# E7 ELECTROMAGNETIC INDUCTION

## OBJECTIVES

### Aims

By studying this chapter you should get to understand the nature of the two kinds of electromagnetic induction, the differences between them and their common features. To demonstrate this understanding you should be able to explain the principles of electromagnetic induction and discuss examples of each of the two types.

### Minimum learning goals

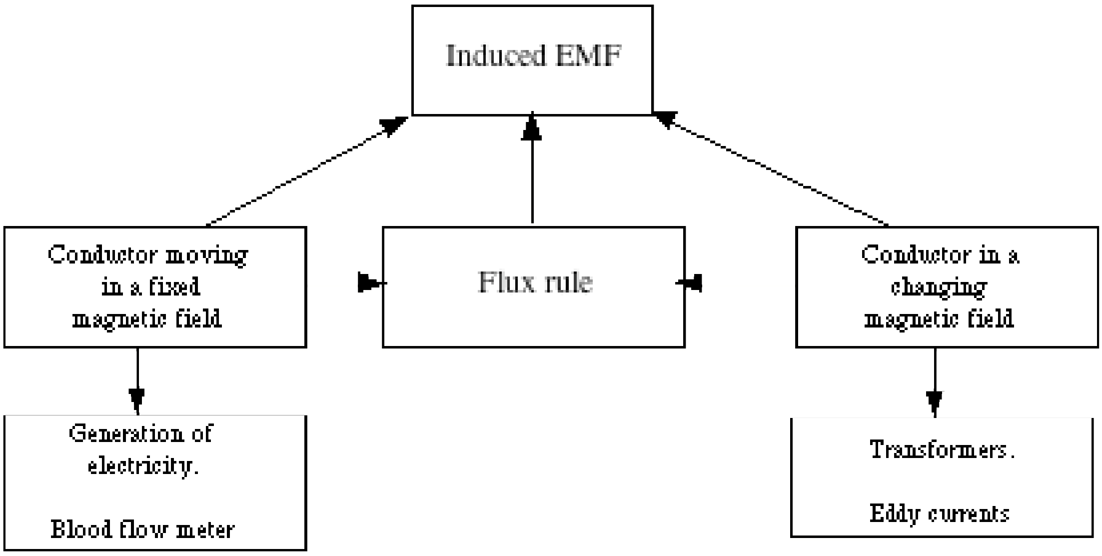
When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms

   *electromagnetic induction, EMF, induced EMF, motional EMF, magnetic flux, transformer, primary, secondary, eddy current.*

2. Describe and explain the two processes of electromagnetic induction
3. State and apply the result $\mathcal{E} = vBl$ for a moving conductor and specify the sense of the EMF.
4. Recall and use the formula for the EMF in a fixed plane conductor induced by a changing magnetic field perpendicular to the plane of the conductor .
5. State, explain and apply the flux rule (Faraday's law) relating induced EMFs of both kinds to rate of change of flux.
6. Describe and explain the origin of eddy currents.
7. Sketch, and explain the principles of operation of, a simple electric generator, an electromagnetic blood flow meter and a transformer.

### CONCEPT DIAGRAM



# PRE-LECTURE

## 7-1 INTRODUCTION - INDUCED EMF

Electromagnetic induction is a term used to describe the production of EMFs by two apparently quite different mechanisms: (1) the movement of a conductor through a region of space where there is a magnetic field and (2) the existence of a changing magnetic field in some region of space. In the first case, charged particles within a moving conductor experience magnetic forces which produce a charge separation which in turn creates a potential difference. In the second mechanism, a changing magnetic field creates an electric field - even in empty space. This induced electric field is not an electrostatic field because its field lines don't start and end on charges - but if some conducting matter is brought into the space the induced electric field can produce a charge separation and a measurable potential difference.

You should recall what is meant by EMF - see for example §5-2 in chapter E5. A source of EMF is something that is capable of giving energy to a system of charged particles by pulling the positive and negative particles apart. The value of the EMF is defined to be the energy supplied per charge. When the separated charges are in equilibrium and there is no current in the system, the potential difference produce by the separated charges is equal to the EMF.

It turns out that both of the effects called electromagnetic induction can be described by the same mathematical law, which we call Faraday's law after the person who probably did most to elucidate electromagnetic induction experimentally.

# LECTURE

## 7-2 VOLTAGE INDUCED IN A MOVING CONDUCTOR

When a length of wire is moved or deformed in a magnetic field, a voltage appears between the ends of the wire. The existence of the voltage depends essentially on the movement of the wire: when the wire is not moving there is no voltage. However the factors which determine the magnitude and sense of the voltage are not obvious. They are explored by means of a number of experiments in the video lecture.

### Demonstrations

A brass rod is driven at a steady speed between the poles of a strong magnet. The rod, the magnetic field and the direction of motion are all perpendicular to each other. When the rod is moved through the magnetic field there is an **induced voltage**; when the motion stops the voltage disappears.

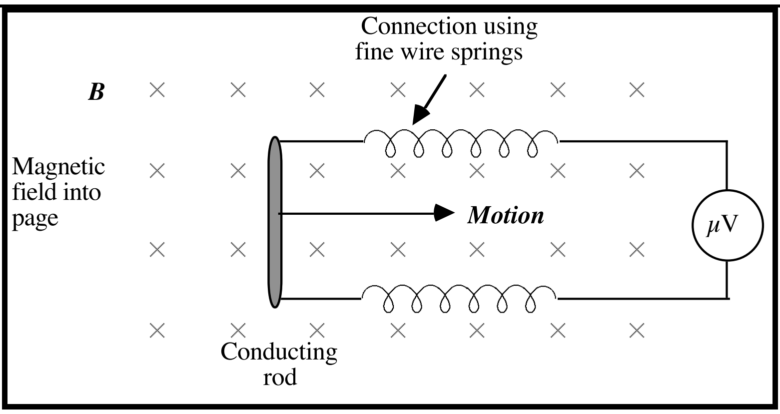


**Figure 7.1 Demonstration of motional EMF**

When the velocity is reversed the same voltage, in the opposite sense, appears. When the conductor is then replaced with one of half the length, and it is driven through the magnetic field at the same velocity as before, the voltage registered is half the previous value. Careful measurements confirm that for the same speed and magnetic field the induced voltage is proportional to the length of the conductor.

For the next experiment, the conducting rod is attached to a pendulum, and the induced voltage is displayed on a chart recorder. As the rod moves backwards and forwards through the magnetic field, the voltage indicated by the chart recorder has its greatest magnitude when the speed is greatest and is zero at the extremes of the swing when the speed is zero. As the pendulum loses energy, its speed at the middle of the swing decreases and the amplitude of the induced voltage falls correspondingly.

The experiment is repeated with a shorter pendulum using the same initial displacement. The speed and corresponding induced voltage are greater. Careful measurements establish that the magnitude of the induced voltage is proportional to the speed of the conductor, and the sign of the voltage reverses when the conductor moves in the opposite direction.

The same pendulum apparatus is used to show that the induced voltage depends on the magnetic field strength. To do this, the pole pieces of the magnet are removed, increasing the gap and reducing the magnetic field strength to about half its previous value. When the pendulum is released with the same amplitude of swing as before, the chart recorder shows that the amplitude of the induced voltage is reduced. Other quantitative experiments show that, for a given speed, the induced voltage is proportional to the magnitude of the magnetic field.

### The voltage induced in any moving conductor

Combining the results of all these and other experiments yields the result that the voltage $V$ induced in a conductor of length $l$ moving with speed $v$ perpendicular to the length of the conductor and perpendicular to the magnetic field is

$$V = vBl$$

where $B$ is the strength of the magnetic field. If the velocity makes an angle $\theta$ with the field, the induced voltage is reduced by a factor of $\sin\theta$, so

$$V = vBl\sin\theta$$

The potential differences appearing across the conductors in these experiments are a consequence of the **induced EMF** within the conductor.

### Theory of motional EMF

Consider a metal rod moving through a magnetic field, like the one used in the demonstrations.

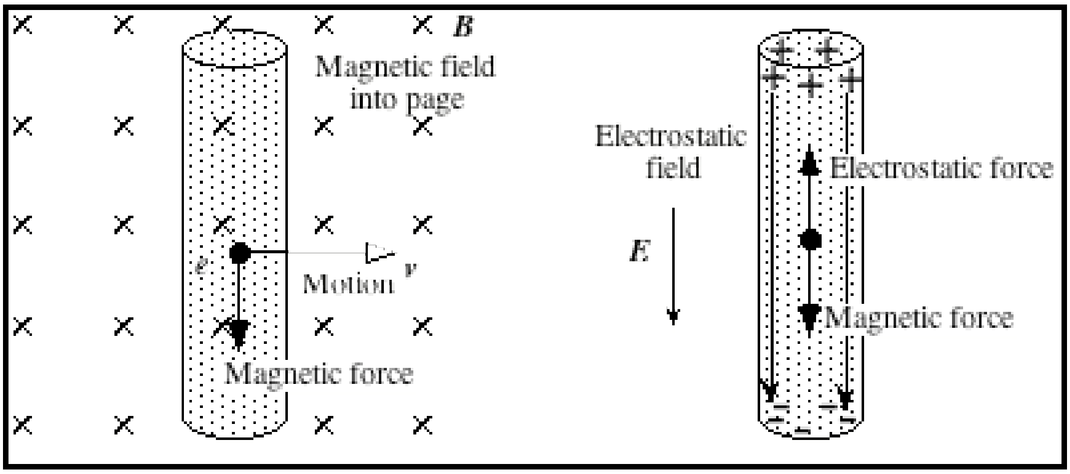


**Figure 7.2 Origin of motional EMF**

The conduction electrons in the rod, which are moving through the magnetic field, experience a magnetic force which is perpendicular to both their motion and the field. This force pulls electrons towards one end of the rod (left side of figure 7.2). Electrons will start to move, under the influence of the magnetic force and other interactions with particles in the rod. The effect of that movement is to produce a charge separation which immediately creates an electrostatic field that tries to pull the electrons back (right hand diagram in figure 7.2). An equilibrium is soon established in which there is a balance between the electrostatic and magnetic forces. Consider an electron somewhere in the rod (not at the ends). The electrostatic and magnetic forces on it must be balanced, so $eE = evE$. Cancelling the charge $e$ you can see that the magnitude of the electrostatic field is

$$E = vB.$$

There is a potential difference $V$ associated with the electrostatic field. Since the field is equal to the (negative) potential gradient we can say that the potential difference ($V$) between two points separated by a distance $l$ and the magnitude of the average electrostatic field are related thus:

$$E = \frac{V}{l} .$$

By comparing the two expressions for the electric field we get a relation for the voltage, $V = vBl$. Since we are considering a case where there is no current, the potential difference must be equal to the EMF which caused it so we can say that

$$\mathcal{E} = vBl . \qquad \text{... (7.1)}$$

Remember that this relation applies only to long thin objects with their long axis, their velocity and the field all mutually perpendicular.

## 7-3 APPLICATIONS OF MOTIONAL EMFS

### Generation of electrical power

The design of electrical generators, which depend on EMFs induced in conductors moving through regions of magnetic field, is a highly specialized branch of engineering. Demonstrations in the lecture illustrate how the simple idea of moving a wire near a magnet has been exploited and refined to produce efficient generators of hundreds of megawatts of electrical power.

### An electromagnetic blood flow meter

In studies of cardiovascular physiology, one of the most fundamental quantities is the flow rate of arterial blood. One method for measuring blood flow rate in experimental animals was developed in Australia, and is now used by physiologists world-wide. It uses the EMF induced in the flowing blood.

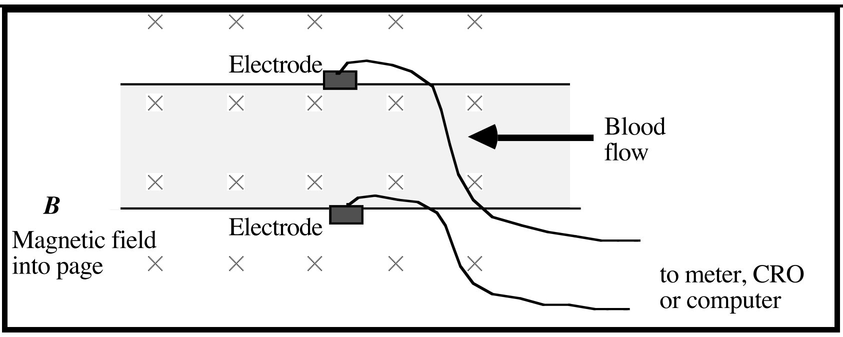


**Figure 7.3 Principle of the electromagnetic flow meter**

A magnetic field (of about $10^{-3}$ T) is set up across the artery, and connection is made to two small electrodes in contact with the artery (the line joining the electrodes is perpendicular to the magnetic field). From equation 7.1 the average speed of the blood is thus

$$v_{\text{blood}} \approx \frac{V}{Bd}$$

there being some uncertainty as the speed varies radially in the artery.

The probe of such an instrument consists of a plastic moulding which contains coils to generate the magnetic field and electrodes to sense the EMF. See figure 7.4.

The probes are made in various sizes, as the artery has to fit properly to make good contact with the electrodes. The probes can be calibrated quite accurately: the observed EMF is readily converted to blood flow rate in the practical unit of $\text{mL.min}^{-1}$. The instrument can either show the mean flow rate on a meter, or display the variation during the pulse on an oscilloscope or chart recorder.

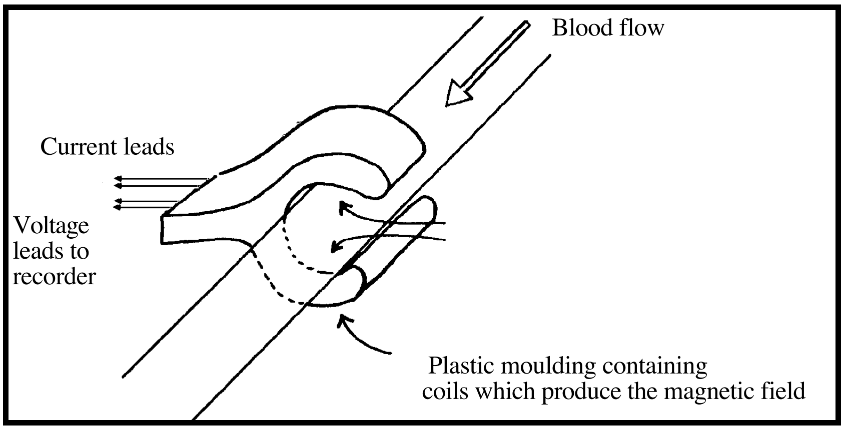


**Figure 7.4 Construction of the blood flow meter**

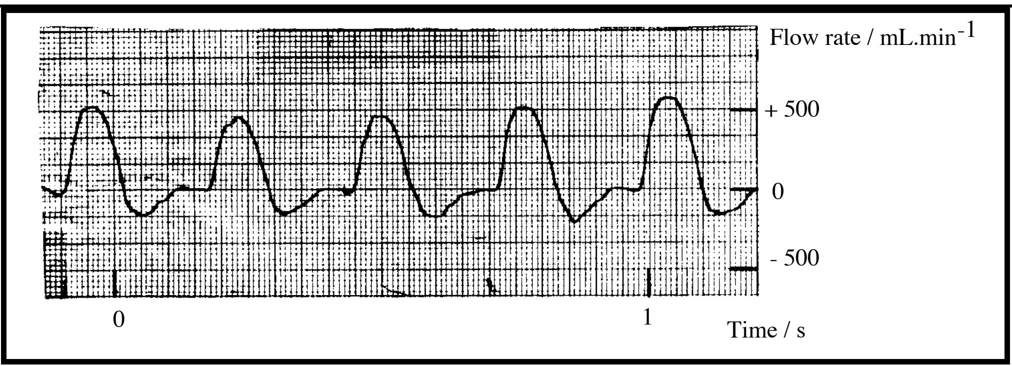


**Figure 7.5 Recording of blood flow in the aorta of a small mammal**

## 7-4 VOLTAGES INDUCED BY CHANGING MAGNETIC FIELDS

### Moving coils and moving magnets

When a coil of wire is moved near a magnet, a motional EMF is induced in the coil. This has already been explained in terms forces on moving charges. However if the coil is held stationary, and the magnet is moved (in the opposite direction with the same speed) exactly the same voltage is induced, even though the conductor is at rest. That is a bit of a puzzle because the explanation of the induced EMF given in §7-2 depends explicitly on having a moving conductor.

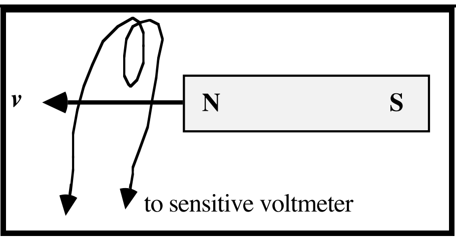


**Figure 7.6. EMF induced by a changing magnetic field**

Nevertheless the effect is real. It turns out that it is the *relative* velocity of the coil and magnet that matters. However, think of this second experiment from the point of view of an observer at the coil who cannot see the magnet. That observer would relate the induced EMF to an increasing magnetic field at the coil. This argument suggests (but does not prove) that EMFs can be produced by a changing magnetic field.

### Demonstrations: experiments with coils and changing currents

A digitally controlled function generator is used to produce a current which varies with time in a controllable way. The current is passed through a coil of about 100 turns, and is displayed on the upper pen of a chart recorder. A second, smaller diameter, coil with a similar number of turns is placed coaxially inside the larger coil. This inner coil is connected to the lower pen of the chart recorder.

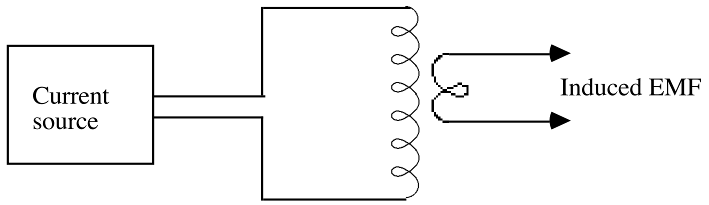


**Figure 7.7 Apparatus for studying EMFs from changing fields**

By adjusting the current *I* in an irregular manner by hand, it is shown that an EMF is induced in the inner coil when, and only when, the current on the outer coil is changing. **The changing magnetic field of the outer coil induces an EMF in the inner coil.**

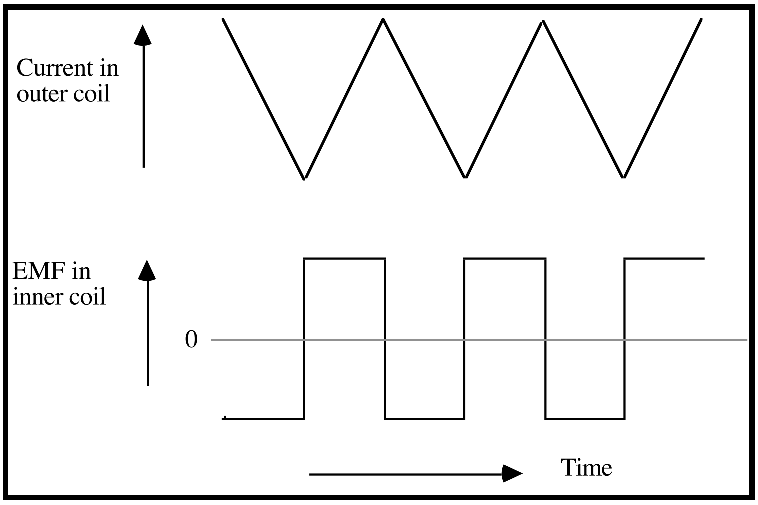


**Figure 7.8 Changing current and the induced EMF**

The function generator is next set to increase steadily, and then decrease the current, between fixed limits. The experiment shows that while the current in the outer coil is rising linearly, the induced EMF remains constant. When the current reverses, the induced EMF reverses.

Runs with the current changing twice and four times as rapidly show corresponding increases in the induced EMF. A further run in which the current is held briefly constant at its extreme values confirms that it is the rate of change of current, rather than the current itself, which determines the induced EMF.

These demonstrations show that the induced EMF in a coil is proportional to the rate of change of the magnetic field:

$$\mathcal{E} \propto \frac{dB}{dt} .$$

To produce an easily measured induced EMF in a single-turn coil, a more rapidly changing current is passed through the large coil. A sinusoidally varying current (at about 400 Hz) is used, and the output waveform is displayed on a CRO.

With this arrangement of the apparatus, three points are demonstrated.

- The voltage induced in a coil depends on the coil area
- The sense of the measured voltage changes when the coil is turned over.
- The induced voltage is proportional to the number of turns in the coil.

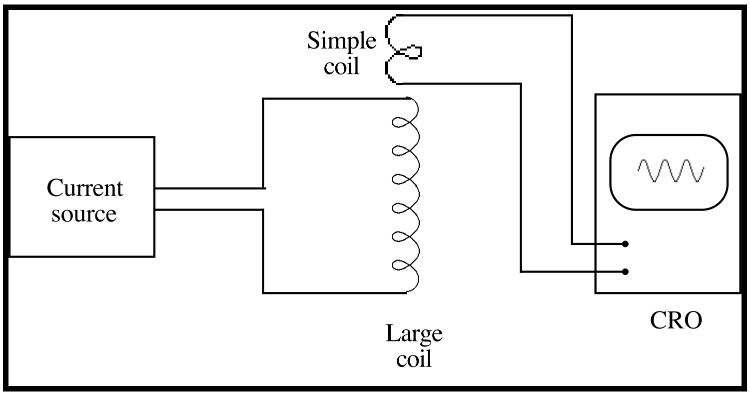


**Figure 7.9 Apparatus to display EMFs**

All these experiments are summarised by saying that if there is a uniform changing magnetic field perpendicular to a plane coil, the EMF induced in the coil is proportional to the product of the number of turns in the coil, its area and the rate of change of magnetic field inside the coil.

$$\mathcal{E} = \text{const} \times NA\,\frac{\mathrm{d}B}{\mathrm{d}t}\ .$$

In fact, as is discussed in the post-lecture, the constant in the above expression is 1 so

$$\mathcal{E} = NA\,\frac{\mathrm{d}B}{\mathrm{d}t}\ . \qquad \text{... (7.2a)}$$

## 7-5 THE FLUX RULE

For a single turn flat coil perpendicular to a uniform but changing field the induced EMF can be written

$$\mathcal{E} = \frac{\mathrm{d}(AB)}{\mathrm{d}t} \qquad \text{... (7.2b)}$$

where $AB$, the product of the coil area and the component of the magnetic field perpendicular to the area, is called the **magnetic flux** ($\Phi$) through the coil.

To define magnetic flux in general, first imagine a surface, small enough so that you can say that the magnetic field doesn't vary over the surface. Find the component of the magnetic field perpendicular to the surface; that gives $B\cos\theta$, where $\theta$ is the angle between the field and a direction perpendicualr to the surface. The flux through the surface is the product of that component and the surface area, $\Delta A$:

$$\Delta\Phi = B\,\Delta A\cos\theta \qquad \text{... (7.3)}$$

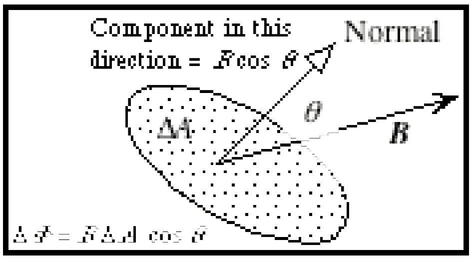


**Figure 7.10 Magnetic flux**

To find the flux through a larger surface, you just repeat the process and add (or integrate) the flux for each piece. The SI unit of magnetic flux is the called the weber (Wb).

To visualise the meaning of magnetic flux, think of it as the number of magnetic field lines passing through the area.

The equations (7.2a,b) for the EMF induced in a flat coil represent a special case of the general law, known as the flux rule or Faraday's law:

$$\mathcal{E} = N\,\frac{\mathrm{d}\Phi}{\mathrm{d}t}\ ; \qquad \text{... (7.4)}$$

the EMF induced in a coil is equal to the rate of charge of magnetic flux through one turn, multiplied by the number of turns in the coil.

This way of thinking is very powerful because it provides the link between EMFs induced in moving conductors and EMFs induced by changing magnetic fields.

## 7-6 APPLICATIONS OF EMFS FROM CHANGING MAGNETIC FIELDS

Devices, instruments and machines which depend on induction of voltages by changing magnetic fields are commonplace. Some examples are: metal detectors, traffic control sensors, hi-fi cartridges, video and audio tape recorders, cash dispensing machines, electric guitars and microwave ovens. Two examples, transformers and eddy currents, are considered in a little more detail.

### Transformers

The simplest **transformer** consists of two separate coils of wire, insulated from each other, both wound on the same iron core. In circuit diagrams a transformer is represented with the coils and core separated for clarity as in figure 7.11.

An alternating voltage applied to one winding, the **primary**, causes an alternating current, thereby producing an alternating magnetic flux in the core. This alternating flux in the core induces a voltage in the **secondary** winding. The secondary voltage depends on the ratio of the number of turns in the primary and secondary coils. For an ideal transformer,

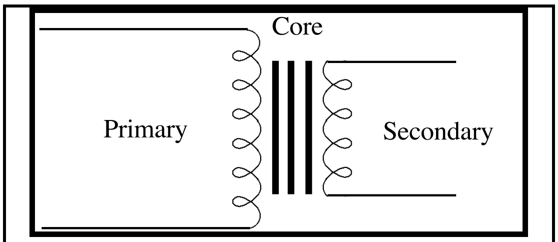


**Figure 7.11 A transformer**

$$\text{secondary voltage} = \text{primary voltage} \times \frac{\text{no. of secondary turns}}{\text{no. of primary turns}} \; .$$

Physically, transformers range from the huge oil-cooled ones used to step up the voltage of electrical power for economical transmission to tiny air-cored ones used in the tuned circuits of radio receivers. Apart from their function in changing AC voltages and in matching power sources to their loads, transformers play an important role in supplying electrical energy to devices without any direct conducting connection. Such isolation may literally be of vital importance when electrical equipment is used in treating and monitoring patients in hospital.

### Eddy currents

In a changing magnetic field, voltages will be induced not only in coils, but in any conductor in the field. As a result loops of current called **eddy currents** will be set up in the conductor. Often they are a nuisance; for example the cores of transformers and motors have to be made of insulated layers (laminations) of iron to prevent eddy currents in the core, and consequent waste of energy.

Once an eddy current exists, it experiences a force in the magnetic field, producing striking and useful effects. In the video lecture, a swinging sheet of non-magnetic metal (aluminium) is rapidly brought to rest by a magnet. This effect, called **eddy current damping**, is produced by the magnetic force on the eddy currents.

The same mechanism explains the operation of one common type of speedometer. A rotating magnet induces eddy currents in an aluminium disk. The magnetic force on these currents produces a torque on the disk, and turns it. A spring exerts a restoring torque on the disk so that the angular deflection is proportional to the angular velocity of the magnet.

# POST-LECTURE

## 7-7 THE FLUX RULE

In this chapter we have considered separately two distinct types of electromagnetic induction. While these two phenomena appear to be physically different they are closely related. Indeed one law, known as the **flux rule** can be used to correctly calculate the EMF, whether it is induced by a changing flux linkage or a conductor moving in a magnetic field. The basis for the rule is worth studying because it gives a deeper understanding of electromagnetic induction than is possible in the video lecture.

Consider a pair of conducting rails connected at one end to form a U-shape. The plane of this conductor is perpendicular to a uniform magnetic field (figure 7.12).

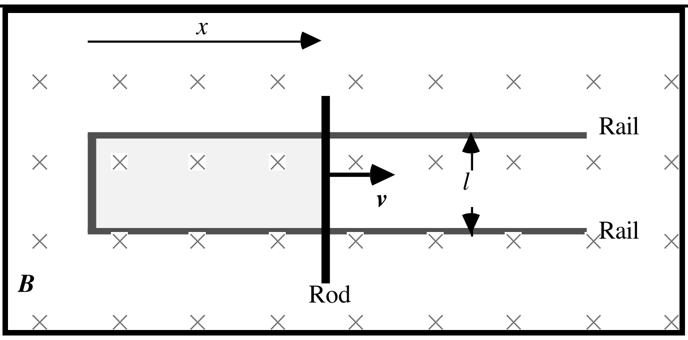


**Figure 7.12 Motional EMF and the flux rule**

The crosses indicate that the magnetic field, magnitude *B*, is directed down into the page. A conducting rod (solid line) rests on the parallel conductors, making contact with them.

Now think what happens when the conductor is moved at velocity **v** to the right. We calculate the EMF induced in the shaded conducting loop by two methods.

1) The rod is moving so the EMF induced in it by its motion across the magnetic field is

$$\mathcal{E} = Blv$$

and this is thus the EMF induced in the loop.

2) The conducting loop around the shaded area is a single turn coil in the magnetic field *B*. The magnetic flux linking this coil is

$$\phi = Bxl$$

where $xl$ is the area enclosed by the conducting loop.

The EMF can be calculated from the rate of change of this linking flux:

$$\mathcal{E} = \frac{d\phi}{dt} = \frac{d(Bxl)}{dt} .$$

Since *B* and *l* are constant, this yields

$$\mathcal{E} = Bl\frac{dx}{dt} = Blv .$$

In this example, the induced EMF can be worked out either from the EMF in a moving conductor (equation 7.1) or from the changing flux linkage (equation 7.4). This leads to a statement of the flux rule for calculating induced EMFs: **the induced EMF is equal to the rate of change of magnetic flux linking the circuit.**

Remember that when we are dealing with coils with many turns (*N* say) the flux links each turn, so the EMF is *N* times that for a single turn.

## 7-8 QUESTIONS

**Q7.1** In the video lecture, one demonstration shows that a voltage of 50 $\mu$V is induced between the ends of a rod of length 40 mm when it moves at 10 mm.s$^{-1}$ perpendicular to its own length between the pole pieces of a powerful magnet. Calculate the strength of the magnetic field.

**Q7.2** Sketch the essential parts of an electromagnetic blood flow meter. One such instrument generates a transverse magnetic field of $1.0 \times 10^{-3}$ T. When it is used on an artery of internal diameter 5.0 mm, the induced EMF is 2.5 $\mu$V. Calculate the blood flow rate assuming a plane velocity profile.

**Q7.3** A coil is moved at a constant velocity towards one pole of a bar magnet. When the coil is 0.10 m from the magnet, the EMF induced in the coil is 40 mV. The coil has 20 turns and area $1.0 \times 10^{-5}$ m$^2$. What EMF would be induced in the following experiments?

- The coil is held stationary, and the magnet is moved with the same velocity away from the coil at the moment when separation is again 0.10 m.
- Both coil and magnet are moved in the same direction, at constant separation of 0.10 m.

When the EMF of 40 mV is induced in the stationary coil, what is the average rate of change of magnetic field at the coil?

**Q7.4** Explain how the strength of an alternating magnetic field can be measured using a small coil having a large number of turns (a search coil).

Such a coil has 1000 turns and a diameter of 20 mm. In a 50 Hz magnetic field an AC voltage of 1.0 V (peak-to-peak) is observed. What is the amplitude of the 50 Hz magnetic field?

**Q7.5** Briefly explain two uses for transformers. If the magnetic flux in a particular transformer is changing at the rate of 1.0 T.m$^2$.s$^{-1}$, what EMF would be induced on a winding of 500 turns?

**Q7.6** One type of metal detector works by inducing eddy currents in the treasure and then detecting the magnetic field of these currents. Which of the following do you think it would discover most readily, given that gold has a higher resistivity than silver:

1) 1 kg of gold Spanish doubloons in an old sack
2) a 1 kg lump of gold,
3) a 1 kg lump of silver?

Give your reasons.

**Q7.7** A flat sheet of copper is placed between the poles of a strong magnet. When somebody tries to pull it out, there is a strong force resisting the motion. Explain what is happening. How can the force be reduced?