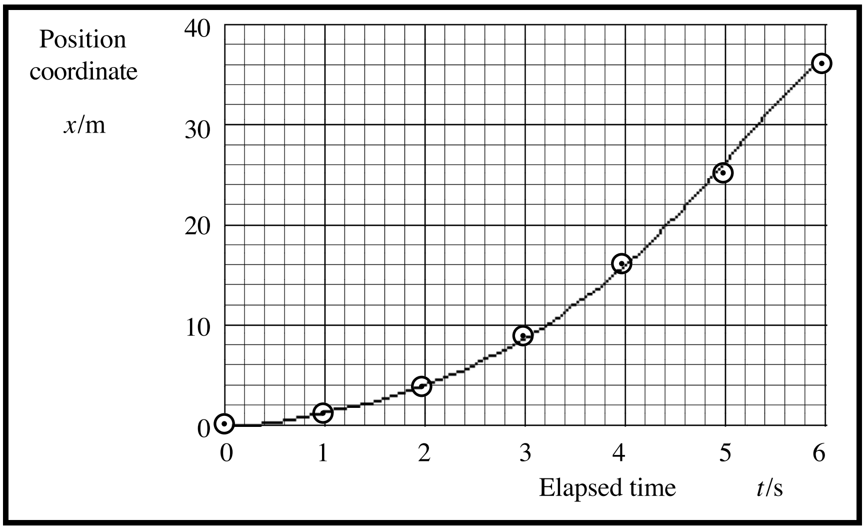
ANSWERS

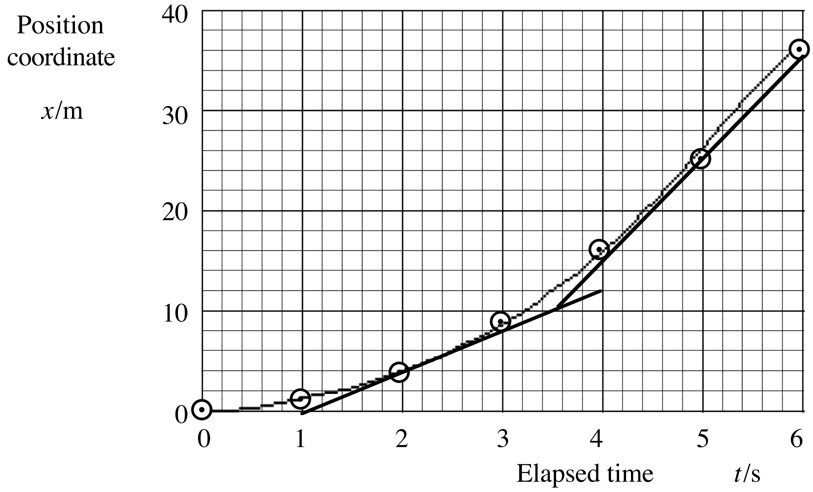
CHAPTER FE1

1.1



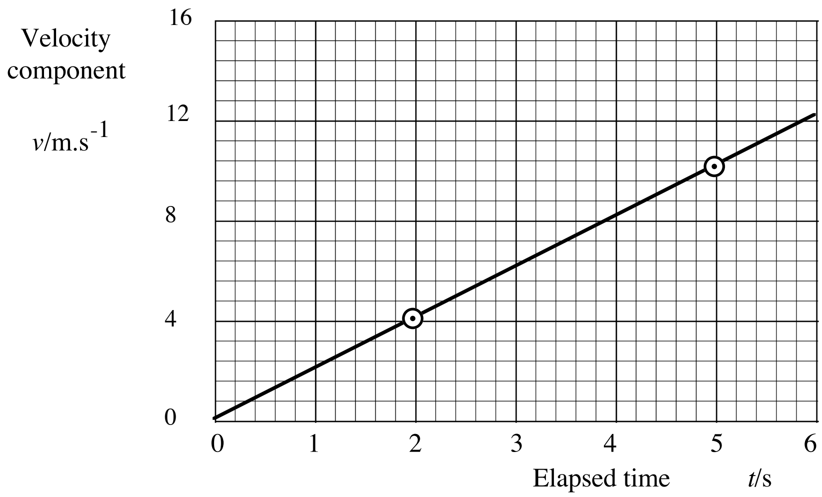
1.2 Average velocity = $\dfrac{\text{change in position}}{\text{time taken}} = \dfrac{25\ \text{m} - 4\ \text{m}}{5.0\ \text{s} - 2.0\ \text{s}} = 7.0\ \text{m.s}^{-1}$.

1.3 Draw tangents to the graph at 1.0 s and 5.0 s.



Measure the slopes of these tangents to get v at the times concerned: 4.0 m.s^{-1}, 10 m.s^{-1}.

1.4 Plot the values from Q1.3 to get this graph.



The slope of this graph is 2.0 m.s^{-2}

1.5 a) <u>0 - 1 s</u>: The car is being pushed. The magnitude of its velocity (i.e. its speed) increases at a constant rate. The acceleration is about 1.0 $m.s^{-2}$.

<u>1 - 5 s:</u> The car is rolling downhill. Its speed is still increasing but at a slower rate. The acceleration component is smaller, about 0.5 $m.s^{-2}$.

<u>5 - 10 s.</u>: The car is in gear, being driven at constant velocity. The acceleration is zero.

<u>10 - 15 s:</u> The brakes are applied. The forward velocity component decreases and the component of acceleration is negative.

As the brakes are eased off, the forward velocity decreases at a lower rate, i.e. the magnitude of the (negative) acceleration component is less.

Finally as the brakes are applied hard, the velocity component decreases at a greater rate until the car stops; it has a negative acceleration component with a greater magnitude.

b) At the instant when the velocity is zero, the displacement from the starting point (the position co-ordinate) is a maximum. When the velocity component is negative, the extra displacement during each short time interval is also negative so the position component decreases. Compare the displacement-time graph (figure 1.12) and the velocity-time graph (figure 1.11).

When the displacement is zero the area above the time axis equals the area below the time axis. So the positive and negative contributions to the displacement add to give a total of zero.

1.6 0 - 1 min: constant acceleration component until the car reaches 40 $km.h^{-1}$.

1 - 2 min: constant velocity component (40 $km.h^{-1}$).

2 - 3 min. constant acceleration component until the car stops.

3 - 4 min: the car now travels in the opposite direction with a constant acceleration until it reaches 80 $km.h^{-1}$.

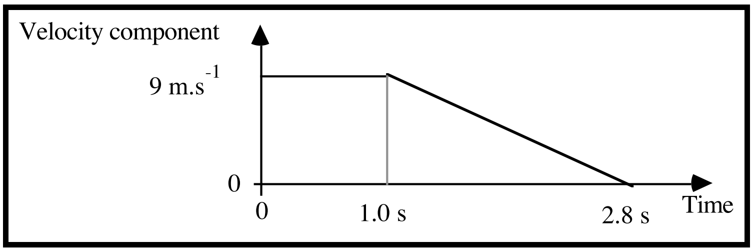
4 - 5 min: constant acceleration component in the direction opposite to the velocity until the car stops.

Areas above and below the time axis are the same so the total displacement for the trip is zero. In other words the car has arrived back at its starting point.

1.7 120 $km.h^{-1}$.

The answer is *not* 80 $km.h^{-1}$. You cannot average the speeds, since the time intervals for each half of the journey are unequal. The driver has taken 15 minutes of the allotted 20 minutes to cover the first 10 kilometres. The car must travel three times as fast for the last 5 minutes.

1.8 a)



$$\text{Time to stop car} = \frac{\text{change in velocity component}}{\text{constant acceleration}}$$

$$= \frac{-9\ m.s^{-1}}{-5\ m.s^{-2}} = \frac{9}{5}\ s.$$

Displacement is given by the area between the graph and the time axis (rectangle plus triangle).

$$\text{Displacement} = (1\ s) \times (9\ m.s^{-1}) + \frac{1}{2} \times \left(\frac{9}{5}\right) s \times (9\ m.s^{-1}) = 17\ m.$$

Since the time is very approximate, call this 20 m.

b) For an initial speed of 18 $m.s^{-1}$, the time to stop the car is $\frac{18}{5}$ s and the displacement is 50 m. (All calculations follow the same scheme as the 9 $m.s^{-1}$ situation.)

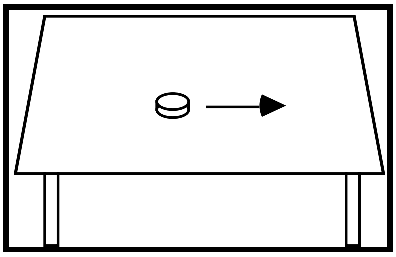
Notice the three-fold increase in stopping distance for a doubling in speed.

1.9 a) Football, butterfly, etc.

b) Car speeding up or slowing down on a straight road, etc.

c) The tip of the blade on an electric fan, etc.

1.10



a) It will move faster to the right. (It has an acceleration to the right.)

b) It will slow down. If the force is large enough or applied for long enough, the object will slow down, stop momentarily, and speed up in the opposite direction (to the left). (It has an acceleration to the left.)

c) It will move in a curve towards you with increasing velocity component towards you. The sideways component of velocity remains unaltered.

1.11 a) The force is zero.

b) The force is directed to the right.

c) The force is sometimes to the right, sometimes to the left.

d) The force is directed radially inward towards the centre of the circle.

e) The force is more complicated than the one in (d). It is equivalent to a force directed towards the centre of the circle (to keep the object on its circular path) together with a force tangential to the circle (which changes the speed of the object).

A ball, on the end of a string, being swung in a *vertical* circle is an example of this type of motion.

1.12 a) At $t = 0$, the object is at $x = A$.

$$v = B + 2Ct + 3Dt^2.$$

$$a = 2C + 6Dt .$$

b) At $t = 0$, the object is at $x = A$.

$$v = -\omega A \sin(\omega t) .$$

$$a = -\omega^2 A \cos(\omega t).$$

1.13 a)

$$v = ct + v_0 .$$

$$x = \frac{1}{2} ct^2 + v_0 t + x_0 .$$

b)

$$v = -\frac{1}{2} kt^2 + v_0 .$$

$$x = -\frac{1}{6} kt^3 + v_0 t + x_0 .$$

CHAPTER FE2

2.1 a) The gravitational force on an object is its weight.

The electromagnetic force that the object exerts on a nearby charged object is zero for the following reason. Each electrically charged particle which goes to make up the object does exert a force on the nearby charge. This force is either repulsive or attractive depending on whether the two charges, the one on the object and the other are like or unlike. Usually the object is electrically neutral; i.e. the amounts of each type of charge within it are equal, so the repulsive and attractive forces effectively cancel each other. Gravitational forces, on the other hand, are always attractive and they can add to give a large total attractive force.

An example of an object which is not electrically neutral is a comb or ball point pen which, after being rubbed against hair or fabric, can pick up pieces of paper. In this case the electrostatic force is greater than the gravitational force.

A related example is a magnet which can lift an iron object. In this case the magnetic force is greater than the gravitational force.

b) Besides the difficulties associated with the small size of the nucleus, it was more difficult to break up the nucleus because of the strength of the nuclear forces.

c) Nuclear forces are comparatively unimportant. Electromagnetic forces are most important since they hold organisms together and determine the chemical processes within them. For large organisms gravitational forces are also important.

2.2 a) There exist forces between molecules.

b) Each molecule exerts an attractive force on neighbouring molecules. If the load on the wire is increased, the molecules are pulled further apart and the attractive force increases.

c) The force to pull the brick apart is greater. Does not this suggest that molecular or electromagnetic forces are stronger than the gravitational force?

2.3 a) Gravitational force = mass × acceleration due to gravity .

On the Earth, weight = $2 \text{ kg} \times 9.8 \text{ m.s}^{-2}$

= 20 N, approximately.

On the Moon, weight = 1/6 of this

= 3 N, approximately.

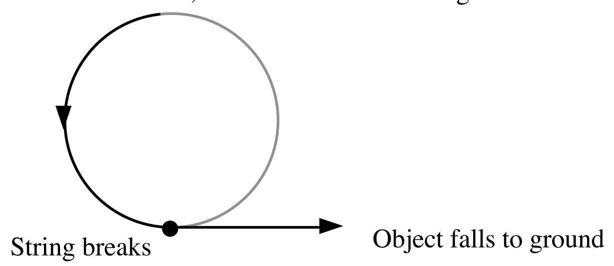
b) The extension of the spring is proportional to the gravitational force and so is proportional to the mass. The acceleration due to gravity is known and the scale on a spring balance is usually calibrated in terms of mass.

It could not be used to read masses on the moon without changing the values marked on the scale.

2.4 a) The force exerted on the object by the string is equal to the centripetal force;

$$F = m\frac{v^2}{R} ,$$

so if v is doubled, F will be four times as great.

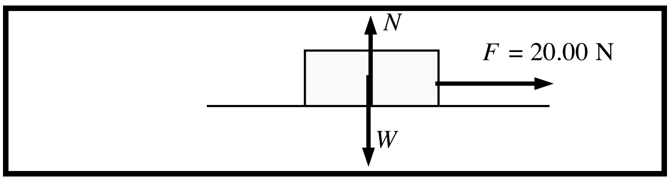


This is a top view. In fact the object falls to the ground.

b) The molecular forces that hold the wheel together.

c) The gravitational force exerted on the satellite by the earth.

2.5 a)



Horizontal component of total force = F to the right

Horizontal acceleration = $\frac{F}{m}$

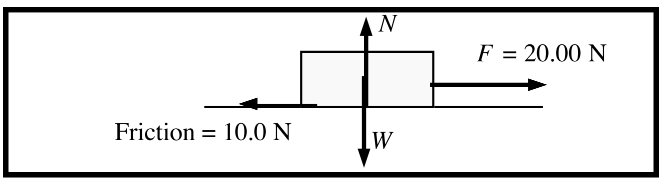
= 10.00 m.s^{-2} .

b) Vertical component of total force = $N - W$ upwards

The vertical acceleration is zero so the vertical component of the total force must be zero,

i.e. $N - W = 0$ or $N = W$.

c)

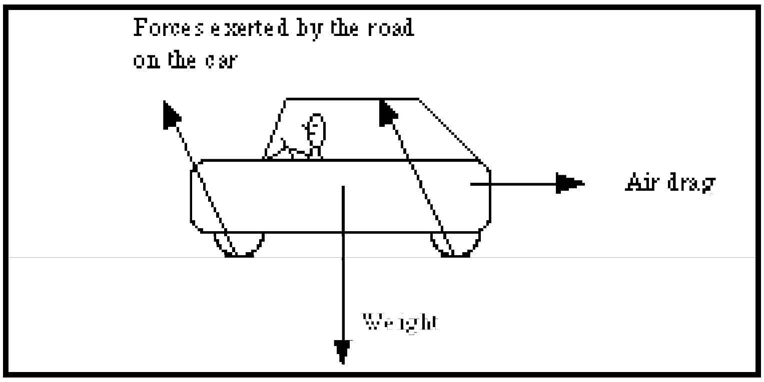


Horizontal component of total force = 20.00 N - 10.00 N

= 10.00 N.

Horizontal acceleration = 5.00 m.s^{-2}.

2.6 a)



The diagram above applies to a four-wheel drive car. When the car accelerates forward or cruises at constant velocity, the force exerted by the road on the driving wheels points forward. .

The contact force exerted by the road on the car (located at the car's wheels) can be described as a normal (vertical) component, magnitude N, and a horizontal frictional component, magnitude F. The frictional component is directed forward when the car is moving with constant velocity or when it is accelerating forward. The frictional force may be directed backwards when the brakes are applied and the car is slowing down (i.e. accelerating backwards).

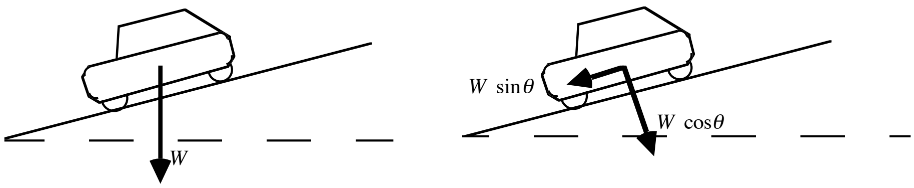
As long as the road is flat and horizontal the total vertical force is zero, so $N = W$.

If the car is moving at constant velocity, the total horizontal force is zero; so $F = D$, the magnitude of the air resistance (drag) force.

If the car is accelerating forwards, there is a net force component $F - D$ forwards.

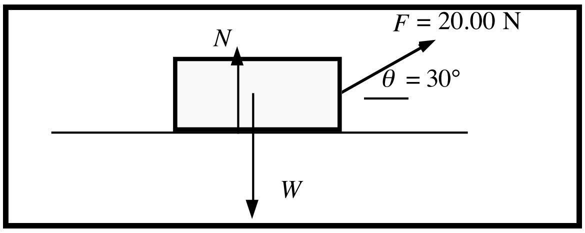
b) The total force that the road exerts on the car must provide a component directed towards the centre of the curve. This can be achieved by having a sideways component of the frictional force or by banking the road so that the normal contact force has a horizontal component, or by a combination of both.

c)



The downhill component is $W \sin\theta$.

2.7 a)



Horizontal component of total force,

$$F_H = F\cos(\theta) = 20.00\text{ N}\cos(30°) = 17.32\text{ N}.$$

Vertical component of total force, $F_V = N + F\cos(90° - \theta) - W$.

b) Horizontal component of acceleration $= \dfrac{F_H}{m} = 8.66\ \text{m.s}^{-2}$.

The vertical acceleration is zero because the vertical component of the pulling force, 20.00 N cos (90° - 30°) = 10.00 N, is smaller than the weight, 2.00 × 9.8 N = 19.6 N. The contact force N adjusts as F_V changes and is just sufficient to prevent a downward acceleration.

(... continued over)

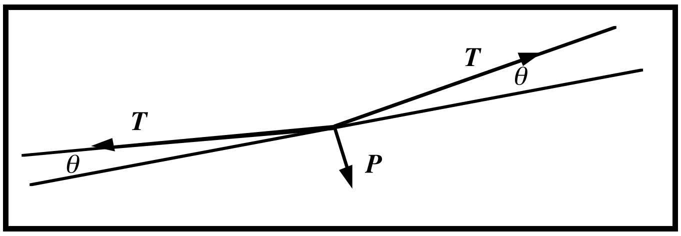
c) If F = 40.00 N in the same direction, the vertical component of the pulling force would be 20.00 N which is greater than the weight, 19.6 N. The upward acceleration would be 0.2 $m.s^{-2}$. The contact force disappears.

d) Only the horizontal motion would be changed. The horizontal acceleration would be less.

2.8 a) Total force component ↑ = 4.00 N + 8.00 N cos (90° + 30°) = 0 N.
Total force component → = 8.00 N cos 30° = 6.93 N.

b) A single force of 6.93 N in the ← direction or any number of forces which combine to give a single force equal to this.

2.9 a)



The magnitude of the force on the car equals the tension in the rope, T (and so is the magnitude of the force on the tree). If the car has not quite started to move, the resultant force on the piece of rope at the bend in the middle is zero. Taking force components perpendicular to the applied force:

$$P - T\cos(90°-\theta) - T\cos(90°-\theta) = 0 .$$

This gives $$T = \frac{P}{2\cos(90°-\theta)} .$$

If θ is small, $\cos(90°-\theta)$ is small and T is large, much larger than the applied force P.

b) If the rope stretches, θ may become too large and $\cos(90°-\theta)$ would no longer be small. The rope may break.

2.10 a) $$N = mg + ma .$$

Here a is the component of the acceleration in the direction vertically up. If the acceleration is downwards then a is negative and N is less than its usual value. The person's "apparent weight" is less than mg .

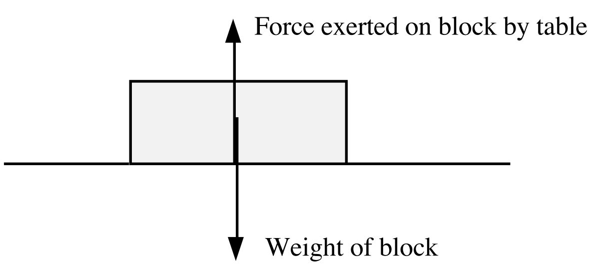
b) If the downward acceleration is equal to the acceleration due to gravity, then N is zero. This is "weightlessness".

"Weightlessness" also occurs in an orbiting spacecraft. The astronaut and the spacecraft both have the same acceleration towards the earth.

c) It may have something to do with the fact that the contact forces between the walls of the stomach and its contents will change. What do you think?

CHAPTER FE3

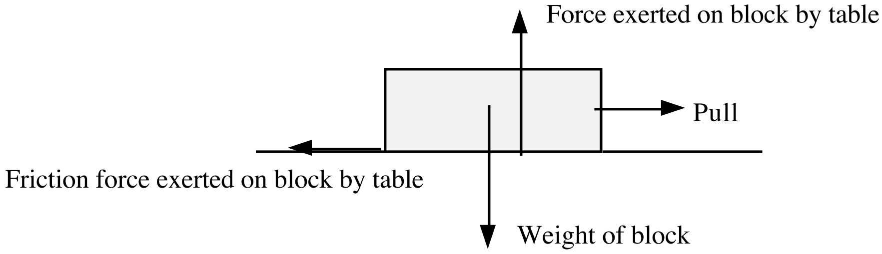
3.1 a)



The total vertical force must be zero. Hence the magnitude of the force exerted on the block by the table must be equal to the weight of the block. The sum of the horizontal force components must also be zero.

b) A frictional force of the same magnitude opposes the horizontal pull.

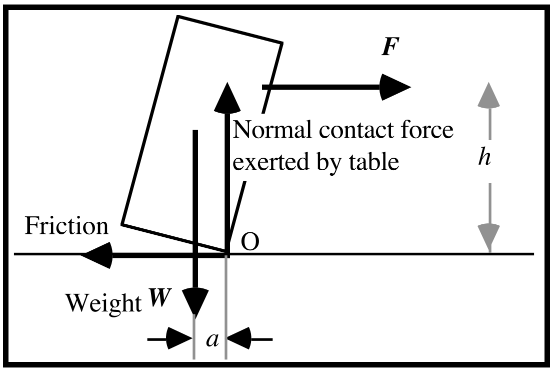
3.2



The total vertical force must be zero and the total horizontal force must be zero.

(Note that the location of the normal contact force has shifted a little. The reasons for this will be explored in this chapter but basically it happens because the total rotational effect of all four forces must be zero. Once you heave learned about torque you will see that the net torque of all four forces is zero.)

3.3 a)



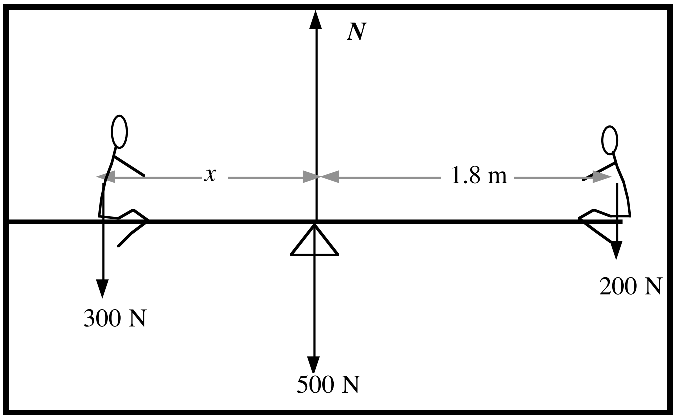
Suppose that the block has just started to tip or, in other words, rotate about point O. Consider torques to find whether, in fact, it will tip.

The vertical component, N, of the force exerted by the table and its horizontal frictional component both have zero torques about O. The block will tip if the torque about O due to the applied force F is greater than the torque about O due to the weight W, i.e. if Fh is greater than Wa.

The block could be pulled along if the force F were applied closer to the base so that Fh was no larger than Wa.

b) The rope should be tied near the base of the tractor, as in the second picture. When the tractor is pulling the rope, the rope exerts a force on the tractor which acts rather like the force in (a) above. If the point of application is too high the tractor will flip over backwards.

3.4 a)



The pivot exerts a force N on the seesaw:

$$N = (300 + 500 + 200)\text{N} = 1000 \text{ N} .$$

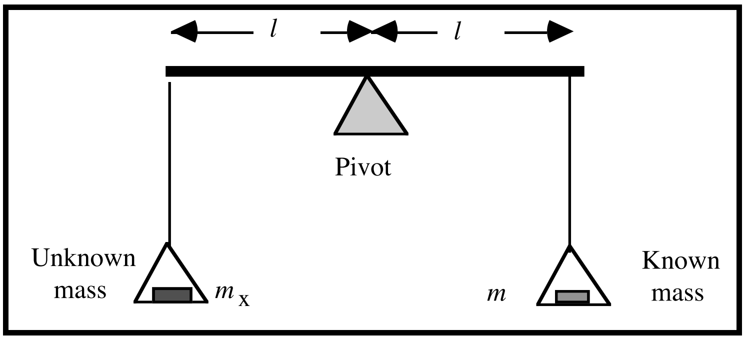
b) It is convenient to take torques about the pivot:

$$(200 \text{ N}) \times (1.8 \text{ m}) - (300 \text{ N}) \times x = 0 ;$$

$$x = 1.2 \text{ m} .$$

You could arrive at the same answer by taking torques about any point in the picture.

3.5



If the arms of the balance are equal in length, the condition that the total torque about the pivot is zero gives:

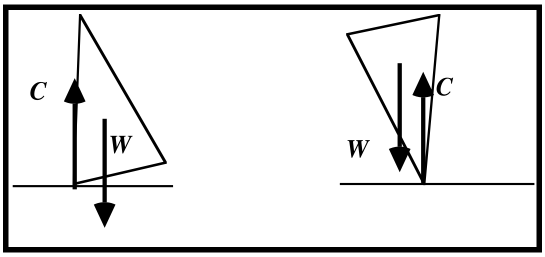
$$m\,g\,L - m_x\,g\,L \;=\; 0\ .$$

The length of the arms and the acceleration due to gravity can be cancelled out so

$$m_x \;=\; m\ .$$

This result does not depend on the particular value of the acceleration due to gravity.

3.6



The forces acting on the cone are its weight ***W*** and a contact force ***C*** exerted by the table In the left hand diagram the weight acting through the centre of gravity of the cone provides a torque about the contact point which returns the cone to its upright position, while in the right hand diagram the torque about the contact point due to the weight makes it topple.

For neutral equilibrium, the weight always acts through the point of contact between the cone and the table.

3.7 a) No. Examples include doughnuts and food bowls.

b) The centre of gravity of any object can be found by suspending it from two different points. Suspending it from one point establishes a line that the centre of gravity must lie on. Suspending it from another point establishes a second line. The centre of gravity is where the two lines intersect.

3.8 a) About 0.75 m. b) About 10 m.

3.9

Buoyant force = weight of displaced air

= density of air × volume × g

$= \rho_a V g$.

Weight of balloon = density of helium × volume × g

$= \rho_h V g$.

Downward force required $= \rho_a V g - \rho_h V g = (\rho_a - \rho_h) V g$

= 10.8 N.

3.10

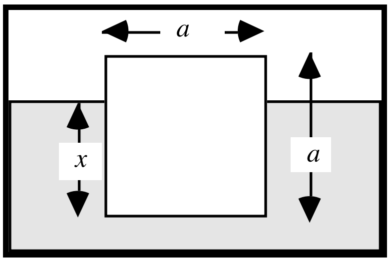
Buoyant force = weight of displaced air

= density of air × your volume × g .

Your volume ≈ $10^{-1}\ \mathrm{m}^3$.

∴ buoyant force ≈ 1 N (≈ weight of an apple).

3.11 a)



Suppose that the sides of the ice cube have length a and the cube is submerged to a depth x.

Weight of ice cube = density of ice × volume of ice × g

$= \rho_i a^3 g$.

buoyant force = density of water × volume displaced × g

$= \rho_w a^2 x g$.

For equilibrium:

weight - buoyant force = 0 ;

$\rho_i a^3 g - \rho_w a^2 x g = 0$;

$$\frac{x}{a} = \frac{\rho_i}{\rho_w} = \frac{917 \text{ kg.m}^{-3}}{1000 \text{ kg.m}^{-3}} .$$

The fraction submerged is 0.917 .

Why do ice cubes from the refrigerator float higher than this in water?

b) The water level remains the same because this mass of ice will turn into the same mass of water which will, of course, exactly fill the submerged volume.

3.12 The steel hull of a ship is thin. It is easy to achieve the condition for equilibrium : the weight of the steel, air, cargo etc. equals the weight of the displaced water.

3.13 a) The buoyant force on the less dense warm water due to the surrounding denser, colder water is greater than its weight so it rises.

b) Water that is colder than 4°C is less dense so it will rise to the top of the lake.

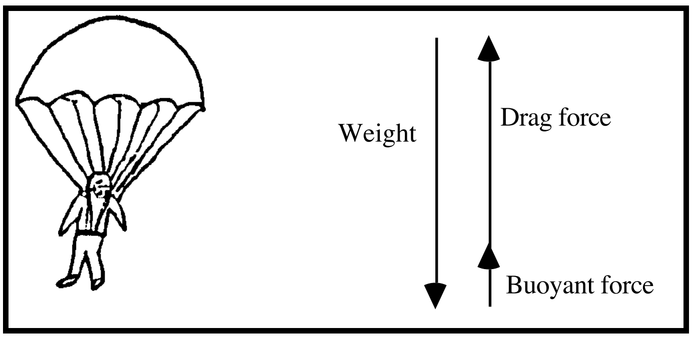
If the lake froze from the bottom up, fish would be left high and dry!

CHAPTER FE4

4.1 By substitution in the formula given, the sedimentation rate is 8.8×10^{-7} m.s^{-1}.

4.2 i) The other vertical forces are the buoyant force and the combined weight of the person and parachute.

ii)



At terminal velocity, the total downward force is zero.

so weight - buoyant force - $\lambda v_T^2 = 0$

$$v_T = \sqrt{\frac{\text{weight - buoyant force}}{\lambda}}$$

(... continued over)

iii) In the sky diving position: v_T = 62 m.s^{-1} (= 220 km.h^{-1}).

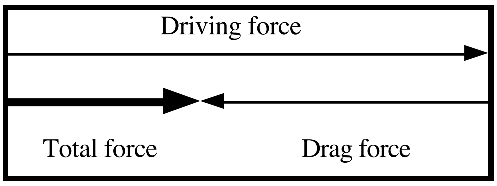
With parachute opened: v_T = 5.7 m.s^{-1} (= 20 km.h^{-1}).

iv) Average downward acceleration $= \dfrac{5.7 \text{ m.s}^{-1} - 62 \text{ m.s}^{-1}}{1 \text{ s}}$

$\approx -0.06 \times 10^3 \text{ m.s}^{-2}$.

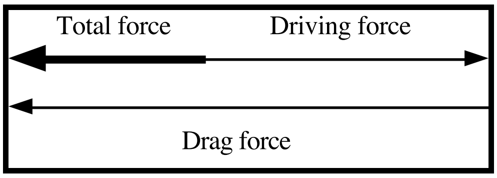
The magnitude of the average acceleration is about 6 times the acceleration due to gravity.

4.3 i) Initially the drag force is equal and opposite to the driving force since the car is travelling at constant velocity. When the accelerator pedal is depressed, the driving force becomes larger than the drag force.

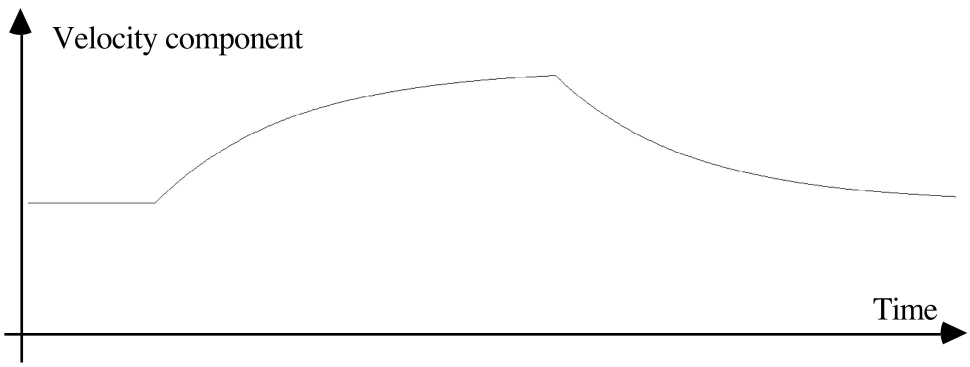


There is a total force acting to accelerate the car. As the speed of the car increases the drag force increases, reducing this total force. The magnitude of the acceleration decreases and the car ultimately begins to travel at a higher constant speed, with the drag force again balancing the driving force.

ii) When the accelerator pedal returns to its original position the drag force is larger than the driving force.

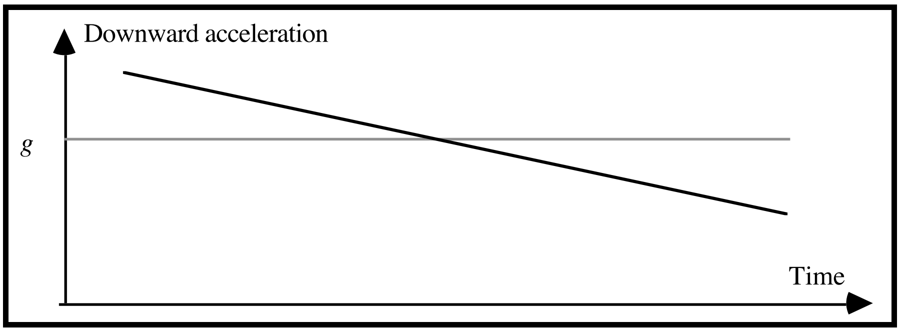


The total force acts to slow the car down, thereby reducing the drag force until it again balances the driving force.

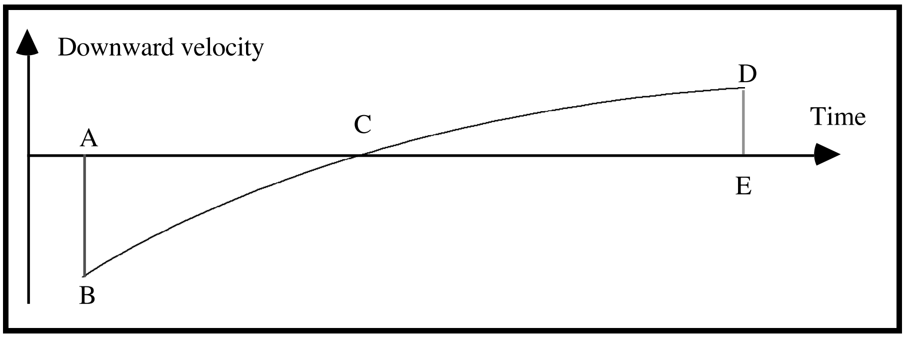


4.4 When the object is rising, the drag force due to the air resistance acts vertically downwards, i.e. in the same direction as gravity. The resultant downward acceleration therefore starts off at some value greater than g and gradually decreases to g at the time when the object reaches its maximum height.

When the object begins to fall, the drag force changes direction and begins to increase in magnitude. This means that the downward acceleration component decreases continually as shown below. (The detailed shape is unimportant as long as the acceleration component decreases.)



The downward velocity component as a function of time looks like this.



Since the downward acceleration is continually decreasing, the slope of the velocity curve decreases with time.

The area ABC in the graph represents the height reached by the object. The area CDE will, of necessity, equal the area ABC. (Why?)

Because these two areas must be equal and because of the shape of the velocity-time graph, the time taken to fall (CE) is greater than the time taken to rise (AC).

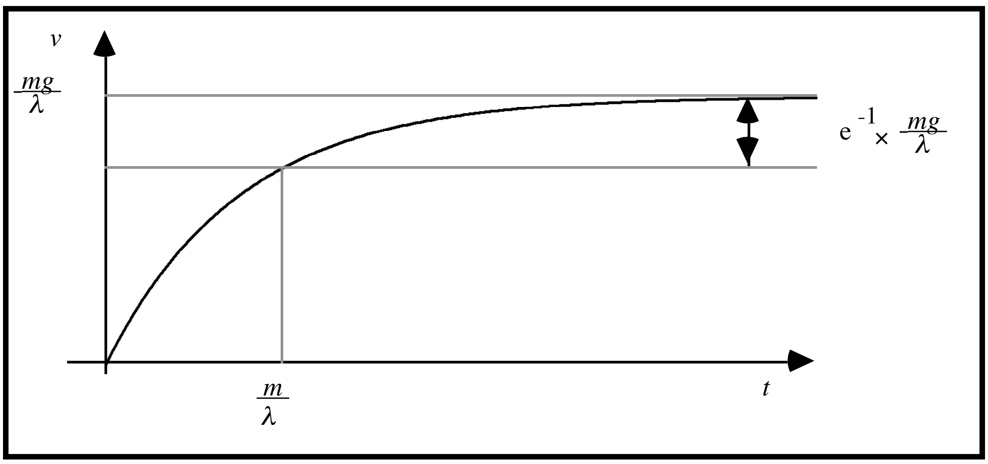
4.5 i) As $t \to \infty$, $e^{-(\lambda/m)t} \to 0$ so $v \to \dfrac{mg}{\lambda}$.

ii) At $t = 0$, $e^{-(\lambda/m)t} = 1$ and $v = 0$.

iii) Now $\dfrac{mg}{\lambda} - v = \dfrac{mg}{\lambda} e^{-(\lambda/m)t} = e^{-1}\left(\dfrac{mg}{\lambda}\right)$

when $t = \dfrac{m}{\lambda}$.

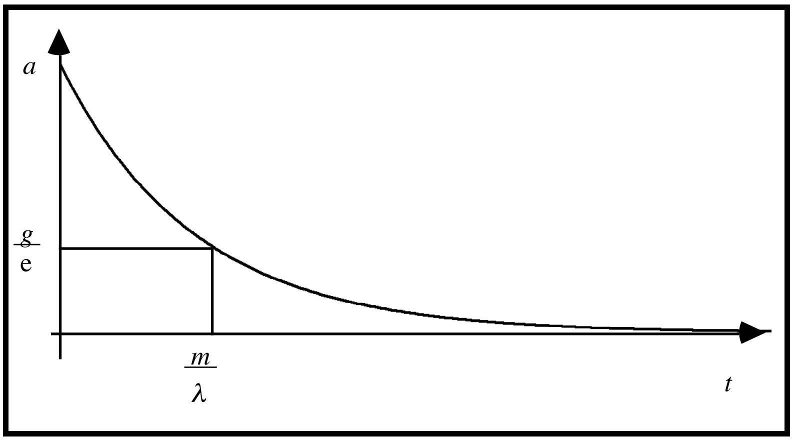
iv)



v) Since $$\frac{d}{dt}\,e^{-(\lambda/m)t} = -\frac{\lambda}{m}\,e^{-(\lambda/m)t}$$

then $$a = \frac{mg}{\lambda}\,\frac{\lambda}{m}\,e^{-(\lambda/m)t} = g\,e^{-(\lambda/m)t}\,.$$

vi)



vii) $$mg - \lambda v = mg\,e^{-(\lambda/m)t}$$
$$= ma = m\frac{dv}{dt}\,.$$

viii) Terminal velocity occurs when
$$mg - \lambda v_T = 0\,;$$

i.e when $$v_T = \frac{mg}{\lambda} \quad \text{as in (i).}$$

ix) The difference, $v_T - v = \frac{mg}{\lambda}\,e^{-(\lambda/m)t}$, never reaches zero, so in the model v never reaches v_T. In the real world a point is reached where the difference $v_T - v$ is comparable to the changes in velocity caused by individual molecules colliding with the object. The mathematical model is then inadequate.

x) $$\frac{da}{dt} = -\frac{\lambda}{m}\,g\,e^{-(\lambda/m)t} = -\frac{\lambda}{m}\,a\,.$$

$$\therefore \quad m\frac{da}{dt} = -\lambda a\,.$$

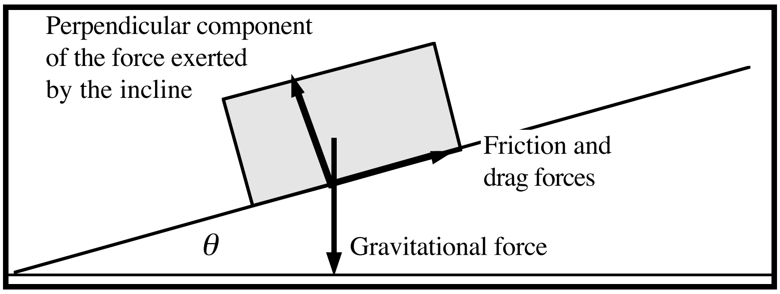
4.6 The second person has one chance in nine of making the same move as the first. If the first moves are the same there is a further one chance in nine of getting the same second move. Thus there is 1 chance in 81 of having the first two moves identical.

The chance of the second person making the same first ten moves as the first person is 1 in 9^{10}, i.e. 1 in 3.5×10^9.

4.7 The time taken for a molecule to diffuse a distance d is proportional to d^2. Each CO_2 molecule takes $10^4 \times 25$ seconds or about 70 hours to diffuse 1 metre.

CHAPTER FE5

5.1 i)



ii) The only forces doing work are those which have a non-zero component along the line of motion, i.e. the gravitational force and the friction and drag forces.

iii) The gravitational force is conservative. The other two forces doing work are non-conservative. (If the buoyant force were non-negligible, the gravitational force would have to be replaced by the effective gravitational force, i.e. the weight minus the buoyant force. The buoyant force is also conservative.)

iv) The component of the gravitational force parallel to the line of motion is $mg\cos(90° - \theta) = mg\sin(\theta)$. Therefore the work done on the object is $mg\sin(\theta)d$. (This work is positive since the component of the force and the displacement are in the same direction.)

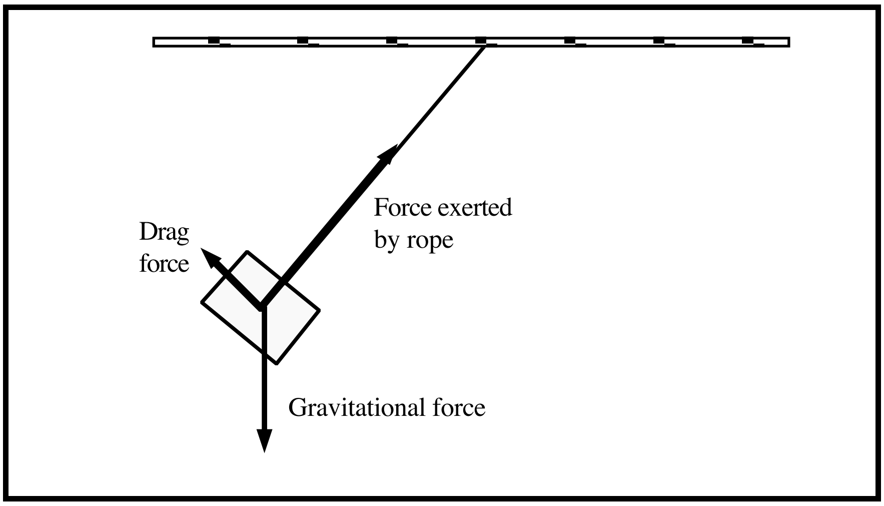
Since there are no non-conservative forces acting on the object, the increase in its kinetic energy is equal to $mg\sin(\theta)d$.

v) Since the force F is in the direction of motion, the work done by F is Fd.

The component of the gravitational force parallel to the line of motion is $mg\cos(90° + \theta) = -mg\sin(\theta)$ so the work done by it is $-mg\sin(\theta)d$.

The total work done on the object is $(F - mg\sin(\theta))d$ which is positive since F is greater than $mg\sin(\theta)$. This work equals the increase in kinetic energy, since there are no non-conservative forces present.

5.2 i)



ii) The gravitational force and the drag force are doing work on the object. The force exerted by the rope is always at right angles to the motion so it does no work.

iii) The gravitational force is conservative. The drag force is non-conservative.

5.3 i) The forces exerted by the object and incline on one another are internal to the system. So are the gravitational forces. There are no external forces acting on the system so it is isolated.

Note that the system chosen in this question differs from that chosen in question l, i.e. just the object itself. The choice of system is important; if the system is not isolated, conservation of mechanical energy cannot be used.

ii) Mechanical energy conservation implies that change in total mechanical energy is zero;

$$\therefore \quad \text{change in KE} = -\text{change in PE of system}$$
$$= m\,g\,d\,\sin(\theta)$$

since the object *rises* a *vertical* displacement of $-d\sin(\theta)$ (i.e. it falls a vertical distance $d\sin(\theta)$).

5.4 i) Since the gravitational forces are internal to the system, the only external force is that exerted by the string on the object. Since this force is always acting at right angles to the motion of the object it does no work. There are no external forces doing work on the system so it is isolated.

ii) Using conservation of mechanical energy:

$$\begin{aligned}\text{maximum change in KE} &= \text{-maximum change in PE of system}\\ &= -mg\,(-h)\\ &= mgh.\end{aligned}$$

Since initially the KE is zero, maximum KE occurs when the object is at the lowest point of its swing, and has a value mgh.

The object rises until the KE is completely transferred to PE, i.e. to a vertical height h above the lowest point of the swing.

iii) If non-conservative forces are present, they do *negative* work on the system. (Why?) These forces continually transfer mechanical energy to non-mechanical energy. Since the PE depends only on the positions of the objects, the change in PE from the starting point to the lowest point is always the same. The KE at the lowest point must therefore be less than mgh. Similarly since the object has a lower KE at the lowest point and it continues to lose mechanical energy while rising, the maximum height reached on the other side is less than h .

5.5 During the motion the drag force is doing negative work on the object thereby reducing the total mechanical energy of the Earth-object system. Since the PE of the system depends only on the height of the object, at any given level the speed of the object when it is rising is larger than when, at some later time, it is falling. This means that the average speed is less on the downward path and the object takes longer to fall than to rise.

5.6 The energy supplied in a day must equal the increase in PE of the water-Earth system i.e. 1.4×10^5 J .

5.7 i) The possum increases its height above the ground at a rate v . It must be doing work at a rate of mgv .

ii) The fastest speed at which the possum can climb is given by the speed at which mgv equals the maximum power P.

$$\therefore \qquad v = \frac{P}{mg} = 2.0\ \text{m.s}^{-1}\ .$$

The possum takes 10 seconds at least to climb the tree.

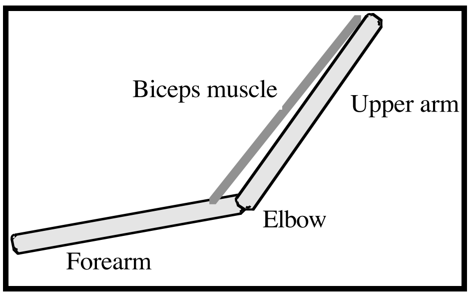
5.8 i) At equilibrium, when the applied force just supports the load, the total torque about the pivot must be zero, so $mga = Fb$. A force larger than $F = \frac{mga}{b}$ must be applied to raise the load.

ii) If we ignore the curved paths as stated, the end of the lever moves in the direction of the applied force. The work done by the applied force is therefore $\frac{mga}{b} \times d$, where d is the distance travelled by the end. Thus $d = \frac{hb}{a}$. (In this case, this result could also be obtained from geometrical considerations. The energy balance approach however is also applicable to more complicated machines.)

(Note that if $a < b$ so that $F < mg$ then $d > h$ as expected.)

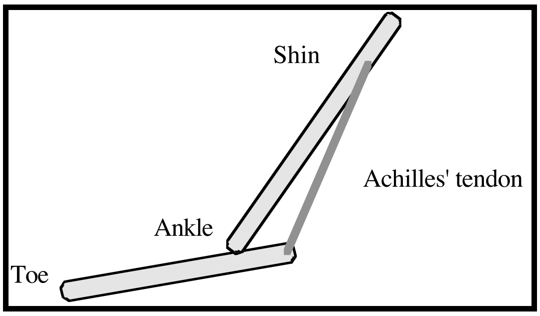
5.9

The end of the biceps muscle moves only a short distance when the forearm is raised. To lift an object, the biceps muscle must provide a force considerably larger than the weight of the load. This is the exact opposite of the machine described earlier.



Many of the muscles in the body are always under tension. For example, the Achilles' tendon which enables us to stand on tip toe is always taut; it always exerts a compressive force on the shin bone several times larger than the weight of the body.

When the foot pivots about the toe, the end of the Achilles' tendon moves slightly further than the bottom of the shin, so the force provided by the tendon is less than the force exerted by the shin bone on the foot. (It is however much larger than the weight of the body.)



5.10 i) Since the gravitational force is at right angles to the direction of motion it does no work.

ii) The work done by the constant gravitational force, over the path from B to C, is mgh. This is also the total work done over the path A → B → C.

iii) The work done for each of the horizontal segments is zero. The total work done is therefore $mgh_1 + mgh_2 = mgh$ where h_1 and h_2 are the distances of D below A, and of F below E respectively.

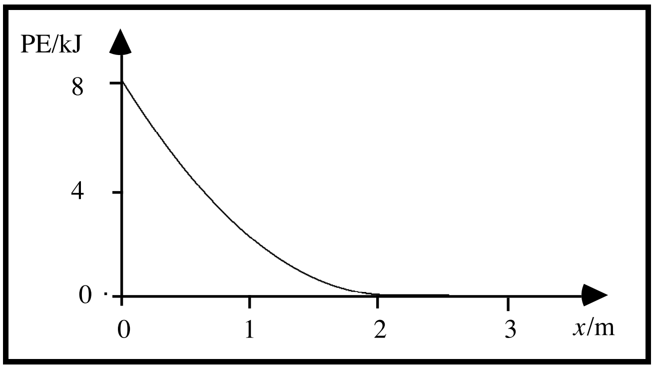
5.11 i) The potential energy at B is also zero since there is no force acting between B and A.

The area under the graph between C and B represents

$$\frac{1}{2} \times (2.0 \times 10^3 \text{N}) \times (0.5 \text{ m}) = 5 \times 10^2 \text{ J}$$

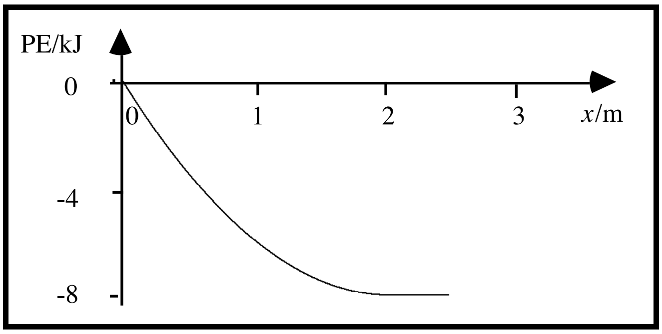
so the PE at C is 5×10^3 J. The PE is positive since the force and the displacement are in the same direction when the object moves from C to B. The PE's at D, E and G are 2×10^3 J, 4.5×10^3 J and 8×10^3 J respectively.

ii)



iii) If the object has the minimum KE to just strike the wall, it will have zero KE at G. The total mechanical energy of the system would therefore be 8×10^3 J. At A where the PE is zero, the kinetic energy must be 8×10^3 J.

iv) The difference in PE between any two points would not be altered so the PE's at B, C, D, E and G would be -8×10^3 J, -7×10^3 J, -6×10^3J and 0 J respectively. The PE graph would be like this.



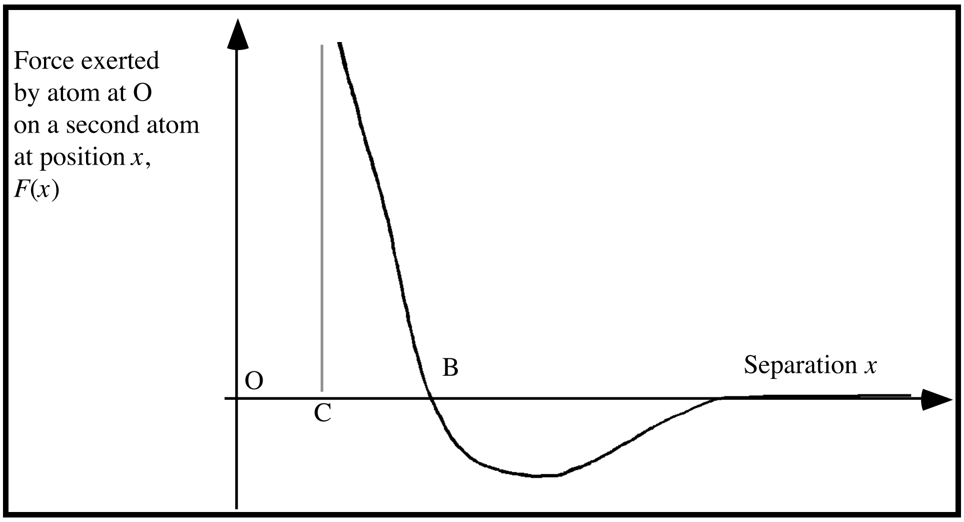
The minimum KE for the object to strike the wall is unaffected by arbitrary choices of the zero of PE.

5.12 If the PE were defined to be zero at $x = 0$, the PE at any likely separation of the atoms would be infinitely negative. It would be impossible to find the changes in PE which occur when the atoms change their separation within the region of interest.

The PE is negative at A since the separation is increased from x to ∞. The displacement and the force are in opposite directions, so the work done is negative.

B corresponds to the point where the force curve crosses the axis. The force on either side of B is directed towards B so a stationary object placed at B will remain there.

C corresponds to the point shown on the $F(x)$ diagram below. The area under the curve from C to B equals the area above the curve from B to infinity because the work done by the force in moving from C to infinity is zero.



5.13 1.6 W.

CHAPTER FE6

6.1 Centripetal acceleration $= \dfrac{v^2}{R} = \omega^2 R$.

6.2 About 3.5 rad.s^{-1}.

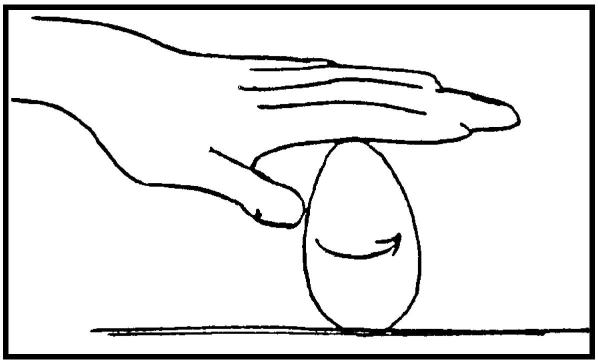
6.3 45 m.s^{-1}.

6.4 Rotational KE is proportional to ω^2, so in this problem the angular velocity must increase four times, to twenty revolutions per second.

6.5 Yes. However, in the absence of friction, there is no torque to change the angular velocity of the sphere about its centre of gravity. The KE of rotation is a constant - the gravitational PE is completely transferred to KE of translation, as in the case of an object sliding without friction.

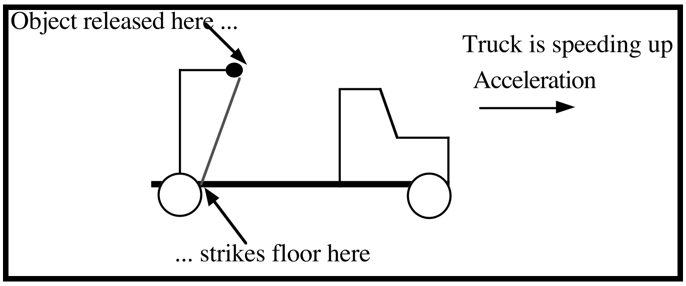
6.6 A raw egg will slow down more quickly because energy is dissipated as negative work done by viscous forces within the fluid inside the egg.

A more spectacular test is to stop the egg from spinning momentarily by placing your hand on it.



A raw egg will resume spinning; a hard-boiled egg will not. Try this for yourself.

6.7 a) The object will land at a point on the floor of the truck behind the point of release.



b) To an observer at the side of the road, the object falls normally. Its horizontal velocity remains constant and equal to the velocity of the truck at the moment of release. Since there is no horizontal acceleration there is no need to invent a pseudoforce to account for it. The accelerated vertical component of the motion is explained adequately in terms of gravity.

6.8 i) Bird's eye view:
When the car is stopping the friction force between you and the car seat will be in the direction of the car's acceleration - backwards. But if this force is too small to give you an acceleration equal to the car's acceleration, you have a forward velocity greater than the car's velocity, so you will move forward faster than the car.

When the car is turning a corner, the analysis is essentially similar. The force exerted by the seat on your backside needs to be directed towards the centre of curvature of the car's path. If the force is not big enough, you will retain some of your straight-ahead motion while the seat goes in the curved path, leaving you behind. You tend to go straight ahead.

ii) Accelerating frame of reference:
When the car is stopping, the friction force between you and the car seat is insufficient to counteract the pseudoforce throwing you forward.

When the car is turning a corner the friction force is too small to counteract the centrifugal (pseudo) force which is throwing you to the side of the car.

6.9 a) The magnitude of the centrifugal force on a particle of mass m in a centrifuge is equal to $m\omega^2 R$ which is $\omega^2 R/g$ times the weight of the particle.

$$8000 \text{ revs per minute} = \frac{2\pi \times 8000}{60} \text{ rad.s}^{-1} ;$$

so

$$\frac{\omega^2 R}{g} = \left(\frac{2\pi \times 8000 \text{ rad.s}^{-1}}{60}\right)^2 \times \frac{110 \times 10^{-3} \text{ m}}{9.8 \text{ m.s}^{-2}}$$

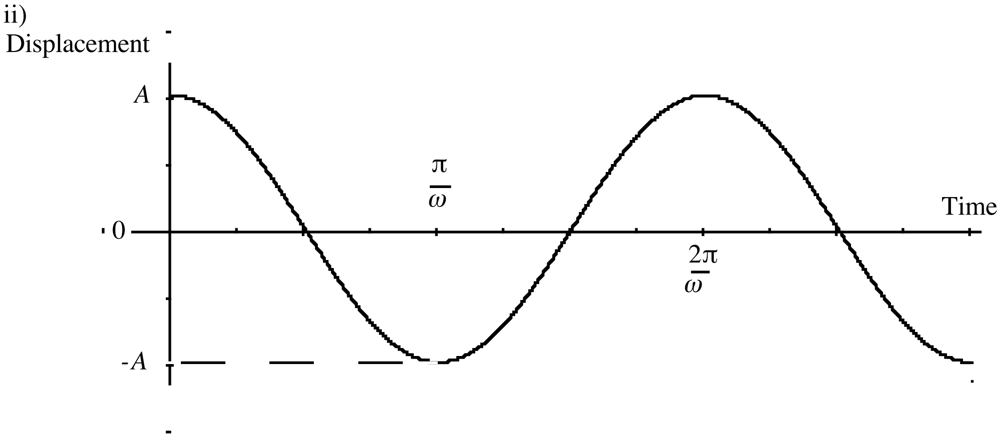
$$= 7.9 \times 10^3 ;$$

i.e. the centrifugal force is about 8000 times the weight.

CHAPTER FE7

7.1 i) At $x = A$.

ii)



iii) The largest value of $|x|$ is A which occurs at $t = 0, \frac{\pi}{\omega}, \frac{2\pi}{\omega}, \ldots$ etc.

iv) Since $\frac{d}{dt}[\cos(\omega t)] = -\omega \sin(\omega t)$, then $\frac{dx}{dt} = -A\omega \sin(\omega t)$.

v) Speed is a minimum (zero) when $\sin(\omega t) = 0$, i.e. at $t = 0, \frac{\pi}{\omega}, \frac{2\pi}{\omega}, \ldots$ etc. The slopes of the displacement-time graph at these times are zero.

vi) Since $\frac{d}{dt}[\sin(\omega t)] = \omega \cos(\omega t)$, then $\frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t)$.

vii) Since $\sin\left(\omega\left(t + \frac{2\pi}{\omega}\right)\right) = \sin(\omega t + 2\pi) = \sin(\omega t)$

and $\cos\left(\omega\left(t + \frac{2\pi}{\omega}\right)\right) = \cos(\omega t + 2\pi) = \cos(\omega t)$,

the displacement, velocity and acceleration will all have the same values again at a time interval $T = \frac{2\pi}{\omega}$ later.

7.2 i) Force exerted by breeze (both driving and damping), elastic forces in the trunk of the tree (restoring).

ii) Elastic forces in the Earth's interior (restoring), non-conservative forces exerted by flowing material in the Earth's core, etc.(damping).

iii) Gravitational force (restoring), forces exerted on the child by both the seat and the suspending ropes (driving), air drag (damping).

iv) Pressure forces exerted by the air due to the sound waves' impinging on the ear drum (driving), elastic force in the ear drum (restoring), force exerted by bones etc. behind the ear drum (damping).

v) Weight and buoyant force together provide a restoring force (and a driving force if the water is moving as in a wave), drag force (damping).

vi) Forces exerted on each atom by the surrounding atoms (restoring and damping).

7.3 (Note that we need two forces, one at each end of the spring, to stretch it; one force would simply accelerate the spring.)

We want the period, which can be found if we know the frequency. To find the frequency, try starting from

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{or} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

If we knew both k and m then the problem would be solved. We do know m, so we now want to find k, the force per displacement. We know the value of displacement for one value of force so that gives k and the problem is essentially solved.

Call the magnitude of one force F and the amount of stretch e. Then, assuming that the spring is linear, then $F = ke$, where k is the force constant of the spring. So substitute $k = F/e$ into the expression for f. That gives:

$$f = \frac{1}{2\pi}\sqrt{\frac{F}{me}}$$ and the period is the reciprocal of that: $$T = 2\pi\sqrt{\frac{me}{F}}.$$

Finally put the values in: $$T = 2\pi\sqrt{\frac{(0.32\ \text{kg}) \times (0.062\ \text{m})}{2.4\ \text{N}}} = 0.57\ \text{s}.$$

Since we weren't told that the spring is linear (k may not be constant), we treat this as an order of magnitude problem and say that the answer is half a second.

You can now see the advantage of working with symbols instead of numbers. To find out what would happen if the mass were increased by a factor of 10, you just look at the formula that you derived and see that if m is increased 10 times then the period must be multiplied by $\sqrt{10}$ or about 3. The answer is that either the spring will break or the new period will be about 1.5 s. Even if you haven't worked through the formula you ought to be able to intuit that a greater mass will produce a more sluggish system, which will have a longer period, so if you had worked out that the period was less then you would know that you had made a boo-boo.

7.4 i) Let h be the depth of the bottom face below the surface.

At equilibrium, weight of pontoon = buoyant force.

$$\therefore \quad \rho L A g = \rho_W h A g .$$

$$\therefore \quad h = 0.5 \text{ m}.$$

ii)

$$\text{Total upward force} = \text{buoyant force - weight of pontoon}$$
$$= \rho_W(h + x)Ag - \rho L A g$$
$$= \rho_W x A g .$$

iii)

$$\text{Total upward force} = -\rho_W x A g .$$

iv) If there were no non-conservative forces present, there would always be a restoring force on the pontoon proportional to its displacement from, and directed towards, its equilibrium position. The pontoon would undergo a vertical SHM.

In reality, there is a drag force acting, which causes the oscillation to die out.

v) The equation of motion is

$$\rho LA\, a = -\rho_W x A g$$

$$a = -\frac{\rho_W g}{\rho L} x .$$

Thus $\omega_N = \sqrt{\dfrac{\rho_W g}{\rho L}} = 4.4 \text{ s}^{-1}$ which corresponds to a frequency of 0.70 Hz.

7.5 Using the result that when KE = 0, the PE is $\frac{1}{2} m\omega_N^2 A^2$, and the conservation of total mechanical energy:

$$\text{maximum KE} = \frac{1}{2} m\omega_N^2 A^2 ;$$

$$\therefore \quad \text{maximum speed} = \omega_N A$$

7.6 i) The natural frequency $\omega_N = \sqrt{C} = 1.25 \times 10^{-3}$ rad.s^{-1} so the period is 5.00×10^3 s. The time for a one-way trip is half the period, i.e. 2.50×10^3 s or about 42 minutes.

ii) Using answer 7.4, the maximum speed is 3.9×10^2 m.s^{-1} (over 1400 km.h^{-1}).

The equation of motion given in the question actually holds for any straight tunnel drilled through the earth no matter how far the end points are apart. It would take only 42 minutes, for example, to get from Sydney to Melbourne or from Sydney to London (if you don't mind getting a bit warm on the way).

7.7 For resonance to occur there must be a system that can oscillate at one or more natural frequencies and a driving force which has a large Fourier (sinusoidal) component at one of these natural frequencies. The wind instruments (including the human voice mechanism) contain columns of air which oscillate at a set of natural frequencies which depend on the positions of the various stops etc. These instruments are driven by blasts of air or vibrating reeds which have large Fourier components at one of these natural frequencies. See *Scientific American*, October 1960, pp 144-154 and July 1973, pp 24 - 35.

CHAPTER FE8

8.1

$$\frac{\text{Height of larger cylinder}}{\text{height of smaller cylinder}} = 2.$$

$$\frac{\text{Diameter of larger cylinder}}{\text{diameter of smaller cylinder}} = 2.$$

$$\frac{\text{Base area of larger cylinder}}{\text{base of smaller cylinder}} = 4.$$

$$\frac{\text{Curved surface area of larger cylinder}}{\text{curved surface of smaller cylinder}} = 4.$$

$$\frac{\text{Volume of larger cylinder}}{\text{volume of smaller cylinder}} = 8.$$

$$\frac{\text{Mass of larger cylinder}}{\text{mass of smaller cylinder}} = 8.$$

8.2 a) As seen in the lecture the height through which the centre of gravity of an animal (or a person) is raised is independent of scale length. The × 2 person has no advantage over the × 1 person. (In a proper high jump event the × 2 person has an advantage since the centre of gravity is further above the ground.)

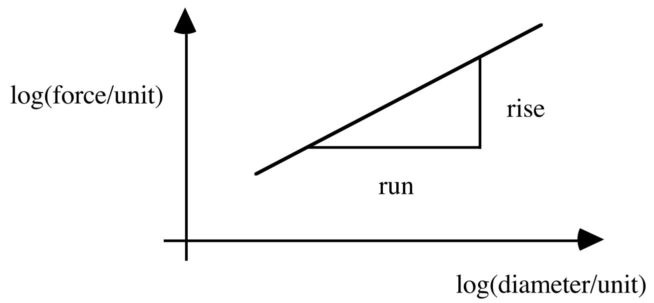
b) The force exerted by the muscles is proportional to L^2. This means that the × 2 person will be able to lift 4 times as much weight as the × 1 person. (Superheavyweight weight-lifting records are larger than lightweight records.)

c) The force exerted by the muscles is again 4 times greater for the × 2 person. However, the athlete has 8 times as large a mass to accelerate. The forward acceleration is thus only half as much. (Sprinters and rugby halves are not usually noted for their height.)

d) The force exerted by the muscles is again 4 times greater for the × 2 person. The × 2 person has a longer arm which does 8 times as much work on the shot as that of the × 1 person. The initial kinetic energy of the shot is 8 times as large but since the mass is also 8 times as large, both shot-putters impart the same initial velocity to the shot.

The vertical and horizontal components of the motions are independent. The larger shot has twice the distance to fall to ground so its time of flight is $\sqrt{2}$ times longer. The horizontal distance travelled will be $\sqrt{2}$ times as far. (The breeding programme again succeeds in spite of the officials' producing the larger shot.)

8.3



For the hypothesis to be OK we need a straight-line graph of the correct slope. If you can fit a straight trend line to the data (i.e. if you are satisfied that a smooth curve could not be a better fit) then measure the geometrical slope of the graph by measuring rise and run with a ruler. If the ratio of these measurements is 2 then the idea that $F = kd^n$ with $n = 2$ is supported.

8.4 The internal volume of blood forced from the heart in each pulse is equal to the volume of the heart. The rate of supply of blood is therefore proportional to the pulse rate $\times L^3$. Since this rate of supply varies as L^2, the pulse rate must vary as L^{-1}.

8.5 The semitrailer has 30 times as much mass to lift up, that is 30 times as much work to do in climbing the hill. It does work at a rate 3 times greater than the car. This means that it will take 10 times as long to climb the hill.

If the scaling used to go from cars to semitrailers slavishly followed the rules of the animal kingdom, as for example, in the transition from a fox terrier to a labrador, a semitrailer of length 9 m would have a mass of 27 000 kg (which is not far off the laden mass) and develop a power of 675 kW (which is far more than the actual power output). Under these conditions we could apply exactly the same analysis as in the lecture to compare speeds of cars and semitrailers climbing hills. We would then come up with the result that the car can travel three times as fast, a result that anyone who has sat behind a semi doing five kilometres an hour up a hill will dispute. The answer is obviously too low, but that is not surprising, because the real input data were ignored.