

ANSWERS TO REVIEW QUESTIONS

IMPORTANT NOTE: READ THIS FIRST.

There are three different kinds of answer here.

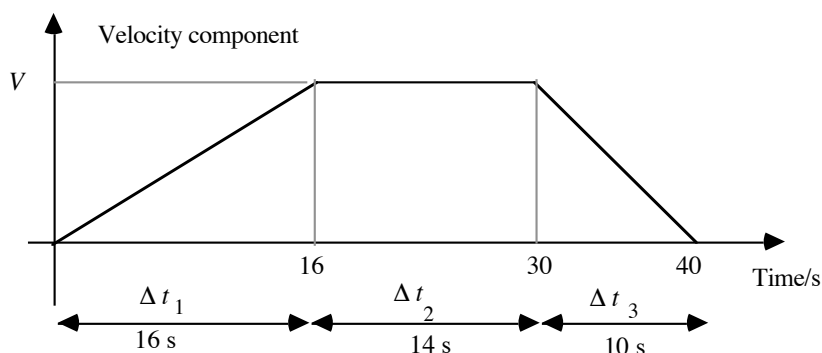
- The usual form for quantitative questions is a short entry giving the final numerical answer. Sometimes there will be also a brief indication of how to get to that result. The amount of detail given is usually much less than that required in a model answer. For descriptive answers ("bookwork") references to the text may be all that is given. These entries have no special label.
- **Model answers** are written out in full as a guide to the kind of response which would get full marks in an exam. However, a model answer given here, memorised and presented in an exam, may not score very well because it lacks originality.
- **Notes** are intended to indicate the features of a good answer or give a commentary on which an answer might be based. They often contain background information which would not need to be reproduced in a model answer.

1 Hawk wins. The distance travelled by each car is represented by the area under the graph. At the time when the race finishes, the area under Hawk's curve is greater so it has travelled further.

2 i) 1.6 m.s^{-2} . ii) 25 m.

3 **Model answer**

a)



b) At $t = 35 \text{ s}$, the acceleration component a_3 is given by the slope of the graph:

$$a_3 = \frac{0 - V}{\Delta t_3}$$

where V is the maximum value of the velocity component. This can be found from the first stage of the motion where the acceleration is

$$a_1 = \frac{V - 0}{\Delta t_1}$$

Putting these together we get

$$a_3 = -a_1 \frac{\Delta t_1}{\Delta t_3} = -0.50 \text{ m.s}^{-2} \frac{16 \text{ s}}{10 \text{ s}} = -0.80 \text{ m.s}^{-2}.$$

The negative value signifies that the direction of the final acceleration is opposite to that of the velocity.

c) In this case, since the direction of the velocity does not reverse, the distance travelled is equal to the final displacement which is given by the area under the graph. This consists of two triangular portions and a rectangular piece, giving the displacement as

$$x = \frac{1}{2} V \Delta t_1 + V \Delta t_2 + \frac{1}{2} V \Delta t_3$$

The value of V can be found from the slope of the first part of the graph:

$$a_1 = \frac{V}{\Delta t_1}$$

so

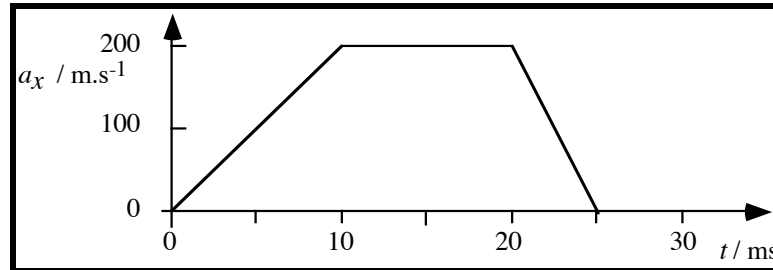
$$V = a_1 \Delta t_1 .$$

Substitute this:

$$\begin{aligned}
 x &= a_1 \Delta t_1 \left(\frac{1}{2} \Delta t_1 + \Delta t_2 + \frac{1}{2} \Delta t_3 \right) \\
 &= 0.50 \text{ m.s}^{-2} \times 16 \text{ s} \times (8 \text{ s} + 14 \text{ s} + 5 \text{ s}) \\
 &= 216 \text{ m}.
 \end{aligned}$$

Since only two-figure precision is justified the distance travelled is 0.22 km.

- 4 a) Using the equation of motion, $F = ma$, find the acceleration component from the force component.



- b) i) Maximum acceleration, $a_{\max} = \frac{F_{\max}}{m} = 200 \text{ m.s}^{-2}$
 (This also gives the value on the graph).
 ii) Maximum velocity will occur at $t = 25 \text{ ms}$ and is given by the area under the graph.
 Maximum velocity component

$$\begin{aligned}
 v_{\max} &= \frac{1}{2} a_{\max} \times (10 \text{ ms}) + a_{\max} \times (10 \text{ ms}) + \frac{1}{2} a_{\max} \times (5 \text{ ms}) \\
 &= a_{\max} \times (17.5 \text{ ms}) \\
 &= (200 \text{ m.s}^{-2}) \times (17.5 \times 10^{-3}) \text{ s} \\
 &= 3.5 \text{ m.s}^{-1}.
 \end{aligned}$$

- 5 a) Since the car is travelling in a straight line the speed (shown on the graph) is equal to the car's forward component of velocity.

Component of average acceleration

$$\begin{aligned}
 &= \frac{\text{change in velocity component}}{\text{time taken}} \\
 &= \frac{0 \text{ m.s}^{-1} - 10 \text{ m.s}^{-1}}{2.2 \text{ s} - 0.2 \text{ s}} \\
 &= -5.0 \text{ m.s}^{-2}.
 \end{aligned}$$

- b) The distance travelled is equal to the displacement, which is represented by the area under the graph:

$$\begin{aligned}
 \text{distance} &= (10 \text{ m.s}^{-1}) \times 0.2 \text{ s} + \frac{1}{2} (2.0 \text{ s}) \times (10 \text{ m.s}^{-1}) \\
 &= 12 \text{ m}.
 \end{aligned}$$

- c) Total time = reaction time + braking time.

$$\text{Drunk's reaction time} = 0.4 \text{ s}.$$

$$\begin{aligned}
 \text{Drunk's braking time} &= \frac{\text{change in velocity component}}{\text{acceleration component}} \\
 &= \frac{0 \text{ m.s}^{-1} - 20 \text{ m.s}^{-1}}{-5.0 \text{ m.s}^{-2}} \\
 &= 4.0 \text{ s}.
 \end{aligned}$$

$$\text{Total time} = 4.4 \text{ s}.$$

- d) Distance travelled = $20 \text{ m.s}^{-1} \times 0.4 \text{ s} + \frac{1}{2} (4.0 \text{ s}) \times (20 \text{ m.s}^{-1})$
 $= 48 \text{ m}.$

6 Model answer

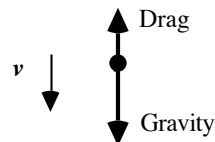
- a) The component of acceleration in the vertically down direction can be found by drawing a tangent to the velocity-time graph. Doing this at time zero gives

$$a = \frac{-5 \text{ m.s}^{-1}}{0.5 \text{ s}} = -10 \text{ m.s}^{-2}.$$

- b) The vertical displacement is found by measuring the area "under" the velocity-time graph. There are about 41 squares between the time axis and the curve up to the contact time of 12 s. Each square represents $(-5.0 \text{ m.s}^{-1} \times 1.00 \text{ s})$ or -5.0 m , so the total displacement is $-41 \times 5.0 \text{ m}$, i.e. about -0.2 km . The prey is 0.2 km below the starting point..

c)

The forces acting are gravity, vertically down, and a drag force in the direction opposite to the falcon's velocity. As the falcon's speed increases, so does the drag force, until eventually (after about 5 s) the magnitude of the vertical drag force is equal to the falcon's weight. From then on, there is no acceleration so the vertical component of velocity remains constant.



- d) At $t = 5 \text{ s}$, the velocity component has reached its constant value, so $a = 0$, and the net force is zero.

7 Notes

- a) 0.1 m.s^{-2} (slope of $v - t$ graph).
 b) 1.2 m (area under $v - t$ graph).
 c) Draw a diagram showing the push and a retarding force, exerted by the road, in the opposite direction.
 d) 50 N .

8

- a) $F_{\text{max}} = 45 \text{ N}$ from graph.

$$a_{\text{max}} = 90 \text{ m.s}^{-2}.$$

- b) 72 ms .
 c) Find the area "under" the graph, which represents the **change** in the velocity divided by the mass. The area is about 20 squares and each square represents $10 \text{ ms} \times 10 \text{ N}$.

So the change in speed = $\frac{20 \times 10 \text{ ms} \times 10 \text{ N}}{0.50 \text{ kg}} = 4.0 \text{ m.s}^{-1}$.

9 Notes

See §§6-4, 6-6, 6-8. In the lab frame the forces are contact forces exerted by the surrounding fluid ("buoyancy" and drag separately or combined) and the perhaps the weight (which is negligible). Centripetal force must not be included as a separate force and the centrifugal force does not exist in the lab frame. If centripetal force is mentioned it must be equal to the sum of the radial components of the contact forces. If the sum of the contact forces is less than $\frac{mv^2}{r}$ then the particle spirals outwards.

In the rotating frame the forces are the real contact forces and a fictitious centrifugal force. The resulting path is approximately a straight line.

- 10** 0.60 kN in the direction along the wire; 0.80 kN along the wire.

Notes

The force exerted on the support at A by the wire is equal in magnitude to the tension in the wire and also to the force exerted by that wire on junction C. Call this magnitude T_A . Similarly, let T_B be the tension in the wire attached to B.

The three forces on C balance. Taking components:

$$T_A \cos 53^\circ + T_B \cos 37^\circ = 1000 \text{ N (vertical);}$$

$$T_A \cos 37^\circ = T_B \cos 53^\circ \text{ (horizontal).}$$

Solving these equations gives the values above.

- 11 a) Gravitational, weak nuclear, electromagnetic, strong nuclear.
 b) Attractive : gravitational, strong nuclear, weak nuclear.
 Either: electromagnetic.
 c) Gravitational: solar system.
 Weak nuclear: β decay process.
 Electromagnetic: intermolecular forces.
 Strong nuclear: fission processes.
- 12 Beginning near the top-left and travelling clockwise the lengths are: 4 cm - 6 cm; 5 cm - 5 cm; 8 cm - 2 cm; 2.5 cm - 7.5 cm.

0.1 N.

Taking the masses of the wires into account will move the points of suspension closer to the geometric centres.

- 13 a) Buoyant force is about $10^{-3} \times (\text{your weight}) = 0.7 \text{ N}$, say. No.
 b) Density of alcohol = $0.8 \times 10^3 \text{ kg.m}^{-3}$.
 [Use: upthrust = ρVg where ρ is liquid density, V is the volume of displaced liquid.]

14 625 mm.

15 700 N. 0.75 m from the head.

[Calculate torques about the centre of gravity.]

16 300 N vertically down, 700 N vertically down.

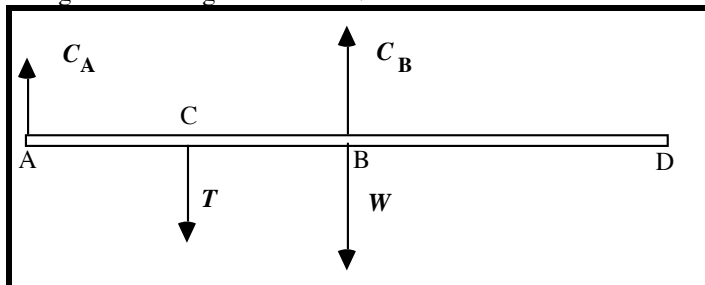
[There are four forces acting on the system of plank plus man: the plank's weight downwards, the man's weight downwards, and two supporting forces exerted by the trestles upwards. At equilibrium both forces and torques must balance.]

0.25 m past the right-hand trestle.

[When the plank begins to tip, the supporting force at the left-hand trestle becomes zero.]

- 17 i) 1.0 kg.
 ii) 0.33 kg.

18 a) Diagram showing forces acting on the PLANK.



C_A , C_B are contact forces exerted by the supports. T is the pull of the attached string.

W is the weight of the plank (pull of the Earth on the plank).

From equilibrium of the 50 N weight: $T = 50 \text{ N}$.

Force equilibrium of plank:

$$\begin{aligned} C_A + C_B &= T + W \\ &= 50 \text{ N} + 200 \text{ N} \\ &= 250 \text{ N} . \end{aligned}$$

Torque equilibrium about point B:

$$C_A \times 2.40 \text{ m} = T \times 1.20 \text{ m} ;$$

$$C_A = 25 \text{ N}$$

and hence

$$C_B = 225 \text{ N} .$$

- b) For the plank to be just about to tilt, contact at A must be just about to be lost: i.e. $C_A = 0$.

Let the weight of the extra object be W_D .

For torque equilibrium about B:

$$W_D \times 2.40 \text{ m} = T \times 1.20 \text{ m} ;$$

$$W_D = 25 \text{ N.}$$

- 19 a) It is essential to note that T , the tension in the light flexible cord, is everywhere the same.

Equilibrium of W : $T = 100 \text{ N} .$

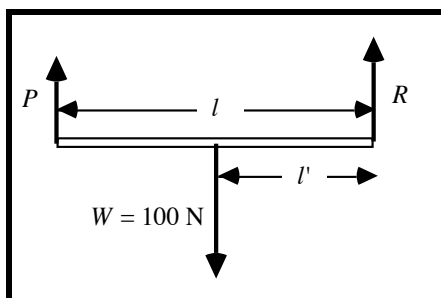
Equilibrium of P_1 , taking horizontal components of force:

$$50 \text{ N} = T \cos \theta + T \cos \theta ;$$

$$\text{So } \cos \theta = \frac{1}{4} ; \theta = 76^\circ .$$

- b) A 50 N force to the left.

- 20 Draw a force diagram



Consider equilibrium (with P at 75 N).

The torques about the hinge must balance : $75 \text{ N} \times l = 100 \text{ N} \times l' .$

$$\therefore l' / l = 3/4$$

i.e. the centre of gravity is 3/4 of the way from the hinge to the far end.

- 21 a) Balance of forces :

Horizontal: $T \cos 18^\circ = R ;$

$$T \sin 18^\circ = S + 35 \text{ N} .$$

Balance of torques about the point where T crosses the arm:

$$35 \text{ N} \times 0.20 \text{ m} = S \times 0.15 \text{ m} .$$

(The torques associated with forces T and R are both zero.)

So $S = 47 \text{ N} ;$

$$T = \frac{82 \text{ N}}{\sin 18^\circ} = 0.26 \text{ kN} .$$

- b) $S = 47 \text{ N} ;$

$$R = T \cos 18^\circ = 0.25 \text{ kN} .$$

- 22 Buoyant force = weight of displaced sea-water

$$= mg$$

$$= V d g$$

where V is volume of sea water = volume of object ;

d is the density of sea water.

But from the data

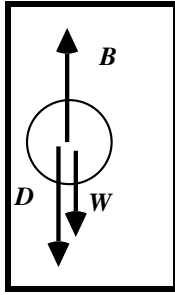
$$\text{buoyant force} = (0.060 \text{ kg}) g$$

$$\text{so } V = \frac{0.060 \text{ kg}}{1.10 \times 10^3 \text{ kg.m}^{-3}} = 5.5 \times 10^{-5} \text{ m}^3 .$$

- 23 See chapter FE4.

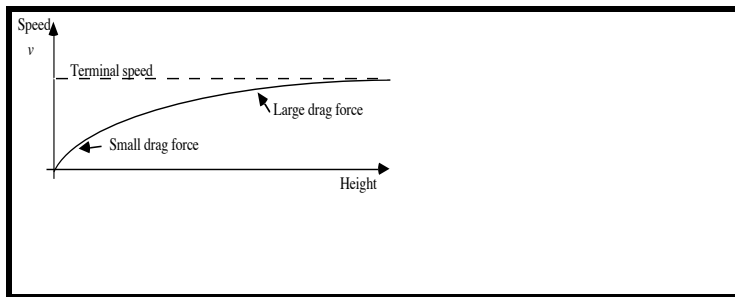
- 24 i) Buoyant force: arises from more collisions on the bottom of the object than on top. The force is in the upwards direction.
 ii) Drag force: arises from more collisions on the front of the object than on the back. The force is in the direction opposite to the motion.
 iii) Diffusion force : arises from more collisions in the denser fluid region than in the less dense region. The force is directed towards less dense region.

25 a)



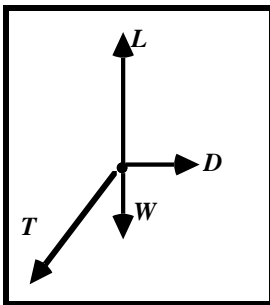
mg : weight
 B : buoyant force
 D : drag force

b)



- 26 a) Gravitation, drag, buoyancy.
 b) $d_1 g V$.
 c) See figure 4.3.

27



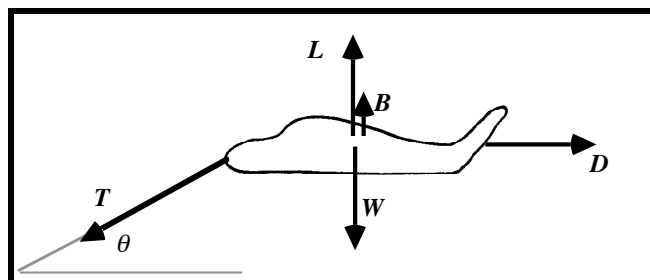
$$D = T \cos 60^\circ ;$$

$$L = W + T \cos 30^\circ .$$

So lift = 330 N ;
 drag = 104 N .

- 28 i) 1.7×10^{-10} m to 5.0×10^{-10} m. (Outside this range the potential energy would need to be greater than the total energy, that is the kinetic energy would have to be negative, an impossible situation.)
 ii) 1.7×10^{-10} m or 5.0×10^{-10} m (KE = 0).
 iii) 1.5×10^{-17} J (at $d = 3.0 \times 10^{-10}$ m).
 iv) 0.5×10^{-17} J. (The total mechanical energy must be increased to 0.)

29 a)



W is the weight of the glider; T is the pull from the rope, D is air drag, L is lift force exerted by the air, B is buoyancy (negligible).

- b) The angle θ of the rope to the horizontal is given by

$$\sin\theta = \frac{400 \text{ m}}{500 \text{ m}} .$$

$$T = 1.5 \text{ kN}; W = 2.0 \text{ kN} .$$

Equilibrium: Force balance equations:

horizontal components: $T \cos \theta = D ;$

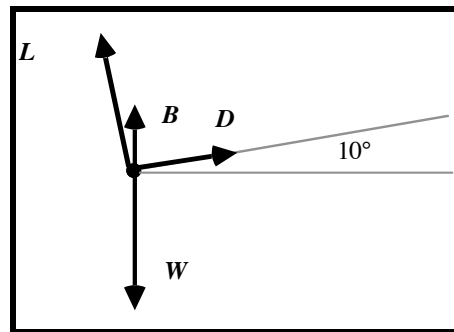
vertical components: $L = T \sin\theta + W .$

So $D = 1.5 \text{ kN} \frac{300}{500} = 0.9 \text{ kN} ;$

and $L = 1.5 \text{ kN} \frac{400}{500} + 2.0 \text{ kN} = 3.2 \text{ kN} .$

30 a)

B buoyancy (negligible),
 D drag,
 L lift,
 W weight



- b) Equilibrium

Force balance equation:

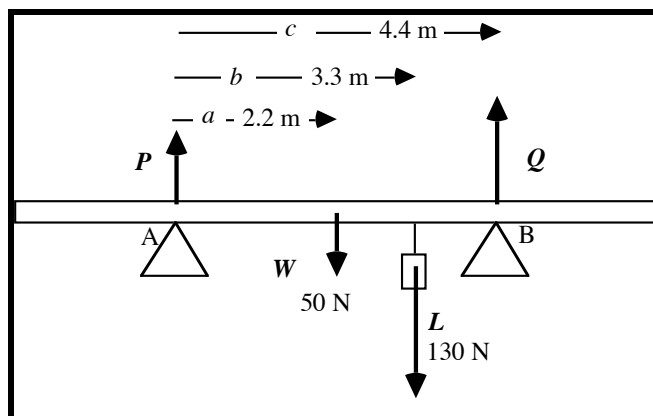
Vertical: $L \cos 10^\circ + D \cos 80^\circ = W .$

Horizontal: $L \sin 10^\circ = D \cos 10^\circ .$

So $\frac{L}{D} = \cot 10^\circ = 5.7 .$

31 **Model answer**

- a) Consider the forces acting on the system which consists of the plank and the suspended object. These are the weight W of the plank, the weight L of the object, and two contact forces P and Q exerted by the supports.



Taking components of forces vertically up, the equilibrium condition for forces is

$$P + Q - W - L = 0 \quad \dots (1)$$

Torques about any axis must also balance. Taking torques about A and anti-clockwise positive:

$$-W a - L b + Q c = 0. \quad \dots(2)$$

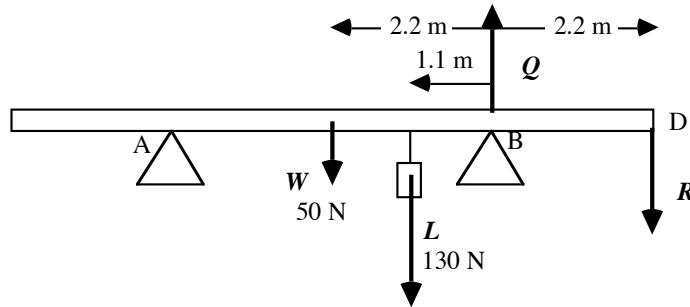
Solve (2) for Q :

$$Q = \frac{W a + L b}{c} = \frac{50 \text{ N} \times 2.2 \text{ m} + 130 \text{ N} \times 3.3 \text{ m}}{4.4 \text{ m}} = 122 \text{ N}$$

Putting this back into (1) gives

$$P = L + W - Q = 58 \text{ N}$$

- b) Now include the new object in the system and call its weight R .

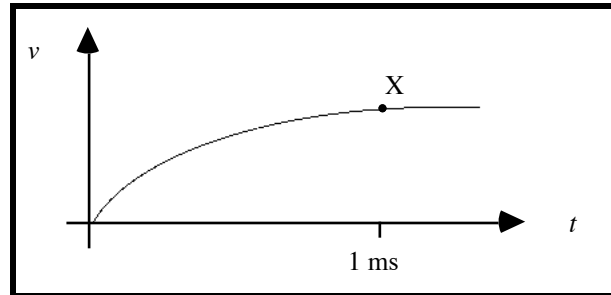


The contact force P at A becomes zero when the contact is broken, i.e. when the plank is just about to tip. Take torques about B and apply the torque condition:

$$W (2.2 \text{ m}) + L (1.1 \text{ m}) = R (2.2 \text{ m})$$

i.e.
$$R = W + \frac{1}{2} L = 50 \text{ N} + 65 \text{ N} = 115 \text{ N}.$$

- 32 a)



We can see that the pushing stage is completed at $t = 1.0 \text{ ms}$ (point X on the velocity-time graph) because there is a roughly constant velocity (constant slope on the height-time graph) after that.

The velocity at 1.0 ms is 1.3 m.s^{-1} which gives an average acceleration of

$$\frac{1.3 \text{ m.s}^{-1}}{1.0 \text{ ms}} = 1.3 \times 10^3 \text{ m.s}^{-2}.$$

- b) 9 cm. [Kinetic energy at X equals potential energy at top of jump.]

33 **Energy approach :**

Original $KE >$ final KE , because some energy will be dissipated by the drag. So the object will travel up faster than it comes down, which means it takes longer to come down than to go up.

OR **Force approach**

As the object travels up it will be slowed down by the combined effects of gravity and drag. On the way back it will be accelerated by a net force equal to the difference between gravity and drag. This means it will take less time to go up (be stopped) than to come down.

- 34 a) i) $mgh = 1.20 \times 10^3 \text{ kg} \times 9.8 \text{ m.s}^{-2} \times 15 \text{ m} = 0.18 \text{ MJ}.$
 ii) Since $h = 0$ then $mgh = 0.$
 iii) $mgh = 1.20 \times 10^3 \text{ kg} \times 9.8 \text{ m.s}^{-2} \times (-200) \text{ m} = -2.4 \text{ MJ}.$
 b) They will all be increased by 2.4 MJ.
 c) By the conservation of mechanical energy principle

$$\Delta KE = -\Delta PE \quad [\text{take PE as } 0 \text{ at the bottom}].$$

$$\frac{1}{2} m v^2 - 0 = -(0 - mgh) ;$$

$$\begin{aligned} v &= \sqrt{2gh} \\ &= \sqrt{2 \times 9.8 \text{ m.s}^{-2} \times 160 \text{ m}} \\ &= 56 \text{ m.s}^{-1}. \end{aligned}$$

- 35** a) 1 fm (when the neutron "hits a brick wall").
 b) Change in potential energy of system between 2.0 fm and 1.5 fm

$$= -0.5 \text{ fm} \times 4 \times 10^3 \text{ N}$$

$$= -2.0 \times 10^{-12} \text{ J}.$$

$$\text{So increase of KE} = 2.0 \times 10^{-12} \text{ J}$$

$$\text{and total KE at 1.5 fm} = 4.0 \times 10^{-12} \text{ J}.$$

- 36** a) Increase in kinetic energy = decrease in potential energy.

$$\frac{1}{2} m v^2 = mgh ;$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 9.8 \text{ m.s}^{-2} \times 10 \text{ m}} = 14 \text{ m.s}^{-1}.$$

- b) If 20% of the energy is lost in the collision, only 80% of the original gravitational potential energy can be recovered.

So at the top of the rebound

$$mgh' = 0.8 mgh ;$$

$$h' = 0.8 \times 10 \text{ m}$$

$$= 8 \text{ m}.$$

- 37** i) $V_B = 0$, $V_C = -8.0 \times 10^{-12} \text{ J}$; $V_D = -8.0 \times 10^{-12} \text{ J}$.

[Change in potential energy = area under curve.]

- ii) $8.0 \times 10^{-12} \text{ J}$.

[which gives total energy at D equal to total energy at A].

- 38** 15 m.s^{-1} .

Notes

Centripetal force = $180 \text{ N} = \frac{mv^2}{r}$. Solving this yields a value of 30 m.s^{-1} for the speed of the stone as it leaves the wheel. That is equal to the speed of the top of the wheel, which is going twice as fast as the whole car. (The bottom of the wheel, in contact with the ground, is momentarily at rest.)

The shape of the stone's path will be parabolic.

- 39** a) $\frac{1}{2} m v^2 = 20 \text{ J}.$

$$v = 20 \text{ m.s}^{-1}.$$

- b) $\omega = \frac{v}{r} = 20 \text{ rad.s}^{-1}.$

- c) 40 N.

- 40** i) $4 \times 10^{-2} \text{ m.s}^{-2}.$

- ii) 3 N.

- iii) The total centripetal force comes from the combination of gravity and the contact force exerted by the Earth.

- 41** a) $\omega = \frac{8 \text{ m.s}^{-1}}{400 \text{ m}/2\pi} = 0.13 \text{ rad.s}^{-1}.$

- b) $a = \frac{(8 \text{ m.s}^{-1})^2}{400 \text{ m}/2\pi} = 1.0 \text{ m.s}^{-2}.$

- c) The force will be directed towards the centre of the track and could be

- a sideways frictional force of the ground on the runner's shoe, or
 - a component of the normal (perpendicular) contact force exerted by a banked track on the runner,
- or
- a combination of both.

- 42 The magnitude of the centripetal force on the discus is equal to the tension in the arm.

$$\omega = \frac{v}{r} = \frac{8 \text{ m.s}^{-1}}{0.5 \text{ m}} = 16 \text{ rad.s}^{-1} .$$

$$T = \frac{mv^2}{r} \\ = \frac{8 \text{ kg} \times (8 \text{ m.s}^{-1})^2}{0.5 \text{ m}} = 1.0 \text{ kN}$$

43 a) i) $T = \frac{60 \text{ s}}{33\frac{1}{3}} = 1.8 \text{ s} .$

ii) $\omega = T = 3.5 \text{ rad.s}^{-1} .$

iii) $v = \omega r = 3.5 \text{ rad.s}^{-1} \times 0.12 \text{ m} = 0.42 \text{ m.s}^{-1}$

- b) The tangential velocity.

c) $F = \frac{mv^2}{r} = \frac{5 \times 10^{-3} \text{ kg} \times (0.42 \text{ m.s}^{-1})^2}{0.12 \text{ m}} = 7 \text{ mN} .$

- 44 i) [Terminal velocity curve : see figure 4.3.]

- ii) [Simple harmonic motion curve (damped) : see figure 7.5.]

45 $f = \frac{1}{2\pi} \sqrt{\frac{g}{x}} .$

- i) 5 Hz.

- ii) Resonance would mean too much movement of the seat.

- iii) A stiffer spring to move the natural frequency well beyond 8 Hz.

- 46 For an object of mass m undergoing SHM the frequency of oscillation is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} .$$

The spring constant k can be found by measuring x , the extension of the web, when a load of known weight W is placed on it.

$$k = \frac{W}{x} .$$

So provided that k and f are independently measured, m can be found.

- 47 a) From the graph $T = 5.0 \text{ ms} .$

so $f = \frac{1}{T} = 0.20 \text{ kHz} .$

- b) Speed is a maximum as the particle goes through the equilibrium position.

So for the equilibrium position $t = 1.25 \text{ ms}, 3.75 \text{ ms}$

c) $a_{\max} = \omega v_{\max}$

$$= 2\pi f v_{\max} = 2\pi \times 0.20 \times 10^3 \text{ Hz} \times 2.5 \text{ m.s}^{-1}$$

$$= 3.1 \text{ km.s}^{-1} .$$

- 48 a) A loss of $1000 \text{ N} \times 1.8 \text{ m}$ i.e. $-1.8 \text{ kJ} .$

- b) Kinetic energy of Tosca → potential energy of stretched trampoline

→ kinetic energy of Tosca

→ gravitational potential energy of Tosca

→ kinetic energy of Tosca.

Recycle through this sequence with dissipation of energy (as heat energy) accompanying all energy interchanges.

- c) No. It is harmonic (or periodic) motion, but not *simple* harmonic motion.

- d) In equilibrium

$$kx = W ;$$

$$(1 \times 10^4 \text{ N.m}^{-1})x = 1000 \text{ N} ;$$

$$x = 0.1 \text{ m} .$$

- 49 Surface area varies as L^2 . Mass and volume vary as L^3 . So (surface area) $^{1/2}$ scales as (mass) $^{1/3}$ or surface area scales as (mass) $^{2/3}$;

$$\frac{\text{surface area of 75 g egg}}{\text{surface area of 50 g egg}} = \left(\frac{75 \text{ g}}{50 \text{ g}}\right)^{2/3} = 1.3.$$

- 50 a) Mass scales as L^3 .

$$\therefore \frac{\text{mass of large fish}}{\text{mass of small fish}} = (1.5)^3 ;$$

$$\text{mass of large fish} = 670 \text{ kg.}$$

- b) Strength of line scales as L^2 so

$$\frac{\text{diameter of stronger line}}{\text{diameter of weaker line}} = \left(\frac{\text{strength of stronger line}}{\text{strength of weaker line}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{\text{mass of larger fish}}{\text{mass of smaller fish}}\right)^{\frac{1}{2}}$$

$$= 1.5^{\frac{3}{2}} .$$

$$\text{So diameter of stronger line} = 1.8 \text{ mm} .$$

- 51 a) $\omega^2 x_0$. [Maximum value of the sine term is 1.]

b)

$$F = ma$$

$$\propto L^3 \cdot \omega^2 L$$

$$\propto \omega^2 L^4 .$$

- c) $F \propto L^2$. (Force varies with cross-sectional area of muscle.)

So $\omega^2 L^4 \propto L^2 ;$

$$\omega^2 \propto L^{-2} ;$$

$$\omega \propto L^{-1} .$$

52 Model answer

- a)

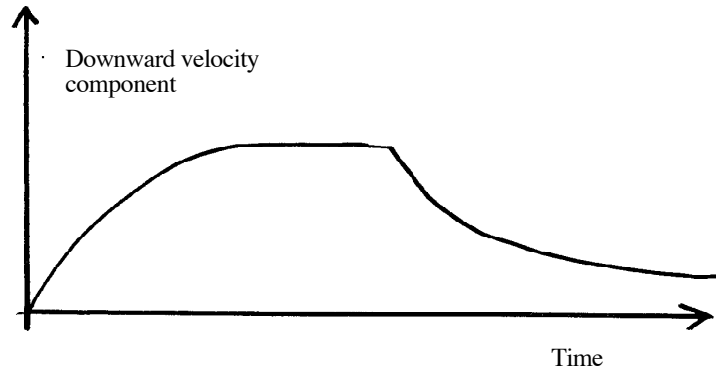
The main forces on the skydiver are her weight W vertically down and a drag force in the direction opposite to the motion (up). The weight remains constant, but the drag increases with increasing speed. The total force starts off being equal to W but decreases until drag and weight are equal. The speed increases until $D = W$ but at a decreasing rate.



- b) As soon as the parachute opens the drag force increases greatly because there is a much greater area over which the air drag operates. So the total force is vertically up.

(... continued over)

- c) The downward velocity decreases and approaches a new terminal value.



53 Model answer

- a) The rate of energy supply in an animal scales as L^2 , where L is the scale factor, a ratio of lengths in similarly shaped bodies. Provided that the rate of energy supply remains constant and that this rate is proportional to the rate of consumption of oxygen from the lungs, then the volume V of air in the lungs, the rate R of consumption of oxygen from the lungs and the time T submerged are related by

$$V = R T$$

Since V scales like L^3 and R scales like L^2 then T must be proportional to L . So the larger animal can stay down longer.

- b) The work done against the drag force D as the whale travels a distance s is equal to Ds . So the average rate of working against the drag is $\frac{Ds}{T}$ or Dv where v is the whale's speed. This can be equated to the rate of energy supply:

$$Dv = R$$

Using the information that the drag force is proportional to the product of the surface area A and v^2 we get

$$Av^2 \cdot v \propto R.$$

Now since both A and R scale like L^2 , v^3 is independent of L , so all whales should be able to swim at about the same speed. (Note that this argument is not affected by the precise form of the relation between drag force and speed.)

54 Model answer

- a) Conventional weighing involves balancing gravitational force on an object ("weight") against a supporting or contact force exerted by the weighing instrument. In a spaceship the astronauts and the ship are normally in free fall and no such contact forces exist. Although gravitational force exists it cannot be balanced against a contact force.
- b) If the astronaut were attached to a one end of a spring with known spring constant then her mass can be determined by measuring the period T of oscillation of the system. Then use the relation

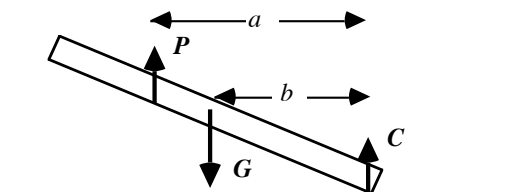
$$T = 2\pi\sqrt{\frac{m}{k}}$$

to calculate the mass m .

55 Model answer

- a)

The forces on the canoe are the supporting force P exerted by the man, the gravitational force G (weight) and a contact force C , which must be vertical if the man pushes vertically and the canoe is in equilibrium.



- b) For equilibrium both forces and torques must balance. Torques can be calculated about any axis. Choose one through the contact point. Then

$$Pa = Gb$$

so
$$P = G \frac{b}{a}$$

where a and b are horizontal distances shown on the diagram. Since these distances are proportional to distances measured along the canoe

$$P = 425 \text{ N} \times \frac{3.0 \text{ m}}{3.8 \text{ m}} = 0.34 \text{ kN.}$$

- c) Lifting at the bottom end only helps to reduce the value of the contact force C . It cannot affect the value of P .

56 Model answer

- a) The weight W of a cylinder of fixed shape and composition is proportional to the mass which, in turn, is proportional to the volume which is proportional to the third power of any linear dimension. So putting h for the height of the cylinder, the scale factor is

$$L = \left(\frac{h_2}{h_1} \right) = \frac{1.0 \text{ m}}{0.20 \text{ m}} = 5$$

So
$$W_2 = W_1 L^3 = 20 \text{ N} \times 5^3 = 2.5 \text{ kN.}$$

- b) The force F required to break a glued joint scales as the contact area, i.e. as L^2 . So

$$F_2 = F_1 L^2 = 125 \text{ N} \times 5^2 = 3.1 \text{ kN.}$$

Since this is greater than the weight of the larger cylinder the glue will support it.

[This result does not imply that the same glue will support an arbitrarily large cylinder.]