Aims
This chapter is an introduction to some of the important concepts used in later chapters. The two major themes are the relativistic view of mass and energy and the quantum mechanical ideas of wave-particle duality.

You should aim to understand the idea of the equivalence of mass and energy and the need to introduce a new formula for kinetic energy. The idea of momentum, although not explicitly examinable by itself, will help you to understand some of the ideas about particle interactions in later chapters.

You should also aim to appreciate how wave and particle pictures are both used to describe different aspects of the behaviour for both electromagnetic radiation (including light) and material particles. You should gain some familiarity with schemes for naming and classifying particles and interactions between particles. However the details of fundamental particle theory (§1-8) are not examinable.

Minimum learning goals
When you have finished studying this chapter you should be able to do all the following.

1. Explain, use and interpret the terms
   atom, nucleus, electron, proton, neutron, nucleon, neutrino, nuclide, atomic number, mass number, free particle, rest mass, rest energy, total energy (for a free particle), momentum, atomic mass unit, electronvolt, Planck's constant, de Broglie waves, photon, photoelectric effect.

2. (a) State, apply and discuss Einstein's relation between mass and energy and the variation of mass with speed (equations 1.1, 1.2, 1.3).
    (b) State and apply the relation between kinetic energy and rest energy (equation 1.4).

3. Recall and apply formulas for
   (a) the speed of light in terms of frequency and wavelength (equation 1.7);
   (b) the energy of a photon in terms of wave frequency (equation 1.6);
   (c) the momentum of a photon in terms of energy, wave frequency or wavelength (equations 1.8, 1.9).

4. (a) Describe examples of particle behaviour and wave behaviour.
    (b) Describe experiments which show that particles have both particle and wave properties.
    (c) State and apply the de Broglie formula for wavelength (equation 1.9).
1-1 THE COMPOSITION OF MATTER

Atoms
According to the current view, solids, liquids and gases, the varieties of matter we meet every day, consist of atoms or groups of atoms called molecules. The atom is the basic unit of a chemical element. Each atom consists of a small, extremely dense, positively charged central nucleus surrounded by a cloud of negatively charged electrons. The charge of the nucleus is carried by protons, each of which has a positive charge (e) with the same magnitude as the electron's charge.

All atoms of the same chemical element have the same number of protons in the nucleus and, in its normal non-ionised state, each atom has the same number of electrons, making it electrically neutral. The number of protons is called the atomic number (symbol Z), so each chemical element has a unique atomic number. The structure and chemical behaviour of atoms can be described in terms of the arrangements of electrons within the atom and their interactions with each other and the nucleus.

The sizes of atoms vary in a complex way with atomic number; the diameters of most atoms lie in the range 0.1 nm to 0.5 nm.

The nucleus
The nucleus is very small compared with the size of an atom. Practically all of the nucleus of the an atom is confined within a roughly spherical region roughly $10^{-5}$ times smaller than the whole atom, that is about 5 fm ($5 \times 10^{-15}$ m) in diameter. Thus, nuclear matter is incredibly dense, about $2 \times 10^{17}$ kg.m$^{-3}$ or about $10^{14}$ times the density of water.

According the current model, the nucleus of every atom consists of protons and neutrons, which together are known as nucleons. A species of nucleus with a given mixture of protons and neutrons is called a nuclide.

Although all the atoms of a chemical element have the same number of protons in the nucleus, different atoms of the same chemical element may have different numbers of neutrons. In other words an element can consist of several nuclides with the same atomic number (Z) but different neutron numbers (N). The total number of nucleons in a nucleus is equal to the mass number ($A = Z + N$) of the atomic species or nuclide. Nuclides of the same element, with the same atomic number but different mass numbers, are called isotopes of the same element.

1-2 MASS AND ENERGY

Probably the best known equation in all of science is Einstein's relation between energy and mass:

$$E = mc^2 \quad \text{...(1.1)}$$

Here $c$ is the speed of light in vacuum. The equation is a prediction from the special theory of relativity, which is a theory about space, time, light and matter that was worked out at the beginning of the twentieth century by Albert Einstein. You don't have to understand that theory in order to appreciate the mass-energy relation. The interpretation is that mass and energy are essentially the same thing. Although we conventionally think of mass and energy as separate physical quantities, relativity theory says that any thing which has mass also has an equivalent amount of energy. A definite amount of energy is always associated with a definite mass. By analogy with currency conversions, the equation specifies the 'exchange rate' between energy and mass (but, unlike money exchange, the conversion factor, $c^2$, is a constant fixed by nature).

An important aspect of the equivalence of mass and energy is that whenever an object acquires kinetic energy it also acquires mass! In other words: mass depends on speed. Clearly this conclusion contradicts the assumption of newtonian physics that the mass of a fixed amount of matter is constant, so we need to be explain why nobody ever noticed the difference before. The
answer is that the difference becomes noticeable only at very high speeds - close to the speed of light. At low speeds the variation of mass with speed is very small.

The equivalence between mass and energy applies even when an object is at rest: although it then has no kinetic energy, it still has energy associated with its mass. The value of a body's mass when it is stationary is called its rest mass \( m_0 \) which is associated with a rest energy \( E_0 \):

\[
E_0 = m_0c^2.
\]

The relation between mass \( m \) and speed \( v \) is

\[
m = \gamma m_0
\]

where

\[
\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

The factor \( \gamma \) is often called the relativistic factor.

An important consequence of the mass-energy relation is that the familiar formula for kinetic energy, \( \frac{1}{2}mv^2 \), is no longer valid at high speeds. It is easy to work out the correct form, using the results above. To do that consider a free particle moving at a high speed. A free particle is defined as one that has no forces acting on it - a reasonable approximation in much of atomic and nuclear physics. The total energy, \( E \), of the particle is, by Einstein's relation, equal to \( mc^2 \). This must be equal to the sum of the rest energy \( m_0c^2 \) and the kinetic energy, \( K \). So

\[
K = mc^2 - m_0c^2 = (\gamma - 1)m_0c^2.
\]

i.e.

\[
K = (\gamma - 1)E_0
\]

This formula gives answers quite different from the old, 'non-relativistic', formula when the particle's speed is a substantial fraction of the speed of light, but at low speeds, the answers are essentially the same - as they must be unless we believe that the old physics was a huge mistake. (To see how the old and new formulas relate mathematically read Appendix 1.)

To appreciate why you may not have noticed the equivalence of mass and energy before it may help to do a few simple calculations.

Example

Calculate the relativistic factors for electrons moving at 0.500\( c \), 0.100\( c \) and 0.001\( c \).

For \( v = 0.500 \, c \),

\[
\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - (0.500)^2}} = 1.155.
\]

For \( v = 0.100 \, c \),

\[
\gamma = 1.0050.
\]

For \( v = 0.0100 \, c \),

\[
\gamma = 1.000050.
\]

You can see that for low speeds, \( v \ll c \), the relativistic factor is very close to being exactly 1 but it becomes large as \( v \) approaches \( c \). For a car moving at 20 m.s\(^{-1} \), for example, the factor is equal to 1 to better than 1 part in \( 10^{14} \). This means that for low speeds the relativistic change in mass that occurs when an object moves is essentially undetectable. People often ask how one can know when to use relativistic or non-relativistic formulas. The answer is that if you are not sure then relativistic formulas (such as 1.4) are always correct and the only disadvantage in using them is that they are a little more complex to use than approximations such as \( \frac{1}{2}m_0v^2 \).

The formulas for mass and kinetic energy give a very significant prediction. If you put \( v = c \) into the expression for \( \gamma \), you will see that it takes an infinite value, so that both mass and energy would also be infinite. The interpretation of that ridiculous result is that you can't have any material thing going that fast - there is a universal speed limit for material objects (and signals also) of \( 3.00 \times 10^8 \, \text{m.s}^{-1} \).
Conservation of mass and energy

Since mass and energy are equivalent, the separate principles of conservation of energy and conservation of mass become one principle, which is sometimes referred to as conservation of mass-energy. In applying the principle it does not matter whether you calculate mass or energy, provided that you count it all. There is no need to do separate calculations for mass and energy.

1-3 MOMENTUM

Kinetic energy is an important property of a moving object but you can't describe direction of motion or change in direction using energy. To include direction of a body's motion we define a new quantity - its momentum. The magnitude \( p \) of a particle's momentum is just the product of its mass \( m \) and its speed \( v \). The direction of the momentum is the same as the direction of motion, i.e. the direction of the velocity. This definition works for both relativistic and non-relativistic speeds, provided that the mass is defined as it was above.

For all particles:

\[
    p = m v = \gamma m_0 v
\]

and for slow particles with \( v \ll c \)

\[
    p = m_0 v.
\]

One important use of the idea of momentum is in the study of interactions between particles. In a system of particles where the total external force is zero or negligible, the total momentum of the system remains constant. Because momentum is a vector quantity, calculations of momentum use components of momentum or velocity. (Such calculations are not required in this course but the principle will be helpful in understanding some kinds of particle interactions.)

1-4 UNITS

Although the SI units of mass and energy are the kilogram and the joule, a lot of atomic and nuclear physics uses the non-SI units, unified atomic mass unit (symbol u) and the electronvolt (eV). The conversions are as follows:

\[
    \begin{align*}
    u & = 1.660 \, 566 \times 10^{-27} \text{ kg} , \\
    \text{eV} & = 1.602 \, 189 \times 10^{-19} \text{ J} .
    \end{align*}
\]

These units have sizes appropriate for atomic and nuclear physics and chemistry. The atomic mass unit has been defined so that the mass of one atom of carbon-12 is exactly equal to 12 u, so the masses of atoms range from 1 u to about 250 u. The electronvolt is defined as the change in potential energy of an electron when it moves through a potential difference of exactly one volt. Energy levels of atoms are conveniently expressed in electronvolts, while energies associated with nuclear processes are typically several megaelectronvolts.

The energy corresponding to one atomic mass unit, given by Einstein's mass-energy relation, is

\[
    (1.000 \, 000 \, 000 \, u)c^2 = 931.5016 \text{ MeV}.
\]

Thus the rest energy of an electron is about \( \frac{1}{2} \) MeV, and the rest energies of a proton, a neutron and a hydrogen atom are all about 1 GeV. Some more exact values are given in table 1.1.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Rest mass/u</th>
<th>Rest energy/MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron, positron</td>
<td>0.000 549</td>
<td>0.511</td>
</tr>
<tr>
<td>proton</td>
<td>1.007 276</td>
<td>938.280</td>
</tr>
<tr>
<td>neutron</td>
<td>1.008 665</td>
<td>939.573</td>
</tr>
<tr>
<td>hydrogen atom</td>
<td>1.007 825</td>
<td>938.790</td>
</tr>
</tbody>
</table>

Table 1.1. Rest masses and energies
1-5 WAVES AND PARTICLES
By the beginning of the twentieth century the physicist's conception of nature was based on two abstractions from reality - the wave and the particle. The recently discovered electron had been shown to be charged and to have a definite mass - its deflection in electric and magnetic fields was just what would have been expected for a charged particle (although the charge and mass were extremely small by usual standards, $1.6 \times 10^{-19}$ C and $9.11 \times 10^{-31}$ kg).

The electron was believed to be a constituent of atoms. However, application of newtonian mechanics and the laws of electricity and magnetism could not explain the observed line spectra emitted by atoms. In fact, if these laws were applied to the atom, such atoms should be unstable and last for less than a nanosecond. Other phenomena such as the radiation from heated bodies and the recently discovered radioactivity had features which were inexplicable using concepts that were current at the time.

In the new view, developed since 1900, the concepts of wave and particle as separate entities must be abandoned. In reality nothing behaves exactly like a 'classical' wave or a classical particle. In some cases, such as electron in a TV tube, the behaviour of an electron matches the idea of a classical particle. In others, the wave description is more appropriate; electrons exhibit diffraction patterns when they bounce off crystals, for example. A similar situation holds for light; interference and diffraction are explained using the wave idea. On the other hand, the photoelectric effect is explained by saying that light consists of particles. But the two kinds of explanation don't mix. For example, in a diffraction experiment with electrons, any attempt to investigate the positions of individual electrons (i.e. applying particle concepts) destroys the diffraction pattern (the wave aspect). In the intermediate region where neither a particle nor a wave description alone is really appropriate, we use the very mathematical theory of quantum mechanics, which reduces to the simpler ideas of waves and particles in appropriate limiting cases. While nobody doubts that quantum mechanics works, its interpretation is still a matter of great argument. Since our conceptions of particles and waves are derived from watching things like falling apples and ripples on ponds, one should not be too surprised to find that those concepts do not help much in picturing what is happening on the sub-atomic scale. On the other hand quantum mechanics can describe and make predictions about a great deal more of the phenomena occurring in the universe than we can using the large scale theories of mechanics (described in the Forces and Energy unit) or the classical theory of electromagnetism (described in the Electricity unit).

1-6 PHOTONS
Many aspects of the behaviour of light and other varieties of electromagnetic radiation can be explained using a wave model. This model is most successful in explaining how light propagates through space, and how interference and diffraction effects occur. It is not so successful in explaining all aspects of the interaction of electromagnetic radiation with matter - we need a particle theory of light as well as a wave theory.

To explain some experiments, which will be described shortly, Einstein conceived electromagnetic radiation as travelling, localised packets of energy, which we now call photons. If the frequency of the radiation viewed as a wave is $f$, then there is only one possible magnitude for the energy content of a photon:

$$E = hf$$

Here $h$ is a universal constant of nature, called Planck's constant:

$$h = 6.63 \times 10^{-34} \text{ J.s} = 4.14 \times 10^{-15} \text{ eV.s}.$$

In atomic and nuclear work we do not measure the frequencies of electromagnetic waves directly. Instead we know their wavelengths, $\lambda$, (in air or vacuum) which are related by the equation:

$$c = f\lambda$$

So, in terms of wavelength the energy of a photon is
The combined constant $hc$ which occurs here is used so often that it is worth noting its value in units which are commonly used for atomic-scale energies (eV) and wavelengths of light (nm):

$$hc = 1.24 \times 10^3 \text{ eV.nm} = 1.24 \text{ keV.nm}.$$  

In the quantum model emission or absorption of electromagnetic radiation can take place only by emission or absorption of photons. So energy transfers are discontinuous - in contrast to the classical picture of electromagnetic radiation in which a wave transfers energy continuously to a charged object.

It has long been known that electromagnetic waves carry momentum as well as energy. So you might expect that a photon should also have momentum. Its magnitude is given by the relation:

$$p = \frac{hf}{c}. \quad \text{(1.8)}$$

Another way looking at this is to relate the wavelength (in vacuum) and momentum. Using the relation $c = f\lambda$, we get

$$\lambda = \frac{h}{p}. \quad \text{(1.9)}$$

It is important to remember that whenever a photon interacts with matter, it is destroyed. In most cases the photon transfers all its energy to the matter. Sometimes a new photon with lower energy may be created.

**The photoelectric effect**

In the late nineteenth century it was found that when light was shone on to the surface of some metals they emitted electrons. However, to get electrons out of a particular metal the light had to have elementary wave components with frequency greater than some critical value. You could have as powerful a beam of light as you liked below that critical frequency and no electrons would come out. Contrariwise, as weak a beam of light as you liked above that frequency would produce electrons. All this is totally inconsistent with the idea that the effect is due to electrons slowly but continuously absorbing energy from the incoming electromagnetic wave. In 1905 Albert Einstein suggested that energy propagation in an electromagnetic wave should be regarded as taking place in localised packets which we now call photons. He then gave the following explanation for the photoelectric effect. A solid metal consists of a crystal lattice of metal ions interpenetrated by a gas of electrons. The electrons are held inside the crystal at the boundaries by the electrostatic attraction of the positive ions. To get one electron out of the crystal requires a certain minimum energy, $\phi$, called the work function of the material. Typical values of the work function range from 1 eV to 6 eV. If we now assume that electrons can absorb light only in packets of energy (of size $hf$) we see that unless $hf > \phi$ the electrons can not get out of the crystal, no matter how intense the original beam (i.e. however rapidly photons arrive). If the photon energy is greater than $\phi$, the excess energy could appear as kinetic energy of the emitted electron.

### 1.7 DE BROGLIE WAVES

Just as we need a wave model and a particle model to make sense of light, it turns out that the properties of material 'particles', such as electrons, cannot be fully described by a classical particle model. There are cases when a wave model seems to be better. For example when electrons are fired through a thin crystal they produce a diffraction pattern of just the kind that classical waves give. Moreover, the spacing in this pattern enables us to find the associated wavelength.

The wavelength, $\lambda$, associated with an object whose momentum has magnitude $p$ is given by the equation

$$E = \frac{hc}{\lambda}.$$
\[
\lambda = \frac{h}{p}
\]  
... (1.9)

where \( h \) is Planck's constant. This equation was suggested in 1924 by Louis de Broglie who reasoned from special relativity theory. He suggested that, since light exhibits particle properties, it may be that matter could exhibit wave properties. The relation above, between wavelength of de Broglie waves and momentum of a material particle, is the same as that connecting the wavelength of an electromagnetic wave and the momentum of its photon.

For non-relativistic speeds you can write the de Broglie equation in terms of the particle's rest mass \( m_0 \) and its speed \( v \):

\[
\lambda = \frac{h}{m_0 v}
\]

but for photons there is no way of expressing momentum in terms of rest mass.

The simple idea of de Broglie waves was soon developed into the modern theory of quantum mechanics, first by Erwin Schrödinger who introduced the idea of a wave-function which could be used to describe atomic-scale systems and then by Max Born, who developed an interpretation of the wave function in terms of probabilities of observing a system in various states.

**Electron diffraction**

The wave nature of electrons was shown experimentally in 1927 by Davisson and Germer in America and G.P. Thomson in England. They reflected beams of electrons off crystalline solids and observed characteristic interference patterns associated with the diffraction of waves; as well as the main peak in reflected intensity when the angle of reflection equals the angle of incidence, they observed maxima at other angles. Since then, many people have done experiments on diffraction of electron beams by single and multiple slits. These effects cannot be explained by thinking of electrons as particles.

**Wave-particle duality**

You may wonder how a decent scientific theory can regard something as being sometimes a wave and sometimes as a particle. Why is it not always one or the other? The resolution of the paradox lies in a theory of measurement which needs more detail than we can give here, but briefly one answer is that if you stick to what can be measured, there is never a conflict between the two models. Here is one example.

Think about Young's twin-slit interference experiment. The existence of the interference-diffraction pattern can be described using wave theory; each wave goes through both slits and when the separate parts come together again, interference effects occur. What happens if the light is so weak that only one photon at a time arrives at the observing screen? This experiment has been done and the answer is that photons are seen to arrive at many different points. When the dim light is first turned on, there is no interference pattern. Photons are detected at various well-defined places on the screen and a pattern gradually builds up. As more photons arrive it is found that their distribution is not uniform. There are many photons at the positions where wave theory predicts a maximum intensity and no photons at places where wave theory says that there should be no light! Wave theory correctly predicts the statistical distribution of many photons but it cannot predict where any individual photon will show itself. Furthermore, it makes no sense to ask which of the two slits any individual photon went through! If you actually try to do an experiment to answer that question, you will destroy the interference pattern. The only way to find out if a photon goes through a particular hole is to put a detector at the hole. However to detect a photon is to destroy it - if you find a photon at one of the slits it will never reach the screen. The conclusion is that we can use wave and particle models for different purposes, but any question that leads to a contradiction between the models is
essentially unanswerable. Modern quantum theory avoids conflicts between the wave and particle models by using abstract mathematics instead of concrete pictures of 'waves' and 'particles'.

1-8 THE UNCERTAINTY PRINCIPLE
Closely related to the problem of wave-particle duality is the famous uncertainty principle formulated by Werner Heisenberg. The principle says that it is not possible to know precise values of all the dynamical variables, such as position, time, energy and momentum, of a system or a particle. For example it is not possible to know both the exact momentum and the exact position for a particle. If we write the uncertainty in the particle's $x$-coordinate of position as $\Delta x$ and the uncertainty in the $x$-component of its momentum as $\Delta p_x$, then the product of those uncertainties has a certain irreducible minimum value:

$$\Delta x \Delta p_x \geq \frac{h}{2\pi}$$

where $h$ is Planck's constant. There is also a restriction on knowing exact values of energy and time for any system:

$$\Delta E \cdot \Delta t \geq \frac{h}{2\pi}.$$  

These statements do not refer to the fact that it is difficult to measure anything with perfect accuracy; they actually say that it is not even possible to know the values exactly. Note the complementary nature of the uncertainties. For example if you know a particle's energy with great precision, $\Delta E$ is small, so that $\Delta t$ must be large. That means that you don't really know much about the time at which the particle had its accurate value of energy. On the other hand if you ask what is going on during an extremely short time interval $\Delta t$, you haven't got much of a clue about the value of the energy. Similarly, if you know a particle's velocity (and hence its momentum) very closely, you don't know much about where it is!

If you look at some examples using the value of Planck's constant, you will see that the uncertainty principle does not upset everyday human-scale calculations, but it is important on the scale of atoms and nuclei. For example, the time required to be sure that you have an energy accurate to $1 \mu J$ (which is pretty accurate on a human scale) is only about $10^{-31}$ s, an incredibly short time. On the other hand, if we say that the energy of an atom is known to the nearest 0.01 eV, we could not be sure about what is happening to the atom over a time scale of about a tenth of a picosecond ($10^{-13}$ s) - not such a short time for an atom.

1-9 PARTICLES AND THEIR INTERACTIONS
A simple view is that matter consists of three kinds of material particle, electrons, protons and neutrons. Detailed studies of the interactions among these particles, especially at high energies, has revealed the existence of many other particles, while theoretical studies indicate that there may be still more to be discovered. Other particles which will be mentioned in this course include positrons, neutrinos, and quarks. Some particles, such as electrons and protons, can exist on their own - as free particles - whereas others, such as quarks, can exist only in combination with other particles. Some properties of the free particles considered in this course are summarised in table 1.2.
<table>
<thead>
<tr>
<th>Particle</th>
<th>Symbol</th>
<th>Rest mass</th>
<th>Rest energy</th>
<th>Charge</th>
<th>Examples of occurrence or production</th>
</tr>
</thead>
<tbody>
<tr>
<td>proton</td>
<td>p</td>
<td>1.0 u</td>
<td>0.9 GeV</td>
<td>+e</td>
<td>• Bound in nuclei.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• A free proton is a hydrogen nucleus or a hydrogen ion.</td>
</tr>
<tr>
<td>neutron</td>
<td>n</td>
<td>1.0 u</td>
<td>0.9 GeV</td>
<td>0</td>
<td>• Bound in nuclei.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• Released by nuclear fusion and fission.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• Free neutrons are unstable.</td>
</tr>
<tr>
<td>electron</td>
<td>e⁻</td>
<td>0.0005 u</td>
<td>0.5 MeV</td>
<td>-e</td>
<td>• Bound in atoms.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• Released when an atom is ionised.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• Created in beta-minus decay of a nucleus.</td>
</tr>
<tr>
<td>positron</td>
<td>e⁺</td>
<td>0.0005 u</td>
<td>0.5 MeV</td>
<td>+e</td>
<td>• Created in beta-plus decay of a nucleus.</td>
</tr>
<tr>
<td>photon</td>
<td>γ</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>• Low energy photons are produced in atomic processes.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• X rays (intermediate energy)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• High energy photons are produced in nuclear processes.</td>
</tr>
<tr>
<td>electron neutrino</td>
<td>νₑ</td>
<td>0 (?)</td>
<td>0 (?)</td>
<td>0</td>
<td>• Created in beta-plus decay of a nucleus.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(&lt; 60 eV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>antineutrino</td>
<td>νₑ</td>
<td>0 (?)</td>
<td>0 (?)</td>
<td>0</td>
<td>• Created in beta-minus decay of a nucleus.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(&lt; 60 eV)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 1.2  Properties of some free particles**

Here, \( e \) is the elementary charge, \( 1.6 \times 10^{-19} \text{ C} \).

Some of the particles in table 1.2 are known as **antiparticles** of other particles: the positron is the antiparticle of the electron, and the antineutrino is the antiparticle of the neutrino. The electron and the positron have exactly the same rest mass and exactly opposite charges. When an electron and a positron interact, they disappear completely leaving only two or three photons which carry away all the original energy (rest energy plus kinetic energy) of the particle-antiparticle pair.

Note that photons have zero rest mass; since photons are particles of light or other electromagnetic radiation which travels at the speed of light, they can never be at rest - all their energy must be kinetic energy. They also have momentum. Photons interact with charged particles but not with other photons.

According to what is called the 'standard model' of particle physics, all matter is composed of fundamental families of particles called **quarks** and **leptons**. The model indicates that there may be four families of quarks and four associated families of leptons, but only three complete families have been observed so far. The composition of the ordinary matter in our environment (the Earth) can be understood in terms of one family of leptons, comprising the electron and its associated electron...
neutrino together with their antiparticles, and one family of quarks, the up quark and the down quark together with their antiparticles.

As far as we know, neutrinos and antineutrinos have zero rest mass, or if their rest mass is not zero it is extremely small. They do, however, carry energy and momentum. Once they have been created, neutrinos interact only very weakly, i.e. rarely, with nuclear matter. They can, for example, pass right through the earth without interacting with anything! Consequently they are very difficult to detect. There are three known kinds of neutrino, one associated with the electron (the electron neutrino) and two others with unstable particles called the muon and tauon (mu and tau neutrinos). Each kind has its own antiparticle.

The two kinds of particles, protons and neutrons, which comprise the nuclei of atoms are known collectively as nucleons. As far as we know, electrons and neutrinos are truly fundamental particles; they have no internal structure. Nucleons, on the other hand, are considered to be made up of quarks. The quarks that exist within nucleons are classified into two families which are called, for no very good reason, "up" and "down". Up quarks have an electric charge of $+2e/3$ and down quarks carry charge of $-e/3$. A proton contains two up quarks and a down quark, giving it a total charge of $+e$, while the neutron has one up and two down quarks, a charge of zero. Although the quarks exist within nucleons, it appears that nucleons cannot be taken apart to form free quarks.

**Fundamental interactions and exchange particles**

Table 1.3 lists the four fundamental types of interaction or 'force' that we introduced in chapter FE2.

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Acts on:</th>
<th>Carrier or exchange particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitational</td>
<td>Everything</td>
<td>Graviton</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>Electrically charged particles</td>
<td>Photon</td>
</tr>
<tr>
<td>Weak force</td>
<td>Leptons: particles such as</td>
<td>Vector boson</td>
</tr>
<tr>
<td></td>
<td>electrons and neutrinos</td>
<td></td>
</tr>
<tr>
<td>Strong force</td>
<td>Quarks</td>
<td>Gluon</td>
</tr>
<tr>
<td>Residual strong</td>
<td>Hadrons: particles like</td>
<td>Mesons</td>
</tr>
<tr>
<td>force</td>
<td>neutrons and protons</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1.3 Interactions and their carrier particles**

The classical or newtonian model of force explains interactions in terms of action at a distance. That idea does not work at the level of fundamental particles. Instead, the current model explains particle interactions using the concept of exchange particles rather than forces. The idea is that particles interact by sending out and receiving other particles. This interactions are represented in a semi-pictorial way using sketches called Feynmann diagrams (figures 1.1 and 1.2). In these diagrams there is an inescapable resemblance to the tracks left by two dirty billiard balls, both moving, showing their paths before and after collision. The value of the diagrams extends much further than showing particle tracks; physicists use the diagrams to do calculations. This is similar in some ways to drawing a vector diagram showing the forces acting on an object and then writing down the equations giving the net force or some other quantity.

Figure 1.1 shows the electromagnetic interaction between two electrons. The same idea extends, in principle, to the other three types of force.
"Classically" we describe the electromagnetic force with the idea of electric and magnetic fields: the charge on a particle produces an electric field which acts on other charged particles nearby. In the exchange force picture one envisages a swarm of photons (the particles of light) in the space between the charged particles. The photons are real if radiation is emitted, or virtual if there is no radiation. (The virtual photons are trapped in the system of the two interacting electrons.) The emission or absorption of a photon (real or virtual) by one charged particle is accompanied by a force acting on that particle so it is this process that transmits the force from one charged particle to the other.

The electromagnetic force is carried by a single type of exchange particle - the photon. For the weak force, up to three types of particle can be involved. One of these is shown in figure 1.2.

A down quark (d) decays to an up quark (u) accompanied by the emission of an electron and an electron antineutrino. The exchange particle is a vector boson (W\(^{-}\)). This is the same interaction as the decay of a neutron. It is conventional to draw antiparticles going backwards in time.
Use the data table on the inside front cover.

**Q1.1** Estimate the de Broglie wavelength for a beam of electrons that have moved through a potential difference of 15 kV. Compare the answer with a typical wavelength for visible light and comment. Would you expect to notice the wave properties (interference and diffraction) of 15 keV electrons in a TV tube?

**Q1.2** Would the wave nature of particles need to be taken into account to provide a reasonable description of the following phenomena?
   a) A 10 kg cannon ball is shot at a gap of 1.0 m wide in a wall. The speed of the cannon ball is 100 m.s\(^{-1}\).
   b) A beam of visible light passes through the prongs of a fork.
   c) An electron of kinetic energy 10 keV passes through a slit of width 1.0 nm.
   d) An electron of kinetic energy 10 eV passes through a slit of width 1.0 nm.

**Q1.3** Neutron diffraction studies of crystal structure are now carried out routinely, just like x-ray diffraction. What wavelength do we associate with a neutron whose kinetic energy is 0.025 eV? Such a neutron is non-relativistic.

**Q1.4 a)** Calculate the relativistic factor for an electron that has been accelerated through a potential difference of 100 kV.
   b) Calculate the speed of this electron.
   c) Calculate the mass of an electron when it is moving at this speed.

**Q1.5** What is the kinetic energy of a proton travelling at \(2.90 \times 10^8\) m.s\(^{-1}\)? Express the answer in electronvolts. How does the kinetic energy compare with the rest energy?

**Q1.6** An electron in a television set is accelerated through 20 kV. What is the percentage increase in its mass?

**Q1.7** In a nuclear reaction, the sum of the rest masses of the products is \(1.8 \times 10^{-34}\) kg less than the sum of the rest masses of the initial ingredients. Assuming that all the excess energy is concentrated in a high speed electron (a \(\beta\) particle), calculate that electron’s kinetic energy and speed.

**Q1.8** **Pair production.** In the intense electric field near a nucleus, it is possible for a photon to turn into an electron and a positron (a particle of the same mass as the electron but with opposite charge). Assuming that the energy given to the nucleus in this process may be neglected, find the minimum value of the radiation frequency needed for pair production to occur.

**Q1.9** When a positron and an electron collide, they annihilate and the energy is often released as two photons with equal energies.
   (i) Calculate the energy in joules and electronvolts when a positron and an electron annihilate.
   (ii) Calculate the wavelength of the radiation.

**Q1.10** The wavelengths of light in the visible spectrum range from about 400 nm to about 650 nm.
   (i) What colours do wavelengths of 400 nm and 600 nm correspond to?
   (ii) Calculate the energies of 400 nm and 600 nm photons.
   (iii) Does an ultraviolet photon have more or less energy than a visible light photon?

**Q1.11** Estimate the number of photons emitted in one second from the following sources, given that each is radiating power at 1.0 mW at the wavelength in question:
   (i) a sodium lamp, \(\lambda = 589\) nm,
   (ii) a helium-neon laser, \(\lambda = 630\) nm,
   (iii) an x-ray tube emitting a line spectrum at 75 pm.

**Discussion Questions**

**Q1.12** Would you expect quantum mechanical effects to be more important at the high frequency end of the electromagnetic spectrum (radio waves) or the low frequency end (gamma rays and x rays). Why?

**Q1.13** Ordinary light does not affect our skin much, but ultraviolet light can be quite harmful. Could that have anything to do with photon energies?

**Q1.14** It is not correct to say that "matter cannot be created or destroyed". What can you say instead?

**Q1.15** An electron and a proton are travelling at the same speed. Which has the shorter wavelength?
AN2

ATOMS

OBJECTIVES

Aims
In studying this chapter you should aim to understand the modern view of atomic structure in terms of energy levels and transitions. You should view the sections on spectra and lasers as important illustrations of the principles of atomic physics. Try to relate this chapter to anything you may have learned about the structure of atoms from chemistry courses.

Minimum learning goals
When you have finished studying this chapter you should be able to do all of the following.
1. Explain, use and interpret the terms state (of an atom), quantum number, energy level, energy band, principal quantum number, exclusion principle, excited state, ground state, shell, subshell, ionisation, ionisation energy, line spectrum, emission spectrum, absorption spectrum, radiative transition, spontaneous emission, stimulated emission, laser.
2. Describe and explain the composition and structure of atoms in terms of nuclei, electrons and energy levels.
3. (a) Describe and explain the emission and absorption of photons by atoms.
   (b) State and apply the relations among energy levels, photon energy and frequency of radiation emitted or absorbed by atoms.
   (c) Explain the origin of line spectra.
4. Describe and discuss how a helium-neon laser works.

2-1 STRUCTURE AND STATE OF AN ATOM

The original idea of an atom was that it was the ultimate indivisible particle of matter. That picture was abandoned with the discovery of the electron, a constituent of matter, in the late nineteenth century. By 1913 the work of Ernest Rutherford and Niels Bohr had led to a new model in which an atom was considered to consist of a small, extremely dense, positively charged nucleus surrounded by negatively charged electrons (figure 2.1). The space occupied by the electrons, and hence the size of the atom, is very large compared with the size of the nucleus. A hydrogen atom (the simplest atom) would be pictured as a single electron orbiting around its nucleus, in this case a single proton, rather like a planet orbiting around the sun. The electron is negatively charged, the proton is positively charged and the attractive electrostatic interaction binds them together.

![Figure 2.1 Rutherford-Bohr scheme for an atom](image)

Although the original Rutherford-Bohr model pictured the electrons as particles in orbit around the nucleus, that view soon became untenable when it was realised that the wave nature of
electrons could not be ignored. According to the theory of quantum mechanics developed during the 1920s, it is not possible to know both the position and velocity (or momentum) of a particle, so the planetary model of the atom had to be abandoned. Although visual models of atoms can never be entirely correct, quantum mechanics gave a better visualisation of an electron in an atom as a fuzzy cloud which represents the probability of finding the electron in various parts of the space. Nevertheless the nucleus of an atom, where most of the atom's mass is located, can be thought of as being confined to a very small fraction of the atom's total volume, while the electrons occupy the remaining space. Figure 2.2 shows some possible "maps" of the single electron cloud in a free hydrogen atom. The most heavily shaded areas show where the amplitude of the wave associated with the proton-electron system has its greatest value. These areas also indicate where the electron is more likely to be found.

![Figure 2.2](image)

**Figure 2.2  Electron density maps for a hydrogen atom - schematic**
The single electron is represented as a fuzzy cloud which is related to the probability of finding the particle in a particular region of space. These perspective sketches show three different possibilities for a hydrogen atom with \( n = 2 \).

Even though atomic electrons do not have definite positions and velocities, the physical state of an atom can be specified by quoting values of physical quantities such as energy and the components of a quantity called angular momentum, associated with each of its electrons. For each electron in an isolated atom each of four quantities can take only certain restricted values; the quantities are said to be quantised. Those allowed values can be described by quoting the values of four distinct quantum numbers for each electron in the atom. So the state of each electron is completely determined by a total of four quantum numbers.

Many important properties of electrons in atoms can be described in terms of just two of the four quantum numbers. These are the principal quantum number \( n \) which must be a positive integer \( (n = 1, 2, 3, ...) \) and the orbital angular momentum quantum number \( l \) which can take integer values between 0 and \( n - 1 \).

<table>
<thead>
<tr>
<th>Shell label</th>
<th>( n )</th>
<th>Subshell label</th>
<th>( l )</th>
<th>Capacity of subshell</th>
<th>Capacity of shell</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>s</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>L</td>
<td>2</td>
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<td>2</td>
<td>6</td>
</tr>
<tr>
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<td>1</td>
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</tr>
<tr>
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<td>f</td>
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</tbody>
</table>

**Table 2.1  The shell structure of atoms**
Electron states can be grouped, according to their quantum numbers, into shells and subshells. All states with the same value of $n$ constitute a shell, while all those states with the same pair of values for $n$ and $l$ make a subshell.

A fundamental principle of quantum mechanics, the **exclusion principle**, says that there cannot be more than one electron in any state. This principle limits the number of electrons which can occupy any subshell to the number of available states in the subshell. There is also a simple numerical rule which says that the number of states in a subshell with quantum number $l$ is equal to $2(2l + 1)$. Table 2.1 and figure 2.3 show how the electron states are organised into shells and subshells.

![Shell structure of a sodium atom](image)

Shells are often specified by capital-letter labels rather than the principal quantum number. The K shell has $n = 1$, L means $n = 2$ and so on. Similarly a set of lower case letters s, p, d, f, .. is used to label the value of $l$. A common way of specifying a subshell is to combine the value of $n$ with the label for $l$. For example the 3p subshell contains all the states with $n = 3$ and $l = 1$.

The way that electrons are distributed among the shells and subshells depends on the energies associated with the electron states, as described below, but the general idea is that the configuration with the lowest energy is the most stable one.

Many properties of atoms can be understood in terms of their shell structure. For example, atoms with completely filled shells are particularly stable and chemically unreactive, whereas atoms with unfilled shells bond readily to other atoms.

### 2-2 ENERGY LEVELS

Many atomic processes can be understood in terms of the energies associated with the binding of each electron to the whole atom. Associated with each electron in a stable atom there is a definite value of energy. That value depends in a complex way on the quantum state occupied by the electron, the charge of the nucleus and on the distribution of the other electrons in the atom.

By convention, the energy of every bound electron is negative. This is simply a consequence of taking the reference point for the potential energy of two interacting particles at infinite separation. For two particles which attract, such as a positively charged nucleus and a negatively charged electron, the potential energy at any finite separation must, therefore, be negative. (See also, for instance, the example of a diatomic molecule in §5-10 of *Forces and Energy.*) Although each electron also has kinetic energy, which must be positive, it turns out that as long as the electron remains bound in the atom the sum of its potential and kinetic energies is always negative. In practice, it is not feasible to distinguish between the internal kinetic energy and potential energy while an electron is bound in an atom. The important quantity is their sum, which we refer to simply...
as the energy, \( E \), of the electron state. Although it is common practice to refer to the energies of individual electrons, you should remember that since the potential energy arises from the interaction between each electron and the rest of the atom (including the other electrons), the energy is really a property of the whole atom. The energy, \( E \), of an electron state does not include kinetic energy associated with the motion of the whole atom.

Atoms, like other natural systems, have a tendency to go to a state of minimum potential energy. So the lowest energy for an isolated atom which contains \( Z \) electrons is that for which the lowest \( Z \) electron-energy states have been filled and all the higher energy states are empty. This situation is called the ground state of the atom - the lowest possible energy state. Any atom which contains an electron whose energy is higher than that of an unoccupied state is said to be excited and it cannot be expected to remain in that state for long. In general, an excited atom is unstable and will return to its ground state by losing energy somehow.

In the simplest atoms, which have only a few electrons, the energies of the electron states increase with increasing values of the principal quantum number \( n \), so that the higher shells have higher energies. In more complex atoms, however, that simple trend breaks down and the energies are more usefully associated with subshells, which generally contain electrons with similar, but not absolutely identical, energies. (For example in atoms with high atomic number the electrons in the 3d subshell have higher energies than those in the 4s subshell.)

The minimum energy required to remove an electron from an atom is called the ionisation energy. The ionisation energy is equal to the minus the energy of the electron state within the atom: \(-E\). Note that since \( E \) is always negative, ionisation energy is positive. A related idea is the atomic binding energy, which is the amount of energy you would have to supply to remove all the electrons from the atom.

The allowed energies associated with the electrons within an atom can be shown on a one-dimensional plot called an energy-level diagram, such as figure 2.3 or 2.4, in which each allowed energy (or group of closely-spaced energies) is represented by a line. Electrons occupying the energy levels can be shown on the diagrams as dots.

**Example: a hydrogen atom**

Figure 2.4 shows some of the allowed energy levels for an isolated hydrogen atom. The lowest energy level, the ground state, has principal quantum number \( n = 1 \), and \( E = -13.6 \text{ eV} \). Since a hydrogen atom contains a single electron, only one of the allowed states can be occupied at one time.

Fairly accurate values of the energy levels, \( E \), of an isolated hydrogen atom can be calculated from the value of the principal quantum number, \( n \):

\[
E = -\frac{E_0}{n^2}
\]

where \( E_0 = 13.6 \text{ eV} \), the ionisation energy for the ground state. Thus the energy for the first excited state, \( n = 2 \), has \( E = -3.4 \text{ eV} \). Other levels have higher energies. The principal quantum number can take any positive integer value, but as \( n \) increases, the energy levels get much closer together, and the energies of all bound states remain less than zero.

![Figure 2.4 Energy levels for an isolated hydrogen atom](image-url)
Molecular energy levels

A molecule consists of several atoms bound together. A proper treatment of molecular energies regards all atoms and all their electrons as one system. A typical molecule has many more possible energy states than a single atom. Consequently, since every electron in a molecule must occupy a different state, many molecules have sets of very closely spaced energy levels.

Energy bands in crystals

In a system of many interacting atoms, such as a crystal, the exclusion principle, that no two electrons can have exactly the same state, applies to the whole system. Since the system contains a huge number of electrons there must also be a huge number of different states. As a consequence there must be a very large number of different allowed energies. These energy states form groups of very closely spaced levels, called energy bands. Within each band the energy levels are so close together that in practice they seem to be continuous. The bands correspond roughly to the subshells in an isolated atom. Between the bands there are gaps where there are no allowed energy states.

![Energy level bands in crystals](image)

Shaded areas indicate occupied states. In a conductor electrons already in the conduction band need only a tiny amount of energy to reach an unoccupied state. In an insulator there is a forbidden energy gap between the top of the highest filled band and the conduction band, so very few electrons get enough energy to move up.
2-3 IONISATION

Normally an atom is electrically neutral, having a total of \( Z \) negatively charged electrons and a nucleus with a positive charge of \( Ze \). However, it is possible for an atom to absorb enough energy (the ionisation energy) to remove one or more of its electrons completely. The atom is then **ionised** and the remaining positively charged atom is a positive **ion**. It is also possible to form negative ions by adding extra electrons to some kinds of atom. To ionise an atom by the removal of an electron requires the absorption of energy by the atom. If the electron is removed from a ground state, the energy absorbed by the atom must be at least as great as the ionisation energy of that electron.

![Figure 2.7 Ionisation of a hydrogen atom by a photon](image)

Thus, for example, if a hydrogen atom in its ground state, \(-13.6 \text{ eV}\), absorbs a 14.0 eV photon it will be ionised. The remaining 0.4 eV of energy supplied will become additional kinetic energy shared between the ion and the electron.

Ionisation can also be caused by collisions with charged particles such as electrons, other ions or energetic neutral atoms.

![Figure 2.8 Energy level changes in ionisation by a photon](image)
2-4 RADIATIVE TRANSITIONS

One of the ways in which an atom can change its internal energy state is by the emission or absorption of a photon. Such changes are called radiative transitions. However a radiative transition between bound states can occur only if the photon energy is exactly equal to the difference in energies between the two states.

An atom can absorb a photon only if the photon’s energy is equal to the energy difference between an occupied state and a higher, unoccupied state. In that case, the photon can be completely destroyed and its energy transferred to the atom by raising the electron to the higher energy level. See figure 2.8.

The energy $E$ of the photon and the frequency $f$ of its associated electromagnetic wave must be related to the initial atomic energy level $E_i$ and the final level $E_f$:

$$E = hf = E_f - E_i$$

![Figure 2.9 Excitation or scattering of a photon by an atom](image)

For example if a 10.2 eV photon strikes a hydrogen atom which is in its ground state ($E_i = -13.6$ eV), the atom may be raised into its first excited state ($E_f = -3.4$ eV). On the other hand a 10.1 eV photon or a 10.3 eV photon cannot be absorbed by the atom (figure 2.9).
As already mentioned, if the photon's energy is greater than the ionisation energy of an atomic electron, that electron can be completely ejected to become a free electron, leaving the ionised atom behind.

Whenever an atom absorbs a photon it ends up either ionised or in an excited state. An atom in an excited state is unstable and will soon lose energy. One way of losing that energy is by successive spontaneous emission of one or more photons. Again, the only possible radiative transitions for the excited neutral atom are those between allowed energy levels. In this case the photon energy, wave frequency and atomic energy levels are related by:

\[ E = hf = E_i - E_f. \]

A highly excited atom can often return to its ground state either directly or by a sequence of transitions between energy levels. Each photon energy is equal to the difference in energy levels for the transition.

2-5 OTHER MECHANISMS OF CHANGING ENERGY STATES

Radiative transitions are not the only way that an atom can change its internal energy state. Collisions with other atoms, ions or charged particles can also alter an atom's internal energy.

For example if an electron whose kinetic energy is 10.8 eV collides with a hydrogen atom which is in its ground state (-13.6 eV), the atom can be raised into its first excited state (-3.4 eV). This requires only 10.2 eV so the remaining 0.6 eV would appear as kinetic energy, shared between the scattered electron and the motion of the atom as a whole.
2-6 ATOMIC AND MOLECULAR SPECTRA

Radiative transitions between energy levels within atoms and molecules account for the existence of line spectra in both visible light and invisible electromagnetic radiation. Radiative transitions between atomic energy levels produce spectra characteristic of the atoms of each element. Similarly the spectra of molecules are made up of groups of many lines close together, giving the appearance of bands.

The principles can be illustrated by considering the spectra formed by free atoms. Each spectral line corresponds to one type of transition between energy levels. An electron in an excited state can move into another state at lower energy, emitting a photon with a well defined energy. For a typical atom there are many possible excited states and a large number of possible transitions. Figure 2.13 shows a few of the possible transitions which give rise to the characteristic spectrum of lithium - a comparatively simple atom.

Figure 2.13  Energy levels for lithium and some possible transitions
The number part of each subshell label is the value of the principal quantum number $n$. The letters s, p, d, f represent values of the quantum number $l$. (See table 2.1.). Wavelengths of the radiation emitted are marked on the arrows.
There are two kinds of complementary atomic spectra: emission spectra and absorption spectra. Emission spectra are formed when excited atoms emit photons with well-defined energies. Absorption spectra are due to the selective removal of photons from a beam of radiation that has a continuous distribution of wave frequencies and photon energies.

An atomic emission spectrum can be produced by any mechanism which supplies energy to excite free atoms. Common examples are electrical discharges in gases and the action of flames. When solid matter is introduced into a flame, some of its molecules can be split into individual excited atoms or ions. (This is an instance of excitation by collisions.) The atoms then radiate at definite frequencies as they return to their ground states.

![Figure 2.14 Emission spectrum produced by a flame](image)

Since atoms of a particular element have a unique set of energy levels, each element gives a characteristic emission spectrum of bright lines, which can be used to identify the element.

An atomic absorption spectrum can be produced by sending an intense beam of radiation through a sample of material containing free atoms. Again the free atoms could be produced in a flame. The spectrum appears as a set of dark lines corresponding to the frequencies of radiation absorbed when the absorbing atoms are raised to higher energy states. Many molecular absorption spectra can be produced by sending white light through solutions of the compounds under study.

![Figure 2.15 Making an absorption spectrum](image)

Most of the light passes through the sample. Only those elementary waves with frequencies corresponding to differences in energy levels for free atoms or molecules in the sample are absorbed. This gives an absorption spectrum of characteristic dark lines or bands in the otherwise continuous spectrum.
An accessible example of an atomic absorption spectrum is found in sunlight. When it is
examined closely the continuous spectrum of light is seen to be interrupted by numerous dark lines,
called Fraunhofer lines. These lines can be grouped into sets which are characteristic of various
light elements that are present in the outer layers of the Sun's atmosphere. These elements absorb
parts of the continuous spectrum produced in deeper layers of the Sun. The element helium was
identified from absorption lines in the solar spectrum before it was discovered on Earth.

2-7 STIMULATED EMISSION AND LASERS

Stimulated emission

An excited atom left alone will lose energy by spontaneously emitting a photon. Since spontaneous
emission is fundamentally a random process it is not possible to predict when an individual excited
atom will decay. Some atoms will remain in their excited states longer than others. On the other
hand, a collection of excited atoms will remain excited for a statistically predictable average time.
There is another emission process called stimulated emission in which an excited atom is
triggered to emit its photon instead of decaying randomly. The stimulus must be a photon whose
energy is exactly equal to the energy of the emitted photon. As we have already seen the value of
this energy is determined by the difference in the energy levels of the atom. Stimulated emission
will occur whenever a suitable photon passes close enough to an excited atom. Since the stimulating
and emitted photons have equal energies, the frequencies of their associated elementary wavelets are
also identical. Furthermore the two waves are exactly in phase and they travel in the same direction
as the incident photon. In short, the stimulating photon and the emitted photon are identical in every
respect. Stimulated emission is the key process in the operation of lasers.

The helium-neon laser

The word 'laser' is an acronym derived from Light Amplification by Stimulated Emission of
Radiation. Laser light is very monochromatic and highly coherent. Those properties are achieved
using the process of stimulated emission to create an avalanche of identical photons. As more and
more photons are released by stimulated emission from excited atoms each new photon can
stimulate more atoms to emit their photons. In order to maintain this process there are two essential
requirements. Firstly there must be a plentiful supply of suitably excited atoms and secondly the
avalanche of photons must be confined sufficiently so that further stimulated emissions can occur.
We will consider these requirements below.

![Figure 2.16 Structure of a gas laser](image)

A helium-neon laser contains a mixture of helium and neon gases in a container which is fitted
with a pair of parallel mirrors. Energy is supplied to the gas mixture by passing an electrical current
through it. The current ionises the gas and, by a sequence of collision processes, excites the neon
atoms. The energy required to lift a neon atom to its excited state is obtained by collision with an
excited helium atom, whose excitation energy is just below the energy required by the neon (the
extra energy comes from kinetic energy) and the helium atom returns to its ground state.

\[
\text{He}^* + \text{Ne} \rightarrow \text{He} + \text{Ne}^*.
\]

(The asterisk indicates an excited state.) This process is indicated on the left side of figure 2.17.
The helium atoms, in turn, get their excitation energy in collisions with electrons produced in the electrical discharge. These collision processes must occur quickly enough to keep most of the neon atoms in their excited state.

**Figure 2.17  Energy levels and transitions in a He-Ne laser**

In this diagram energy levels are referred to the ground state as zero. A helium atom is raised to an excited state (1) and transfers its energy to a neon atom which becomes excited (2). Stimulated emission follows (3). The neon atom then decays (4) very rapidly back to its ground state (5) ready for the next cycle.

Once the neon atoms have been excited they are stimulated to emit laser light. Initially the stimulating photons come from spontaneous decays, but once the laser process has been started, most of them will come from previously stimulated transitions. Since each photon released can stimulate more transitions, the number of photons can build up very rapidly. To keep the laser process going, the beam of photons is confined by two accurately parallel, highly reflective, mirrors at opposite ends of the discharge chamber. Since each stimulated photon travels in the same direction as the photon which triggered it, the number of photons bouncing back and forth along the axis of the laser cavity soon builds up to produce a very intense beam of light inside the laser. Those photons which do not travel parallel to the axis are lost from the beam and take no further part in the laser action. A small part of the intense beam between the mirrors is let out of the laser by having one of the mirrors not quite perfectly reflecting (say 99.9% reflectivity). Even the small fraction of the beam which comes out is still very intense.

The radiative transition which produces the laser light from the neon atom does not leave the atom in its ground state - if it did there would be many neon atoms able to re-absorb the photons in the laser light. So any neon atoms in a state which could remove laser photons need to be got rid of very quickly. The lower state of the lasing transition is another unstable state which then decays very rapidly in two stages back to its ground state as shown in figure 2.17. (The two photons emitted in these transitions have no role in the laser action.)
AN2: Atoms

<table>
<thead>
<tr>
<th>A</th>
<th>C</th>
<th>D</th>
<th>B</th>
</tr>
</thead>
</table>

Start with 4 excited atoms.

A photon of just the right energy strikes A ...

and stimulates A to produce an identical photon.

The two photons travel together...

until one of them stimulates C to produce another identical photon.

Further amplification occurs when D is stimulated to emit.

Atom B was not affected and will decay spontaneously.

**Figure 2.18  Photon avalanche in a laser**

A photon arrives at excited atom A and stimulates it to emit an identical photon. Excited atom B emits its photon spontaneously and the photon is lost. One of the photons from A stimulates C to emit a photon and D is also stimulated to emit an identical photon.

The laser action is sustained by returning neon atoms from their ground state back to the excited state, through further collisions with excited helium atoms.

**Properties of laser light**

The important features of laser light are as follows.

- It is spectrally very pure, being much closer to the ideal of pure monochromatic light than light from spectral lamps. For example, the sodium D line from an ordinary sodium vapour lamp has a bandwidth (or frequency spread) of 1 GHz but a gas laser can easily attain a bandwidth of only 1 MHz.
- The light is highly coherent. Whereas coherent light produced by splitting waves (as in Young's experiment or in thin films) loses its coherence as the path difference between the beams is increased, laser light maintains its coherence much longer.
- Because of its coherence laser light can be made highly collimated, i.e. a parallel beam remains parallel to a much greater degree than a conventional light beam.
- Very high power concentrations can be achieved, i.e. the irradiance of laser light can be made very large.

**FURTHER READING**

Q2.1 A hydrogen atom is in the -3.4 eV excited state.
   (i) Calculate the energy a photon must have to excite the atom into the -1.5 eV excited state.
   (ii) What is the wavelength of this photon?
   (iii) What part of the electromagnetic spectrum does this correspond to?

Q2.2 A hydrogen atom in the -1.5 eV excited state falls to the ground state. Calculate the wavelength of the photon emitted.

Q2.3 A laser is powered by a discharge lamp running from a capacitor bank that stores 4 kJ. The conversion to laser light is 0.2% efficient and the laser pulse length is 50 ns. What is the power in the laser pulse? What is the irradiance if the area of the beam is $2 \times 10^{-6} \text{ m}^2$?

Q2.4 The emission spectrum of sodium has two closely spaced lines at wavelengths of 588.995 nm and 589.592 nm. Both lines are due to transitions which end up at the same level. Calculate the difference in the energies of the upper levels.

Q2.5 Describe the sequence of energy level transitions of a neon atom in a helium-neon laser. Why is it important that should be more than one radiative transition?

Q2.6 Some lasers operate at wavelengths outside the visible range. For example a methyl fluoride laser involves stimulated emission between molecular energy levels about 3 meV apart. Estimate the wavelength of the radiation. What part of the spectrum does it lie in?

Discussion questions

Q2.7 Absorption and emission spectra of the same element do not usually contain the same prominent spectral lines. Why not?

Q2.8 The materials on the inside of fluorescent light tubes absorb ultraviolet light and subsequently emit visible light. What do you think of a claim from an inventor who says he has found a material that absorbs visible light and emits ultraviolet?

Q2.9 Atoms in the gaseous state emit line spectra but in solids they emit continuous spectra. How do you account for the difference?

Q2.10 In black and white photographic film, the latent images are formed when light splits some atomic bonds between silver and chloride ions. The film is more sensitive to blue light than red light, and hardly reacts at all to infrared. Can you explain that?

Q2.11 At a distance of 1 km the light from a 1 mW laser is much brighter than that from a 1 kW flood lamp. Explain why.
**AN3 THE NUCLEUS**

**OBJECTIVES**

**Aims**

By studying this chapter you should develop your understanding of the structure of nuclei and the decay of unstable nuclei. Energy is a key concept in that understanding. You should also be able to describe and discuss the various types of radioactive decay and you will learn to use the statistical-mathematical description of the decay process.

**Minimum learning goals**

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, use and interpret the terms
   - atomic number, mass number, neutron number, nuclide, isotope, isobar, abundance, isotopic abundance, line of beta stability, valley of beta stability, radioactivity, radioactive decay, radionuclide, positron, neutrino, antineutrino, alpha particle, alpha rays, Coulomb barrier, beta particle, beta rays, orbital electron capture, gamma ray, decay constant, activity (rate of decay), becquerel, specific activity, half life, radioactive dating, daughter nuclide, radioactive series, chain decay, secular equilibrium, fission, chain reaction, fusion.

2. Describe and discuss the structure and stability of nuclei in terms of atomic number, mass number, binding energy and binding energy per nucleon.

3. (a) Describe and discuss the processes of alpha, beta and gamma decay.
    (b) Write decay schemes, including mass numbers and atomic numbers, for each type of decay.

4. (a) State, use and explain equations for the decay of number of nuclei and activity with time (equations 3.1, 3.2).
    (b) Solve simple quantitative problems involving number of nuclei, activity, specific activity, half life and decay constant.

5. (a) Describe and explain the technique of radioactive dating.
    (b) Do simple numerical calculations based on this technique.

6. (a) Describe radioactive chain decays.
    (b) Describe how the activity of a daughter nuclide changes with time to produce a secular equilibrium.

7. Describe the processes of fission and fusion.

**TEXT**

### 3-1 SIZE AND COMPOSITION OF THE NUCLEUS

The sizes of atoms can be found by measuring the distances between planes of atoms in crystals. Typically one finds these distances to be between 0.1 nm and 0.6 nm. The negatively charged part of atoms - the electrons - take up most of that space. In contrast, the positively charged part at the centre of the atom is relatively much smaller in size. To emphasise this point, the term nucleus (meaning a kernel of a nut), was given to it. Nuclear radii are something like a hundred thousand times smaller, being in the range 1.3 fm to 9 fm. (1 fm = 10^{-15} m.)
Composition of the nucleus

Simple nuclear physics experiments show that nuclei contain particles collectively called nucleons. There are two types of nucleon, the proton and the neutron. Both particles have similar masses: about 1 u. The simplest nucleus (hydrogen) is a single proton, while the largest nucleus contains about 260 nucleons.

The neutron has no electric charge whilst the proton has a charge equal in magnitude to an electron's but opposite in sign. Atoms are normally electrically neutral with the positive nuclear charge balanced by the negative charge of the atomic electrons.

We use the following symbols:

\[ Z \quad \text{atomic number} \quad \text{the number of protons in the nucleus.} \]
\[ N \quad \text{neutron number} \quad \text{the number of neutrons in the nucleus.} \]
\[ A = Z + N \quad \text{mass number} \quad \text{the number of nucleons in the nucleus.} \]

A species of nucleus with a given mixture of protons and neutrons is called a nuclide. Values of \( Z \) and \( N \) are used to specify a nuclide which can be represented symbolically as \(^ZAX\). For example \(^{14}_6\)C represents the nuclide carbon-14 which has 14 nucleons, including 6 protons. This notation has some redundancy as \( X \), the chemical symbol for the element concerned, is implied by \( Z \). So the atomic number subscript is sometimes omitted, as in \(^4\)He. In text, isotopes are identified by adding the mass number to the name, e.g. aluminium-27.

All nuclides of the same element have the same atomic number. Nuclides with the same \( Z \) and different \( N \) are called isotopes of the element. Different isotopes of the same element have the same chemical properties. Nuclides with the same \( A \) and different \( N \) (or \( Z \)) are called isobars because they have almost the same mass.

Nuclear size

We have already mentioned that a nucleus is much smaller than an atom. There are several ways of measuring nuclear sizes, including scattering experiments in which high energy particles are bounced off the nuclei in a target. The results of many such experiments give an empirical formula for the radius of a nucleus in terms of its mass number:

\[ r = r_0 A^{1/3}, \]

where \( r_0 \) is a constant with a value of about 1.3 fm. This result is interesting because it indicates that the volume of a nucleus is approximately proportional to the number of nucleons in it. That result suggests that, unlike the electron clouds in an atom, each nucleon occupies a fixed volume within a nucleus. Considering the nucleus to be roughly spherical, its volume is

\[ V = \frac{4}{3} \pi r^3, \]

so

\[ V = \frac{4}{3} \pi r_0^3 A. \]

Example

These results indicate that nuclear matter is incredibly dense. To calculate the density of a nucleus, consider \(^{238}\)U as an example. Using the mass of the nucleus, \( m \approx A \) u, \( u = 1.66 \times 10^{-27} \) kg and the formula for volume above, its density is

\[ \rho = \frac{m}{V} = \frac{A \text{ u}}{\frac{4}{3} \pi r_0^3 A} = \frac{3 \times (1.66 \times 10^{-27} \text{ kg})}{4 \pi (1.3 \times 10^{-15} \text{ m})^3} = 1.8 \times 10^{17} \text{ kg.m}^{-3}. \]
3-2 STABLE AND UNSTABLE NUCLEI

Some naturally occurring nuclides are stable while the others, called radioactive nuclides or radionuclides, can disintegrate spontaneously. There are two kinds of radioactive decay which alter the nature of a nuclide: alpha decay in which a nucleus emits a helium nucleus (or alpha particle, $^4$He) and beta decay which involves the emission of either an electron or a positron.

Naturally occurring samples of non-radioactive elements generally have fixed fractions or isotopic abundances of stable isotopes. For example oxygen contains 99.759% of the nuclide $^{16}$O, 0.037% $^{17}$O and 0.204% $^{18}$O. It is also possible to make unstable (radioactive) nuclides of oxygen, $^{13}$O, $^{14}$O, $^{15}$O, $^{19}$O and $^{20}$O, which decay rapidly to nuclides of other elements.

Chemists order the chemical species of atoms, the elements, in a table of increasing $Z$, the periodic table. With nuclei, further insight comes from adding another dimension, $N$. The result is shown in figure 3.1. Stable nuclides are shown as black squares. Other nuclides, which are unstable to radioactive decay, are within the jagged outline. A 'best fit' curved line (not shown) through the black squares is called the line of beta stability. Beta stability means that the nucleus does not break up by beta decay.

Figure 3.1 Stable and unstable nuclei

Neutron number ($N$) is plotted against proton number ($Z$). Stable nuclei are plotted as black squares. Unstable nuclei are shown as small dots.
The abundance of an element, that is the amount of the element relative to all matter in the universe, depends on the nuclear stability of its isotopes. Notice from figure 3.1 that there are relatively few stable isotopes of elements with odd values of Z, whereas even-Z elements generally have several stable isotopes. For example there is only one stable isotope of the relatively rare element gold (Z = 79) in contrast to lead (Z = 82) which has four stable isotopes. However there is also a general trend in abundance with Z. We find in our solar system and beyond that low Z elements ('light' elements) are much more abundant than high Z ('heavy') elements. The lightest elements, hydrogen and helium were formed, we believe, in the big bang that started the Universe some $10^{10}$ years ago. Stars make heavier elements in their 'nuclear furnaces', but it is only the most massive stars that allow those elements to be incorporated into planets later on. Light stars fade away into white dwarfs when their nuclear fuel runs out; as a result, the heavier elements are 'locked away' for ever, at least for a long time. A massive star can end its life as a supernova, a huge explosion capable of flinging the heavy elements out into space. That material is later recycled into new solar systems. Whether the process occurs in a star as a normal fuel process or in a supernova; heavy elements are made from lighter ones in a step by step operation. Lighter elements are thus more abundant throughout the Universe.

Stability is an important parameter in the $Z/N$ diagram (figure 3.1) and one might well ask how it varies in relation to the beta stability line. The line of beta stability follows the law $Z = N$ for low Z and then deviates to the neutron excess side of the curve for $Z > 20$. This can be explained by saying that small nuclei are most stable, with equal numbers of the two types of nucleons, but in heavier nuclei the mutual electrical repulsion between protons becomes important, contributing to the instability of the nucleus. Large nuclei with $Z = N$ can become more stable by converting protons to neutrons through beta decay (see §3-6).

### 3.3 NUCLEAR BINDING ENERGY

A convenient measure of the stability of a given nuclide is its binding energy $E_B$. The binding energy is the energy that would be needed to take the nucleus completely apart into its separated nucleons. The concept is similar to the atomic binding energy associated with electrons in atoms, but the magnitudes of nuclear binding energies are much greater.

The binding energy of a nucleus X can be calculated, using Einstein's mass-energy relation, from the rest mass $m_X$ of the nucleus and the rest masses of the constituent nucleons:

$$B = (Zm_p + Nm_n - m_X)c^2.$$  \hfill (3.1)

The result of this calculation is always positive because the total mass of the unbound nucleons is always greater than the mass of the nucleus. The extra mass comes from the energy which must be put into the system to pull it apart. Another way of looking at the relation is to say that if you could make Z protons and N neutrons combine to form a nucleus, then the total energy released would be $B$, given by equation 3.1. In practice binding energies are calculated using masses of atoms rather than nuclei, because that is what one usually measures directly. If you put the mass of an atom for $m_X$ you are including the mass of Z electrons, so to compensate you use the mass of a hydrogen atom instead of the proton mass. That gives an answer which is in error by the difference between the atomic binding energy of the Z electrons in atom X and the atomic binding energy of Z hydrogen atoms, but since atomic binding energies are tiny compared with nuclear binding energies, the error is negligible.

The binding energy, $B$, of a nucleus rises almost, but not quite, linearly with the atomic number $A$. To emphasise the departure from linearity we can look at the average binding energy per nucleon, $B/A$. Figure 3.2 shows how the binding energy per nucleon changes with the mass of the nucleus. The biggest changes occur for the light nuclei, but the graph soon flattens out and peaks at $A = 56$, which corresponds to iron. The peak represents the nuclei in the which the individual nucleons which are most tightly bound.
While figure 3.2 shows how the binding energy per nucleon varies along the line of beta stability, it is also interesting to ask how it varies across the line of stability. Imagine a three dimensional graph formed by plotting \(-B/A\) against \(Z\) (that is figure 3.2 turned upside down) and adding another axis to show how \(-B/A\) depends on \(N\) you would see that the line of beta stability follows the bottom of a groove, the so called 'valley of beta stability'. The valley floor at the end of the graph representing large nuclei \((Z > 92)\) is steep enough for alpha radioactivity to occur, but at this stage we will consider only beta decay.

The graph shows the variation in rest energy, referred to an arbitrary zero, for nuclides with the same mass number, \(A = 125\). The lowest point on the graph is for the stable nuclide \(^{125}\)Te. All the other isobars are unstable. The difference in energy between adjacent nuclides increases away from the valley floor.
Figure 3.3 shows the variations (from an arbitrary zero) in the total binding energy for a set of isobars, nuclei with the same mass number, so that the shape of the graph also represents the variation in binding energy per nucleon. It is like a section across the valley of beta stability. In a group of isobars, those that are most tightly bound have the highest values of binding energy per nucleon, or the lowest values of total energy and rest mass. Nuclides with too many neutrons or too many protons are less tightly bound. In beta decay, the number of nucleons in the nucleus does not change, so it is energetically possible for nuclides on the walls of the valley in figure 3.3 to decay to the nuclide with the lowest energy; nuclides on the valley sides are unstable. The stable nuclide lies near the bottom of the valley. Nuclei far from the valley floor break up as soon as they are formed. A typical time scale for such a break-up is \( t = s/v \), where \( s \) is the distance across a nucleus and \( v \) is a typical nucleon's speed within the nucleus. That time is about \( 10^{-21} \) s.

3-4 RADIOACTIVITY

Discovery

In 1896, shortly after Wilhelm Röntgen had discovered x rays, Henri Becquerel tested phosphorescent salts for unknown radiations and found that salts of uranium blackened his photographic plates. This radiation was also found to produce ions in air, just as x rays did. In 1898, Marie Curie noticed that thorium and its salts also had these properties, and coined the name radioactivity to describe them. This radioactivity has nothing to do with the phosphorescence and very little to do with x rays. It was soon realised that radioactivity was a property of the element itself and not of the compound in which it occurred. As we now understand, what happens in a radioactive decay is that the nucleus rearranges, with the release of different kinds of energetic particles.

The term radioactivity is used to describe a nuclear decay process where a nucleus loses energy by the emission of particle(s) such that the time is long compared with the \( 10^{-21} \) s discussed above. Such decays are essentially random - it is quite impossible in principle to predict when any particular nucleus will decay. Statistically, however, a large enough sample of each nuclide decays at a regular, decreasing, rate. One parameter which describes the decay a nuclide is its half life, which is the time interval required for half the original number of nuclei to decay. A more detailed description of decay statistics will be given in §3-8.

Radioactivity is classified into three types, called alpha (\( \alpha \)), beta (\( \beta \)) and gamma (\( \gamma \)). These will be considered in turn.

3-5 ALPHA DECAY

In alpha decay, a nucleus emits a helium nucleus, also called an alpha particle, with the release of kinetic energy. The general decay scheme can be written as:

\[
^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}\text{He} ;
\]

nucleus X decays to nucleus Y by emitting a helium nucleus. (Here \( Y \) is just some nucleus or other, not yttrium.) Notice that the number of nucleons on the two sides of the decay scheme is the same (i.e. the number of nucleons is conserved in the reaction) and also that the total charge on both sides is the same (the total charge is conserved).

The mass/energy must agree on both sides of the reaction scheme. Here we use rest masses and count the energy released (kinetic energy) as one item:

\[
m_{X} = m_{Y} + m_{\alpha} + \frac{E}{c^2}
\]

where \( E \) is the total kinetic energy of the daughter nucleus and the alpha particle.

So the energy release is determined by the difference between the rest mass of the original nucleus and that of the two products. We see that the decay certainly cannot occur unless \( m_{X} > m_{Y} + m_{\alpha} \), which
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The Nucleus 33 is equivalent to kinetic energy $> 0$. (We will see later on that the kinetic energy must be actually greater than about 4 MeV.)

The ejected alpha particle gets a fixed share of the released energy - which is generally between 3 MeV and 10 MeV, depending on the type of nucleus. Most of the energy release goes into the kinetic energy of the alpha particle and a little goes into the kinetic energy of the recoiling nucleus. The reason that the alpha particle takes a fixed share of the energy is that the total momentum of all the particles must also be conserved. A nucleus at rest has zero momentum, and in order for the total momentum to remain zero, the daughter nucleus and the alpha particle must move off in opposite directions. There is only one way to conserve both energy and momentum, so the energy of the alpha particle is quite predictable.

An example of alpha decay is the decay of radium (Ra) to radon (Rn):

$$^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + \alpha \hspace{1cm} (E = 4.78 \text{ MeV}; \text{half life 1620 years}).$$

We need to explain why this alpha decay process takes so long. An alpha particle, being positively charged, is repelled by the product nucleus (the radon nucleus in the example above) and one might expect that this would speed the decay process rather than inhibiting it. In fact it is just the opposite.

To understand the energetics of alpha particle decay, it is convenient to think of the alpha particle trapped inside the product nucleus and trying to get out. Figure 3.4 shows the potential energy of the system. (Review question 28 in Forces and Energy covers a similar situation except that the potential energy is entirely electrical and the scale size is much larger.) For an uncharged particle like a neutron in the vicinity of a nucleus, the potential energy distribution is shaped like a well. However, an alpha particle, being positively charged also experiences an electrostatic or Coulomb repulsion which is the dominant interaction at larger separations. There is a Coulomb barrier outside the nucleus. The strong nuclear force dominates inside the nucleus.

![Figure 3.4 Energetics of alpha particle decay](image)

The solid curve shows the variation of PE with radius. The horizontal line represents the value of the energy level (PE + KE) for the alpha particle. If this level is less than zero, the alpha particle can never get out. If it is greater than zero but less than the height of the barrier, the alpha has to tunnel through the barrier to get out. Values shown are for the alpha decay of radium-226.

We may also show the alpha particle's total mechanical energy on our diagram (figure 3.4). It is partly kinetic and partly potential (we leave out the rest energy in this argument). The mechanical energy,
should remain constant as the alpha particle leaves the nucleus. It would seem impossible for the alpha particle to leave, for in the region between A and B, it has a potential energy greater than its total energy, i.e. a negative kinetic energy - impossible! Our view changes when the alpha is treated as a wave. This wave proceeds out from the nucleus and whilst most of it is reflected back, a little passes through the Coulomb barrier by a quantum-mechanical process called tunnelling. The uncertainty principle also helps to explain the tunnelling. In the short time interval $\Delta t$ that the alpha particle needs to escape, its energy is unknowable by an amount $\Delta E$ which is large enough to let it over the barrier.

This model of the decay process explains the observations well, in particular the relation between the half life of alpha emitters and the energy release given above. If the energy release is great, the alpha particle does not have to penetrate so far through the barrier (distance AB in figure 3.4) and the difference between the potential energy and total energy is relatively small. Both these things give the alpha particle a better chance of getting out so the decay is quicker for higher energy alpha particles. A change of energy release from 4 MeV to 9 MeV causes the half life to drop from $1.4 \times 10^{10}$ years to 0.3 $\mu$s! This relation between half life and energy release explains why we do not find alpha particles with energies much below 4 MeV, the nuclei live for so long that the alpha emission is extremely rare. Figure 3.5 shows how the half life depends on the energy released.

![Figure 3.5 Variation of half life with alpha particle energy](image)

With an energy of 6 MeV, a typical alpha particle has a range in air of 70 mm. In a denser medium, such as water, the range is reduced (approximately) by the ratio of water's density to air's, about 800 to one. This rather short range is important later on when the biological radiation damage of alpha particles is discussed.

### 3.6 BETA DECAY

Nuclides on either side of the line of beta stability can decay by the emission of beta particles i.e. negative electrons for nuclides on the neutron excess side of the line and positive electrons (positrons) for nuclides on the proton excess side. In each case, the decay brings the nuclides nearer the beta stability line. In the first case we may write, for beta-minus emission:
\[ \frac{A}{Z}X \rightarrow \frac{A}{Z+1}Y + e^- + \nu_e \]

For example:

\[ _{8}^{14}C \rightarrow _{7}^{14}N + e^- + \bar{\nu}_e \quad (E = 0.16 \text{ MeV}; \quad \tau = 5730 \text{ years}). \]

If you like you can put values of \( A \) and \( Z \) on the electron symbol, to show how their values balance:

\[ _{8}^{14}C \rightarrow _{7}^{14}N + 0^+e^- + \bar{\nu}_e \]

In the second case, beta-plus emission, a positron (symbol \( e^+ \)) and a neutrino are emitted:

\[ \frac{A}{Z}X \rightarrow \frac{A}{Z-1}Y + e^+ + \nu_e \]

For example:

\[ _{11}^{22}\text{Na} \rightarrow _{10}^{22}\text{Ne} + e^+ + \nu_e \quad (E = 0.54 \text{ MeV}). \]

(In these schemes the subscript \( e \) on the symbol for the neutrinos is a reminder that they are 'electron neutrinos'.)

Again the mass-energy must quite agree on both sides. In terms of rest masses:

\[ m_X = m_Y + m_e + \frac{E}{c^2}. \]

Note that the neutrino's rest mass is considered to be zero. Here \( E \) is the total kinetic energy released, which is shared among the daughter nucleus, the neutrino and the electron.

In beta decay the initial and final nuclides are isobars; the mass number \( A \) does not change. The neutrinos are rather odd particles. The electron neutrino (subscript \( e \)) is produced in beta-plus decay and its antiparticle the electron antineutrino in beta-minus decay. They have zero (or at least very low) rest mass and no electric charge. These properties they share with the photon. However they are quite different from the photon, notably in their interaction with other particles which is almost non-existent! For example, the nuclear reactions which produce the sun's energy output are believed to produce copious amounts of neutrinos but practically all of them which reach the earth pass right through, unnoticed and undetected.

Alpha decay is a two-body decay and there is a definite sharing of the energy release between the two particles. Most of the energy goes to the alpha particle and little to the recoiling product nucleus. On the other hand beta decay is a three-body process and chance enters into the 'carve up' of the energy and momentum released. There is an infinite number of ways to share the energy and still conserve momentum. As with alpha decay, a small fraction of the energy goes to the product (daughter) nucleus. The rest is shared in a seemingly random way between the neutrino and the electron. In one decay the electron may get most of the kinetic energy; in the next it will be the other way round. The effect is to produce a rather broad beta spectrum which is a plot of the distribution of electron energies, like figure 3.6. The energy is plotted on the horizontal axis and the numbers of cases per energy interval of such energies are plotted on the vertical axis. (You can't plot number of particles directly against energy because that would give an infinite total number of particles.) On a similar scale, the alpha spectrum would be a sharp 'spike'.
Beta decay takes place when a nucleon changes from proton to neutron or from neutron to proton.  

\[ p \rightarrow n + e^+ + \nu_e \]

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

The other nucleons are just 'spectators' - they take no part in the beta decay.  

This spectator idea can be carried at least one step further.  There is convincing evidence that nucleons contain particles called **quarks**. Two **flavours** of quarks are involved: the **up quark** (symbol \( u \)) has electric charge \( \frac{2}{3} \) of a proton's and the **down quark** (symbol \( d \)) has an electric charge \( -\frac{1}{3} \) of a proton's.  

Nucleons are made of three quarks:  

\[ p = uud \quad \text{and} \quad n = udd. \]

In this model beta-plus decay is  

\[ u \rightarrow d + e^+ + \nu_e. \]

and beta-minus decay is  

\[ d \rightarrow u + e^- + \bar{\nu}_e. \]

In both cases there are two spectators: one up quark and one down quark.  

The decay process can be represented in a Feynmann diagram, figure 3.7, which shows the two spectator quarks doing nothing. The \( W^- \) is one of the carrier particles for the weak nuclear force.  

It is the complexity (i.e. the three-body nature) of beta decay which leads to long half lives.  They range from milliseconds to \( 10^{16} \) years and longer. As with alpha decay, one can characterise a beta emitting nuclide by its half life and by its energy release.  

Beta particles can be quite penetrating in matter. As with alpha particles, the range increases with energy and is around 1 mm in aluminium for a 1 MeV beta particle. By the end of its range, a beta has lost nearly all its kinetic energy. A special fate awaits the positron at this point. It finds a negative electron and annihilates with it to produce at least two high energy photons of electromagnetic radiation.
Orbital electron capture

Nuclides which emit beta-plus particles can also 'decay' by capturing one of their inner orbital electrons:

\[ ^{A}_{Z}X^{+} + e^{-} \rightarrow ^{A}_{Z-1}Y + \nu_{e} \cdot \]

For example:

\[ \text{^{7}_{4}Be} + e^{-} \rightarrow \text{^{7}_{3}Li} + \nu_{e} \cdot \]

The mass-energy balance is

\[ m_{X} + m_{e} = m_{Y} + \frac{E_{EC}}{c^{2}} \]

The corresponding equation for beta-plus emission is

\[ m_{X} = m_{Y} + m_{e} + \frac{E_{\beta^{+}}}{c^{2}} \]

Taking the difference between these equations gives

\[ -m_{e} = m_{e} + \frac{E_{\beta^{+}} - E_{EC}}{c^{2}} \cdot \]

i.e.

\[ E_{EC} - E_{\beta^{+}} = 2m_{e}c^{2} , \]

If \( E_{\beta^{+}} < 0 \) then positron emission will not occur. However provided that \( E_{EC} > 0 \) electron capture may still occur. \(^{7}\text{Be}\) is an example with \( E_{EC} = 0.86 \text{ MeV} \).

Electron capture is detected by the observation of an x-ray emission line from the product atom which develops a vacancy in one of its electron shells. This vacancy is 'plugged up' by an electron falling into it from further out in the atom, producing an x-ray photon. (For more information about this process see chapter AN6.)
3-7 GAMMA RAYS

In the process of gamma decay a nucleus in an excited state gets rid of its excess energy by emitting a high-energy photon. It often happens after an alpha or beta decay. An example is the beta decay of caesium-137 which produces an excited barium-137 nucleus. The barium then sheds its excess energy by emitting one gamma-ray photon.

\[ ^{\text{55}}_{\text{137}} \text{Cs} \rightarrow ^{\text{56}}_{\text{137}} \text{Ba}^* + e^- + \bar{\nu}_e \]  
\[ (E = 0.52 \text{ MeV}; \tau = 30.0 \text{ years}) \]

and then

\[ ^{\text{56}}_{\text{137}} \text{Ba}^* \rightarrow ^{\text{56}}_{\text{137}} \text{Ba} + \gamma \]  
\[ (E_\gamma = 662 \text{ keV}; \tau = 2.6 \text{ minutes}). \]

The asterisk (*) on the Ba indicates that this nuclide is in an excited state. Just as atoms have excited states with well-defined energies so do nuclei, but the excitation energies of nuclear states are much larger than those of atoms. Transitions between nuclear energy levels result in the emission of gamma ray photons with well-defined energies. The gamma ray, once emitted, is identical to any other electromagnetic photon of its energy, and is often very penetrating.

Example

Figure 3.8 is an energy level diagram (not to scale) for the decay of the nuclide gold-198. It can decay by emitting beta particles, but there are two possible beta decays, which leave the product mercury 198 nucleus in one of two excited states. The subsequent decay of the excited mercury nucleus to its ground state can produce one of three different gamma-ray photons.

The examples above are illustrations of chain or series decay. More details of that topic are given later in §3-10.

3-8 THE STATISTICS OF RADIOACTIVE DECAY

Radioactive decay is a random process; the probability that any given nucleus will decay in a given time interval is quite independent of external influences and what has already happened. The probability of getting a decay in a given time interval is proportional to the number of unstable nuclei present. Starting with a reasonably large number \( N_0 \) of nuclei at some instant of time, which we call \( t = 0 \), the number of nuclei left drops off exponentially with time:

\[ N = N_0 e^{-\lambda t} \]  
... (3.2)

where \( \lambda \) is called the decay constant. Its unit is the reciprocal second, s\(^{-1}\). (A formal derivation of this result can be found in appendix 2.)

It is worth thinking about this law for a few minutes. It does not matter when \( t = 0 \) is. If \( N_0 \) represents the number of nuclei at that time then the law holds.

But the law is not exactly correct! It approaches correctness when \( N \) is very large. To see what that means imagine the last ten nuclei decaying. There are 10, now there are 9, then 8... Finally there is one and it decays too. It is clear that an exponential law which describes a continuous variable cannot apply exactly to a variable which is a discrete number so the law is not being followed at this late stage of the decay.

The rate of decay also changes exponentially, and goes down with the same decay constant.
\[
\frac{dN}{dt} = -\lambda N e^{-\lambda t} = -\lambda N .
\]

Note the minus sign here, which simply indicates that the number of nuclei left is decreasing. To get the rate, \( R \), at which disintegrations occur, called the activity, take the negative of the rate of change of \( N \):

\[
R = -\frac{dN}{dt} = \lambda N \quad \text{... (3.3).}
\]

The activity itself decays exponentially:

\[
R = R_0 e^{-\lambda t} .
\]

It is this decay rate which is often measured with such particle detectors as Geiger counters. The unit for activity is the reciprocal second (s\(^{-1}\)) but, in honour of the discoverer of radioactivity, it is called the becquerel (symbol Bq).

\( 1/\lambda \) is the time required for both the number of nuclei \textit{and} the decay rate to drop by a factor of e\(^{-1} = 0.3679 \). The half life is more commonly used than the decay constant. It is the time (\( \tau \)) required for the activity to drop to half its previous value. The easiest way to write the decay law using the half life is like this:

\[
N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{\tau}} \quad \text{... (3.4)}
\]

and

\[
R = R_0 \left(\frac{1}{2}\right)^{\frac{t}{\tau}}
\]

A little bit of mathematical reasoning can be used to get the relation between half life and decay constant:

\[
\lambda \tau = \ln(2) \quad \text{... (3.5)}
\]

or

\[
\tau = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda} .
\]

The half-life is useful for calculations. Suppose that we start with \( N_0 \) radioactive nuclei. By the definition, after one half life has elapsed only \( \frac{1}{2} \) \( N_0 \) radioactive nuclei remain. After two half lives only \( \frac{1}{4} \) \( N_0 \) remain. After \( n \) half lives the number of radioactive nuclei (and the activity of the sample) has decreased to \( \frac{1}{2^n} \) of its original value. Using this logic we can form a quick estimate of the number of nuclei remaining (or the activity) without resorting to more complex calculations.

**Example**

Given that the half life of the beta emitter iodine-131 is 8.05 days you can easily calculate how much is left at any time. For example, after 30.0 days the fraction left is

\[
\left(\frac{1}{2}\right)^{\frac{30.0 \text{ d}}{8.05 \text{ d}}} = \frac{1}{2^{3.72}} = 0.075.
\]

You can do this calculation easily using the \( y^x \) function of your calculator.

The activity of a sample of radioactive material depends on the size of the sample; the more nuclei you have the more disintegrations you will get. One way of describing activity in a way that does not depend on how much material you have is to specify the activity per mass of the sample, which is called the specific activity.

**Example**

1.00 kg of 1.00% uranium ore contains 10.0 g of \(^{238}\text{U} \). Given that the half life is \( 4.5 \times 10^9 \) years, we can calculate the disintegration rate.
The number of uranium atoms present can be estimated by making the approximation that the mass of an atom is its mass number \((A)\) times 1 atomic mass unit \((u)\). So the number of atoms is

\[
N \approx \frac{\text{mass of uranium}}{238 \times \text{mass of } 1u} \approx \frac{0.0100 \text{ kg}}{238 \times 1.66 \times 10^{-27} \text{ kg}} = 2.53 \times 10^{22}.
\]

Now the activity can be found.

\[
R = \lambda N = \frac{\ln(2)}{\tau} \times 2.53 \times 10^{22}
\]

\[
= \frac{\ln(2) \times 2.53 \times 10^{22}}{4.5 \times 10^9 \times 365 \times 24 \times 3600 \text{ s}} = 1.2 \times 10^5 \text{ Bq}.
\]

In this case the specific activity of the ore is \(1.2 \times 10^5 \text{ Bq.kg}^{-1}\).

### 3-9 RADIOACTIVE DATING

This is done with various radionuclides but carbon-14 stands out as a radioactive clock because it is used to date archaeological artefacts.

The radionuclide \(^{14}\text{C}\) is continually made in the upper atmosphere by the **nuclear reaction**:

\[
_{7}^{14}\text{N} + n \rightarrow _{6}^{14}\text{C} + p.
\]

In this reaction \(^{14}\text{N}\) is the most abundant isotope of nitrogen and nitrogen is the most abundant element in air; \(n\) is a neutron, one of the secondary particles formed when cosmic rays from outer space collide with the nuclei of air atoms and \(p\) is a proton leaving the reaction. As described above, \(^{14}\text{C}\) is beta-active with a 5730 year half life. This gives it plenty of time to mix with the more abundant stable carbon isotopes in the biosphere before decaying appreciably. The mixing ratio is \(1 : 8.0 \times 10^{11}\), atom for atom. When an organism dies (and no \(^{14}\text{C}\) goes to or from its environment after death) then the level of carbon-14 decays.

**Example**

We can use the half life to calculate the specific activity of a sample of pure carbon. The number of atoms in a sample of carbon with mass \(m\) is \(m/m_C\) where \(m_C\) is the average mass of a carbon atom. If the fraction of carbon-14 atoms is \(f\) then the number of carbon-14 atoms in the sample is

\[
N = \frac{f m}{m_C}.
\]

The disintegration rate is

\[
R = \lambda N = \frac{\ln(2) N}{\tau}
\]

so

\[
R = \frac{\ln(2) f m}{m_C \tau}
\]

and the specific activity is

\[
R/m = \frac{\ln(2) f}{m_C \tau}.
\]
Substituting values, we have:

\[ f \approx \frac{1}{8.0 \times 10^{11}}, \]
\[ m_C \approx 12 \text{ u} = 12 \times 1.67 \times 10^{-27} \text{ kg}, \]
\[ \tau = 5730 \text{ y}. \]

This gives:

\[ \frac{R}{m} \approx \ln(2) \frac{1}{8.0 \times 10^{11} \times 12 \times 1.67 \times 10^{-27} \text{ kg} \times (5730 \times 365 \times 24 \times 3600 \text{ s})} \approx 0.24 \text{ kBq.kg}^{-1}. \]

**Figure 3.9  Radiocarbon dating**

**Example**

A wooden artefact, recovered from a tomb, has a carbon-14 activity of 102 Bq.kg\(^{-1}\). Assuming that the rate of carbon-14 production when the wood was alive was the same as it is now and that no carbon-14 was interchanged with the environment, we can estimate the artefact’s age.

We solve:

\[ 102 \text{ Bq.kg}^{-1} = (0.24 \times 10^3 \text{ Bq.kg}^{-1}) e^{-\lambda t}. \]

Taking logarithms of both sides and rearranging we obtain \( \lambda t = 0.856 \) and

\[ t = 0.856 \times \frac{5730 \text{ y}}{\ln(2)} = 7 \times 10^3 \text{ y}. \]

The assumption that carbon-14 is produced at a constant rate has been proved wrong in a series of ages done on wood from long-living trees. The wood’s age can be obtained by counting the annual growth rings. Seasonal variations in rings allow the age of a ring in one tree to be correlated with older or younger trees nearby. This allows the counting process to cover an interval from the present, right back much further than the life-span of any one tree. The small correction derived from this study is used to produce more accurate ages by the carbon-14 method.
3-10 RADIOACTIVE SERIES OR CHAIN DECAYS

In some cases a radionuclide decays to a daughter nuclide which is also radioactive. One example has been given already.

\[
\frac{137}{55}\text{Cs} \rightarrow \frac{137}{56}\text{Ba}^* + e^- + \nu_e \quad (E = 0.52 \text{ MeV}; \, \tau = 30.0 \text{ years})
\]

and then

\[
\frac{137}{56}\text{Ba}^* \rightarrow \frac{137}{56}\text{Ba} + \gamma \quad (E_\gamma = 662 \text{ keV}; \, \tau = 2.6 \text{ minutes}).
\]

In this example, the half life of the daughter \(\frac{137}{56}\text{Ba}^*\) is much shorter than that of its parent, a common situation.

Another example is the alpha decay of radium to radon (a gas) followed by the comparatively rapid gamma and alpha decay of the radon.

\[
\frac{226}{88}\text{Ra} \rightarrow \frac{222}{86}\text{Rn} + \alpha + 4.78 \text{ MeV} \quad (1620 \text{ years}).
\]

\[
\frac{222}{86}\text{Rn} \rightarrow \frac{218}{84}\text{Po} + \alpha + 5.49 \text{ MeV} \quad (3.8 \text{ days}).
\]

This chain continues further, through different paths, eventually ending up at the stable isotope of lead, \(\frac{206}{82}\text{Pb}\).

A complete sequence of chain decays is shown in figure 3.10.

As a radionuclide decays, the number of product nuclei builds up. If we start with \(N_0\) nuclei of A and the product nuclide B is stable the number of nuclei of B builds up and approaches the value exponentially \(N_0\). However if the product B is also radioactive, it will also decay exponentially. What happens depends on the relative values of the half-lives of the members of the chain. Figure 3.11 shows a hypothetical example of a nuclide A which produces a daughter B whose half is about five times as long as A's. The product of B's decay, C, is stable.
Most naturally occurring radionuclides have very long half lives - otherwise they would not still be around. So we consider the case of a parent nuclide, the first member of a series which has a long half-life. On a human time scale its activity, $R_p$, which is equal to the rate of production of its daughter, is practically constant. Then the total rate of change of the number of daughter nuclei is equal to its rate of production minus the rate of decay, $R_D$:

$$\frac{dN_D}{dt} = R_D + R_p = -\lambda_D N_D + R_p.$$ 

We will now take the case where at $t = 0$ there is no daughter; this could happen if a chemical separation had removed all the daughter from the parent at $t = 0$. You can check by back substitution that in this case the solution to the equation is

$$N_D = \frac{R_p}{\lambda_D} \left( 1 - e^{-\lambda_D t} \right).$$ 

Hence the activity of the daughter (plotted in figure 3.12) is:

$$R_D = \lambda_D N_D = R_p \left( 1 - e^{-\lambda_D t} \right).$$

Figure 3.11  A simple decay chain
The half life of the intermediate nuclide B is 5 times the half life of A. Nuclide C is stable.
After a time long compared with $1/\lambda$ (say $10/\lambda$), the decay rate of the daughter becomes constant (like its parent). If the series or chain has several members and if we wait for a time long compared with the longest half life in that chain (parent excluded), then all the decay rates will be the same. Each member in the series decays at the rate it is formed. This is called secular equilibrium and is found for example in ores containing $^{232}_{90}$Th and $^{238}_{92}$U if no daughters escape.

3-11 NUCLEAR REACTIONS

A nuclear reaction is similar in a number of ways to a chemical reaction. In both cases, one often needs to 'push' the reacting atoms together. However, in a nuclear reaction, the orbiting electrons are usually of little importance.

**Fusion**

In a nuclear fusion reaction two nuclei combine to form one nucleus, with the release of energy. An example is the fusion of two isotopes of hydrogen, tritium (hydrogen-3) and deuterium (hydrogen-2), to form helium-4 and a free neutron:

$$\frac{2}{1}H + \frac{2}{1}H \rightarrow \frac{4}{2}He + \frac{1}{0}n + 17.6 \text{ MeV}.$$  

Another example is shown in figure 3.13.

![Figure 3.13  An example of fusion](image)

The act of pushing two nuclei into contact is resisted by the Coulomb repulsion between the nuclear charges (both are positive). It is this repulsion that inhibits the easy extraction of energy from fusion reactions.

Recall the graph (figure 3.2) giving the dependence of the binding energy per nucleon, $E_B/A$, on the mass number, $A$. It has a maximum at around $Z = 26$. Its low value for smaller $Z$ shows that energy can be extracted from the process of combining two smaller nuclei into a larger one. This process is the grand provider of energy in the universe. Most stars use it in part of their lives. A great deal of effort is being spent in laboratories all over the world on the attempt to emulate this, first in a laboratory, then in a power station. The kinetic energy of the products of this fusion reaction will be transformed to heat, which will boil water and the steam will drive turbo-generators producing electricity.

**Fission**

At the other end of the periodic table we have nuclides like $^{235}_{92}$U. We can use them to produce energy by fission, which is the breaking up of a nucleus into two or more smaller nuclei. This is easy to initiate with neutrons, which, being uncharged, can enter nuclei of any size without being repelled by the Coulomb force. The entering neutron excites the uranium nucleus and it splits into two fission fragments which move apart with some considerable speed. Their energy is eventually converted to heat. Each fission also gives out neutrons, on the average 2.5 of them. These neutrons can induce fission in more nuclei. The result is a chain reaction.
An uncontrolled chain reaction can occur if enough uranium-235 is confined in a lump which is large enough to confine plenty of neutrons. Suddenly making a single lump larger than a critical mass produces the kind of explosion that was used in the first "atomic" bombs. In a fission reactor the chain reaction has to be controlled by soaking up excess neutrons in materials called moderators. The energy produced can be used to generate steam which turns the turbines in a power station.

Because a uranium nucleus has more neutrons than protons the fission fragments lie on the neutron excess side of the line of beta stability. Their beta activity is a serious problem, both in the reactor and after the spent uranium fuel is removed. Heavy shielding is needed to protect people from the intense radiation. In addition, some of the $^{235}$U nuclei in the reactor absorb neutrons without breaking up and transuranic nuclei are produced. Some of these nuclei have very long half lives and, like the long lived fission fragments, they pose a serious problem for their long term safe disposal.

**FURTHER READING**
Q3.1 The half life of technetium-99 is 6 hours. What fraction of a sample is left after (a) 6 hours, (b) 2 days, (c) 20 hours?

Q3.2 A sample of wood from the funeral boat of the Egyptian King Sesosrius III has a carbon activity of 0.15 kBq kg\(^{-1}\). Contemporary material has an activity of 0.237 kBq kg\(^{-1}\). When was the tree growing?

Q3.3 The unstable nuclide \(^{213}_{84}\text{Po}\) decays through an intermediate nuclide \(X\) to the stable nuclide \(^{209}_{83}\text{Bi}\). Write down a possible scheme for each of the two decays. Define any symbols you use. What are the atomic number and the mass number of \(X\)?

Q3.4 The iron compression ring for the piston of a car engine has a mass of 0.030 kg. The ring is radiated with neutrons until it has an activity of \(4.0 \times 10^5\) Bq, due to the formation of \(^{59}\text{Fe}\) (half life 50 days). It is immediately installed in a test engine.

(a) What is the activity of the 0.030 kg of iron after the 150 days?

(b) After being used for 150 days, the crankcase oil has an activity of 200 Bq, due to iron worn off the ring. What was the mass of iron worn off the ring during the 150 days of use?

Q3.5 A neutron star is a lump of nuclear matter. It is believed that pulsars are neutron stars. They emit radio waves and other electromagnetic radiation with quantum energies as high as \(10^{15}\) eV. Consider a neutron star with a mass twice that of the sun. The sun's mass is \(2 \times 10^{30}\) kg. Find the radius of the neutron star.

Q3.6 The curie (symbol Ci) is an obsolete unit of activity, which could be defined as the activity of exactly one gram of pure radium-226, which has a half life of 1620 years. Use this information to calculate the conversion factor from curies to becquerels.

Q3.7 Carbon-14 decays to nitrogen-14 by the scheme:

\[ ^{14}\text{C} \rightarrow ^{14}\text{N} + e^- + \bar{\nu}_e \]

The mass of a carbon-14 atom is 14.003 242 u and that of nitrogen-14 is 14.003 074 u. What is the maximum kinetic energy of the electron? (Hint: to calculate the maximum KE ignore the recoil energy of the nucleus.)

Q3.8 1.00 g of pure radium chloride \((^{226}\text{RaCl}_2)\) was dissolved in 1.00 L of water and held in a sealed round bottomed flask. 24 hours later a tube was introduced below the liquid surface and an inert gas (helium) was used to extract \(^{222}\text{Rn}\) from the solution. 90% of this radon nuclide was extracted. What was the activity of the radon collected?

Data: half life of \(^{226}\text{Ra}\) = 1620 years; half life of \(^{222}\text{Rn}\) = 3.83 days.

The stable isotopes of chlorine are:

\(^{35}\text{Cl}\), 76% abundance, mass 34.968 851 u
\(^{37}\text{Cl}\), 24% abundance, mass 36.965 898 u

Q3.9 If the Earth (mass \(6 \times 10^{24}\) kg) could be squeezed so that its density were equal to that of nuclear matter, what would its diameter be?

Discussion questions

Q3.10 A free neutron decays into a proton and an electron. Why do we say that the nucleus contains protons and neutrons rather than protons and electrons?

Q3.11 Isotopes of the same element have the same chemical properties. How can they be separated?

Q3.12 Radium has a half life of about 1600 years. It is thought that the Universe has existed for about 5 gigayears. How come there is still plenty of radium about?

Q3.13 Why are fission fragments environmentally unfriendly?

Q3.14 All nuclei contain integer numbers of protons and neutrons. Why can't the atomic masses of the elements be expressed as integers?

Q3.15 It is possible to manufacture a great variety of radioisotopes using, for example, nuclear reactors. Why are these isotopes rare in the natural world?

Q3.16 Do fission fragments decay by beta plus or beta minus decay? Why?

Q3.17 Does it make sense to define half life for a process which is not exponential? Does it matter when you start counting the time? Why "half"? Why not third life or quarter life or 10% life?
AN4

INTERACTION OF RADIATION WITH MATTER

OBJECTIVES

Aims
From this chapter you should develop your understanding of the various ways that photons, charged particles and neutrons can interact with matter and the concepts, such as mass attenuation coefficient, stopping power and range, that have been invented in order to aid that understanding. These ideas are the basis for the later study of the effects of x rays, gamma radiation and other ionising radiations on living things.

Minimum learning goals
When you have finished studying this chapter you should be able to do all of the following.

1. Explain, use and interpret the terms
   linear attenuation coefficient, mass attenuation coefficient, attenuation length, mean free path, half thickness, density-thickness, build-up, secondary particles, photoelectric effect, atomic photoelectric effect, photoelectrons, photoionisation, absorption edges, Compton effect [Compton scattering], Compton edge, pair production, rate of energy loss, linear stopping power, mass stopping power, minimum ionisation, range, straggling, bremsstrahlung [braking radiation], Cherenkov radiation, elastic scattering.

2. State, explain and apply the exponential attenuation law for a beam of particles suffering all-or-nothing interactions (equation 4.1).

3. State, explain and apply the relations among linear attenuation coefficient, mass attenuation coefficient, density and composition of materials (equations 4.1, 4.2, 4.3, 4.4).

4. Distinguish between absorption of energy and the attenuation of a beam of photons and describe the build-up of secondary particles.

5. (a) Describe and compare the processes (photoelectric effect, Compton effect, Rayleigh scattering, pair production) by which photons interact with matter.
   (b) Describe and explain in general terms how attenuation coefficients and the relative importance of those processes vary with photon energy and explain the origin of absorption edges.
   (c) Explain why pair production occurs only for photons with sufficiently high energy.

6. (a) Describe the processes (excitation and ionisation, bremsstrahlung and Cherenkov radiation) by which a beam of energetic charged particles loses energy as it passes through bulk matter.
   (b) Describe and explain how the speeds, charges, kinetic energies and masses of particles affect their interactions with an absorber.

7. Describe how linear stopping power and mass stopping power depend on particle energy and the composition of the absorber.

8. Describe and explain how the number of particles in a monoenergetic beam of charged particles varies with depth of penetration into an absorber.

9. Describe some examples of how neutrons interact with bulk matter.

TEXT

4-0 INTRODUCTION
This chapter is all about the interactions between radiation and matter. Here radiation is defined broadly to include charged particles such as alpha and beta radiation, beams of charged particles created by
accelerating machines as well as electromagnetic radiation or photons. We even include beams of neutral particles such as neutrons. There are two kinds of interesting questions. Firstly, we can ask questions about what happens to the radiation, how far it travels and how its strength is affected. It will be necessary to define carefully what we mean by strength. The second group of questions is concerned with what the radiation does to the matter; some of those questions will be considered again in chapter AN5.

When a beam of radiation of any kind penetrates matter some of the radiation may be absorbed completely, some may be scattered and some may pass straight through without any interaction at all. The processes of absorption and scattering can be described and explained in terms of interactions between particles. Particles in the beam of radiation strike particles in the material and are either stopped or scattered.

There are two broad kinds of process by which a particle travelling through matter can lose energy. In the first kind the energy loss is gradual; the particle loses energy nearly continuously through many interactions with the surrounding material. In the second kind the energy loss is catastrophic; the particle moves without any interaction at all through the material until, in a single collision, it loses all its energy. The motion of charged particles through matter is characterised by gradual energy loss whereas photon interactions are of the "all-or-nothing" type. In this chapter we will start by considering the interaction of photons with matter and then proceed to look at the absorption of material particles.

4.1 ATTENUATION COEFFICIENTS

If the interactions are of the "all-or nothing" type then the attenuation of a beam of particles with identical energies, all travelling in the same direction, is described by an exponential law. If at some distance into the material \( N_0 \) particles are moving through a slab of material, then after penetrating an extra distance \( x \) it is found that the number of particles in the beam is reduced to

\[
N(x) = N_0 e^{-\mu_l x}
\]

... (4.1).

This exponential attenuation law follows from the fact that, over any short distance, the probability of losing a particle from the beam is proportional to the number of particles left. Where there are many particles many will be lost, but as the number left decreases so does the rate of loss.

![Figure 4.1 Attenuation of a photon beam - schematic](image)

The quantity \( \mu_l \) is known as the **linear attenuation coefficient**; it is a measure of how rapidly the original photons are removed from the beam. A large value of \( \mu_l \) means that the original photons are removed after travelling only a small distance. It is important to remember that the exponential attenuation law does not describe what happens to the energy carried by the photons removed from the beam - it is possible that some of that energy may be carried through the medium by other particles, including some new photons.

**Example**
The linear attenuation coefficient for 200 keV x-rays in lead is $1.0 \times 10^3 \text{ m}^{-1}$. What is the fraction of such photons remaining after penetrating a lead sheet of thickness 2.0 mm?

**Answer**

$$N = N_0 e^{-\mu l x}$$

so the fraction left is

$$\frac{N}{N_0} = e^{-\mu l x} = e^{-\left(1.0 \times 10^3 \text{ m}^{-1}\right) \times \left(2.0 \times 10^{-3} \text{ m}\right)} = e^{-2.0} = 0.135.$$

An alternative way of expressing the exponential attenuation law is to replace the linear attenuation coefficient by its reciprocal:

$$\lambda = \frac{1}{\mu}$$

so that

$$N(x) = N_0 e^{-x/\lambda}$$

where $\lambda$ is known as the **attenuation length** or **mean free path**. It is the average distance travelled by a photon before it is absorbed. The distance over which one half the initial beam is absorbed is called the **half thickness** $x_{1/2}$. It is related to the linear attenuation coefficient and the mean free path by

$$x_{1/2} = \ln(2) \sqrt{\frac{\mu l}{2}} = \ln(2) \lambda = 0.693 \lambda.$$

... (4.2)

Since photons interact with individual atoms, the probability that a photon will interact somewhere within a slab of matter depends on the total number of atoms ahead of it along its path. So the attenuation of radiation depends on the amount of material in the beam's path and not on how it is distributed. It is useful, therefore, to describe the attenuation process in a way that does not depend on the density of material, only on what kind of stuff it is. We can achieve that by defining the **mass attenuation coefficient** $\mu_m$ which is related to the linear attenuation coefficient and the mean free path by

$$\mu_l = \mu_m \rho$$

where $\rho$ is the density of the material. This means that the mass attenuation coefficient is the same for ice, liquid water and steam whereas the linear attenuation coefficients will differ greatly. The total attenuating effect of a slab of given type material can be described by quoting the mass attenuation coefficient, which is characteristic of the material's chemical composition and the photon energy, together with the material's density and its thickness. We now have yet another way of writing the attenuation law:

$$N = N_0 e^{-\mu l x} = N_0 e^{-\mu_m \rho x}.$$  

The product $\rho x$, the **areal density** (mass per area) of a thickness $x$ of the attenuating material, is also called the **density-thickness**. It is often quoted instead of the geometrical thickness $x$. Although the SI unit of density-thickness is kg.m$^{-2}$, the obsolete unit g.cm$^{-2}$ is still used in the literature.
If an absorber is made of a composite of materials the mass attenuation coefficient is readily calculated by adding together the products of the mass attenuation coefficient and the proportion (α) of the mass due to each element) for all the elements present in the material, i.e.

\[ \mu_{m\ [\text{total}]} = \sum (\alpha \mu_{m\ }) . \quad \ldots (4.4) \]

The exponential law will always describe the attenuation of the original radiation by matter. If the radiation is changed, degraded in energy (and not totally absorbed) or if secondary particles are produced then the effective attenuation is less so the radiation will penetrate more deeply into matter than is predicted by the exponential law alone. Indeed it is possible to get an increase in the number of particles with depth in the material. The process is called build-up and has to be taken into account when evaluating the effect of radiation shielding. Examples will be given after we examine the various interaction mechanisms that radiation has with matter.

**Figure 4.3  Build-up of secondary particles**

The primary particles are attenuated with \( \mu_1 = 0.010 \ \text{mm}^{-1} \). Secondary particles are produced in 75% of interactions. The secondary particles have an attenuation coefficient of 0.005 mm\(^{-1}\) and when they interact they are completely absorbed, i.e. there are no tertiary particles produced.
4-2 PHOTONS

Gamma rays, x rays and light are photons with different energies. Recall that the energy of a photon is determined by its frequency ($f$) or wavelength ($\lambda$) and is given by

$$E = hf = \frac{hc}{\lambda}$$

where $h$ is Planck's constant and $c$ is the speed of light in vacuum. The factor $hc$ occurs so often that it is worthwhile listing and using the value of the product as though it were a separate constant. In terms of units commonly used in atomic and nuclear physics:

$$hc = 1.24 \text{ eV} \cdot \mu \text{m} = 1240 \text{ MeV} \cdot \text{fm}.$$  

Photons interact with matter in a variety of ways, depending on their energy and the nature of the material. The three interaction mechanisms are the photoelectric effect (or photoelectric absorption), Compton scattering and pair production. These will be considered in turn.

4-3 PHOTOELECTRIC EFFECT

In the photoelectric effect, a photon interacts either with an electron which is bound in an individual atom or with an electron in condensed matter, usually a solid, which is not bound to an individual atom but may be shared among many atoms.

**Interaction of photons with atoms**

In chapter AN2 we discussed how photons can interact with an atom either by ionising it (ejecting an electron) or by exciting it (leaving all the electrons inside the atom). The atomic photoelectric effect involves the absorption of a photon by an atomic electron which is then ejected from the atom. This can occur only when the incoming photon has an energy greater than the ionisation energy ($E_B$) of the electron to be removed. Since an atom is much more massive than an electron the ejected electron takes practically all the energy and momentum of the photon. The kinetic energy ($K$) of the ejected electron is then $K = hf - E_B$. The ejected electrons are known as photoelectrons and, since the atom is ionised, the process is one form of photoionisation.

![Figure 4.4 Photoelectric absorption](image)

A photon loses all its energy to an atomic electron which is raised to a higher energy level. Note that this is a schematic diagram; it is not possible to locate individual electrons in precise orbits within an atom.

The mass attenuation coefficient for photoelectric absorption decreases with increasing photon energy; i.e. as a general rule high energy photons are more penetrating than low energy radiation. For a fixed value of the energy the attenuation coefficient increases with the atomic number $Z$ of the substance. The relationship is described by the empirical relation:
\[
\mu_m(\text{pe}) \propto E_\gamma^{-3.5} Z^{4.5}
\]

On a graph (figure 4.5) showing how attenuation varies with photon energy, the general decrease of \(\mu_m\) with increasing energy is interrupted by a series of jumps, called absorption edges, which occur at a unique set of energies for each element. An absorption edge occurs when the photon is energetic enough to eject an electron from one of the deep energy levels in the atom (figure 4.4). If a photon's energy is even just a little bit less than that required to raise the tightly bound electron to a vacant energy level, then that photon cannot interact with the inner electron. On the other hand, if the photon's energy were to be made equal to or greater than the required energy, the interaction suddenly becomes possible. So there are sudden changes in interaction probabilities and attenuation coefficients at photon energies corresponding to quantum jumps between filled and unfilled energy levels of atoms in the absorber. The edge with the highest photon energy corresponds to an interaction with an electron in the K-shell. The next set of edges correspond to the group of closely spaced energy levels in the L-shell. The energies of the absorption edges are characteristic of each element and, like spectra, they can be used to identify elements.

![Figure 4.5 Photoelectric mass attenuation coefficient](image)

The graph show how the photoelectric mass attenuation coefficient varies with photon energy for water, argon and lead. The near-vertical lines, called absorption edges, represent the possibility of interactions with inner-shell electrons.

**Interaction of photons with condensed matter**

In many solids the valence electrons are not attached to individual atoms but are shared by all the atoms. As in the case of isolated atoms, photons can either remove electrons from the material or excite them to higher energy levels.

In a metal the energy required to remove the conduction electrons from the material (typically 1 to 3 eV) is less than that required to ionise an isolated atom, so an electron can be removed from a piece of metal by ultraviolet light, for example. This phenomenon is also known as the photoelectric effect. It is the mechanism used to produce an electric current when light enters a photomultiplier (see chapter AN5).
On the other hand most of the electrons in insulators and semiconductors are more tightly bound than the conduction electrons in metals. If the energy of an incident photon in insufficient to release an electron from the surface of the material it may nevertheless be enough to raise the electron to a higher energy level in the material. This process will allow the electron to move and so it increases the conductivity of the material. This effect, known as **photoconductivity**, is exploited in light detecting devices such as light-dependent resistors and photodiodes.
4.4 COMPTON EFFECT

In the Compton effect, the photon scatters from a free electron or a loosely bound atomic electron. The scattered photon has less energy than the incident photon and the excess energy is transferred to the electron. The laws of conservation of mass-energy and conservation of momentum limit the maximum kinetic energy of the photoelectron to a value,

\[ K_{\text{max}} = hf \frac{2g}{1 + 2g}, \]

where

\[ g = \frac{hf}{m_e c^2}. \]

The factor \( g \) is the ratio of the photon's energy (\( hf \)) to the rest energy (\( m_e c^2 \)) of the charged particle doing the scattering. In the case of maximum energy transfer the electron travels forward and the photon is scattered to travel back along the original photon direction. For large photon energies (more than about 5 MeV) all but about 0.25 MeV goes to the scattered electron, an important feature if we are detecting gamma rays.

![Figure 4.9 Compton scattering by a weakly bound electron](image)

The scattered photon has less energy than the incident photon.

The energy distribution of the scattered electrons is shown in figure 4.10. Most of the electrons have energies close to \( K_{\text{max}} \). The sharp decrease at \( K_{\text{max}} \) is known as the Compton edge.

![Figure 4.10 Energy distribution of Compton-scattered photons](image)

Values marked beside the curves are energy of the incident photon.
The mass attenuation coefficient for Compton scattering, $\mu_m^C$, varies as shown in figure 4.11. Both the maximum value of $\mu_m^C$ and the energy at which it occurs increase slowly with increasing atomic number.

4.5 RAYLEIGH SCATTERING

A photon can also scatter from an atom as a whole, neither exciting nor ionising the atom. When that happens the atom will recoil taking some of the energy and leaving the scattered photon with a slightly smaller energy. This process, called Rayleigh scattering, is the main scattering process for very low photon energies, but it is still much less probable than the photoelectric effect. Rayleigh scattering is important for visible light and is responsible for the blueness of the sky (see Light., §2-2.) The mass attenuation coefficient for Rayleigh scattering, $\mu_m^R$, is also shown in figure 4.11.

4-6 PAIR PRODUCTION

In pair production a high energy photon is transformed into an electron-positron pair:

$$\gamma \rightarrow e^+ + e^-.$$

This process cannot happen in free space - it needs the presence of a third body, usually a nucleus, to simultaneously conserve energy and momentum. Since the rest energy of an electron is 0.51 MeV, pair production is energetically impossible for photon energies less than 1.02 MeV. However when pair production becomes possible it soon becomes the dominant interaction process for beams of very high energy photons. The attenuation coefficient for pair production $\mu_m^{pp}$, shown in figure 4.13, varies with photon energy and the atomic number of the absorbing material approximately as
\[ \mu_m[\text{pp}] \propto \rho Z^2 \log \left( \frac{E_\gamma}{E_{\text{ref}}} \right) \]

where \( E_{\text{ref}} \) is some arbitrary reference energy.

**Figure 4.13** Mass attenuation coefficient for pair production

### 4.7 TOTAL ATTENUATION COEFFICIENT FOR PHOTONS

The relative importance of the three main interaction mechanisms depends on the energy of the incident photon and the nature of the absorbing material. In general terms the photoelectric effect is most important at low energies. At higher energies Compton scattering becomes the main energy loss mechanism. At still higher photon energies Compton scattering is less effective. Above 1.1 MeV pair production becomes possible and is the dominant effect for energies greater than a few MeV. We can specify the energies at which relative importance of the effects changes by defining the energy at which they have the same value of the mass attenuation coefficient, \( \mu_m \). Let \( E_{(\text{pe,C})} \) be the energy at which an incident photon loses energy at the same rate by both the photoelectric (pe) and Compton (C) effects. Its value is given by the point at which the photoelectric and Compton curves cross on the graph of attenuation coefficient and energy (top graph of figure 4.14). Similarly \( E_{(\text{C,pp})} \) is the energy at which an incident photon loses energy at the same rate by both Compton scattering and by pair production (pp).

The values of these changeover energies depend on the nature of the material; both \( E_{(\text{pe,C})} \) and \( E_{(\text{C,pp})} \) vary with the atomic number, \( Z \), of the absorbing material, as shown in figure 4.15.

The total mass attenuation coefficient is just the sum of all the contributions from the different effects:

\[ \mu_m[\text{tot}] = \mu_m[\text{pe}] + \mu_m[C] + \mu_m[\text{pp}] . \]

This total mass attenuation coefficient describes the decrease of the original incident radiation. The total linear attenuation coefficient is given by

\[ \mu_l[\text{tot}] = \mu_m[\text{tot}] \rho . \]

Compton scattering produces a photon of reduced energy which nevertheless may interact again. Similarly the electrons and positrons produced in pair production may have large energies which they carry with them deeper into the material. Hence the exponential decrease law using the mass attenuation coefficient underestimates the total amount of radiation energy penetrating materials.
Both the photoelectric effect and Compton scattering can leave ionised atoms behind in the material. The effects of these ions may have important consequences, especially in biological materials. That topic will be considered in chapter AN5.
4-8 INTERACTIONS OF CHARGED PARTICLES WITH MATTER

Charged particles, such as electrons, protons and alpha particles, interact with matter electromagnetically or through one of the two kinds of nuclear interactions, the weak interaction or the strong interaction. The electromagnetic interaction involving collisions with electrons in the absorbing material is by far the most common. Neutral particles such as the neutron can interact only through the nuclear interactions. Thus charged particles can be detected directly by their electromagnetic interactions whereas neutral particles have to suffer nuclear interactions which produce charged particles before their presence can be detected.

Electromagnetic interactions

The electromagnetic interaction consists mainly of two mechanisms: (a) excitation and ionisation of atoms, and (b) bremsstrahlung, the emission of electromagnetic radiation (photons) when a charged particle is severely accelerated, usually by interaction with a nucleus. A third kind of interaction, producing Cherenkov radiation, while playing an important role in the detection of very high energy charged particles, absorbs only a small amount of energy. The contribution of each mechanism depends on the charge, mass and speed of the incident particle as well as the atomic numbers of the elements which make up the absorbing material.

Individual interactions - scattering

Unlike photons, each charged particle suffers many interactions along its path before it finally comes to rest, but only a small fraction of its energy is lost at each interaction. For example, a typical alpha particle might make fifty thousand collisions before it stops. Hence the energy loss can usually be considered as a continuous process. Charged particles are deflected or scattered at each interaction. Although the amount of scattering at each collision may be small, the cumulative effect may be quite a large change in the direction of travel. Occasionally an incident particle will pass very near a nucleus and then there will be a single large deflection. This nuclear scattering effect is most pronounced for light incident particles interacting with heavy target nuclei.

4-9 STOPPING POWER

There are several ways of describing the net effects of charged-particle interactions, the rate of energy loss along the particle's path, \( \frac{dE}{dx} \), being most important. Here \( E \) is the particle's energy and \( x \) is the distance travelled. This rate of energy loss with distance travelled depends on the material and is called the linear stopping power, \( S_l \), of the material:

\[
S_l = -\frac{dE}{dx}
\]

A common unit for linear stopping power is MeV.m\(^{-1}\). In general the stopping power will vary as the particle loses energy so it depends on the charged particle's energy.

The linear stopping power of a material also depends on the density of electrons within the material (and hence on the atomic numbers of the atoms) as well as the energy of the particle. So a more fundamental way of describing the rate of energy loss is to specify the rate in terms of the density thickness, rather than the geometrical length of the path. So energy loss rates are often given as the quantity called the mass stopping power:

\[
S_m = -\frac{dE}{d(\rho x)} = -\frac{1}{\rho} \frac{dE}{dx}
\]

where \( \rho \) is the density of the material and \( \rho x \) is the density-thickness.
4-10 EXCITATION AND IONISATION BY CHARGED PARTICLES
The dominant mechanism of energy loss at lower (non-relativistic) energies is the electromagnetic interaction between the moving charged particle and atoms within the absorbing material. Since the electromagnetic interaction extends over some distance, it is not necessary for the charged particle to make a direct collision with an atom; it can transfer energy simply by passing close by. However, since the internal energy of an atom is quantised, only certain restricted values of energy can be transferred. The incident particle can transfer energy to the atom, raising it to a higher energy level (excitation) or it may transfer enough energy to remove an electron from the atom altogether (ionisation). Although this fundamental mechanism operates for all kinds of charged particles, there are considerable differences in the overall patterns of energy loss and scattering between the passage of light particles (electrons and positrons), heavy particles (muons, protons, alpha particles and light nuclei), and heavy ions (partially or fully ionised atoms of high Z elements). Most of these differences arise from the dynamics of the collision process. In general, when a massive particle collides with a much lighter particle, the laws of energy and momentum conservation predict that only a small fraction of the massive particle's energy can be transferred to the less massive particle. The actual amount of energy transferred will depend on how closely the particles approach and restrictions imposed by quantisation of energy levels. The largest energy transfers occur in head-on collisions.

In non-relativistic newtonian dynamics, when an object A (mass $M$) hits a stationary object B (mass $m$) head-on the laws of dynamics predict that the energy lost by the incident particle is

$$ K_B = 4 \frac{K_A m M}{(M+m)^2}, $$

where $K_A$ is the kinetic energy of the incident particle. For the case $M \gg m$ this becomes

$$ K_B = 4 \frac{K_A m}{M}. $$

The energy transferred is a very small fraction of the incident particle's energy. However when $M = m$ then all the kinetic energy is transferred to the target ($K_B = K_A$) and the projectile stops. This result is strictly true only for particles travelling with speeds much less than that of light (non-relativistic speeds) but similar results are obtained also for relativistic particle speeds. When the collision is not head-on the energy transfer to the target is less and of course the energy loss of the incident particle is correspondingly less as well.

**Energy loss by heavy particles**

When a massive particle collides with an electron the energy lost at each collision is relatively small. For example, a slow alpha particle hitting an electron transfers a maximum of only 0.05% of its energy to the electron. Since head-on collisions are rare, usually the energy loss is much lower. Many collisions are needed to significantly reduce the incident particle's energy. Therefore we can consider the energy loss as a continuous process. Although the energy given to an electron may be a small fraction of the incident energy, it may be sufficient to ionise the atom and for the ejected electron to travel some distance away from the interaction point, leaving a trail of excited and ionised atoms of its own. These 'knock-on' electrons can leave tracks called delta rays. Mostly, however, the knock-on electrons will lose their energy within a very short distance of the interaction point.

The energy dependence of the rate of energy loss (stopping power) by excitation and ionisation of heavy particles for some typical materials is shown in figure 4.16. This graph is a plot of the energy-loss rate as a function of the kinetic energy of the incident particle. Note that the stopping power is expressed using density-thickness units. To obtain the energy loss per path length you would need to multiply the energy loss per density-thickness (shown on the graph) by the density of the material. As for photon interactions, it is found that when expressed as loss rate per density-thickness, the graph is nearly the same for most materials. There is, however, a small systematic variation; the energy loss is slightly lower in materials with larger atomic numbers. The diagram shows the rate of energy loss for the extreme cases
of carbon ($Z = 6$) and lead ($Z = 82$). At high incident energies there is also some variation with density of the same material because a higher density of atomic electrons protects the more distant electrons from interactions with the incident particle. This results in lower energy loss rates for higher densities.

![Mass stopping power and energy](image)

**Figure 4.16** Mass stopping power and energy

For low energies the stopping power varies approximately as the reciprocal of the particle's kinetic energy. The rate of energy loss reaches a minimum, the **minimum ionisation** point, and then starts to increase slowly with further increases in kinetic energy. Minimum ionisation occurs when the particle's kinetic energy is about 2.5 times its rest energy, and its speed is about 96% of the speed of light in vacuum. Although the energy loss rate depends only on the charge and speed of the incident particle but not on its mass it is convenient to use kinetic energy and mass rather than the speed. At minimum ionisation the energy loss is about 0.2 MeV.$(\text{kg.m}^{-2})^{-1}$ ($= 3 \times 10^{-12} \text{J.m}^2.\text{kg}^{-1}$ in SI units), decreasing slightly with increasing atomic number of the absorbing material.

The distance that a particle penetrates a material before it loses all its kinetic energy is called the **range** of that particle. Energy loss along the path is shown in figure 4.17. The rise near the end of the path is due to the increased energy loss rate at low incident energies. At very low speeds the incident particle picks up charge from the material, becomes neutral and is absorbed by the material.

![Stopping power and range](image)

**Figure 4.17** Stopping power and range

For a given material the range will be the nearly the same for all particles of the same kind with the same initial energy. The number of particles as a function of distance along the path is shown in figure 4.18. A small variation in the range, called **straggling**, is due to the statistical nature of the energy loss process which consists of a large number of individual collisions. The actual number of collisions is
always subject to some fluctuation. In spite of that, the average range can be used to determine the average energy of the incident particles.

**Figure 4.18 Range and straggling**

**Energy loss by electrons and positrons**
Electrons and positrons also lose energy by ionisation but there are several differences. There is a slight difference between the interactions of positrons and electrons, resulting in a slightly higher energy loss for the positrons. Both, however, have lower loss rates at high energies than heavier particles travelling at the same speed.

Because of its light mass an electron is easily scattered in collisions with other electrons. The resulting erratic path will be longer than the linear penetration (range) into the material and there will be greater straggling.

### 4-11 BREMSSTRAHLUNG

The name bremsstrahlung comes from the German; the literal translation is 'braking radiation'. It occurs when a charged particle is accelerated - that is whenever its speed or direction of motion changes. The effect is most noticeable when the incident particle is accelerated strongly by the electric field of a nucleus in the absorbing material. An accelerated charged particle radiates electromagnetic energy (photons). Since the effect is much stronger for lighter particles, it is much more important for beta particles (electrons and positrons) than for protons, alpha particles, and heavier nuclei. At particle energies below about 1 MeV the energy loss due to radiation is very small and can be neglected. Radiation loss starts to become important only at particle energies well above the minimum ionisation energy. At relativistic energies the ratio of loss rate by radiation to loss rate by ionisation is approximately proportional to the product of the particle's kinetic energy and the atomic number of the absorber. So the ratio of stopping powers is

\[ \frac{S_{\text{rad}}}{S_{\text{ion}}} = \frac{1}{E'} Z E \]

where \( E \) is the particle's kinetic energy, \( Z \) is the mean atomic number of the absorber and \( E' \) is a proportionality constant; \( E' \approx 800 \text{ MeV} \).
The kinetic energy at which energy loss by radiation equals the energy loss by collisions is called the **critical energy**, $E_c$. Approximately

$$E_c \approx \frac{E}{Z} = \frac{800 \text{ MeV}}{Z}.$$

Another quantity of interest is the **radiation length**, defined as the distance over which the incident particle's energy is reduced by a factor $e^{-1}$ (0.37) due to radiation losses alone. Some typical values are given in table 5.1

<table>
<thead>
<tr>
<th>Material</th>
<th>Critical energy $E_c$ / MeV</th>
<th>Radiation length $L$ / m</th>
<th>Density $\times$ radiation length $\rho L$ / kg.m$^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>102</td>
<td>300</td>
<td>362</td>
</tr>
<tr>
<td>Water</td>
<td>92</td>
<td>0.36</td>
<td>361</td>
</tr>
<tr>
<td>Aluminium</td>
<td>51</td>
<td>0.089</td>
<td>240</td>
</tr>
<tr>
<td>Iron</td>
<td>27</td>
<td>0.018</td>
<td>140</td>
</tr>
<tr>
<td>Lead</td>
<td>9.5</td>
<td>0.0056</td>
<td>64</td>
</tr>
</tbody>
</table>

**Table 4.1 Critical energies and radiation lengths**

**Electron-photon cascades**

Bremsstrahlung by a high energy electron results in a high energy photon as well as a high energy electron. Pair production by high energy photons results in a high energy electron and a high energy positron. In both cases two high energy particles are produced from a single incident particle. Furthermore the products of one of these processes can be the incident particles for the other. The result can be a cascade of particles which increases in number, while decreasing in energy per particle, until the average kinetic energy of the electrons falls below the critical energy. The cascade is then absorbed by ionisation losses. Such cascades, or showers, can penetrate large depths of material.
4-12 CHERENKOV RADIATION

There is another mechanism by which charged particles can produce electromagnetic radiation. When the particle moves faster than the speed of light in the material it generates a shock wave of electromagnetic radiation similar to the bow wave produced by a boat travelling faster than the speed of water waves. Cherenkov radiation does not occur at all if the particle’s speed is less than the speed of light in the material. Even at high energies the energy lost by Cherenkov radiation is much less than that by the other two mechanisms but it is used in radiation detectors where the ionisation along the track cannot be conveniently measured, for example in large volumes of transparent materials. Flashes of Cherenkov light are produced when cosmic rays enter the Earth's atmosphere.

Figure 4.21  Cherenkov radiation
The speed \( v \) of the charged particle is greater than the speed \( c/n \) of light in the material; \( n \) is the refractive index.
4-13 ENERGY LOSSES IN MIXTURES OF MATERIALS

As was the case with the attenuation length for photons, the rate of energy loss for charged particles in a material made up of a mixture of materials can be estimated by adding the individual energy loss rates weighted by the mass fraction of each material present. If you know the mass stopping power (energy loss per density thickness) for each material in the mixture then the total mass stopping power is just the sum of the individual mass stopping powers:

\[ S_m = S_{m1} + S_{m2} + S_{m3} + \ldots \]

In terms of linear stopping powers, you need to know the densities as well and the relation can be written as

\[ \frac{S_l}{\rho} = \frac{S_{l1}}{\rho_1} + \frac{S_{l2}}{\rho_2} + \frac{S_{l3}}{\rho_3} + \ldots \]

where \( \rho \) is the average density (total mass/total volume) and \( \rho_i \) is the density of the \( i \)th component material.

Charged particle tracks

The picture shows tracks left by fast charged particles passing through a nuclear emulsion, which is similar to X ray film but usually thicker. The primary particle, probably a proton, enters from the left and interacts with a nucleus of the emulsion. That interaction produces many new particles out of the high energy of the original proton. The original particle retains a large fraction of its kinetic energy but its track is lost among the many secondary particles. The secondary particles are also fast and on the average they travel in the same direction as the incident particle (because momentum is conserved). Slower particles travel at large angles with respect to the path of the incident particle. Neutral particles are also produced but they do not leave a trail of ionisation along their path. The ionisation along the tracks is fairly uniform, as is the rate of energy loss. However the ionisation is not continuous on a very small scale (smaller than the scale of this picture). The globular appearance of the tracks arises because the sensitive grains of the emulsion are embedded in a non-active substance such as gelatin. Only those grains which are sufficiently ionised produce an image when the film is developed. The straggling tracks near the main tracks are produced by electrons which have been pulled out of the atoms by the passing particle; they are called delta rays because of a fancied resemblance to the letter \( \delta \).
INTERACTION OF NEUTRONS WITH MATTER

Neutrons, being uncharged, do not interact electromagnetically with electrons or nuclei in matter. Instead, the nuclear interaction with nuclei is the most common interaction, but this can occur only if the neutron comes within 1 fm of the nucleus. Hence the attenuation coefficient for neutrons is small and neutrons can penetrate large amounts of matter.

The main interaction processes are elastic scattering, inelastic scattering, radiative capture and other nuclear captures.

**Elastic scattering**

In an elastic scattering process kinetic energy and momentum are both conserved. When a neutron scatters elastically from a nucleus it gives some of its kinetic energy to the nucleus, but the nucleus does not go into an excited state:

$$X + n \rightarrow X + n.$$  

Since the neutron is small compared with most nuclei, it does not lose much energy in each collision and it can take many collisions to lose its kinetic energy.

**Inelastic and capture processes**

This heading covers a number of different mechanisms. In all of them some of the neutron's kinetic energy is transferred to internal energy of the target nucleus which is left in an excited state and later decays by emitting neutrons or gamma radiation.

$$X + n \rightarrow X^* + n ;$$  
$$X^* \rightarrow X + \gamma ;$$  
$$^A_Z X^* \rightarrow ^{A-1}_Z Y + n$$  

For heavy target nuclei fission may be the preferred break-up mechanism, e.g.

$$^A_Z X + n \rightarrow ^{A_a}_Z X_a + ^{A_b}_Z X_b$$

where \( Z = Z_a + Z_b \) and \( A = A_a + A_b + 1 \).

The fission fragments \( X_a \) and \( X_b \) are usually unstable and decay often producing more neutrons.

The kind of reaction that occurs depends strongly on the energy of the neutron. The capture reactions occur much more readily for slow neutrons, the attenuation coefficient depending on velocity as \( v^{-1} \). Thus capture processes are the most important for slow neutrons with kinetic energy less than 0.1 eV. On the other hand, scattering processes are the most important energy loss mechanism for fast neutrons with kinetic energies greater than about 100 keV.
Q4.1 For gamma rays of energy $E$, the linear attenuation coefficient in material A is twice that in material B. Given that 80% of the gamma rays penetrate a given thickness of A, what fraction will penetrate the same thickness of B.

Q4.2 The mass attenuation coefficient of 1 MeV photons in aluminium is 0.0065 $\text{m}^{-2}.\text{kg}^{-1}$. The density of aluminium is $2.7 \times 10^3 \text{ kg.m}^{-3}$.
   a) What is the linear attenuation coefficient of 1 MeV gamma rays in aluminium?
   b) What fraction of incident 1 MeV gamma radiation penetrates 30 mm of aluminium.

Q4.3 The total radiation penetrating to a given depth in a material is sometimes greater than that predicted using the known attenuation coefficient. This is known as "build-up". Use an example to explain what is happening.

Q4.4 The accompanying diagram shows the mass attenuation coefficient due to the photoelectric effect as a function of the energy of the incident photon. Explain the cause of the features labelled K and L.

Q4.5 The diagram below shows the mass attenuation coefficient due to the photoelectric effect as a function of the energy of the incident photon. Sketch the expected dependence for absorption by a material of greater atomic number $Z$.

Q4.6 Pair production by photons has two restrictions. The first is that the energy of the photons must be greater than 1.02 MeV and the second is that pair production can occur only near a heavy charged particle. Explain the reasons behind these restrictions.
Q4.7  The mass attenuation coefficients for photons in lead are shown in the accompanying diagram. Label the curves according to the process they represent.

\[ \mu_m(\text{pe}) = 6 \times 10^{-5} \text{ m}^{-1}.\text{kg}^{-1}, \mu_m(\text{C}) = 1.2 \times 10^{-3} \text{ m}^{-1}.\text{kg}^{-1}, \mu_m(\text{pp}) = 0.005 \text{ m}^{-1}.\text{kg}^{-1}. \]

a) Which process of attenuation is dominant for 10 MeV photons?

b) What is the total attenuation coefficient for 10 MeV photons?

c) Which of the mass attenuation coefficients \( \mu_m(\text{pe}), \mu_m(\text{C}), \mu_m(\text{pp}) \), increase and which decrease if the photon energy is increased to 20 MeV?

Q4.8  The mass attenuation coefficients for 10 MeV photons in lead are:

\[ \mu_m(\text{pe}) = 6 \times 10^{-5} \text{ m}^{-1}.\text{kg}^{-1}, \mu_m(\text{C}) = 1.2 \times 10^{-3} \text{ m}^{-1}.\text{kg}^{-1}, \mu_m(\text{pp}) = 0.005 \text{ m}^{-1}.\text{kg}^{-1}. \]

a) Which process of attenuation is dominant for 10 MeV photons?

b) What is the total attenuation coefficient for 10 MeV photons?

c) Which of the mass attenuation coefficients \( \mu_m(\text{pe}), \mu_m(\text{C}), \mu_m(\text{pp}) \), increase and which decrease if the photon energy is increased to 20 MeV?

Q4.9  A proton of kinetic energy 2.5 GeV passes through a sheet of lead 15 mm thick. The density of lead is 11.4 \( \times 10^3 \) kg.m\(^{-3}\). Use the graph in figure 4.16 to estimate the energy lost by the proton.

Q4.10  You can make a rough estimate of the range of (kinetic energy) 300 MeV muons moving through air by using the energy loss rate at minimum ionisation. This rate is 0.2 MeV.m\(^{-2}\).kg\(^{-1}\) and so the range is 300 MeV/0.2 MeV.m\(^{-2}\).kg\(^{-1}\) = 1500 kg.m\(^{-2}\). The density of air is 1 kg.m\(^{-3}\). How far will the muon travel? Is this an overestimate or an underestimate of the range?

Q4.11  Estimate the minimum the value of the relativistic factor \( \gamma \) at which an electron will emit Cherenkov radiation when it is travelling through (a) water, (b) crown glass and (c) air.

\[ n_{\text{water}} = 1.33, \quad n_{\text{glass}} = 1.52, \quad n_{\text{air}} = 1.0003. \]

Q4.12  a) Explain why "bremsstrahlung" is a more important energy loss mechanism for electrons than for protons travelling through matter.

b) Explain why energy loss rate by "bremsstrahlung" is greater for electrons travelling through lead than for electrons travelling through water.

Q4.13  Use the graphs of figure 4.19 to determine the dominant energy loss mechanism for electrons travelling through lead with energy

(a) 100 keV, (b) 1 MeV, (c) 10 MeV and (d) 100 MeV

Q4.14  The mean free path for pair production is about 9/7 of the radiation length for bremsstrahlung. Calculate the number of electrons and photons emerging from a block of lead 20 mm thick when a single electron with kinetic energy 10 MeV enter the block perpendicularly from the other side. A simple model of the sequence of interactions, known as a shower, assumes that after travelling one mean free path a photon interacts by pair production to yield an electron-positron pair, both particles having the same energy. The electrons and positrons travel, on average, one radiation length without interaction and then produce a bremsstrahlung photon. The photon and the electron, after the interaction, have the same energy. Further it is assumed that the mean free path and the radiation length are the same.

Q4.15  Explain why light elements are more effective at slowing down fast neutrons through the mechanism of elastic scattering.

Q4.16  An alpha particle, a proton and a beta particle are incident on a thick iron shield. All have the same kinetic energy. Which particle penetrates the greatest distance into the iron and which penetrates the least?
Q4.17 According to the theory of relativity nothing can travel faster than light. But Cherenkov radiation is explained by saying that it is produced by a charged particle going faster than light. Explain.

Q4.18 It was stated in the text that for very fast charged particles the ratio of stopping powers by radiation loss (bremsstrahlung) and ionisation was proportional to $Z\varepsilon$. It was not stated whether that was for linear stopping power or mass stopping power. Comment.

Q4.19 What is the dominant mechanism of energy loss by extremely fast charged particles travelling through matter? How does the energy loss mechanism change as the particles slow down? Is there some stage where the total rate of energy loss is a minimum or a maximum?

Discussion Questions

Q4.20 If speed of light is the universal speed limit how can a charged particle produce Cherenkov radiation when it travels faster than light?

Q4.21 Compare the law of attenuation for photons with the radioactive decay law. In what ways are they similar? Why are they similar. In what ways are they different?
Aims
Studying this chapter should give you some appreciation of the variety of radiation detectors available and equip you with some practical knowledge about their characteristics and uses. This chapter also contains practical knowledge about the description and measurement of the effects of radiation on humans. You should learn how the problem of measuring doses is currently handled in terms of physical concepts, such as energy absorption, and empirical constructs, such as quality factors and equivalent dose rates. Although you should study the detailed information about the sources of radiation in the environment, there is no point in trying to memorise detailed statistics. Use the data to develop your appreciation of the problems and the precautions which need to be taken.

Minimum learning goals
When you have finished studying this chapter you should be able to do all of the following.

1. Explain, use and interpret the terms
   gas-filled detector, gas amplification, ionisation chamber, proportional counter, Geiger counter, multi-wire detector, scintillation counter, scintillator, fluorescence, photomultiplier, image intensifier, solid state detector, semiconductor detector, film badge, stochastic process, specific ionisation (exposure), absorbed dose, gray, background, dose equivalent, sievert, quality factor (relative biological effectiveness), dose rate, annual intake, annual limits on intake (ALI), background radiation, cosmic radiation, dose limit.

2. Outline the operating principles of ionisation chambers, proportional counters, Geiger counters, scintillation counters, photomultipliers, semiconductor detectors and film badges.

3. Describe uses of various types of radiation detector.

4. (a) Describe and explain ways of specifying radiation doses.
   (b) State and apply the relation between dose equivalent and absorbed dose.

5. (a) Describe and discuss the significance of various sources of natural background radiation.
   (b) Describe how radionuclides can enter the body from the environment.

6. Describe and discuss the principles of dose limits.

5-1 RADIATION DETECTORS - INTRODUCTION
There are three aspects of radiation detection. The first aspect is the interaction of radiation with matter within the detector's sensitive volume. We have dealt with the physics of those interactions in chapter AN4. The second step is to observe the products of the interaction to get some measurable physical quantity, which is often an electrical signal. That aspect will be dealt with in this chapter. The third stage is the signal processing system, which extracts useful information from the measurements. That aspect can be summarised as a black box of electronics and computers which processes the electrical signal. You will study those topics in your laboratory course.
5.2 GAS-FILLED DETECTORS

Many radiation detectors exploit the ionisation produced by radiation as it passes through a gas. Such detectors are used mainly for detecting radiation which consist of charged particles, although photons can produce relatively small amounts of ionisation in a gas. The sensitive volume of this kind of detector is filled with a suitable gas. Radiation passing through the gas leaves a trail of ionised atoms and free electrons. Those charged particles can be made to move in electric fields. The electric fields draw the ions and electrons to electrodes where their arrival causes an electric current to flow in the signal processing system.

![Diagram of gas-filled detector](image)

**Figure 5.1 General principle of gas-filled detectors**

The exact relationship between the number of ions produced and the current generated depends on how easy it is for the ions and electrons to move, the strength of the electric fields and the geometry of the electrodes. Strong fields accelerate the electrons and ions to energies at which they themselves can ionise the gas, thus producing more electrons and ions which in turn produce even more ionisation. This process of **gas amplification** is most useful in producing a measurable current from a few ions. However ions and electrons may recombine before they reach the electrodes. How quickly this happens depends on the composition of the gas and its pressure.

For a given geometry the way a gas-filled detector behaves depends on the potential difference between the electrodes. See figure 5.2. The smaller the field (or the applied voltage) the longer it takes for an ion to get to an electrode, and the greater is the probability of its recombining. If the electric field within the gas is too weak, ions and electrons can recombine before they are swept to the electrodes, so there will be only partial collection of the charge and no gas amplification. This corresponds to the part of the graph to the left of line A in figure 5.2.

Various kinds of gas-filled detectors can be classified according to the parts of the graphs (figure 5.2) where they operate.

**Ionisation chambers**

The **ionisation chamber** is operated with the applied voltage in the range A to B on figure 5.2. The electrodes are usually parallel plates and the electric field is weak. There is no gas amplification, but most of the ions produced by the incident radiation are collected. An ionisation chamber can be operated in one of two modes. In one mode each ionising particle produces a small output pulse which takes some time to decay. In the more usual mode, used for detecting high levels of x rays or gamma rays, a small continuous current gives a measure of the rate at which ionising particles pass through the chamber.
Figure 5.2 Charge collected by a gas-filled detector
Curves are shown for the detection of typical alpha and beta particles. Details of the curves depend on particle energy. The lines A, B, C, D and E delineate the different operational modes of gas-filled detectors (see text). The applied voltage scale, although typical, is arbitrary since it will vary with the size and geometry of the detector.

Proportional counters and Geiger counters

Proportional counters and Geiger counters are often cylindrical (figure 5.3); the anode (the positive electrode) is a wire on the axis of a hollow cylindrical cathode (the negative electrode). The electric field is strongest near the central wire \( E \approx \frac{k}{r} \). The potential difference is chosen such that for most of the counter's volume the electric field is too low for gas amplification. When an ionising particle passes through the sensitive volume, the resulting electrons and ions are swept towards the electrodes. Substantial gas amplification takes place only near the central wire. One advantage of this design is that the signal varies little with the position of the path followed by the incident ionising particle.

Figure 5.3 Structure of a Geiger counter
A charged particle produces many ion pairs along its path; so many ions are created even in small counters. Thus every charged particle entering the sensitive volume of the counter will be counted. On the other hand photons have all-or-nothing interactions and only a small fraction will interact inside the sensitive volume of the counter. The size of that fraction depends on the energy of the photons. If the number of incident photons is large this does not matter; the incident flux can still be measured quite accurately. However if we wish to count every photon, gas-filled counters are not suitable; more substantial counters such as a scintillation counter, are needed and, even then, not every photon will be counted.

**Proportional counter**

In the region B to C on figure 5.2 some gas amplification takes place. The larger the electric field the shorter is the distance required for the electrons to gain sufficient kinetic energy to generate extra ions by collision with the surrounding gas molecules. The charge collected at the electrodes is larger than that in an ionisation chamber and it is roughly proportional to the number of ions produced by the incident radiation. Each ionising particle produces a short electrical pulse whose amplitude depends on the detected particle's energy.

**Geiger counter**

In the high voltage region D to E on figure 5.2 the gas amplification process spreads sideways, mainly through photons generated in the secondary collisions, so that ionisation can be produced over the whole sensitive volume of the counter. The gas amplification avalanche is terminated when the density of secondary ions becomes large enough so that their fields, when added to the applied field, reduce the effective field to less than that required to sustain gas amplification. The output current no longer depends on the number of original ions, but is saturated to a value determined by the detector geometry and the applied voltage. Each particle detected produces an electrical pulse whose amplitude is practically independent of the particle's energy. The pulses can be counted electronically or the pulse rate can be displayed on a meter. A detector operated in this way is known as a Geiger counter (or Geiger-Muller counter). A Geiger counter is essentially just a proportional counter operated at a higher voltage.

**Multiwire chambers**

Usually the anode of a gas-filled detector is a thin wire. The electric field near a wire is intense so most of the gas amplification takes place close to the wire. In a **multi-wire detector** the anode consists of a number of parallel wires. There are two important advantages of such a detector. Firstly, there is improved uniformity in the response of the detector to radiation entering it at different places. Secondly, by measuring the current on each wire individually, it is possible to determine the location where the ionisation was produced so information about the particle's track can be obtained.

![Multiwire detector](image)

**Figure 5.4 Multiwire detector**

The sensing wires may be connected together (as shown) or each wire may have its own voltage supply, amplifier and signal processing circuits.
5.3 CLOUD CHAMBERS AND BUBBLE CHAMBERS
In cloud chambers and bubble chambers the location of the ionisation produced by the incident particles is made visible and can be photographed. In a cloud chamber the ionisation produced acts as a centre for condensation of a supercooled vapour. Droplets are formed which when allowed to grow can be illuminated by an electronic flash and photographed. Bubble chambers contain a superheated liquid which is ready to boil but does not do so because there are no centres for the bubbles to start. The ions produced by incident radiation provide such centres and the liquid starts to boil along the track followed by the incident particle. Cloud and bubble chambers have been used mostly in particle physics research and are now considered to be obsolete.

5.4 SCINTILLATION COUNTERS
Scintillation counters work by producing flashes of light - scintillations - in response to the passage of charged particles or high energy photons through specially chosen solid or liquid materials called scintillators. When an energetic particle enters the scintillator a large number of atoms are raised to excited states with higher energy levels. The excited atoms return to lower energy states by emitting photons in the visible or near ultraviolet regions of the spectrum. This process is called fluorescence. If the substance of the detector is transparent to these low energy photons, they can escape from the detecting material to be picked up by a light detector. So a charged particle or a high-energy photon penetrating the detector can be detected by the flash of light that it causes. A special kind of light detector, the photomultiplier (see below), is used to produce a voltage pulse in response to the pulse of light. These pulses can be counted and their sizes can also be analysed electronically to give information about the energies of the particles being detected.

Inorganic and organic scintillators
Two basic kinds of detector materials are used - organic and inorganic. The difference in their behaviour results from the different ranges of energy levels excited by the incident radiation. Organic scintillators, which usually contain aromatic hydrocarbon molecules, respond very quickly - fluorescence occurs within a few nanoseconds. But they are not very efficient, producing only a few photons for each incident particle.

Inorganic scintillators are slower, fluorescing in about 250 nanoseconds. However they are more efficient than organic materials for detecting gamma radiation because of their greater density and higher average atomic number. A very common example of an inorganic scintillator is a crystal of sodium iodide containing some thallium, NaI(Tl).

Photomultipliers
A photomultiplier responds to a light pulse by producing a pulse of electric current. Figure 5.5 shows the structure of a typical photomultiplier. Photons enter through a window, hit the photoemitting cathode and, through the photoelectric effect, release electrons. An electric field set up between the cathode and the first of a series of electrodes called dynodes accelerates the photoelectrons. The dynodes are made of a material which, when hit by an electron, will emit more than one secondary electron by a mechanism similar to the photoelectric effect. The secondary electrons are pulled to the next dynode where the amplification process is repeated until the current is collected on the final anode. A typical twelve-dynode photomultiplier will produce a few million electrons from a single photoelectron released at the photocathode.

Not every photon will release an electron at the photocathode; on the average only one in four will do so. The ratio of the average number of photoelectrons released to the number of incident photons is called the quantum efficiency of the photocathode.
**Image intensifiers**

The current from a photomultiplier depends on the total amount of light falling on the photocathode; it does not depend on the positions where photons hit the photocathode. If patterns of light are to be amplified an image intensifier is used. An image intensifier works in the same way as a photomultiplier. It is not a particle detector but may be used to amplify weak light produced on x-ray screens and similar detectors. Initially an image is focused onto the photocathode. The number of photoelectrons released at a given place depends on the intensity of light at that place. The electrode structure of the image intensifier is such that electric fields produced guide the electrons from each region of the cathode along a unique path to a fluorescent scintillating screen, where they interact to produce a flash of light. On their way through the image intensifier the electrons numbers are multiplied and their energy can be increased so that the intensity of light emitted from the screen is greater than the light intensity falling on the photocathode.

**5.5 SOLID STATE DETECTORS**

In crystalline materials electrons in the highest energy states are not attached to individual atoms but are shared by all the atoms. The energy levels of these electrons have a broad band structure with gaps between the bands. For materials of interest here all the levels up to the top of one band are filled but the next higher band, above a gap, is nearly empty. Incident radiation can raise an electron from a level in the filled valence band to a level in the conduction band. Both the electron in the conduction band and the hole left in the valence band can now move around.

**Semiconductor detectors**

In semiconductors with gap widths of 1 eV to 2 eV the number of electron-hole pairs created by radiation in the vicinity of a junction between two different materials is detected directly by electrical means. Electric fields in the junction region sweep the electrons and holes to the anode and cathode respectively, producing an electric current. So semiconductor detectors work like solid ionisation chambers. They have the advantage that they can be quite small, so they can be used for accurate position measurements.

**Film badges**

Another kind of solid state detector is photographic emulsion. The active material is a grain of silver bromide. Incident radiation raises an electron from an energy level in the valence band to an energy level in the conduction band, leaving a hole in the valence band. Both the electron and hole diffuse through the silver bromide grain and become trapped at irregularities in the crystal structure of the grain. If a single grain accumulates more than about four electron-hole pairs it becomes possible to change the silver bromide crystal to a pure silver crystal by photographic development.
5-6 BIOLOGICAL EFFECTS OF IONISING RADIATION

The effects of radiation on living things arise from the ionisation produced by the particles. Cell damage is initiated by ions (both negative and positive) along the track of an ionising particle. The damage is magnified by free radicals generated by the ions.

It is convenient to divide up the effects of ionising radiation into two classes: those due to massive doses of radiation over a small time (less than about 14 days) and those due to much smaller doses. In this second class, the exposure time could be years and the probability of a disease depends on the dose received. The word stochastic is applied to these effects. For example if one is exposed to cigarette smoke or a toxic chemical, then there is a likelihood of contracting a cancer, which may be fatal. The greater the exposure, the more probable is the cancer. It is the same with radiation. Ionising radiation can induce various cancers with a probability which increases with radiation dose.

Other stochastic effects that can be caused both by poisons and radiation are of a genetic type. If the reproductive cells of an organism are affected by certain poisons or subjected to ionising radiation then the organism's descendants may suffer mutations. If, as in humans, the reproduction is sexual, then the mutation rate is greatly enhanced if both the male and female are poisoned or irradiated.

It has to be stressed that the two types of radiation induced stochastic effects: namely, genetic and non-genetic (cancers), would only slightly lift the incidence of the disorders concerned, most of the disorders being due to causes other than radiation.

5-7 MEASURES OF THE INTENSITY OF IONISING RADIATION

We are concerned here with the physical effects of radiation rather than chemical or biological effects. The first need is for a physical measure of radiation intensity which is capable of being correlated with radiation's biological effects. The measure chosen, called the absorbed dose, is the ratio of the energy deposited by ionisation to the mass of living tissue.

\[ D = \frac{E}{m} \quad \text{... (5.1)} \]

The SI unit for absorbed dose is the joule per kilogram, which for radiation purposes is called the gray (symbol Gy). The obsolete unit for dose, the rad, was defined as 100 ergs per gram. The conversion relation is 1 rad = 0.01 gray, exactly. It is, in principle, possible to measure radiation dose by the temperature rise it produces in the tissue. For 1 gray given in a short episode, the temperature rise would be only 0.000 25 K. Since 1 gray is a significant dose, measuring radiation doses this way is clearly impractical!

The obvious practical way to measure radiation dose is by recording the ionisation produced by the radiation. This can be done for gaseous media using an ionisation chamber. The chosen gas is held in a container with two electrodes which can pick up the ions of the appropriate charge: electrons or negative ions to the anode and positive ions to the cathode. The physical quantity used to describe the ionisation would be electric charge per mass of the medium, which is called specific ionisation or exposure (SI unit, coulomb per kilogram). (An obsolete unit of exposure, the roentgen is equivalent to \(2.58 \times 10^{-4}\) C.kg\(^{-1}\).)

We now find the relation between this measure and the absorbed dose. It takes about 35 eV to produce each pair of ions; one positive, the other negative. This figure of 35 eV varies little from one absorbing medium to another and it also depends little on the nature of ionising particle: electron, alpha particle etc. The ratio of energy deposited to the ionisation charge produced is 35 eV per electron charge or

\[ \frac{35 \times 1.6 \times 10^{-19} J}{1.6 \times 10^{-19} C} = 35 \text{ J.C}^{-1} \]

So a specific ionisation of 1.0 C.kg\(^{-1}\) is equivalent to an absorbed dose of about 35 J.kg\(^{-1}\) or 35 Gy.

The ionisation chamber is the standard instrument for measuring radiation. There are problems with its sensitivity as the absorbed dose due to the "normal" background of 0.1 microgray per hour will
produce an ionisation current of less than 1 fA (10^{-15} \, \text{A}) in a reasonably sized chamber. These currents are comparable with leakage currents over insulators, making measurement difficult. High pressure chambers are often used to get around this problem but they are bulky and heavy.

The Geiger and scintillation counters are more sensitive. However, as the names suggest, they count individual particles as they arrive instead of measuring the ionisation current. (The scintillation counter can also give the energies of the particles it counts). Both types of counter require calibration against an ionisation chamber.

The Geiger counter is often used to measure gamma ray doses. The scintillation counter is a better instrument although it is more expensive. Gamma rays have to interact to be registered and the probability of interaction in the sensitive volume of the counter decreases for higher gamma ray energies. One then might think that a Geiger counter exposed to high energy gamma rays would underestimate the radiation dose. There is fortunately a compensating effect because the secondary electrons have an energy commensurate with the gamma ray energy. For higher energies the electrons travel further and each produces more ions and deposits more energy into tissue. A similar thing happens with ionisation chambers as they, like the counters, are almost transparent to gamma ray photons of reasonable energy like 100 keV or so. At this energy a Geiger counter has a probability of around 1% of stopping a gamma ray photon and recording it. For higher energies the figure is smaller.

For beta particles one may also use Geiger counters where the efficiency for counting may approach 100%. One then needs to calculate the dose per count and this is not too difficult a task.

A similar method can by used with alpha particles but another problem arises. Alpha particles have a short range in tissue and other media. Gray for gray the alpha particle is observed to do much more biological damage than a particle like the electron. The density of ions along the particle's track is much higher for the alpha particle than it is for electrons. This density is proportional to the square of the charge so it is four times higher for an alpha particle than for an electron of the same speed. In addition, the speeds of alpha particles are much smaller than those of the electrons, the energy loss per track length is even higher for alpha particles. It is the high ionisation density along its track that makes an alpha particle so damaging.

To take this into account a dose unit called the sievert (symbol Sv) is used. It is a measure of the dose equivalent which is related to the absorbed dose by

\[ H = D \times Q \cdot N \]  ... (5.1).

H is the dose equivalent and D is the absorbed dose. Q is called the quality factor and allows for effects such as the density of ionisation described above. (The quality factor corresponds to an obsolete quantity called relative biological effectiveness or RBE.) N is a constant introduced by the International Commission on Radiological Protection (ICRP). The ICRP has assigned its value as one sievert per gray for the present. The ICRP has also proposed a definition of \( Q \) in terms of stopping power for the ionising particles in water (figure 5.6). Water has been chosen as the reference material since it is similar to living tissue as far as ionisation loss is concerned. The range of stopping powers for typical alpha and electron energies are also shown on figure 5.5. Note that electrons (beta radiation) typically have a quality factor of 1, while alpha particles, which produce heavy ionisation, can have values of \( Q \) in the range 10 to 20. Gamma rays have quality factors around 1 or less depending on the photon energy.

Some publications contain references to dose equivalent expressed in the obsolete unit, rem. The conversion is 1 rem = 0.01 Sv.
It is possible to make models which describe the movement of radionuclides in the body. The models are based on measurement and take into account how particles and solutions move in the body and any possible chemical binding effects. For relatively long-lived radionuclides, the rate of excretion is clearly of great importance. The model can be used to find the annual intake of a radionuclide which would give a certain equivalent dose. Estimates of this sort are done by the ICRP. The ICRP also specifies the annual limits on intake (ALI) for various radionuclides. Varying vulnerability of various body organs is taken into account. The use of the word "limits" means that the ALI is linked to the general limits on radiation dose equivalents published by the ICRP. These general limits will be given later.

As an example of interest the ICRP give a figure of 200 Bq for the ALI of plutonium-239. This is for inhalation of most compounds of plutonium; the ALI for the oxide is 500 Bq.

5-9 NATURAL BACKGROUND RADIATION

Natural background radiation comes from many sources, including cosmic rays and various radionuclides in the environment.

Cosmic radiation

High energy charged particles come from somewhere outside the solar system and bombard the Earth's atmosphere. Many secondary particles result from collisions of the cosmic-ray particles with atoms of the atmosphere. At ground level there is a general flux of the secondary particles of about 300 particles per square metre per second. They constitute a source of ionising radiation which increases with altitude (and to a limited extent with latitude).

Carbon-14

This radionuclide is made in the high atmosphere by neutron bombardment of nitrogen-14, the most common nuclide in the earth's atmosphere. The neutrons are secondary particles from cosmic ray collisions. Carbon-14 is a beta emitter (its nuclei emit negative electrons). The half life of 5730 years gives it plenty of time to diffuse down into the biosphere where its decay electrons can irradiate all living tissue. The accounting of the various background sources must include carbon-14, because it is a universal constituent of all living organisms, including people.

Potassium-40

Potassium-40 is an electron emitter with a half life of $1.3 \times 10^9$ years. Like uranium-238 and thorium-232, it is a relic from the formation of the solar system. It is believed that the sun (and the solar system)
started to condense after a "push" from the explosion of a nearby supernova, a big star that collapsed after running out of nuclear fuel. Huge quantities of neutrons are produced in the violent event which starts like an implosion and ends as explosion. The neutrons are pushed into the lighter nuclides (like iron) one by one. This process, together with beta decay, adds neutrons and protons and builds heavy nuclei like uranium. The half life of potassium-40 is comparable with the age of the solar system.

**Uranium and thorium**

Uranium-235, uranium-238 and thorium-232 are alpha emitters; their half lives are respectively: $7.04 \times 10^8$ years, $4.47 \times 10^9$ years and $1.41 \times 10^{10}$ years. They are relic nuclides from the explosion which "blew" them into the solar system. Their decays in each case lead through a chain of radionuclides. For example for thorium-232 we have the chain of decays shown in figure 3.9.

Nearly all rocks (and it seems most other objects in our environment!) contain thorium-232 and uranium-238 at the parts per million level. As far as radiation sources are concerned; we should first look at the secondary electrons produced by gamma rays given out by the daughter nuclides. These may be either inside or outside the body when they decay and are consequently termed internal or external sources. Internal sources could also irradiate the body with alpha particles and electrons. In this respect, by far the most serious radionuclide is radon-222. It has a long enough half life (3.8 days) for significant amounts to diffuse out of the ground where it is formed by the decay of its parent radium-226. Radon-222 then decays in the air to polonium-218 which attaches on to dust particles. If the dust is inhaled, the polonium-218 is likely to go to and stay in the lungs, where its decay and the decay of its daughters will cause serious irradiation. Life outdoors is quite safe; but humans like to live indoors in shut-in rooms, and what is worse, in brick and concrete buildings. The levels of irradiation in some of these situations are given in table 5.1.

The total annual dose is a little over 2 mSv of which about half comes from being indoors. The "normal" background level of approximately 1 mSv.y$^{-1}$ (or 0.1 $\mu$Sv.h$^{-1}$) is the level as assessed outdoors. Note that the cosmic radiation level increases with altitude. The figure given is probably appropriate to near sea level (where most people live in Australia).

The levels of radon-222 are given in table 5.2. These figures can be converted to dose levels through the following relation: 1 Bq.m$^{-3}$ gives an annual dose equivalent of 0.1 mSv if a subject is continuously exposed.

As well as these natural radiation levels we also expose ourselves to medical x rays and fallout. In the article cited above, these are estimated respectively at 0.2 and 0.02 milligray per year in the UK and 0.72 and 0.04 milligray per year in USA. As well as this there is occupational exposure: around 0.03 milligray per year.

**The ICRP’s recommended radiation limits**

The ICRP has set a maximum dose limit for the public of 5 millisieverts in one year and a maximum dose for occupational exposure to radiation: of 50 millisieverts in one year.

The ICRP has also enunciated a most important principle: ALARA (As Low As is Reasonably Achievable), (see ICRP publication No 26, page 14). This means that one should assess each radiation reduction proposal by using a "cost benefit" analysis. "Costs" are the sum total of all the negative aspects of an operation and include damage to health and the environment as well as monetary costs. It would be pointless, for example, to improve the shielding of a worker so as to reduce his or her radiation dose by 1% when that worker could wipe it out a hundred-fold by moving to a brick house.
<table>
<thead>
<tr>
<th>Source</th>
<th>Dose rate / mSv.y(^{-1})</th>
<th>Fraction of total %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra-terrestrial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>External exposure</td>
<td>0.3</td>
<td>15</td>
</tr>
<tr>
<td>Internal Exposure</td>
<td>0.01</td>
<td>0.5</td>
</tr>
<tr>
<td>Terrestrial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>External exposure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outdoor</td>
<td>0.06</td>
<td>3</td>
</tr>
<tr>
<td>Indoor</td>
<td>0.29</td>
<td>14</td>
</tr>
<tr>
<td>Internal exposure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K40 &amp; Rb 87</td>
<td>0.19</td>
<td>9</td>
</tr>
<tr>
<td>U238 series</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outdoor inhalation</td>
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<td>3</td>
</tr>
<tr>
<td>Indoor inhalation</td>
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<td>38</td>
</tr>
<tr>
<td>Ingestion</td>
<td>0.14</td>
<td>7</td>
</tr>
<tr>
<td>Th 232 series</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outdoor inhalation</td>
<td>0.03</td>
<td>1.5</td>
</tr>
<tr>
<td>Indoor inhalation</td>
<td>0.17</td>
<td>9</td>
</tr>
<tr>
<td>Ingestion</td>
<td>0.02</td>
<td>1</td>
</tr>
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</table>

Table 5.1 Annual doses to humans from various sources
(From ICRP publication No 39, 1984, page 2)

<table>
<thead>
<tr>
<th>Site</th>
<th>Concentration of Rn 222 / Bq.m(^{-3})</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inside wood house</td>
<td>10 to 30</td>
<td>4 air changes per hour</td>
</tr>
<tr>
<td>brick house</td>
<td>10 to 80</td>
<td></td>
</tr>
<tr>
<td>concrete house</td>
<td>10 to 170</td>
<td></td>
</tr>
<tr>
<td>USSR:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inside wood house</td>
<td>4 to 16</td>
<td></td>
</tr>
<tr>
<td>brick house</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>adobe house</td>
<td>10 to 400</td>
<td></td>
</tr>
<tr>
<td>slag house</td>
<td>150 to 300</td>
<td></td>
</tr>
<tr>
<td>UK:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>industrial premises</td>
<td>2 to 10</td>
<td>adequate ventilation</td>
</tr>
<tr>
<td>office buildings</td>
<td>7 to 25</td>
<td>poor ventilation</td>
</tr>
<tr>
<td></td>
<td>0.2 to 40</td>
<td>poor ventilation</td>
</tr>
<tr>
<td></td>
<td>2 to 13</td>
<td>air-conditioned</td>
</tr>
<tr>
<td>USA:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inside wood house</td>
<td>0.2 to 10</td>
<td>2 to 6 changes per hour</td>
</tr>
<tr>
<td>inside basement</td>
<td>4 to 35</td>
<td>1 to 3 changes per hour</td>
</tr>
<tr>
<td>inside concrete house</td>
<td>1 to 180</td>
<td></td>
</tr>
<tr>
<td>Outdoors over land</td>
<td>0.07 to 10</td>
<td></td>
</tr>
<tr>
<td>Outdoors over sea (S. Pacific)</td>
<td>0.07 to 0.2</td>
<td></td>
</tr>
<tr>
<td>South Pole</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2 Levels of radon-222 in various situations
Adapted from Atomic Energy in Australia, October 1975, pp 18 -19.
A few comments are in order on the method used by the ICRP to set the limits quoted above. For the radiation levels concerned, carcinogenesis is considered to be the greatest risk from radiation and the ICRP assesses the risk factor for all cancers at 0.01 per sievert. The risk factor for genetic defects (in the first two generations) is assessed at 0.004 per sievert. These figures were obtained after studying the medical records of workers in high radiation areas and people exposed to nuclear weapon explosions.

In the above, we are considering the stochastic effects of radiation. We contrast these with the effects of massive doses (such as 10 sievert). Death often results from exposures to such doses if they are concentrated into a short period of time (days) so that the body's repair mechanisms do not have sufficient time to act.

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**QUESTIONS**

Q5.1 A fair approximation of the number of ions produced when an ionising particle travels through a gas can be made knowing that it takes about 35 eV to produce an ion pair in most gases. Calculate the number of ion pairs produced by an electron, kinetic energy 1.0 MeV, passing through an ionisation chamber 30 mm thick. What is the total charge of electrons? If a hundred such particles pass through the chamber each second and all the ions are collected at the electrodes what is the total current? The density of the gas is 1.3 kg.m⁻³.

Q5.2 Describe the process of gas amplification.

Q5.3 The total attenuation coefficient for a 300 keV photon passing through argon is 0.01 m².kg⁻¹. If 2000 such photons enter an ionisation chamber 300 mm thick how many will interact in the chamber? The density of argon is 1.8 kg.m⁻³.

Q5.4 Draw a block diagram of the three main components of a scintillation counter.

Q5.5 A cylindrical gas-filled ionisation detector is operated as a proportional counter. Electrons are to be collected on the central wire. What is the sign of the potential difference of the central wire with respect to the outer electrode? Draw a cross section of the counter. Identify the region in which gas amplification takes place.

Q5.6 Describe the features and operation of a proportional counter.

Q5.7 What are the similarities and the differences between ionisation chambers, proportional counters and Geiger counters?

Q5.8 What is the purpose of a photomultiplier in a scintillation counter?

Q5.9 The sensitive volume of ionisation chambers, proportional counters and Geiger counters contains gas. Why is a denser material (a liquid or solid) not suitable for use in such detectors?

Q5.10 The sensitive volume of ionisation chambers, proportional counters and Geiger counters has to be enclosed in a gas-tight box. How do the properties of the enclosure - material, wall thickness, geometry, etc. - affect the performance of the detector?

Q5.11 An x-ray tube passes a current of 100 mA at 100 kV for 0.1 s. Given that it converts 0.5% of the energy to x rays and 10% of this is absorbed by a patient whose mass is 70 kg, calculate the absorbed dose.

Q5.12 A 70 kg person accidentally ingests 1 µg of radium-226. Given that each decay of a radium nucleus gives an alpha particle of energy 4.8 MeV (which is totally absorbed) estimate the yearly absorbed dose.

Use 1 eV = 1.6 × 10⁻¹⁹ J; activity of 1 g of radium = 3.7 × 10¹⁰ Bq.
AN6 APPLICATIONS - X-RAYS & NMR

OBJECTIVES

Aims
Firstly, you should aim to be able to explain how x-rays are produced and the main features of x-ray spectra. You should be able to explain those features. Secondly, you should aim to understand, qualitatively, the physical principles of nuclear magnetic resonance.

Minimum learning goals
When you have finished studying this chapter you should be able to do all of the following.
1. Explain, use and interpret the terms x rays, x-ray tube, continuous spectrum, cut-off wavelength, characteristic x rays, Moseley's law, nuclear magnetic resonance [NMR], NMR spectroscopy, NMR imaging [MRI].
2. Describe and explain how x rays are produced in an x-ray tube.
3. Sketch typical plots of x-ray spectra for various accelerating voltages.
4. (a) Describe and explain the origin of the continuous x-ray spectrum.
   (b) Explain the short wavelength cut-off to the continuous spectrum.
   (c) State and apply the relation between the cut-off wavelength and the accelerating voltage (equation 5.1).
5. Explain how characteristic x rays are produced.
6. Describe the principles of NMR, NMR spectroscopy and NMR imaging.

TEXT

6-1 X RAYS
X rays can be described as very energetic photons or short wavelength electromagnetic radiation. They can be produced when fast charged particles (generally electrons) are stopped by matter. Although x rays and gamma rays can have similar energies, the term gamma rays is usually reserved for radiation produced by the decay of excited nuclei.

X rays were discovered by Wilhelm Röntgen in 1895, while he was investigating electrical discharges through gases. Within five years the discovery had already been applied in many areas of the life sciences. Besides the well-known use for examination of fractured and broken bones, very early attempts were made to apply them to cancer treatment.
6-2 PRODUCTION OF X RAYS
In an x-ray tube, a beam of electrons is accelerated through a potential difference in the range 10 kV to 200 kV. The electrons then collide with a target of a heavy metallic element such as tungsten or molybdenum. See figure 6.1. Most of the energy of the electron beam goes into increasing the internal (thermal) energy of the target but a small fraction appears as high energy photons - x rays.

![Figure 6.1 An x-ray tube - schematic](image)

6-3 X-RAY SPECTRA
The energies of the x-ray photons, and hence the wavelengths of the radiation, are spread over a range of values. Figure 6.2 shows the radiated power plotted as a function of wavelength for a typical x-ray tube.

![Figure 6.2 X-ray spectra for a molybdenum target](image)

Note the short wavelength (high frequency) cut-off for each accelerating voltage. Characteristic lines appear only when the accelerating voltage is high enough to dislodge electrons from inner atomic shells.

For lower values of the accelerating voltage and electron energy, the distribution of x-ray intensity with wavelength is a **continuous spectrum** which cuts out sharply at a fixed value of the wavelength at the short wavelength or high frequency end of the spectrum. (See the curves labelled 5 kV, 10 kV, 15 kV and 20 kV in figure 6.2.) If the accelerating voltage is increased this short wavelength cut-off decreases.
There is also an increase in the total x-ray power and a decrease in the average wavelength. At sufficiently high electron energies, a set of characteristic lines may be superimposed on the continuous spectrum. These lines always occur at the same wavelengths for a fixed target material.

**Continuous spectrum**

In an x-ray tube the electrons are first accelerated by a potential difference $V$, which gives each of them a kinetic energy $eV$. About 99% of the electrons that strike the target do nothing spectacular at all. Most of them undergo multiple collisions with the electrons and atoms in the target and, in the course of these collisions, the electrons lose their energy a little at a time and thus merely increase the average kinetic energy of the particles in the target. Thus most of the energy of the electron beam goes into heating the target and the temperature of the target increases. X rays are produced by a few of the fast electrons which penetrate target atoms sufficiently to experience the very strong electric fields near the atomic nuclei. These electrons are strongly accelerated so they can lose most of their energy in one collision by the process of bremsstrahlung - as described in chapter AN4.

**Cut-off wavelength**

The short wavelength cut-off to the continuous spectrum can be explained as follows. An electron cannot lose more kinetic energy in a collision than it started with, and so the maximum energy of the photons produced is equal to $eV$. We can work out the highest frequency $f_0$ (and shortest wavelength $\lambda_0$) of the radiation produced. The maximum energy of an x-ray photon is equal to the kinetic energy of one electron which loses all its energy to the photon, so

$$E = eV = hf_0 = \frac{hc}{\lambda_0};$$

and the cut-off wavelength is

$$\lambda_0 = \frac{hc}{eV} \quad \cdots (6.1).$$

However most of the electrons hitting the target do not lose all their kinetic energy in one hit. An electron might undergo several collisions before emitting an x-ray photon, so the photon's energy will be less than the maximum value. Note that the commonest value of the photon energy is somewhat less than the maximum value, giving a peak intensity at a wavelength greater than the cut-off wavelength.

**Characteristic lines**

If the accelerating voltage is high enough an x-ray line spectrum can be produced. The line spectrum appears as a set of spikes superimposed on the continuous spectrum, like the 25 kV curve in figure 6.2. The spikes, or lines, indicate that a very large number of x-ray photons of definite wavelength (and hence definite energy) are being emitted. The wavelengths of the lines are characteristic of the material of the target so the radiation is called **characteristic x rays**.

The origin of characteristic x rays can be understood in terms of energy levels within the atoms of the target material. The electrons in the outermost shell, which are responsible for the chemistry of the atom, are only loosely bound. It takes only a few electronvolts to remove one of them to ionise the atom. On the other hand, for example, it takes 20 keV to remove a K-shell electron from a molybdenum atom. This much higher binding energy occurs because K-shell electrons are, on average, closer to the nucleus than those in outer shells and also because electrons in the outer shells are, to some extent, shielded from the attractive electrostatic force of the positive nucleus by the inner electrons.

A characteristic x-ray photon is produced when an accelerated electron knocks an electron out of an inner shell in the target atom and another electron subsequently falls into the vacancy. The excess energy of this third electron is emitted as an x-ray photon.
When electrons fall into the vacancies in the K shell, they produce the K series of characteristic x rays. There is a series of lines because the electron which fills the vacancy can come from the L shell, the M shell, the O shell, etc. or even from outside the atom. Within a particular series, transitions are labelled with Greek letters. Figures 6.2 and 6.3 show the Kα and Kβ lines for molybdenum. The single spike labelled Kα in the spectrum of figure 6.2 corresponds to a group of three transitions in figure 6.3. Each of the Kα and Kβ lines is really a set of closely spaced lines, corresponding to the slight differences in energy levels within the L and M shells. (The label Kβ corresponds to the electron falling from the M shell; the Kα from the L shell.) Note that in the case of molybdenum, if the accelerating voltage is less than 20.04 kV, no electron can be removed from the K shell so the Kα and Kβ radiation will not be seen. If an electron is knocked out of the L shell we get the L series of lines, produced by electrons falling from outer shells. (No L series lines appear in figure 6.2 only because their wavelengths are off the scale.)

Because the spacing of the energy levels depends on the particular element, the x-ray spectrum is characteristic of the element (just as is its optical spectrum which corresponds to rearrangements among the higher energy levels, or outer shells, where the energy spacing between levels is much smaller). The frequency of characteristic x rays is described by a simple law called Moseley's law. It states that the frequencies of corresponding x-ray spectral lines (such as the Kα's), as we go from element to element, may be represented by an equation of the form

\[ f = A (Z - \sigma)^2 \]  

... (6.2)

where \( Z \) is the atomic number of the element (the number of protons in the nucleus) and the constants \( A \) and \( \sigma \) depend on the particular line (i.e. they have one value for the Kα line, another for the Kβ etc.). For Kα and Kβ we have, approximately,

\[ A \approx 2.5 \times 10^{15} \text{ Hz}; \quad \sigma \approx 1. \]

The frequency of a particular line (say the Kα) for a particular element depends only on the spacing between the energy levels involved (for the Kα line they are the K and L levels). Moseley's law really tells us about how the spacing of corresponding energy levels depends on \( Z \), the atomic number. In the modern theory of the atom, quantum mechanics, Moseley's law can be derived from basic principles.
6-4 X-RAY ENERGIES

X rays can have a large range of energies, from around 100 keV produced in a Van der Graaff machine, to a high energy limit of around 20 GeV produced in the 3 km long linear accelerator at Stanford, U.S.A. Sometimes these high energy x rays are called gamma rays, but that is just a matter of nomenclature - they are all high energy photons.

The energy, frequency and wavelength of some typical x rays and, for comparison, light are shown in the table.

<table>
<thead>
<tr>
<th>Photon</th>
<th>Energy</th>
<th>Frequency</th>
<th>Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>high energy x ray</td>
<td>1.00 MeV</td>
<td>$2.42 \times 10^{20}$ Hz</td>
<td>$1.24 \times 10^{-12}$ m</td>
</tr>
<tr>
<td>low energy x ray</td>
<td>100 eV</td>
<td>$2.42 \times 10^{16}$ Hz</td>
<td>$1.24 \times 10^{-8}$ m</td>
</tr>
<tr>
<td>light</td>
<td>2.8 eV</td>
<td>$6.7 \times 10^{14}$ Hz</td>
<td>$4.5 \times 10^{-7}$ m</td>
</tr>
</tbody>
</table>

Table 6.1 X-ray energies

6-5 NUCLEAR MAGNETIC RESONANCE

Since the discovery of magnetism, there have been arguments about whether magnetic fields play any direct role in life. Deleterious effects, as well as quack cures and treatments, have been (and still are) attributed to magnetism. The place of magnetism in such things as water divining and bird migration is, surprisingly, still controversial, while some bacteria (e.g. *Aquaspirillum magnetotacticum*) certainly follow the terrestrial magnetic field to their food. In any discussion of the relevance of magnetism in the life sciences today, one application stands head and shoulders over all rivals: nuclear magnetic resonance (NMR). Both in its potential for biochemical analysis and diagnostic medical imaging, NMR is likely to become one of the most powerful tools ever developed. NMR machines will be commonplace in hospitals within a decade.

Principles of NMR

All atomic isotopes with an odd number of nucleons in the nucleus ($^1$H, $^{13}$C and $^{31}$P are biologically significant examples) have a fixed naturally occurring magnetic moment. They behave like tiny bar magnets. In strong magnetic fields such nuclei can, in accordance with the laws of quantum mechanics, take up only a limited number of possible orientations. The proton ($^1$H) for example, can be aligned only parallel or antiparallel with the field.

![Figure 6.4 Possible orientations of a proton](image_url)

Each orientation has a definite magnetic energy in the field. When photons of energy exactly equal to the energy difference between two orientation states are delivered to the sample, resonance occurs. The atomic nuclei flip between the orientation states resonantly, and in doing so absorb energy from the
incident radiation field. *The energy difference, and hence the resonant frequency, is proportional to the magnetic field strength.*

It turns out that if the sample is placed in a strong magnetic field (of order 1 T) the photons of the requisite energy are in the radio range of frequencies. Most chemicals, and in particular the materials that comprise living animals and plants, are transparent to radio waves, and equally important, are not harmed by them at a low energy densities. So, by placing the creature, plant or sample in an appropriately strong magnetic field, and measuring the absorption of the resonant frequency radio waves, it is possible to measure the amount of hydrogen ($^1$H) in the sample. Other atoms can be studied similarly, as they have different resonant frequencies in the same magnetic field. It is this basic process which has been refined to such a remarkable degree in NMR.

### 6-6 NMR SPECTROSCOPY

To understand NMR spectroscopy, consider just two protons in different locations in an organic molecule. The magnetic field at proton A is the vector sum of the externally applied field and the local field of the molecule at point A. In general the local field of the molecule at B will be different. Hence protons A and B are in different total magnetic fields and therefore have different NMR resonant frequencies. So the NMR resonant frequency of each proton in a molecule is different.

Although the magnetic field differences, and hence the energy differences, are small they are readily distinguishable. Frequency is one of the most precisely measurable of all physical quantities. By sweeping the radio frequency through a range (or more usually keeping the frequency constant and sweeping the magnetic field) the resonance of each proton in a molecule is excited in turn. The result is presented as a spectrum which is characteristic of each substance.

Figure 6.5 Protons in a molecule

Figure 6.6 does not show the means for sweeping the magnetic field or the oscillator frequency. Nor does it show the separate coil often used for detecting the resonance.

One of the many applications of NMR spectroscopy is in monitoring the progress of biochemical reactions in living things. The upper trace in figure 6.7 is the NMR spectrum of a peptide in living human red blood cells and the lower trace is the NMR spectrum of its hydrolysis products. By watching the spectrum gradually change from one form to the other, it is possible to measure the progress of the reaction in the patient quite harmlessly.
6-7 NMR IMAGING

In NMR spectroscopy, an important consideration is to make the external magnetic field as uniform as possible throughout the volume of the sample, so that all the molecules are in the same magnetic field and the resulting spectral lines are sharp. By contrast, in NMR imaging, the externally applied magnetic field is deliberately made not quite uniform. Then nuclear magnetic resonance occurs only in those regions where the magnetic field strength is exactly the resonance value. The NMR response then measures the total number of protons in that region.

By changing the applied magnetic field, the number of protons in other regions of the patient is measured. By suitably varying the magnetic field, the region of resonance is scanned through the patient, and corresponding measurement of the strength of the resonance is recorded to build up an image of the scanned region. In practice the production of high quality images is achieved by precise computer control of the scanning magnetic field and computer synthesis of the image. The images can be remarkably clear,
and in many soft-tissue examinations these are much better than x-ray images, which are shadows rather than true images.

It is interesting to note that the production of indirect images by computer synthesis was developed initially in radio astronomy; it is physically impossible to build radio telescopes large enough to form real-time images of sufficient resolution. Modern computer technology has made indirect imaging possible in many other disciplines. In medicine computerised axial tomography (CAT) and more recently NMR imaging have the potential to go beyond the two-dimensional images of astronomy. In medicine, CAT and NMR techniques can build full three-dimensional images.

### QUESTIONS

**Q6.1** Find the short wavelength limit for x rays from

(a) a television tube operated at 10 kV,
(b) an x ray tube operated at 100 kV,
(c) a 20 MeV betatron,
(d) a 1 GeV linear electron accelerator.

**Q6.2** From the energy level diagram for molybdenum calculate the approximate energies of the Kα and Kβ x-ray photons. What are the approximate wavelengths of the two x rays? Compare your results with the experimental curves. What minimum potential difference across an x ray tube is required to excite these x rays? Is this consistent with the experimental curves?