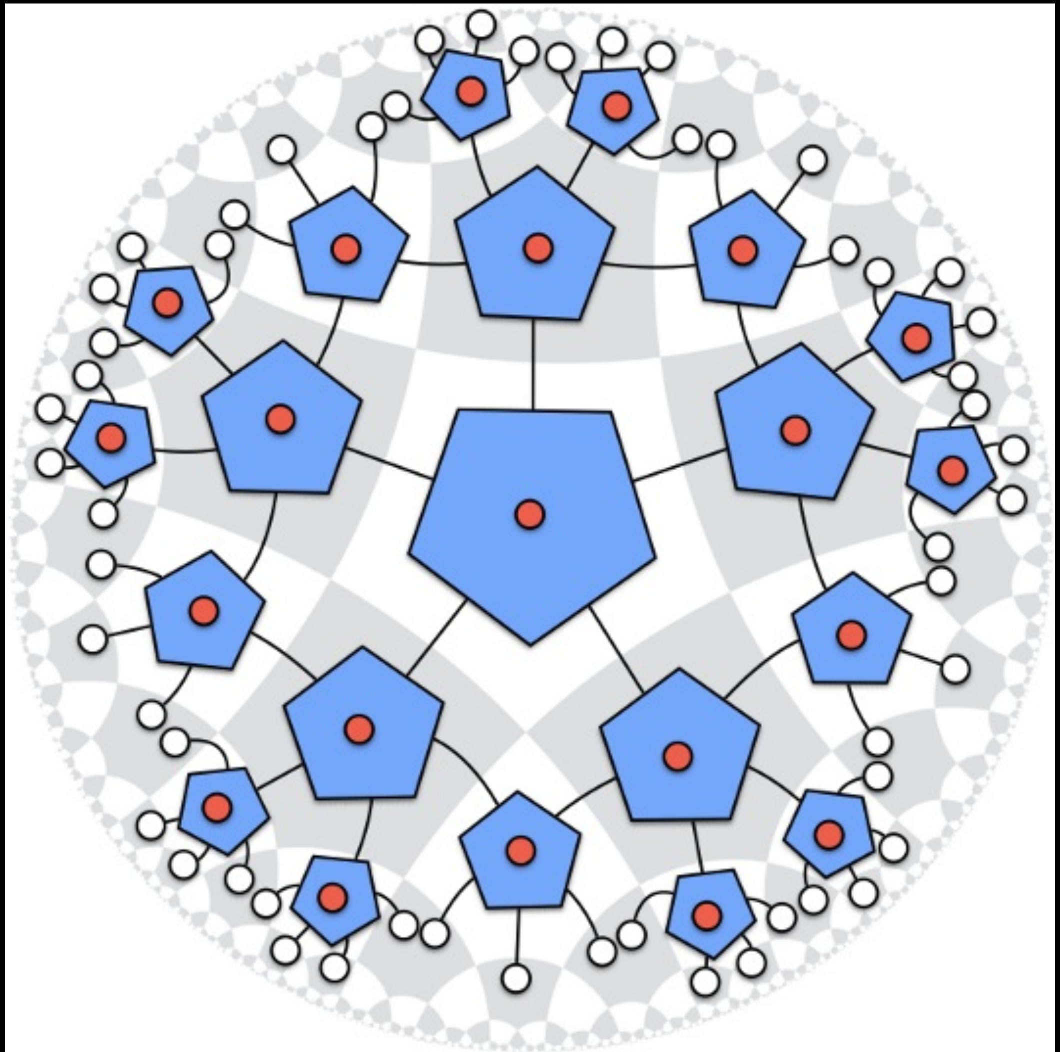
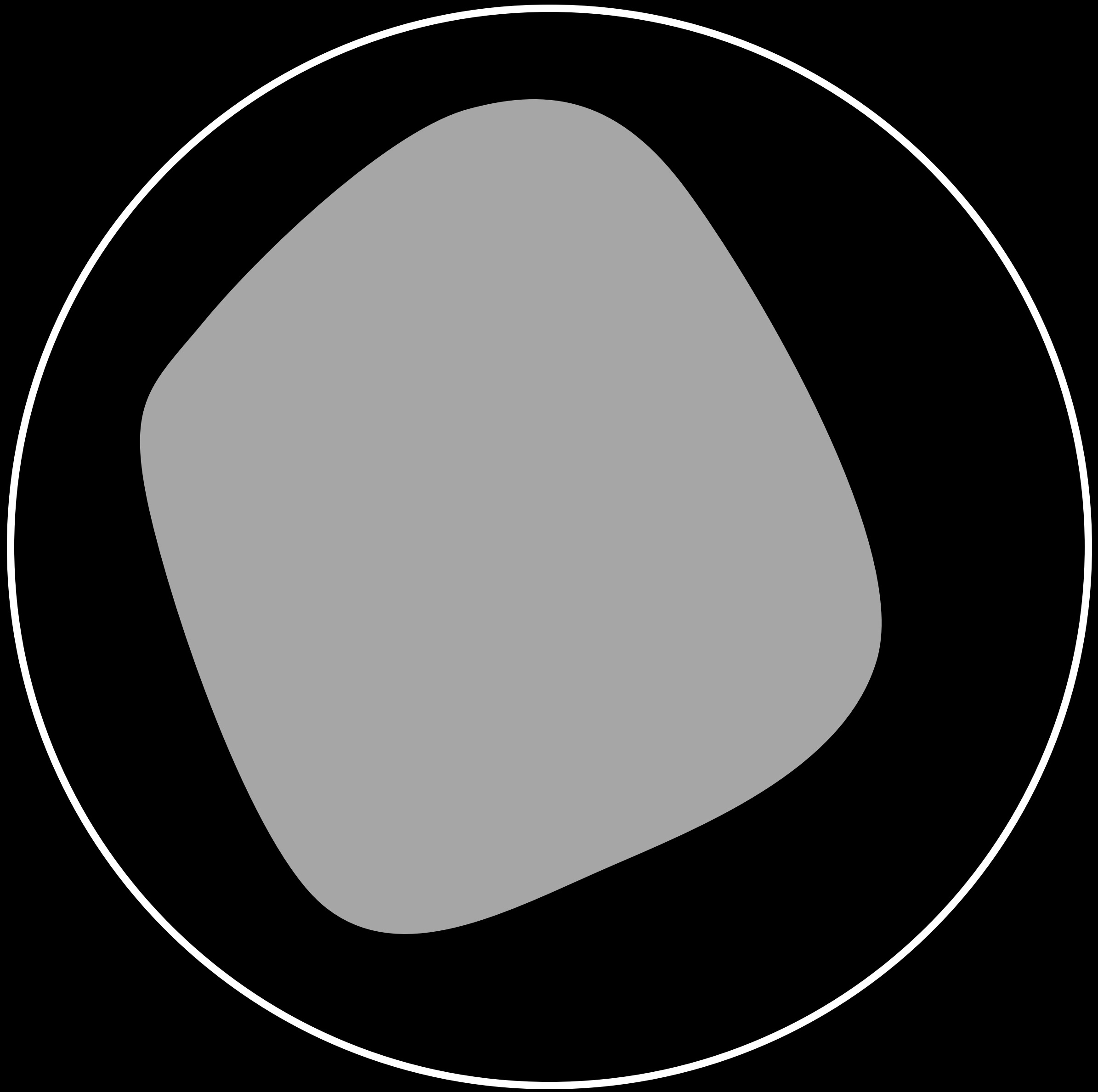
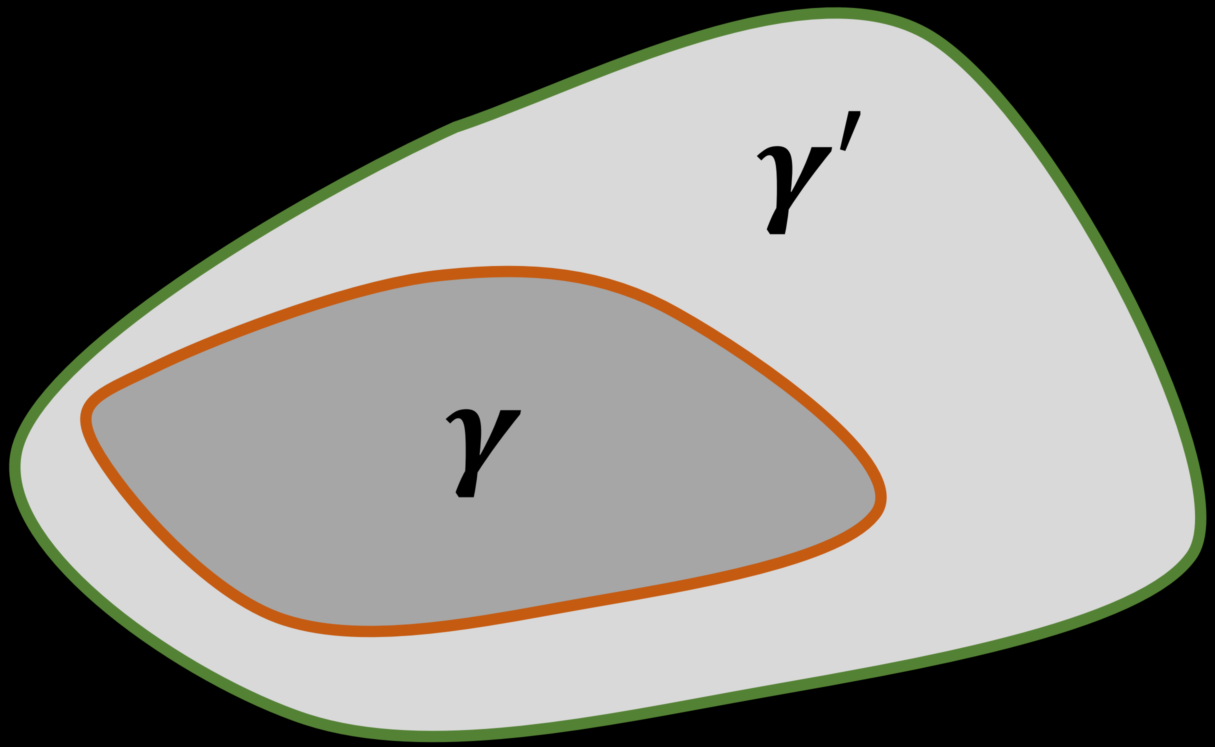
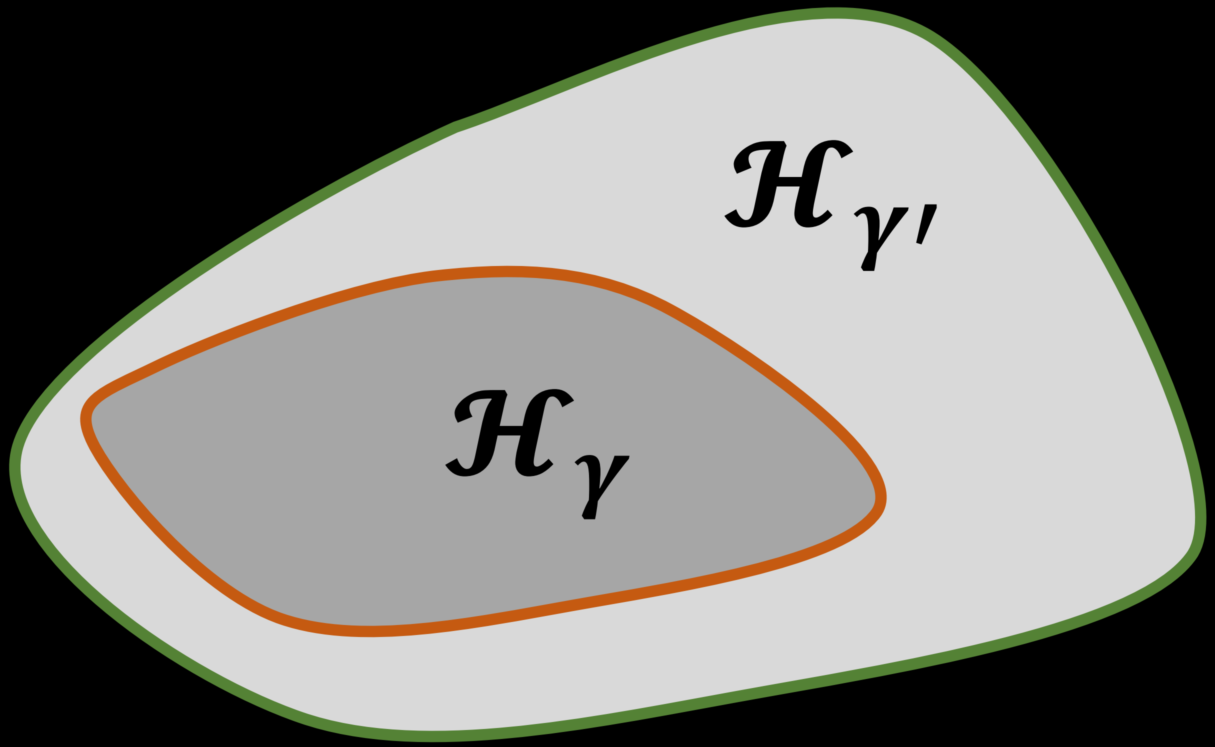


Cutoffs





$$\gamma \leq \gamma'$$



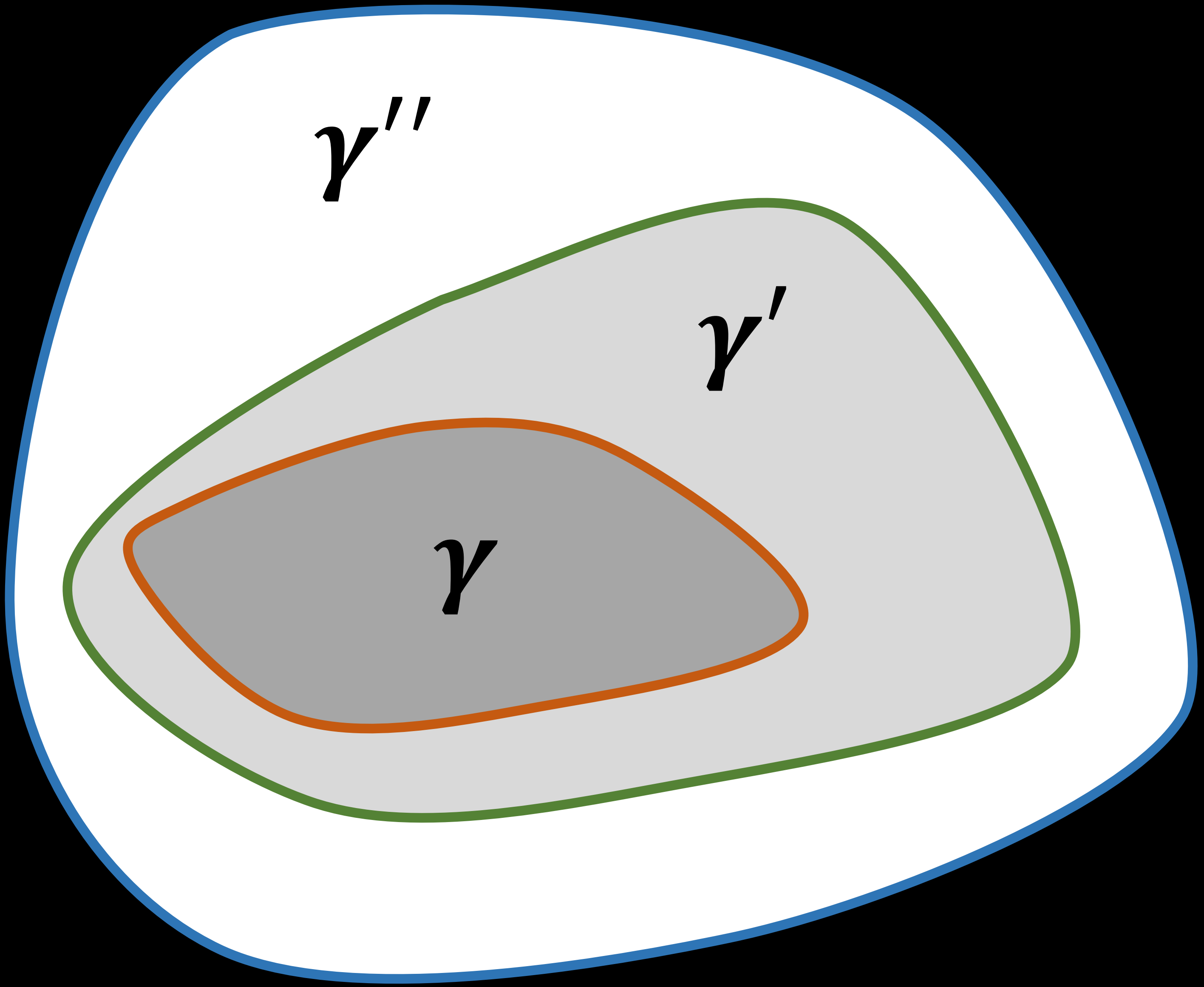
$$\gamma \leq \gamma'$$

$$\mathcal{H}_{\gamma} \hookrightarrow \mathcal{H}_{\gamma'}$$

$$\gamma \leq \gamma'$$

$$\downarrow$$

$$T_{\gamma}^{\gamma'} : \mathcal{H}_{\gamma} \hookrightarrow \mathcal{H}_{\gamma'}$$



$$T^{\gamma''}_{\gamma'}\,T^{\gamma'}_{\gamma} = T^{\gamma''}_{\gamma}$$

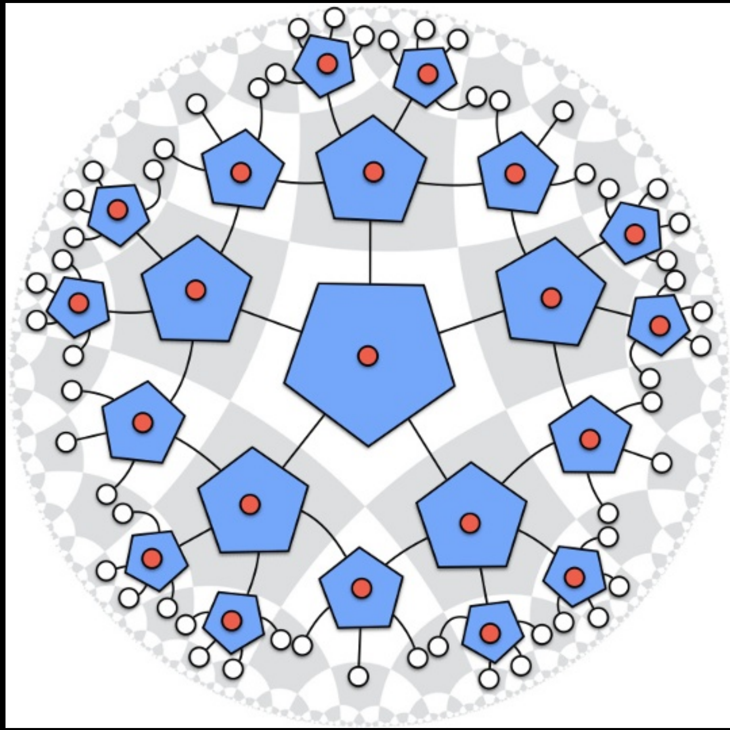
Wishlist for T

1. $T_{\gamma}^{\gamma'}: \mathcal{H}_{\gamma} \hookrightarrow \mathcal{H}_{\gamma'}$ is an isometry

2. $T_{\gamma'}^{\gamma''} T_{\gamma}^{\gamma'} = T_{\gamma}^{\gamma''}$

3. $T_{\gamma}^{\gamma'}$ maps a holographic state
   to a holographic state

$T_{\gamma}^{\gamma'}$ maps a holographic state to a holographic state



$T_{\gamma}^{\gamma'}$ maps a holographic state to a holographic state

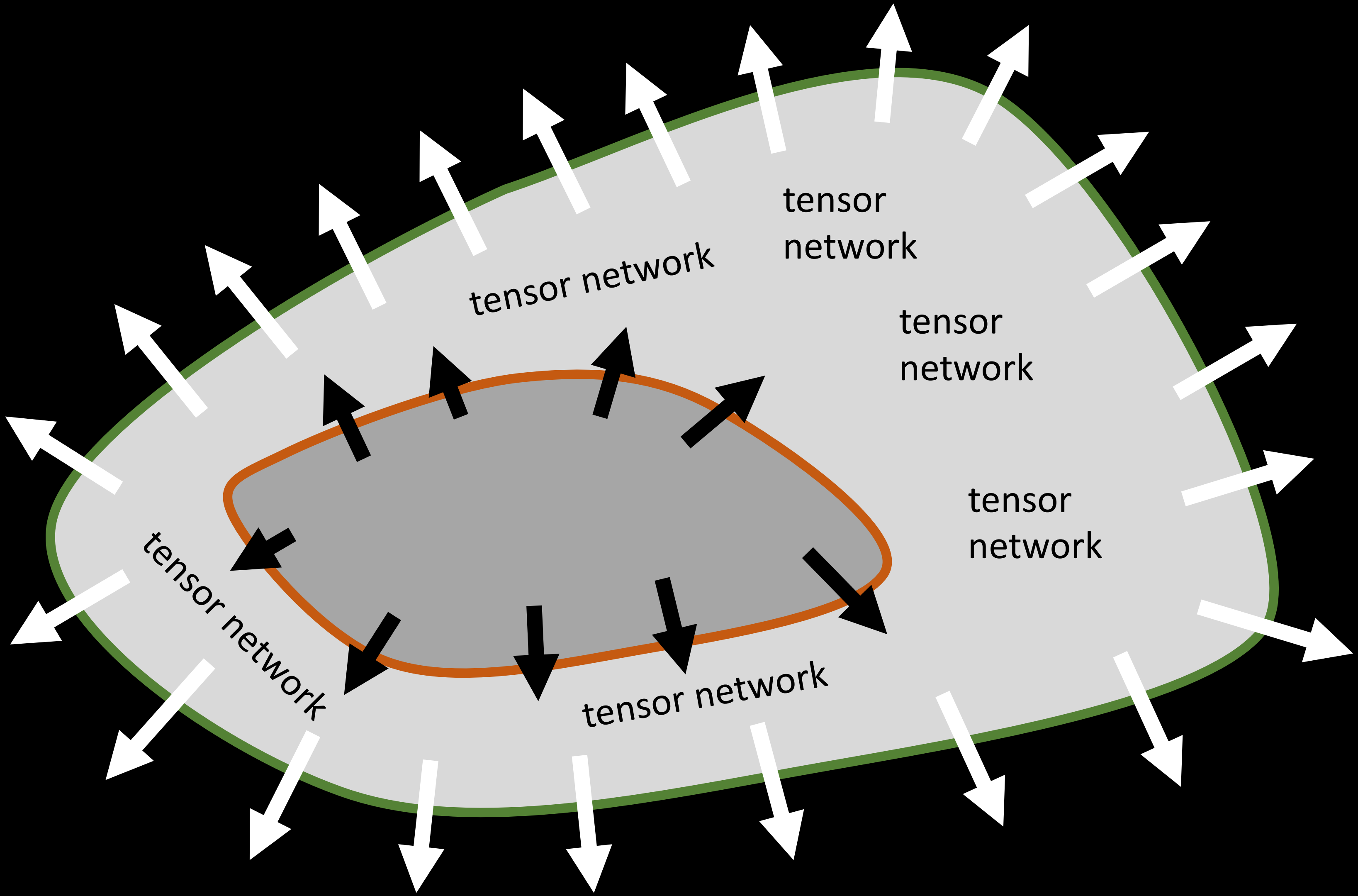
Wishlist for the tensor network

1. $T_{\gamma}^{\gamma'} : \mathcal{H}_{\gamma} \hookrightarrow \mathcal{H}_{\gamma'}$ is an isometry

2. $T_{\gamma'}^{\gamma''} T_{\gamma}^{\gamma'} = T_{\gamma}^{\gamma''}$

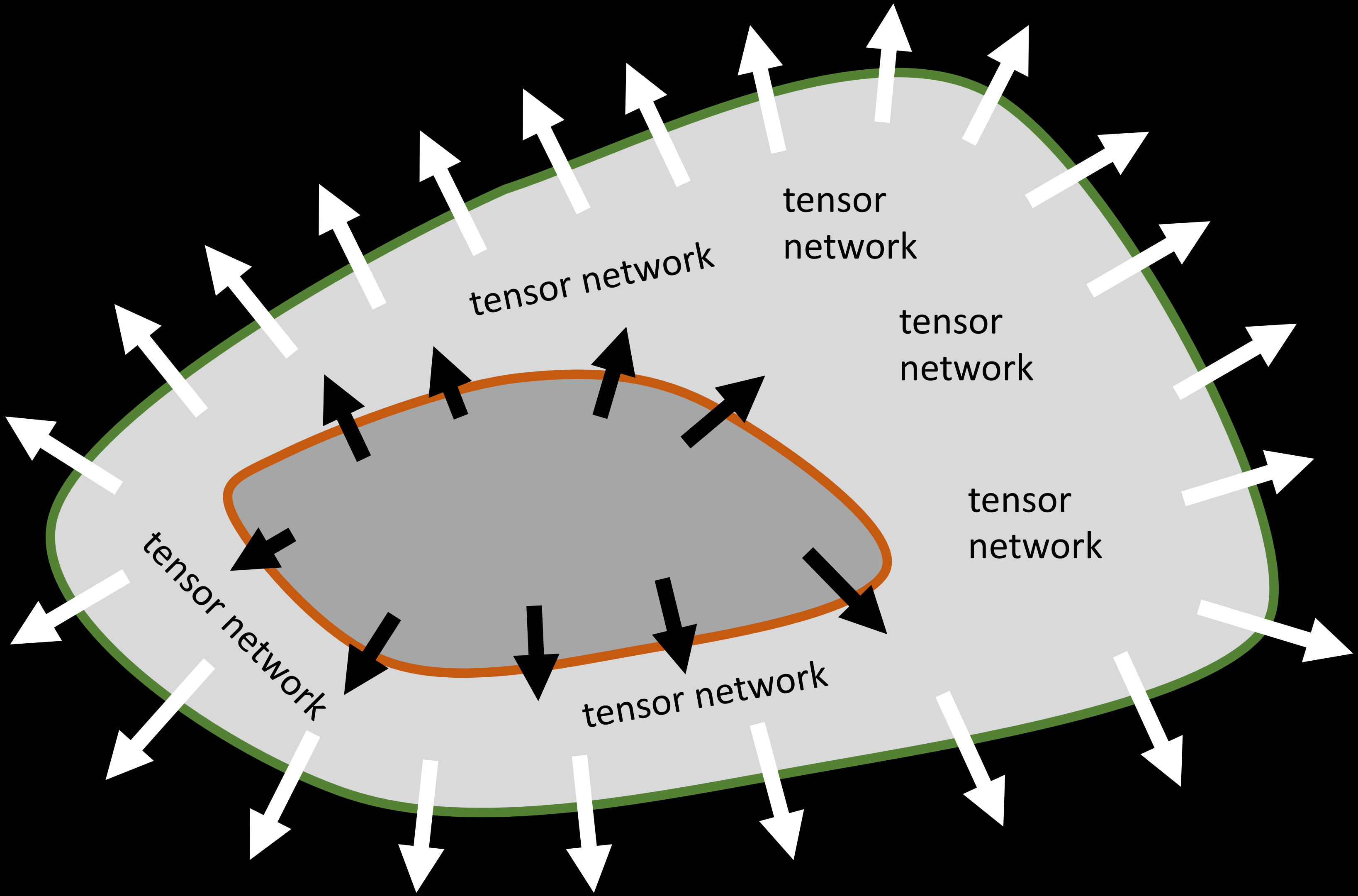
Not so fast!

Which regions γ are allowed?



tensor network
tensor network
tensor network
tensor network
tensor network
tensor network

If $\mathcal{H}$ and $\mathcal{K}$ are Hilbert spaces, and the dimension of $\mathcal{K}$ is smaller than that of $\mathcal{H}$, then there are no isometries from $\mathcal{H}$ to $\mathcal{K}$.


tensor network
tensor
network
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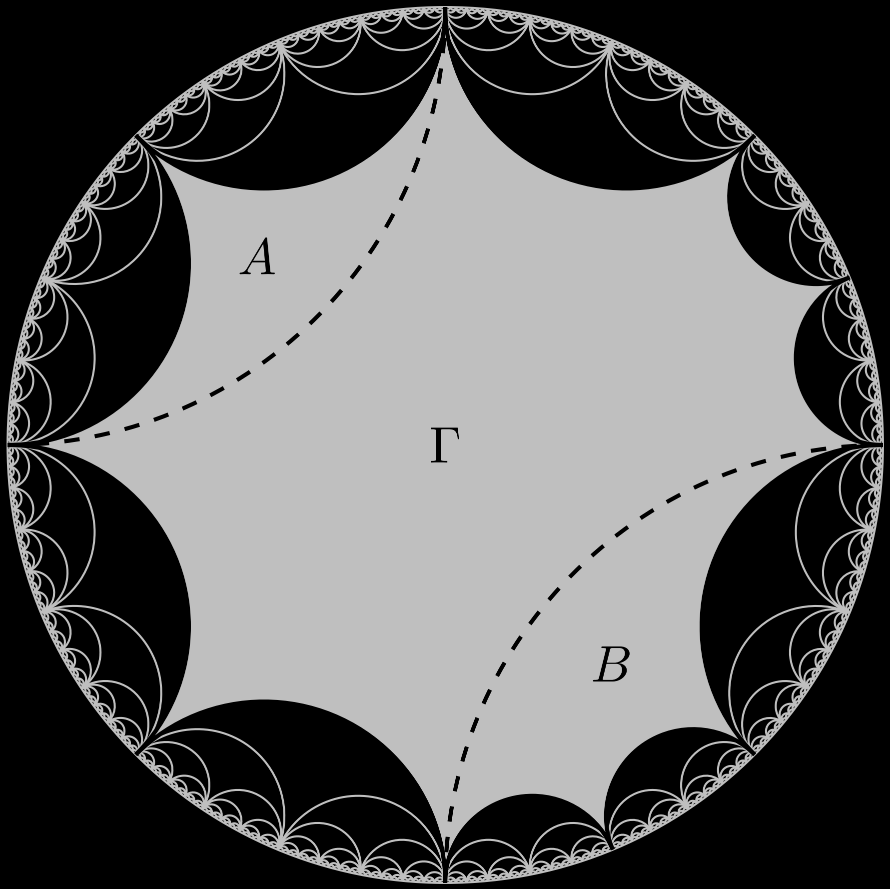
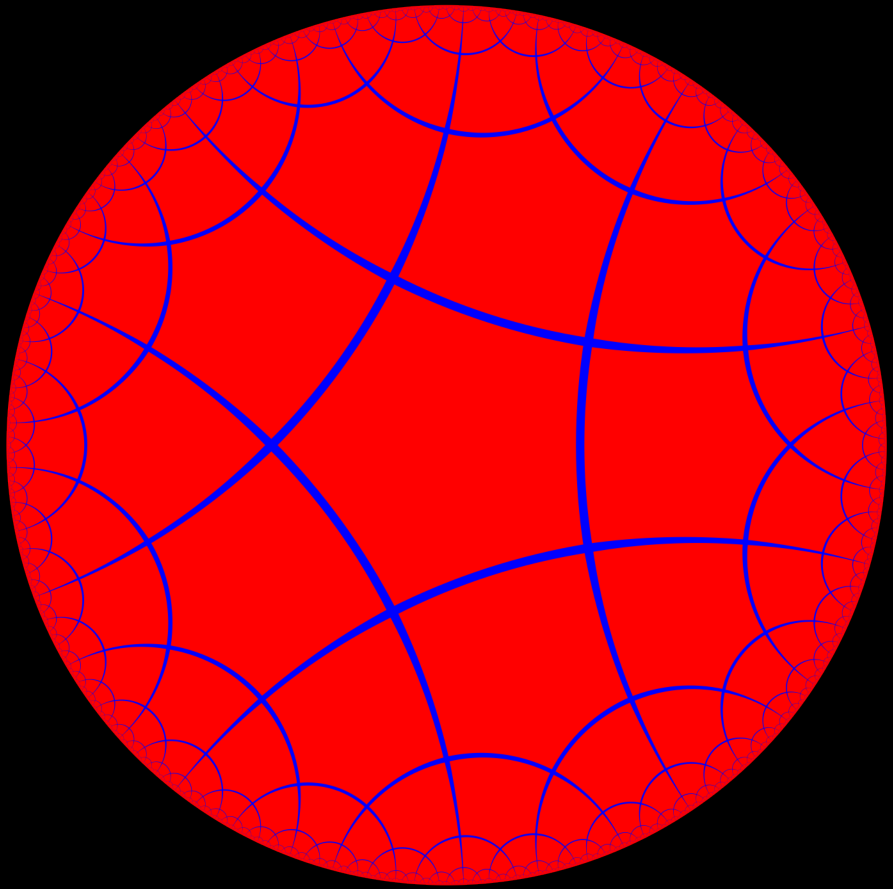
tensor network

Convex regions definitely work, see for yourself:



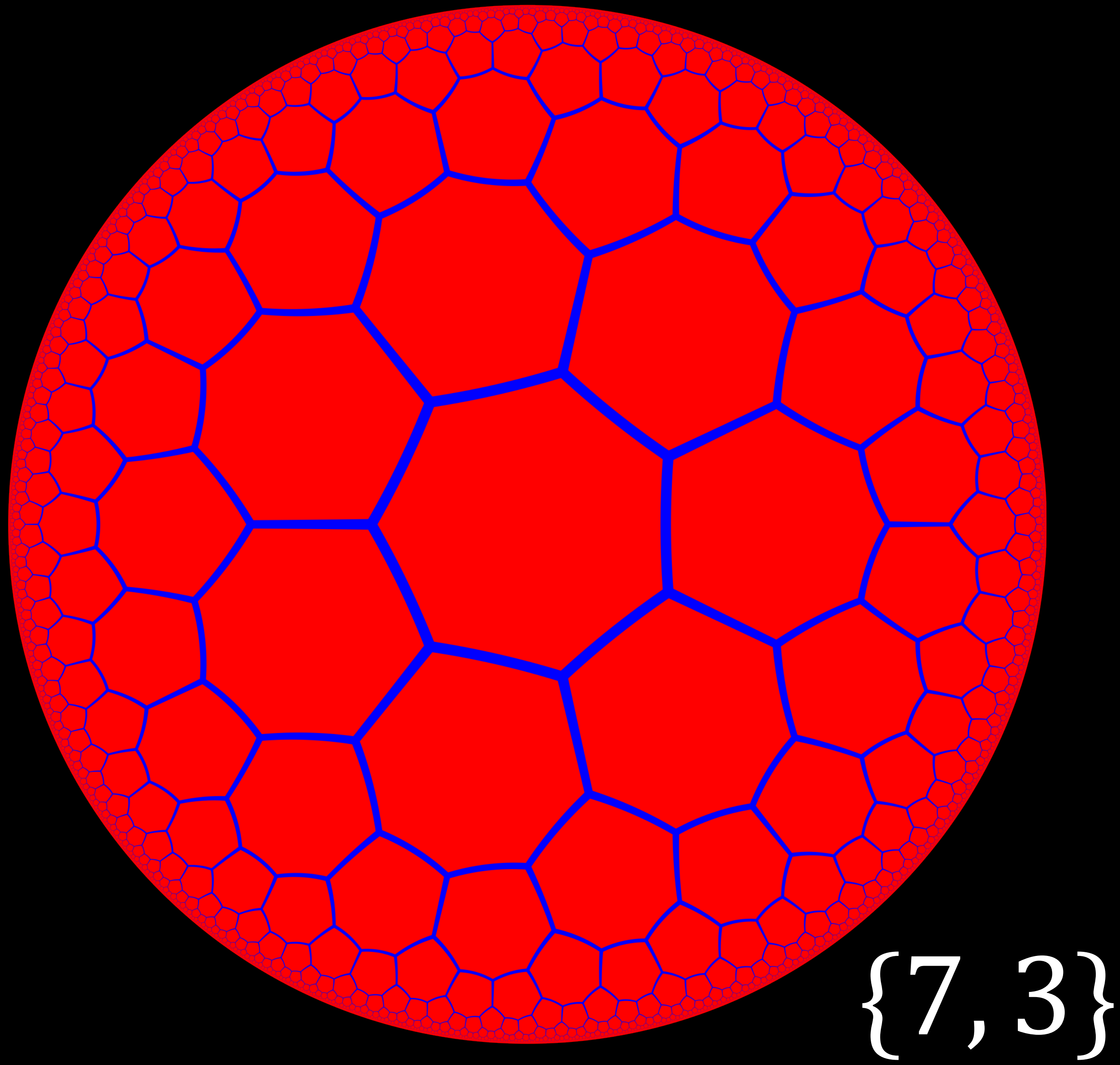
$\{3, \infty\}$



$\{5, 4\}$

BIG problem

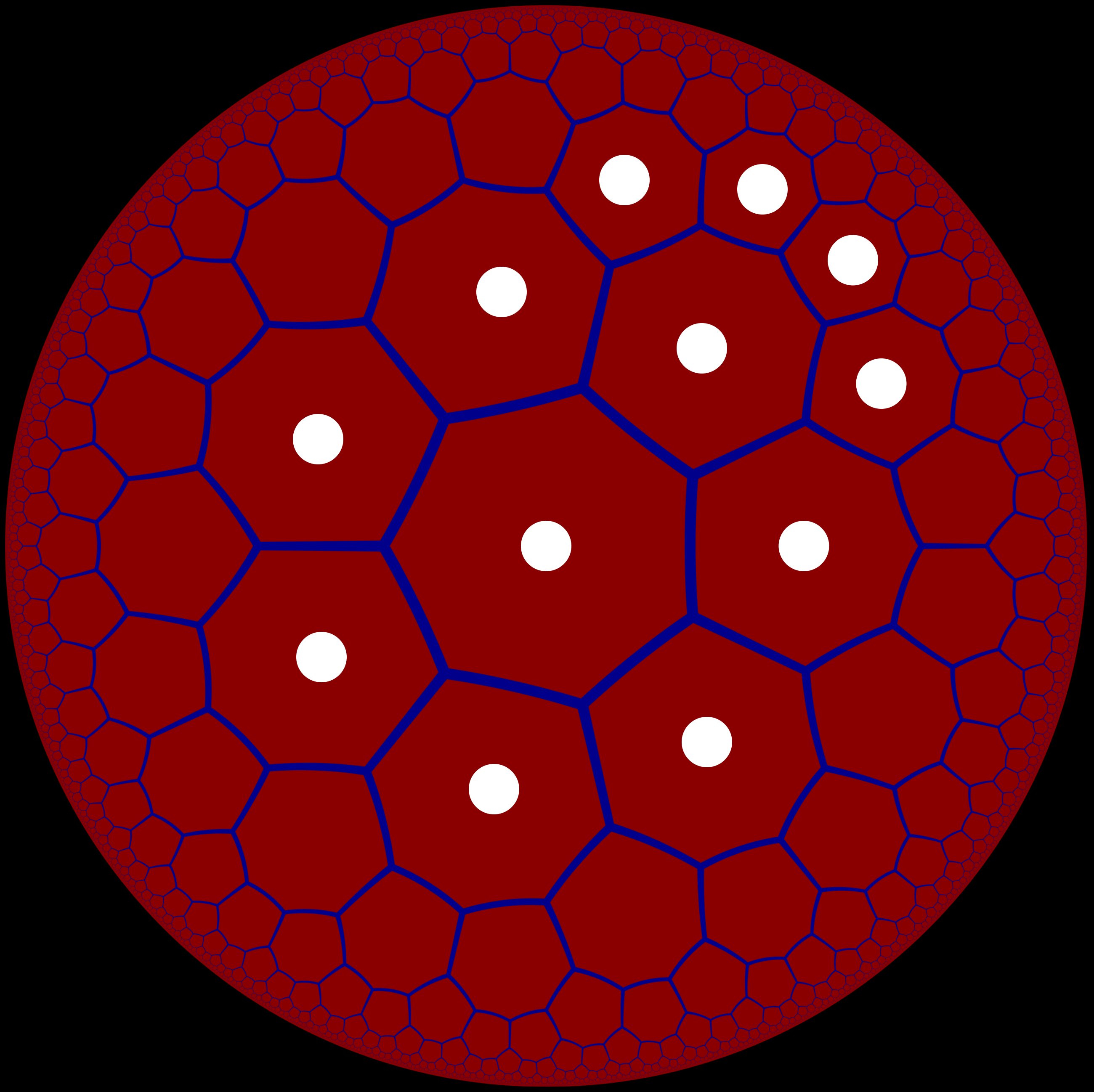
where are the convex subsets?



$\Longrightarrow$ We need a weaker condition than convexity!

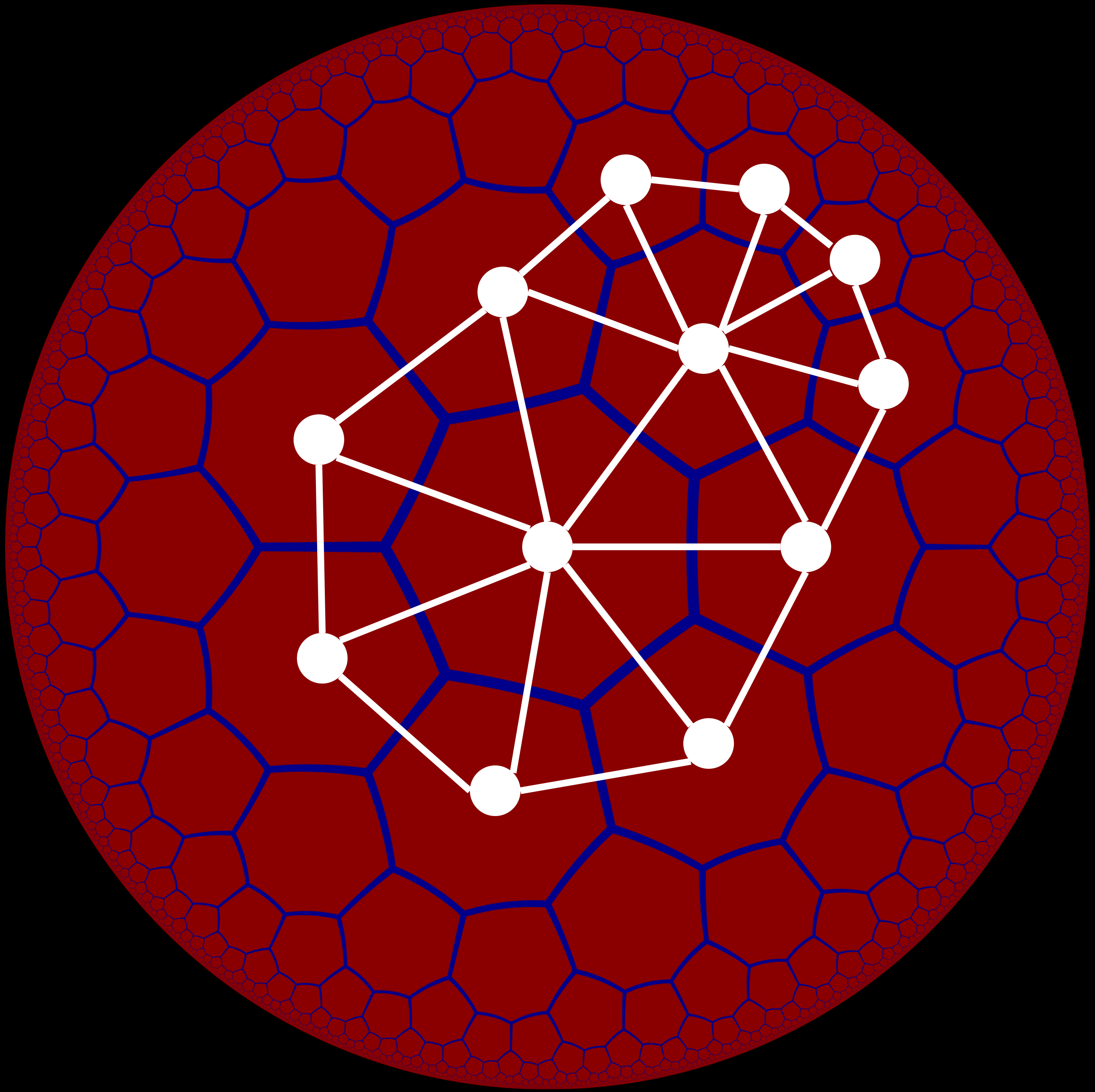
Dual graph

vertices



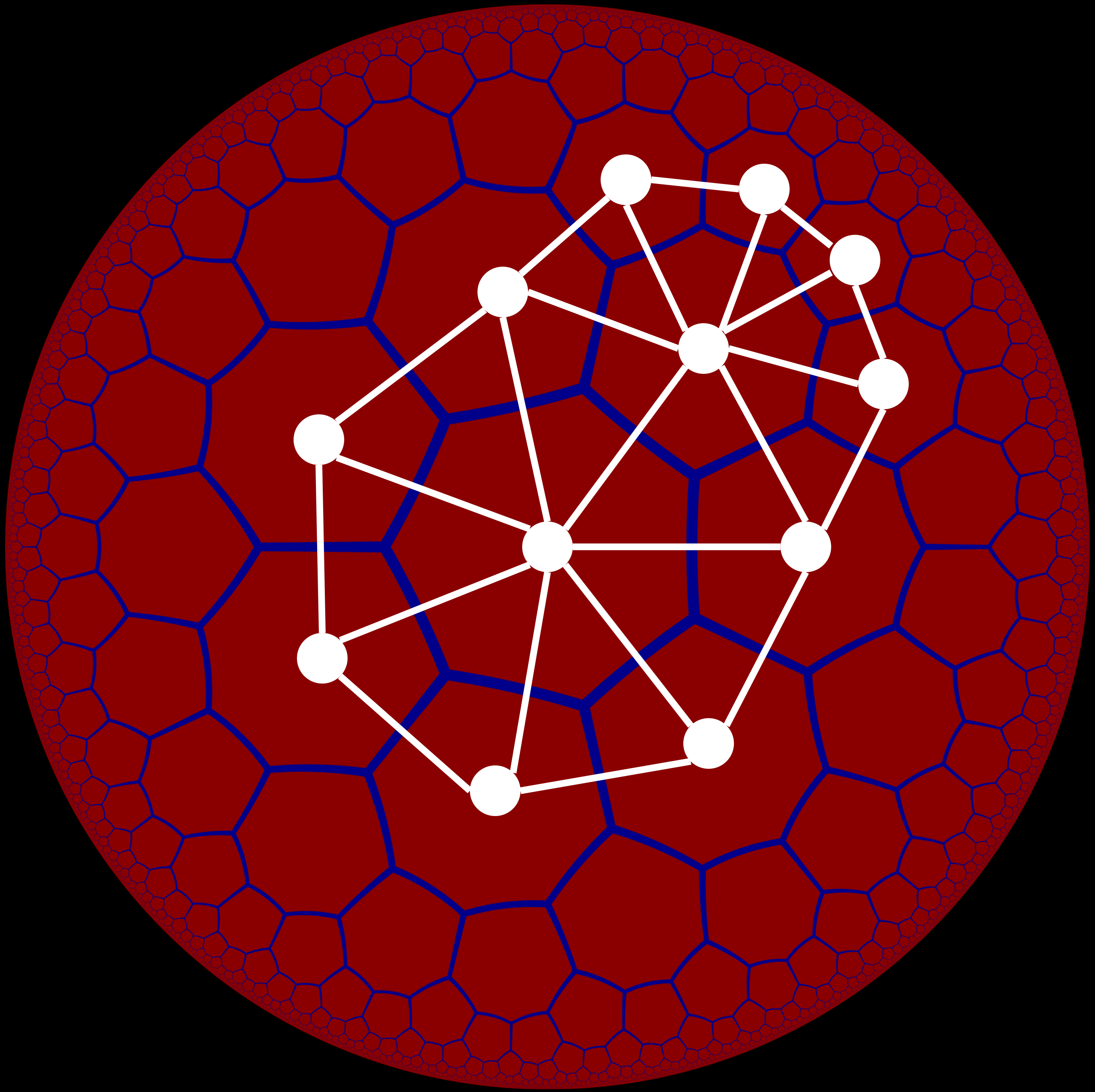
Dual graph

edges

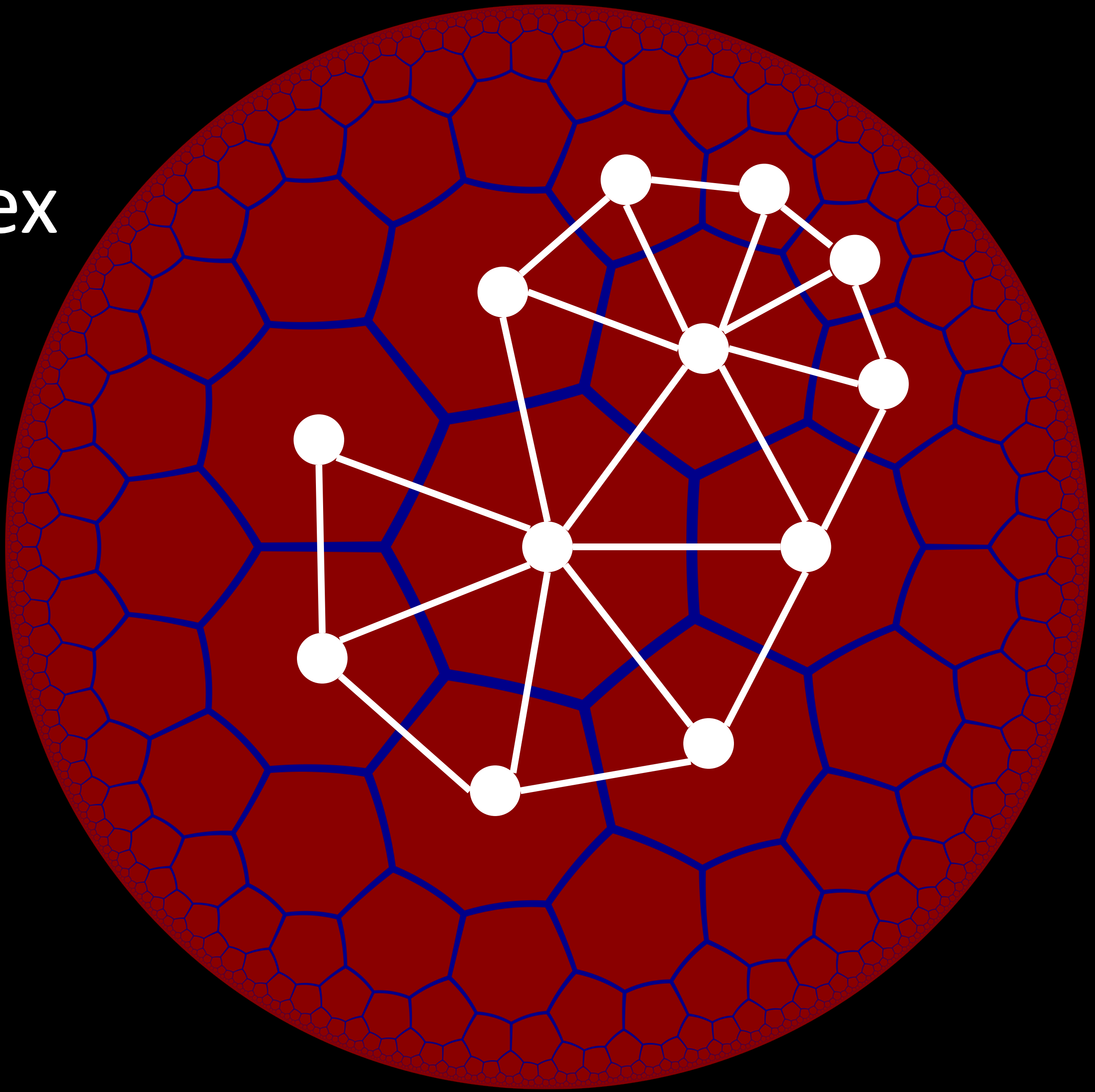


Definition. A subgraph of a graph is *convex* if for every pair of vertices in the subgraph, every shortest path between them lies entirely in the subgraph.

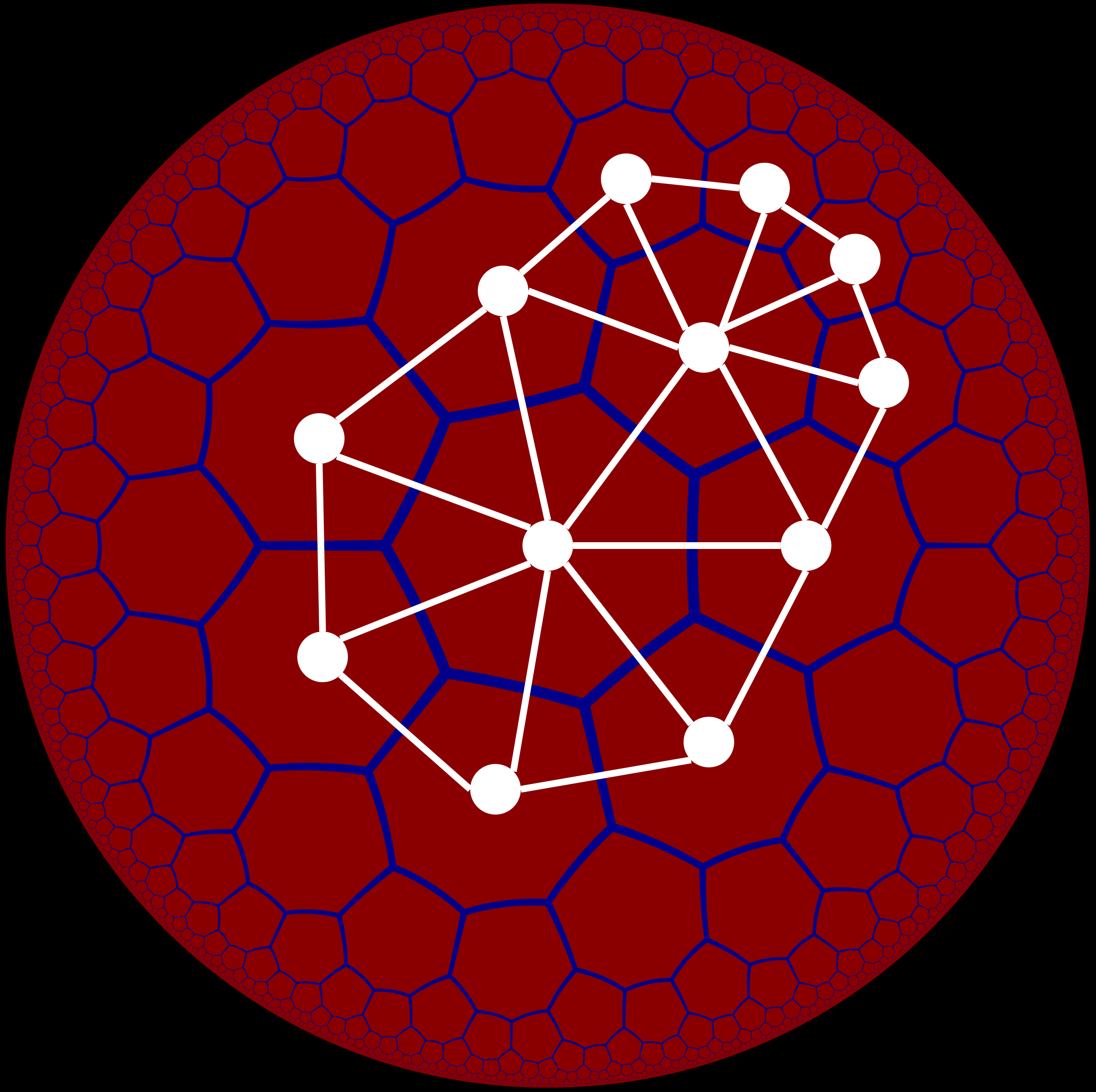
convex



not convex



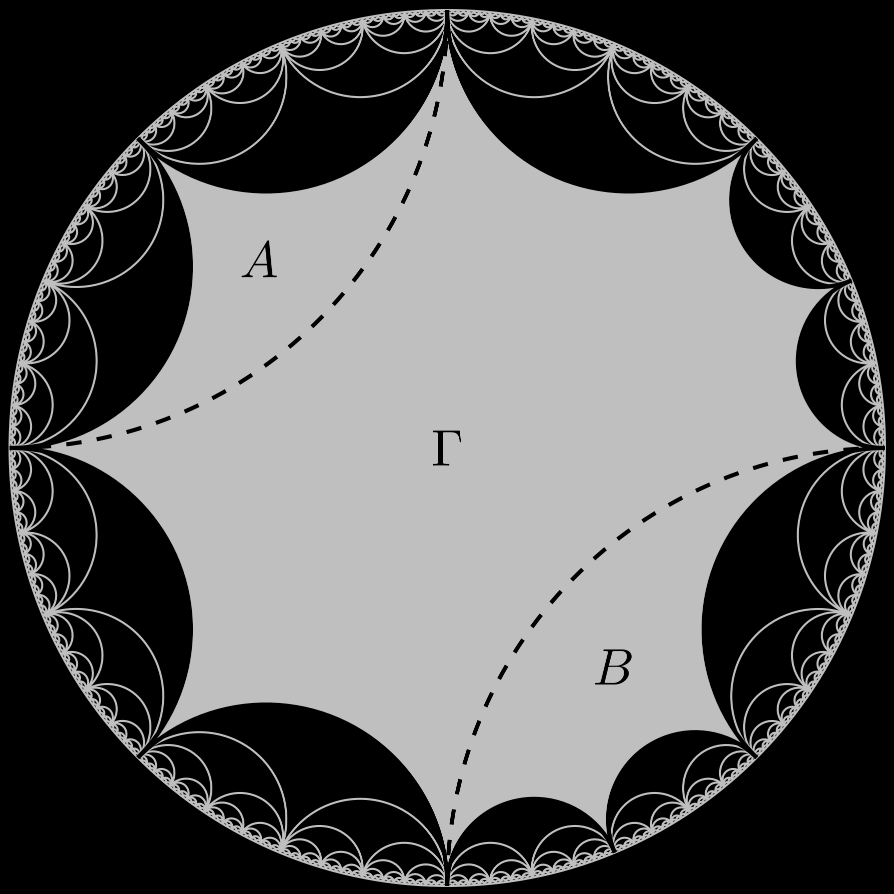
require that a **cutoff** is a region having a *convex dual graph*



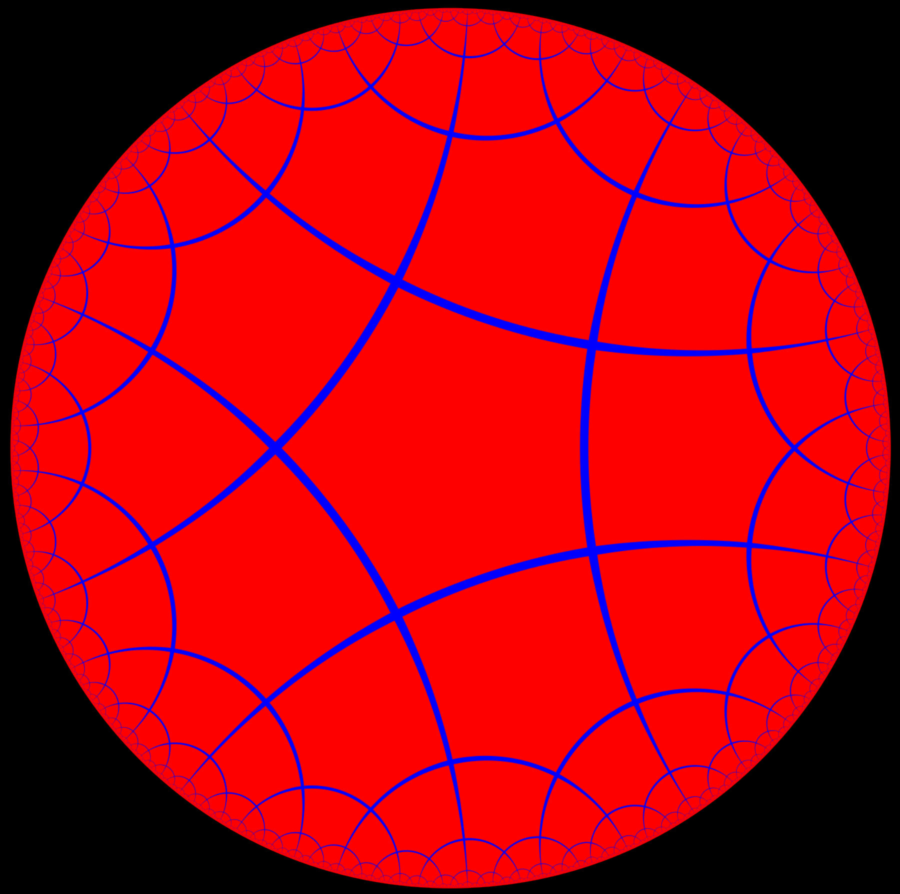
Conjecture. *If an area consisting of a union of finitely many tiles is convex, then it has a convex dual subgraph.*

convex $\implies$ graph-convex

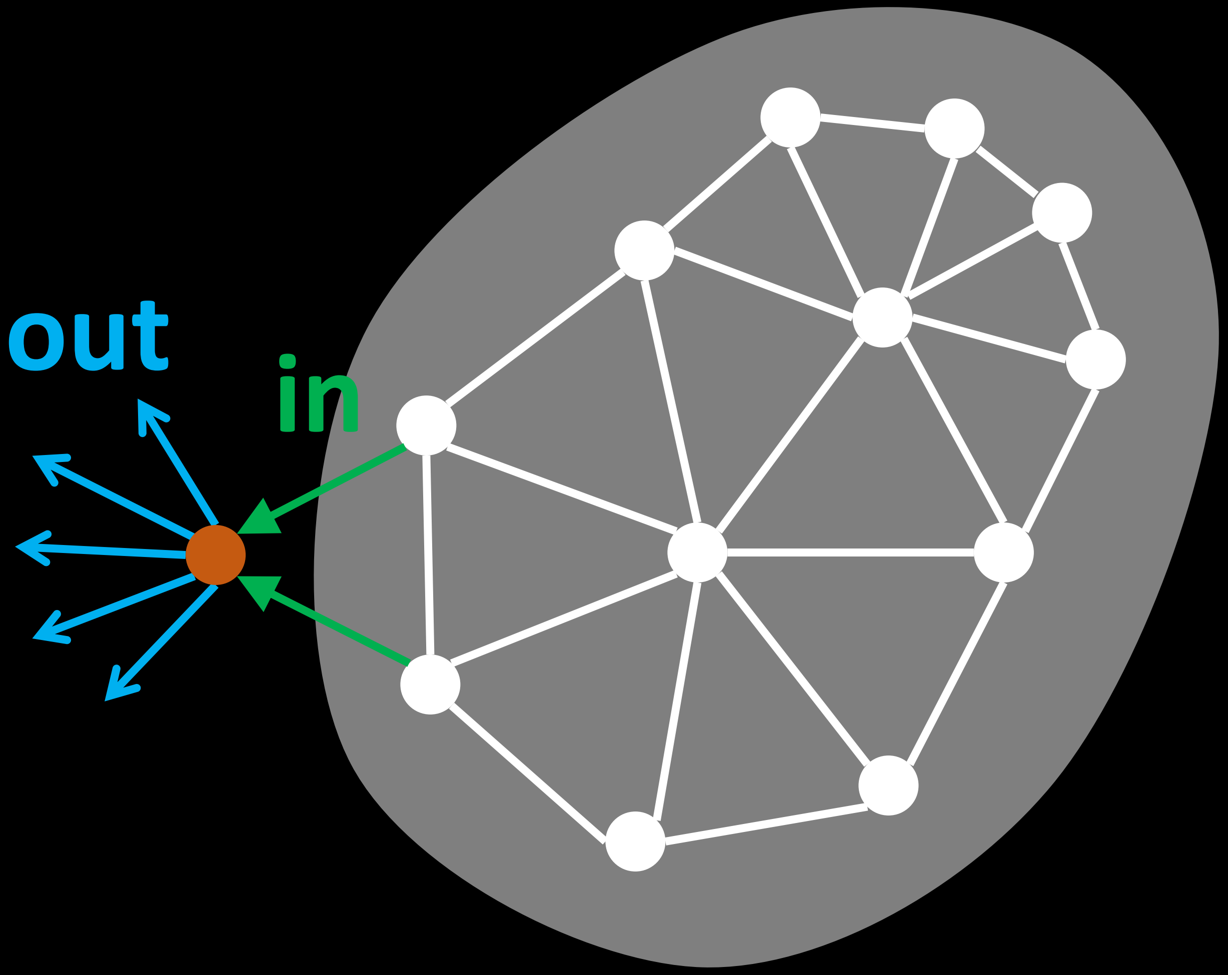
For example here:



$\{3, \infty\}$

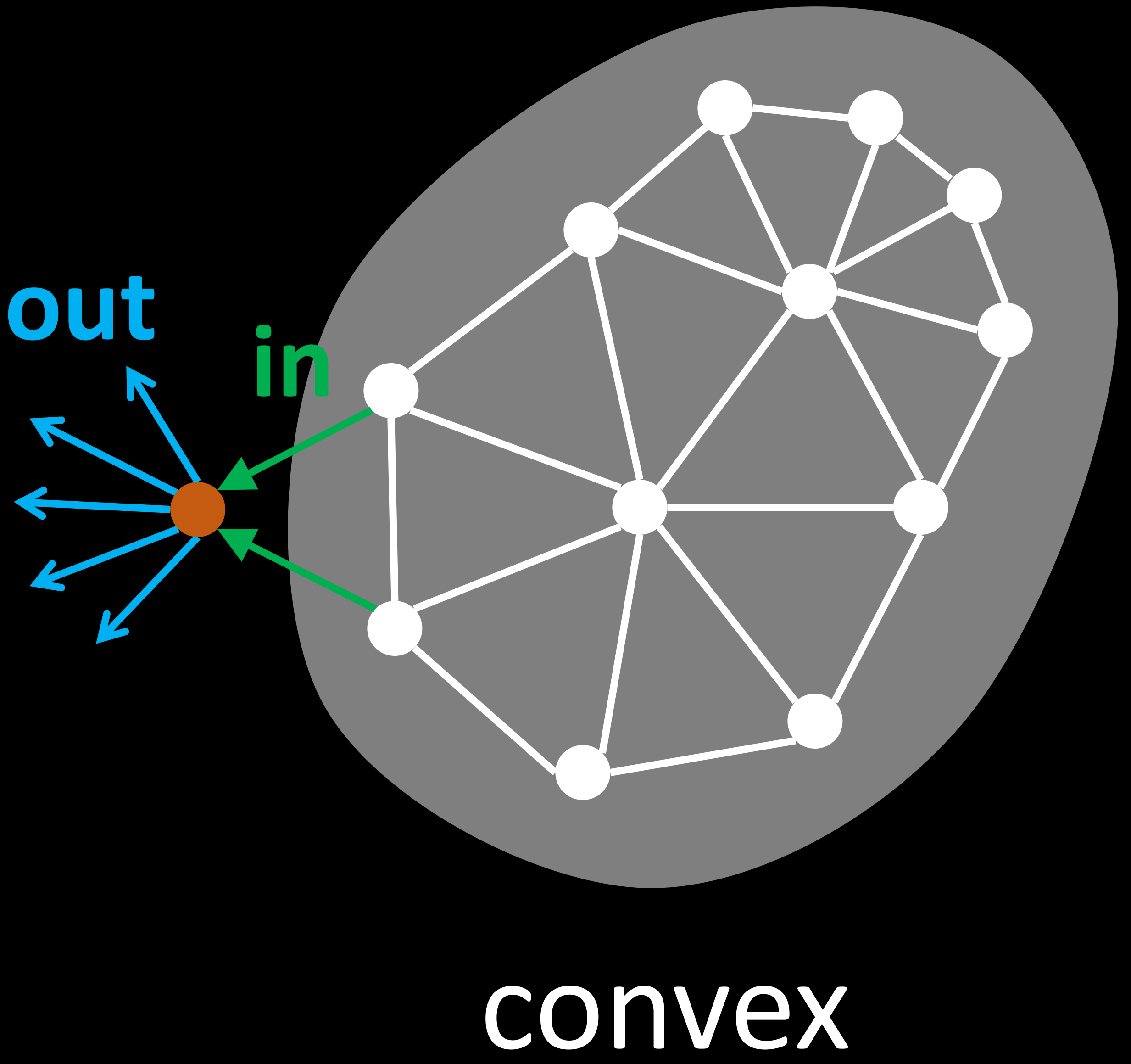


$\{5, 4\}$



convex

Conjecture.
$\#(\text{in}) \leq \#(\text{out})$



Wishlist for the tensor network

↓

Wishlist for the **tensor**

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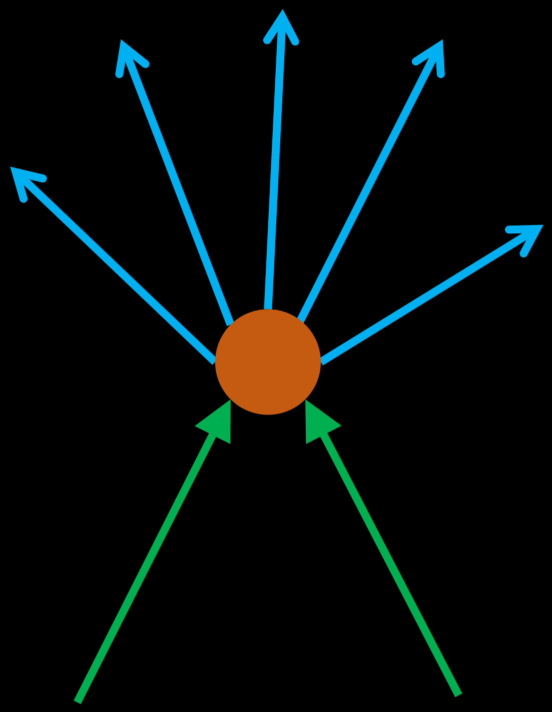
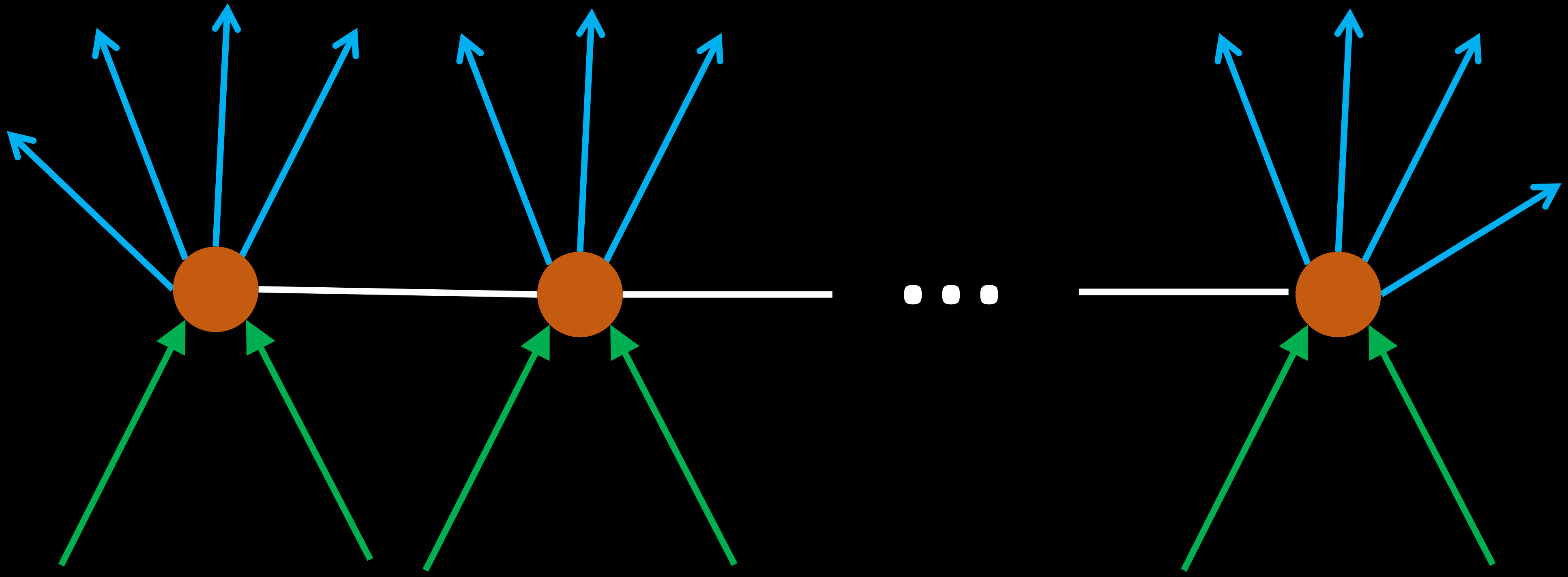


isometry!

Wishlist for the **tensor**



isometry!

☑️ perfect

☑️ blockperfect

🧐 *something more general?*

Definition 2.5. Let $\mathscr{H}$ be a separable Hilbert space, π_0 a unitary representation of the Moebius group on $\mathscr{H}$ which acts properly on a von Neumann algebra $\mathscr{A}$. We denote by $\{\mathscr{A}(I)\}_{I\subset S^1}$ the conformal net constructed in Lemma 2.3. Let us assume that

(i) the spectrum of the generator of rotations of $PSU(1;1)$ is positive (*positive-energy representation*).

(ii) There exists a unique vector $\Omega \in \mathscr{H}$ invariant under $PSU(1;1)$, (vacuum vector).

(iii) Ω is cyclic for the von Neumann algebra $\mathfrak{A} := \left\{ \bigcup_{I\subset S^1} \mathscr{A}(I) \right\}''$ generated by the net $\{\mathscr{A}(I)\}_{I\subset S^1}$.

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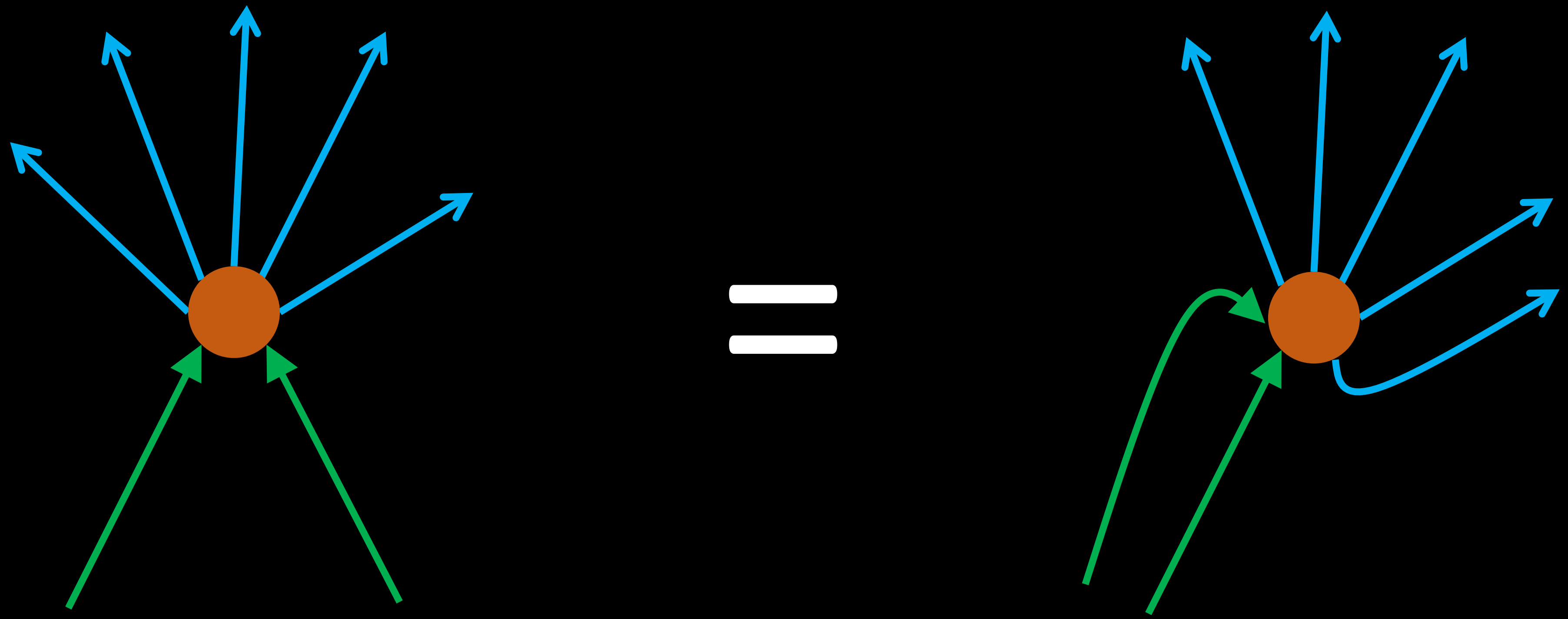
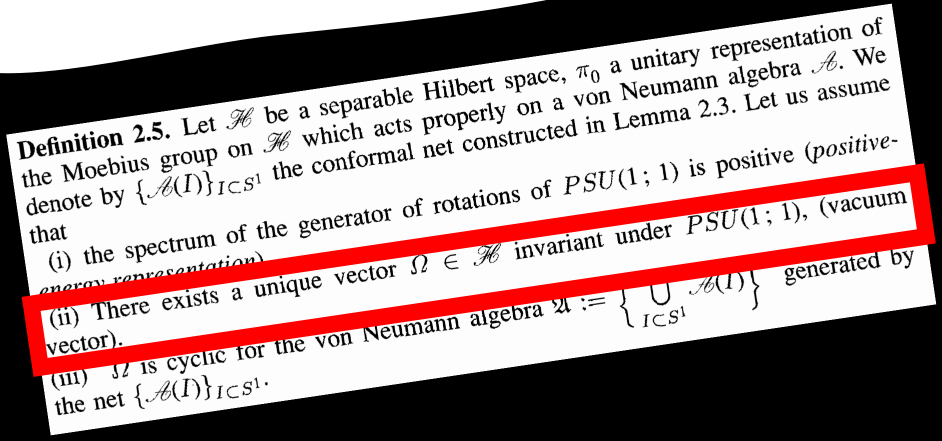
<u>translation:</u> holographic state should be invariant under symmetry group of the tessellation

Wishlist for the tensor



discretely rotation invariant!

Summary



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