The Effective Wavelength of SUSI's 700 nm System

John Davis, 23 March 2007 (Revised 3 April 2007)

1 Background

The effective wavelength of observations made with a broad-band system, whether it be photometric, interferometric or whatever, will depend on the spectral distribution of flux from the source being observed. In the interferometric case it is important to establish the effective wavelength because the value of the uniform disk angular diameter determined from a fit to the observed values of visibility^2 (V^2) scales directly as the wavelength used in the fitting procedure. Any uncertainty in the wavelength translates directly as the same fractional uncertainty in the angular diameter.

There are two possible approaches to determining the effective wavelength. The first is to simply calculate it from the spectral response of the instrument, which itself has to be evaluated. The shortcomings of this approach are that not all the parameters required for the calculations are known with any certainty. The second approach is to measure it from the observed power spectra whose scale has been calibrated with the aid of laser fringes. The question in this case is how reliable, repeatable and accurately can the measurement be made.

It should be possible to get reasonable agreement, at least within the estimated uncertainties, between the two approaches. It has been important to pursue this, in spite of a few red herrings along the way, in order to decide on, and give confidence in, the values we adopt. With this in mind both methods have been employed and the results are discussed with a recommendation for an effective wavelength scale to be adopted.

2 Calculated Values of the Effective Wavelength

My initial erroneous effort to calculate the effective wavelength as a function of stellar spectral type, which I included in the draft of the instrument paper, used the formula appropriate for photometry with which I was familiar. Mike Ireland liked the values I produced but not the formula I used and, as a result of correspondence with Mike and discussions with Bill Tango, I have revised my calculations using an appropriate formula. Because of the way we measure V^2 the calculations should be done in terms of wavenumber (or frequency), rather than wavelength as I did originally.

The formula I have used for the revised calculations is based on material in Mike's thesis and notes produced by Bill. I won't go into details except to note that I have used symbols that I am familiar with for the various quantities so they don't necessarily correspond to the ones in the sources I have used. The formula is:

$$\frac{1}{\lambda_{\mathrm{eff}}} = \sigma_{\mathrm{eff}} = \frac{\int_0^\infty I^2(\sigma)\sigma \mathrm{d}\sigma}{\int_0^\infty I^2(\sigma)\mathrm{d}\sigma} \tag{1}$$

with

$$I^2(\sigma) = T(\sigma)N^2(\sigma)S^2(\sigma) \tag{2}$$

where σ is the wavenumber equal to $1/\lambda$, $T(\sigma)$ is the transfer function equal to $(V_{\mathrm{observed}}(\sigma)/V_{\mathrm{true}}(\sigma))^2$, $N(\sigma)$ is the photon flux from the star per unit wavenumber interval outside the atmosphere, and $S(\sigma)$ is the spectral response of the instrument including reflections, transmisions (including the atmosphere), filter response, detector quantum efficiency etc.

In the following sub-sections I will outline the different sets of data I have used. For each parameter I have produced plots of the data against σ and used the plots to tabulate values for the parameter at intervals of $\Delta\sigma = 0.005\,\mu\mathrm{m}^{-1}$ for σ from 1.250 to $1.550\,\mu\mathrm{m}^{-1}$ (corresponding to a wavelength range from approximately 645–800 nm).

2.1 The Spectral Response

The spectral response of the instrument is the product of the spectral responses of the various components that affect the overall response. This can be written as:

$$S(\sigma) = T_A(\sigma).R(\sigma).T_F(\sigma).T_f(\sigma).Q(\sigma) \tag{3}$$

where all the quantities are a function of wavenumber σ. $T_A(\sigma)$ is the transmission of the atmosphere, $R(\sigma)$ is the product of the reflectances of all mirrors in the system, $T_F(\sigma)$ the transmission of the 700 nm filter, $T_f(\sigma)$ the transmission of the optical fiber feeding the APD, and $Q(\sigma)$ is the quantum efficiency of the APD detector.

The atmosphere has been included here as the relative photon flux distributions are ex-atmosphere. An uncertainty in determining the spectral response in this way is that the reflectances of the mirror coatings have changed with time and predicted responses have had to be used. However, the reflectances of the mirrors with overcoated silver, overcoated aluminium and bare aluminium have all been included along with the transmission of AR coatings. The resulting spectral response is shown in Figure 1.

2.2 Photon Fluxes

I have used the library of optical spectra by Silva and Cornell (Ap.J.Suppl., 81, 865, 1992) to obtain relative photon flux distributions for a number of main-sequence spectral types. The original publication listed fluxes in ergs/Å normalised to 100 at 5450 Å for wavelengths at 5 Å intervals from 3510 Å to 8930 Å for 72 different spectral types. I have these tables with an additional column giving relative photon fluxes (p_λ) as photons.m^{-2}.nm^{-1} again normalised to 100 at 5450 Å. I think this was done by Bill Tango many years ago. I have added

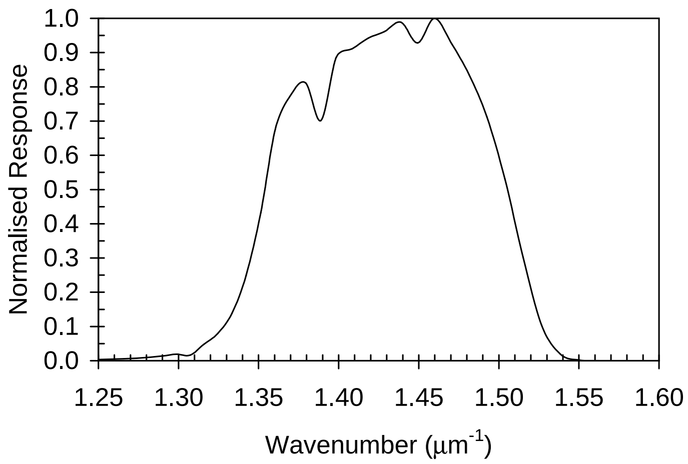

Figure 1: The spectral response of SUSI's 700 nm red system.

two columns, one with wavenumber ($= 1/\lambda$) and one with photons.m^{-2}.μm ($= p_\lambda/\lambda^2$). The latter to give the relative photon flux in wavenumber intervals.

Plots of the relative photon flux per unit wavenumber interval v. wavenumber for spectral types O5 V, B6 V, A13 V, A8 V, F3 IV, G2 IV, G8 IV, K4 V and K5 V have been used to tabulate the fluxes against wavenumber for the range 1.250–1.550 μm^{-1} at intervals of 0.005 μm^{-1}.

2.3 The Transfer Function

The transfer function includes the loss in V^2 due to residual atmospheric effects and instrumental effects. Calculations have been done for a range of seeing effects—from losses for full tip-tilt correction with 1 arcsec seeing to losses for no tip-tilt correction with 2 arcsec seeing. The calculations have assumed r_0 = 10 cm at 550 nm for 1 arcsec seeing and the V^2 losses have been based on the curves published by Tango & Twiss (Progress in Optics, XVII, 239,1980). Again the losses have been tabulated against wavenumber for the range 1.250–1.550 μm^{-1} at intervals of 0.005 μm^{-1}.

2.4 The Calculated Effective Wavelengths

Table 1 lists the calculated values for the effective wavelength for the selected main-sequence spectral types. The 'Maximum' values are for 2 arcsec seeing and no tip-tilt correction and the 'Minimum' values for 1 arcsec seeing and full tip-tilt correction. The mean values are the means of nine values calculated for no tip-tilt correction, full tip-tilt correction and approximately half tip-tilt correction for 1, 1.5 and 2 arcsecond seeing. This is overkill but the calculations had been done so the mean was taken. It corresponds to ~half tip-tilt correction for 1.5 arcsecond seeing which is assumed to be representative of mean conditions for SUSI observations.

Table 1: The calculated values of effective wavelength(λ_{eff}) for main-sequence stars as a function of spectral type. See the text for details of the 'Maximum', 'Minimum' and 'Mean' values.

Spectral Type	$(B-V)_0$	Maximum λ_{eff} (nm)	Minimum λ_{eff} (nm)	Mean λ_{eff} (nm)
O5 V	-0.33	694.04	693.23	693.63
B6 V	-0.15	694.49	693.66	694.07
A13 V	0.05	695.08	694.25	694.66
A8 V	0.25	695.68	694.84	695.26
F3 IV	0.38	695.94	695.10	695.52
G2 IV	0.63	696.62	695.76	696.19
G8 IV	0.74	696.41	695.56	695.98
K4 V	1.05	696.86	695.99	696.42
K5 V	1.15	697.16	696.29	696.72

The mean values of effective wavelength from Table 1 are plotted against $(B-V)_0$ in Figure 2.

3 Measured Values of the Effective Wavelength

The effective wavelength can be determined from recorded sets of fringe scans. The power spectrum of a scan, determined in the data-processing pipeline, is an estimate of $I^2|\gamma|^2$ where$|\gamma|$ is the degree of coherence at the observing baseline. In the pipeline the power spectra are averaged and are used to estimate the fringe visibility. The first moment of the power spectrum is

$$\sigma_1 = \frac{\int_0^\infty I^2(\sigma)|\gamma(\sigma)|^2\sigma \mathrm{d}\sigma}{\int_0^\infty I^2(\sigma)|\gamma(\sigma)|^2 \mathrm{d}\sigma} \tag{4}$$

Mike Ireland measured the effective wavelength for three stars in 2004 based on a calibration of the scan step size made with laser fringes on the following day. The stars were β Car (A2 IV), V337 Cen (K3 IIa) which is heavily reddened ($E(B-V)$ ~0.3), and α Lup (B1.5 III). All were observed with a baseline of 20 m. Two measurements were obtained for β Car and α Lup and only one for V337 Car. It is difficult to compare with the current calculated values because of the differing luminosity classes and reddening. However in the case of the luminosity class IV star, β Car, the values given by Mike have an average of 700.5 nm which is 5.9 nm (~0.85%) larger than the calculated value for $(B-V)$ = 0.0. Mike's IDL script included a correction for γ^2 so that the effective wavelengths are effectively for a point source. It is noted that, in using this script to

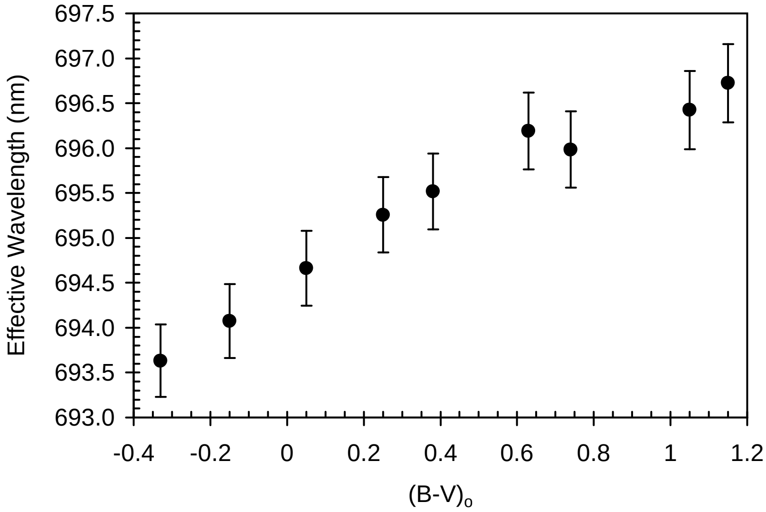

Figure 2: The calculated mean effective wavelength of SUSI's 700 nm red system as a function of $(B-V)_0$ for main-sequence stars. The 'error' bars indicate the Maximum and Minimum values listed in Table 1.

explore the effective wavelength for other observations, this correction produced distorted power spectra at longer baselines and some clearly erroneous values for the effective wavelength. For example, using the new laser calibration discussed below for 7 sets of scans of β Vir at a projected baseline of 54.3 m, where V^2 is $\sim$0.14, the effective wavelength with the programmed baseline correction is 686.8$\pm$7.2 nm and without the correction it is 701.9$\pm$3.1 nm. Estimates of the correction of the latter value to zero baseline (equal to the point source value), suggest an adjustment of the order of -2.6 nm and not the -15.1 nm given by the IDL script. It is also noted that recent experience, discussed below, has shown that it is necessary to take the mean of several scan sets rather than just one or two measurements because of scatter in the measured values.

In view of the difficulty in comparing results for different luminosity classes and different baselines, it was decided to make all observations at 5 m with stars for which $\gamma^2 \sim 1$ and equation (4) reduces to equation (1). The IDL script was modified by commenting out the 'correction' to a point source. A number of bright main-sequence stars covering a range of spectral types was selected. The stars are listed in Table 2 and in each case, except for β Vir, ϕ Leo and σ Leo, consecutive sets of 1000 scans were recorded. Julian North made the first observations specifically for the determination of effective wavelengths (τ Sco and κ Vel). Processing these observations with Mike Ireland's original script (less the point source correction) revealed two things. Firstly, the values for the effective wavelength showed a very significant scatter from one set of scans to the next. The standard deviation in effective wavelength when 10 or more sets of scans were taken in rapid succession ranged from 0.7–1.4 nm. For β Vir, with the observations alternating with calibrators, the standard deviation for the 7 sets of scans on 13 March 2007 was 2.31 nm. Secondly, the mean values for the effective wavelength were larger than the calculated values by $\sim$8 nm ($\sim$1.1%).

The size of the scan step was re-calibrated with the laser on 18 March 2007. Several sets of scans were taken and all gave the same result. The new value is 0.137347 compared to Mike's 2004 value of 0.138657—a decrease of $\sim$0.95%. While the reason for the change is unknown—the same IDL script produced both values—the new value brought the measured effective wavelengths for τ Sco and κ Vel to within <0.3% of the calculated values.

Observations of ι Cen and β TrA were made on 19 March. The calibration of the scan step size was repeated on 20 March giving exactly the same result as on 18 March.

Table 2 lists the stars observed with the measurements of effective wavelength based on the new scan step calibration. Included are the results for β Vir and its calibrators ϕ Leo and σ Leo from observations made at 5 m. The measured effective wavelengths are plotted with the calculated values in Figure 3.

Table 2: The measured values of effective wavelength($\lambda_{\rm eff}$) for selected main-sequence stars measured at a 5 m baseline.

Star	Spectral Type	$(B-V)$	$(B-V)_0$	Date of Obs.	Number of Obs.	$\lambda_{\rm eff}$ (nm)	sem (nm)
τ Sco	B0 V	-0.25	-0.30	070314	18	695.20	0.17
κ Vel	B2 IV-V	-0.19	-0.24	070309	5	695.93	0.48
β Vir	F8 V	0.55	0.52	070313	7	697.26	0.87
ϕ Leo	A7 IVn	0.21	0.20	070313	4	694.83	1.27
σ Leo	B9.5 Vs	-0.06	-0.05	070313	4	694.97	0.84
ι Cen	A2 V	0.04	0.04	070319	10	695.33	0.43
β TrA	F2 III/IV	0.30	0.30	070319	10	695.33	0.22

4 Discussion

The measured values show significant variations in the value of the effective wavelength from one set of scans to the next and the mean values do not follow a smooth curve like the calculated values although the general trend is the same. It is suspected that seeing conditions affect the scatter in the measurements.

The effect of aberrations have not been included in the calculations—it is a 'known unknown'! Any effect from aberrations in reducing measured values of V^2 would tend to increase the values of effective wavelength since they would have a larger effect at shorter wavelengths. On the other hand, Mike said he used a hack of 18%/100nm when matching his calculations to his observations

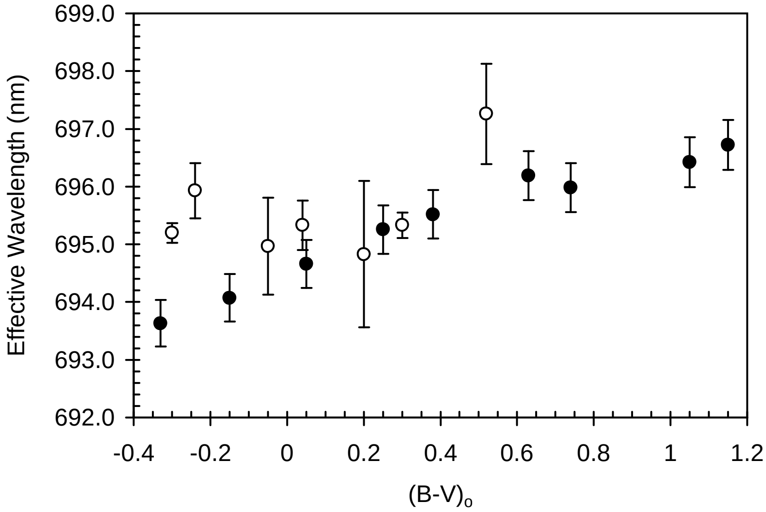

Figure 3: The effective wavelength of SUSI's 700 nm red system as a function of $(B-V)_0$ for main-sequence stars. The filled symbols are the calculated values shown in Figure 2 with the 'error' bars indicating the Maximum and Minimum values listed in Table 1. The open symbols are the measured values listed in Table 2.

which he claimed was consistent with his observed low throughput at 900 nm compared to 700 nm. Any slope of this kind would shift the effective wavelength to lower values. While all this is vague and Mike's calculations were very different from the ones presented here, the effects may largely balance out in light of the fact that there is now no major discrepancy between calculation and measurement. The average difference between the calculated and measured values is only $\sim$0.12% which, given the uncertainties in both methods, may be considered satisfactory.

5 Conclusion

The calculated and measured values of effective wavelength presented may be used to produce a scale of effective wavelength versus $(B-V)$ to be adopted for SUSI observations with the 700 nm filters. These values will only be strictly applicable to main-sequence stars. With the acceptable agreement between the measured and calculated values it is reasonable to propose that values for different luminosity classes, and for reddened stars can be calculated following the procedures used here—possibly with an appropriate increase in the uncertainty.

Experience in calculating the values of effective wavelength as a function of $(B-V)_0$ has shown that changing the various factors that enter the evaluation of the spectral response or transfer function has negligible effect on the shape of the relationship between the two parameters but simply offsets the curve in effective wavelength. For main-sequence stars it is therefore proposed to adopt the shape of the curve given by the calculated values with an increase of 0.8 nm in effective wavelength throughout ($\sim$0.12%) to place it close to the mean difference between the measured and calculated values. This is shown in Figure 4. It is suggested that a conservative uncertainty of $\pm$0.3% be adopted and the boundaries implied by this are also shown in Figure 4. This corresponds to an uncertainty of approximately $\pm$2.0 nm. The effective wavelengths corresponding to the proposed curve are tabulated in Table 3 against $(B-V)$.

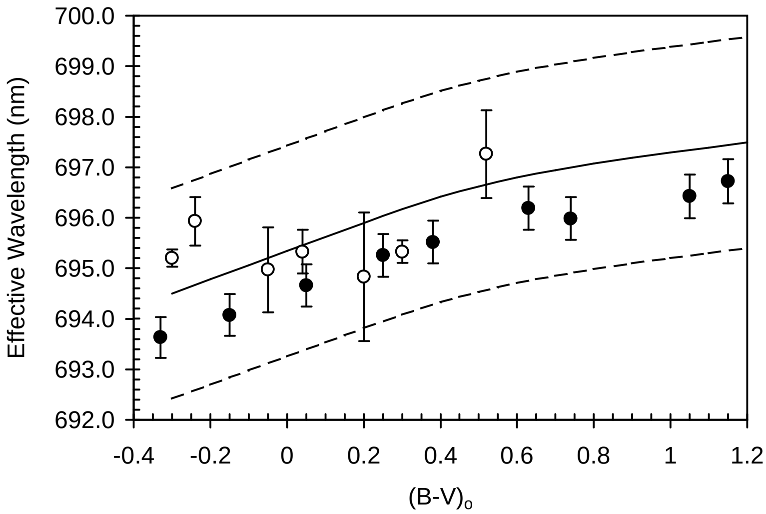

Figure 4: The same as Figure 3 with the addition of the proposed curve for adoption shown as the solid line. The suggested adoption of an uncertainty of $\pm$0.3% is represented by the two dashed lines. The proposed scale of effective wavelength v. $(B-V)_0$ is listed in Table 3.

Table 3: The proposed scale of effective wavelength $\lambda_{\rm eff}$ v. $(B\text{-}V)_{\rm o}$ for unresolved main-sequence stars observed with SUSI's 700 nm system.

$(B\text{-}V)_{\rm o}$	$\lambda_{\rm eff}$ (nm)	$(B\text{-}V)_{\rm o}$	$\lambda_{\rm eff}$ (nm)	$(B\text{-}V)_{\rm o}$	$\lambda_{\rm eff}$ (nm)	$(B\text{-}V)_{\rm o}$	$\lambda_{\rm eff}$ (nm)
-0.30	694.5	0.10	695.6	0.50	696.6	0.90	697.2
-0.20	694.8	0.20	695.9	0.60	696.8	1.00	697.3
-0.10	695.1	0.30	696.2	0.70	696.9	1.10	697.4
0.00	695.3	0.40	696.4	0.80	697.1	1.20	697.5

A The Effective Wavelength from Observations of Resolved Stars

Equation 4 shows that the effective wavenumber determined from the first moment of the power spectrum includes $\gamma(b, \sigma)^2$ which is a function of baseline and wavenumber. I have calculated the difference between $\lambda_{\text{eff}} = 1/\sigma_{\text{eff}}$ and $\lambda_1 = 1/\sigma_1$ for the A8 V spectral type star used in the calculations of effective wavelength. The effective wavelength was calculated for a selected series of values of V^2 at $\sigma = 1.43\,\mu\text{m}^{-1}$ (near the centre of the spectral response). The calculations took into account the variation of V^2 with σ across the spectral response of the system and were done for the case of ~half tip-tilt correction and 1.5 arcsecond seeing.

The results are plotted in Figure 5 and show that there is a significant variation of λ_1 with the value of V^2_{true}—in other words on the position on the transform at which V^2 is measured. The dependence is greatest for low vales of V^2 corresponding to longer baselines (i.e. higher spatial frequencies).

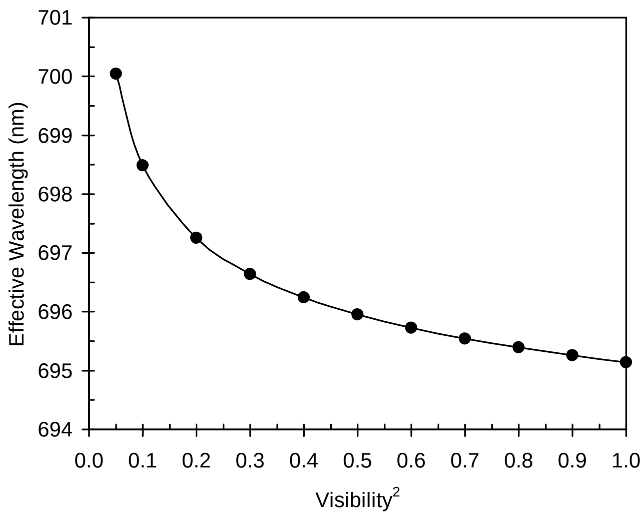

Figure 5: The calculated dependence of uncorrected 'effective wavelength', determined from the first moment of the power spectrum, on V^2_{true} for an A8 V star.

In Figure 6 the fractional difference between the uncorrected 'effective wavelength' (λ_1) determined from the first moment of the power spectrum, and the effective wavelength (λ_{eff}), expressed as a percentage, is plotted against V^2_{true}. Calculations show that this curve holds to within 0.02%, down to $V^2_{\text{true}} = 0.05$, for all main-sequence spectral types from O5 V to K5 V.

NOTE: Bill Tango's recent document includes an equation (4.8) which gives

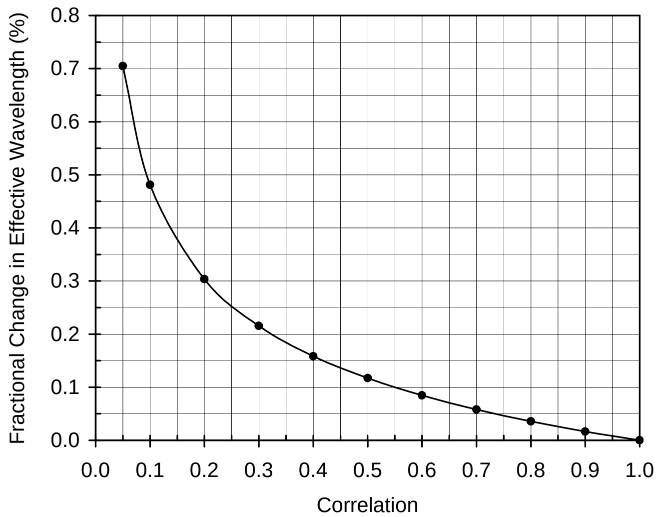

Figure 6: The fractional change (%) in uncorrected 'effective wavelength' (λ_1) determined from the first moment of the power spectrum relative to the effective wavelength ($\lambda_{\rm eff}$), as a function of $V^2_{\rm true}$. Calculations show that this curve holds for all main-sequence spectral types from O5 V to K5 V to within 0.02% down to $V^2_{\rm true} = 0.05$.

a relationship between σ_1 and σ_0 which I have not been able to reconcile with the above calculations—watch this space as Bill and I sort this out.