

FE1

MOTION

OBJECTIVES

Aims

From this chapter you should develop your understanding of the ways that physicists describe motion. You will appreciate the differences between the everyday meanings of common words like velocity and acceleration and their scientific meanings - which can be quite different! As well as learning this new word-language you should also learn how graphs can be used to describe the quantitative details of motion. You will come to appreciate, we hope, that such graphs are much more useful and comprehensive than memorised formulas and equations - which have limited validity in any case.

Minimum learning goals

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms *position, displacement, vector, scalar, component, time, time interval, average velocity, velocity [instantaneous velocity], speed, acceleration, centripetal acceleration.*
2. Plot, interpret and find values of kinematic quantities from graphs of displacement component, velocity component, acceleration component, distance travelled and speed, all plotted against time interval. Solve kinematic problems using graphical techniques.
3. State, apply and explain the relation among centripetal acceleration, radius and speed.
4. State the relation between acceleration and total force and apply the relation qualitatively to some simple examples.

PRE-LECTURE

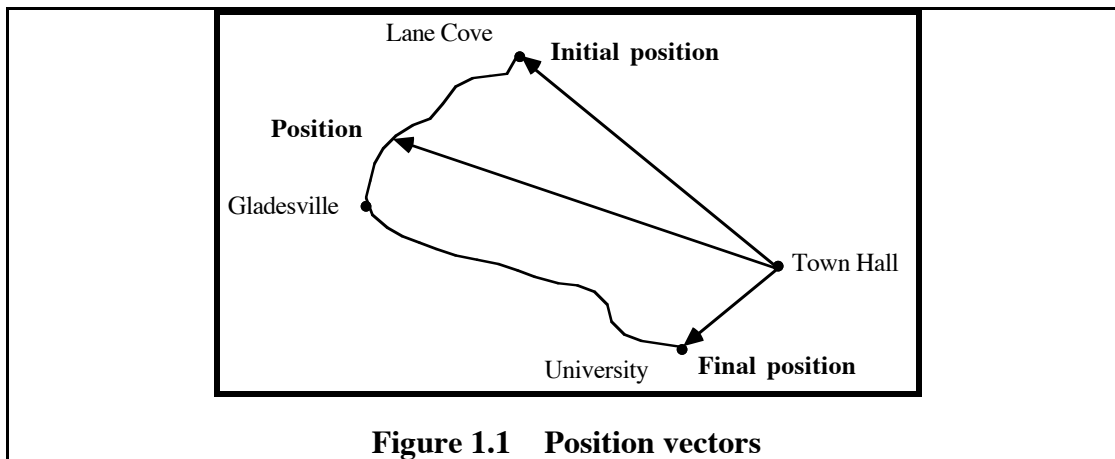
Introduction

This chapter deals with ways of describing the motion of objects. The basic concepts are introduced using examples involving motion in only one dimension (i.e. motion along a straight line). These ideas are then extended to treat motion in two and three dimensions.

1-1 POSITION AND DISPLACEMENT

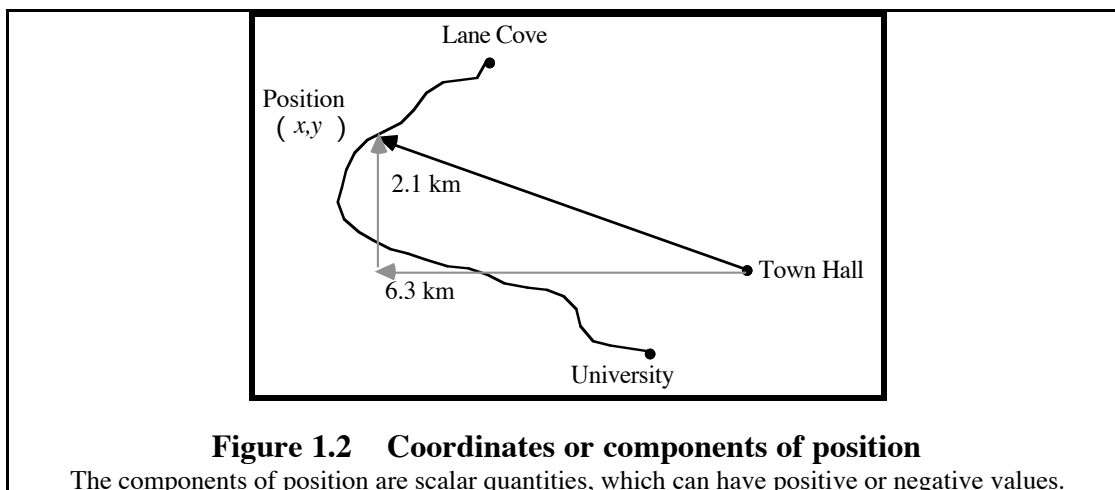
A basic description of the motion of an object can be developed using the ideas of **position** and **time**. As time goes on a moving object changes its position relative to some reference point. As the object moves it also traces out a path, which is usually curved and whose length, measured along the path, is the **distance travelled** by the object.

To illustrate the idea of **position**, consider a car journey from Lane Cove to the University via the Gladesville Bridge (figure 1.1). The car is continually changing its speed and direction of travel. (Remember that going up and down hills also involves changing direction.)

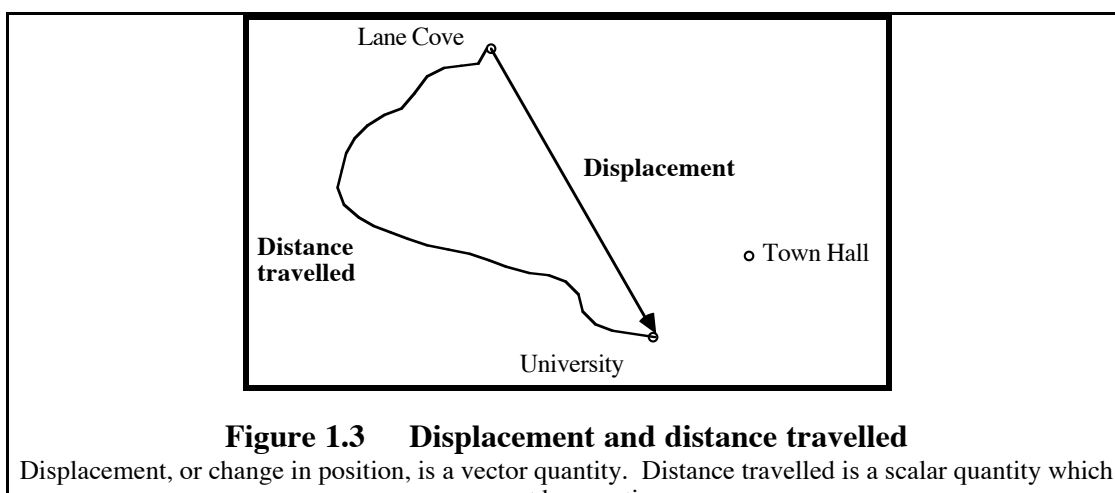


At any instant, we can specify the position of the car from some reference point, the Sydney Town Hall for instance. The position could be described by the length and direction of an imaginary arrow, the car's **position vector**, drawn from the Town Hall to the car.

However it is usually much more convenient to describe position using a set of rectangular coordinates, x, y, z . These three quantities are also called **components**. In this example x could be the easterly component of position, y the northerly component, and z the altitude component. In figure 1.2 x has a negative value (-6.3 km, for example) because the car is to the west, rather than the east, of the origin (Town Hall).



A change in position is called a **displacement**. It can be specified by quoting the changes in each of the three position components. Notice that the magnitude of the displacement and distance travelled are quite different quantities.



1-2 VELOCITY IN ONE DIMENSION

In order to develop the other concepts used to describe motion - velocity and acceleration - we can use one-dimensional examples, with a single position component. The treatment of the other two components in more general cases is exactly the same.

Example 1.1: One-dimensional motion of a car

Consider a car rolling downhill - out of gear. Let's call its position component x and let the time interval since it started to roll be t . Suppose that its motion is described by the following table:

time	position component
t / s	x / m
0.0	0
1.0	1
2.0	4
3.0	9
4.0	16
5.0	25
6.0	36

Q1.1 It is often more revealing to display this information pictorially on a graph. Plot the position component as a function of time elapsed on the graph paper below (figure 1.4).

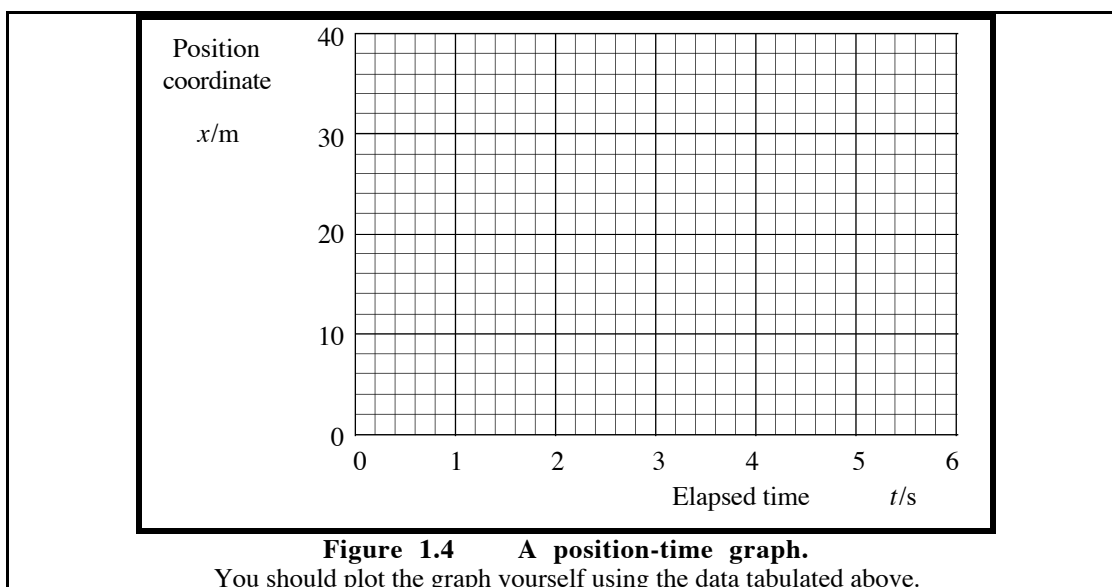


Figure 1.4 A position-time graph.

You should plot the graph yourself using the data tabulated above.

We define **average velocity** over some time interval:

$$\text{average velocity} = \frac{\text{change in position}}{\text{time taken}} .$$

In the general case we would need three components to describe the position, so we would need three components to describe the velocity. In this example we have only one position component and one component of average velocity:

$$\text{average velocity component} = \frac{\text{change in } x}{\text{change in } t} ;$$

$$\text{or} \quad \bar{v}_x = \frac{\Delta x}{\Delta t} \quad \dots (1.1)$$

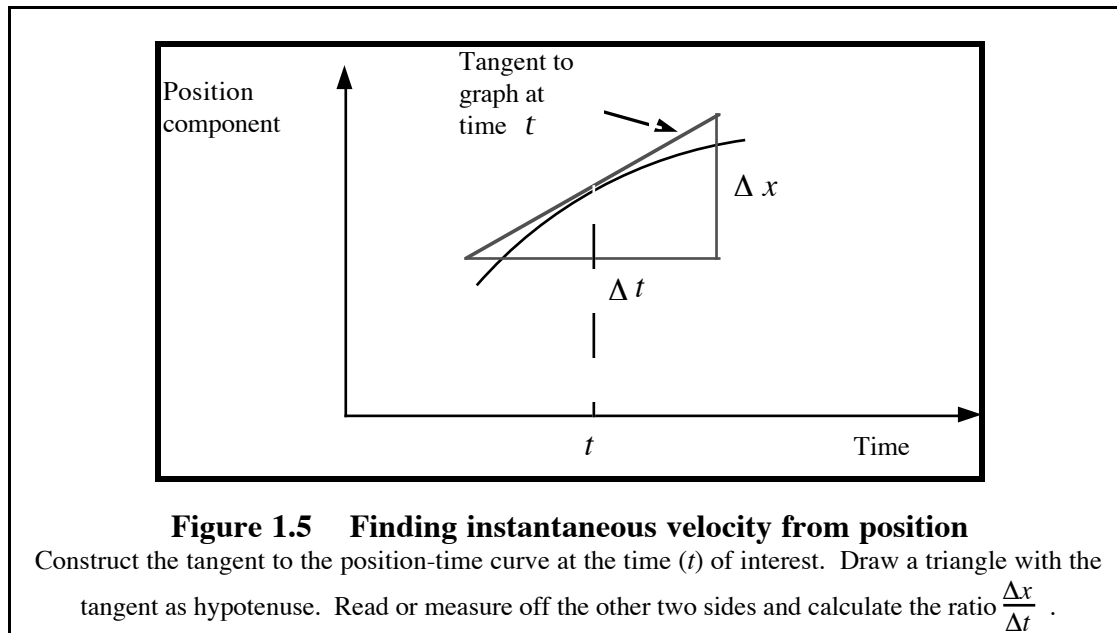
(The bar on top of the symbol v indicates 'average' and the subscript x indicates the component of velocity in the direction corresponding to increasing values of coordinate x .)

The SI unit of velocity is the **metre per second**, symbol $\text{m}\cdot\text{s}^{-1}$.

Q1.2 Use the data of Q1.1 to calculate the component of average velocity over the time interval between 2.0 s and 5.0 s.

The **instantaneous velocity** (or just **velocity** if the context is clear) at any instant is the rate at which an object's position is changing at that instant. For a one-dimensional motion, the velocity

component is given by the slope of the position-time graph. The slope of a graph at a point is equal to the slope of the tangent to the graph at that point, which can be found as shown in figure 1.5.



Q1.3 Use this definition and the data of Q1.1 to estimate the velocity component of the car at $t = 2.0$ s and at $t = 5.0$ s. Note that these values are different from the average velocity calculated in Q1.2.

In our one-dimensional example the component of **instantaneous velocity** is given by the time-derivative of the position component:

$$v_x = \frac{dx}{dt} \quad \dots (1.2)$$

In general, velocity is a **vector** quantity because it has direction. Note that the three components of a velocity are **scalar** quantities. Components themselves do not have direction, but they can take positive or negative values.

In physics the magnitude of a velocity is called **speed** and, since it is a magnitude, a speed cannot be negative.

1-3 ACCELERATION IN ONE DIMENSION

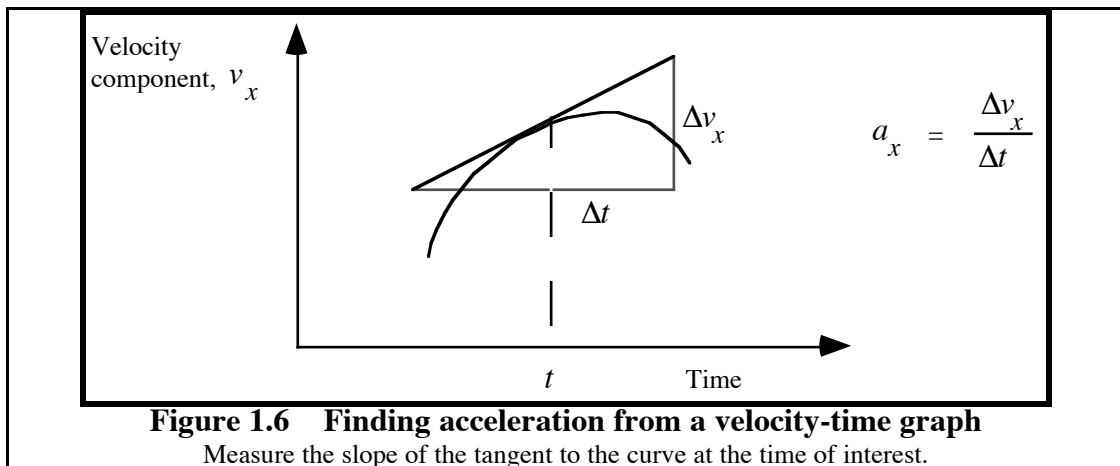
By analogy with the definition of average velocity the **average acceleration** of an object during some time interval is defined as:

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}.$$

This definition can be expressed in terms of components as

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} \quad \dots (1.3)$$

The instantaneous **acceleration** at any instant is defined as the rate at which the velocity is changing at that instant. Acceleration is also a vector quantity so, in general, we need three components to describe it. The **component of instantaneous acceleration** is given by the slope of the tangent to the velocity-time graph (figure 1.6).



In calculus notation,

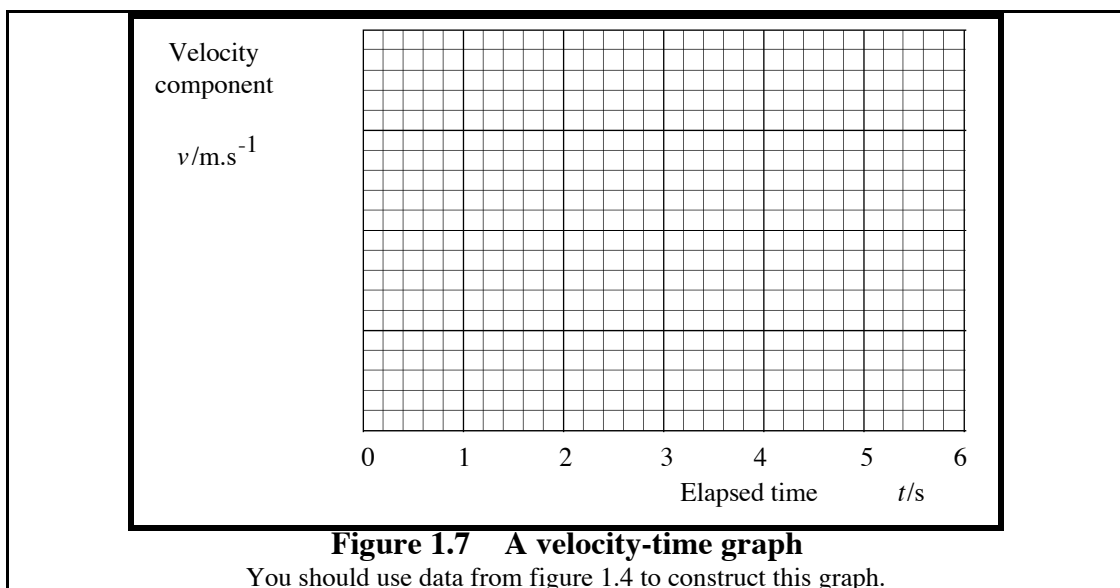
$$a_x = \frac{dv_x}{dt}, \quad \dots (1.4)$$

i.e. acceleration is the derivative of velocity with respect to time.

The SI unit of acceleration is the **metre per second per second**, symbol m.s^{-2} .

To find the acceleration in example 1.1, we would have to find the velocity components at a series of instants, plot these on a velocity-time graph and find the slopes at various times.

Q1.4 Plot the velocity component of the car (example 1.1) at $t = 2.0 \text{ s}$ and $t = 5.0 \text{ s}$ on the graph paper below (figure 1.7).



In this example the acceleration was constant so a straight line through the two points you have just plotted represents the velocity component at all times. Use this graph to find the constant value of the acceleration component.

LECTURE

1-4 MOTION IN ONE DIMENSION

In the following one-dimensional example the velocity component remains positive, which means that the direction of travel remains constant. In this special case the distance travelled happens to be equal to the magnitude of the displacement, or change in position. Remember, however, that this is not true in general.

Example 1.2: Starting a car

- a) A car is started by rolling it downhill out of gear.
- b) When it is going sufficiently fast the gears are engaged. This starts the engine successfully and the car is driven to the bottom of the hill
- c) The car is then stopped.

The following position-time graph (figure 1.8) is obtained.

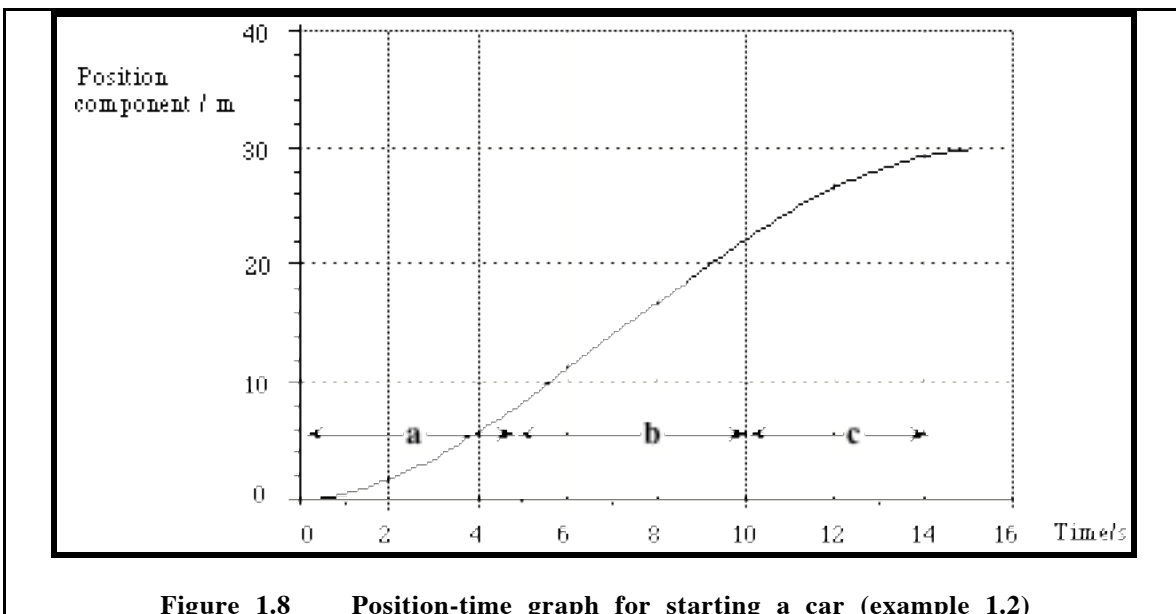


Figure 1.8 Position-time graph for starting a car (example 1.2)

By estimating slopes in figure 1.8 we obtain the velocity-time graph in figure 1.9.

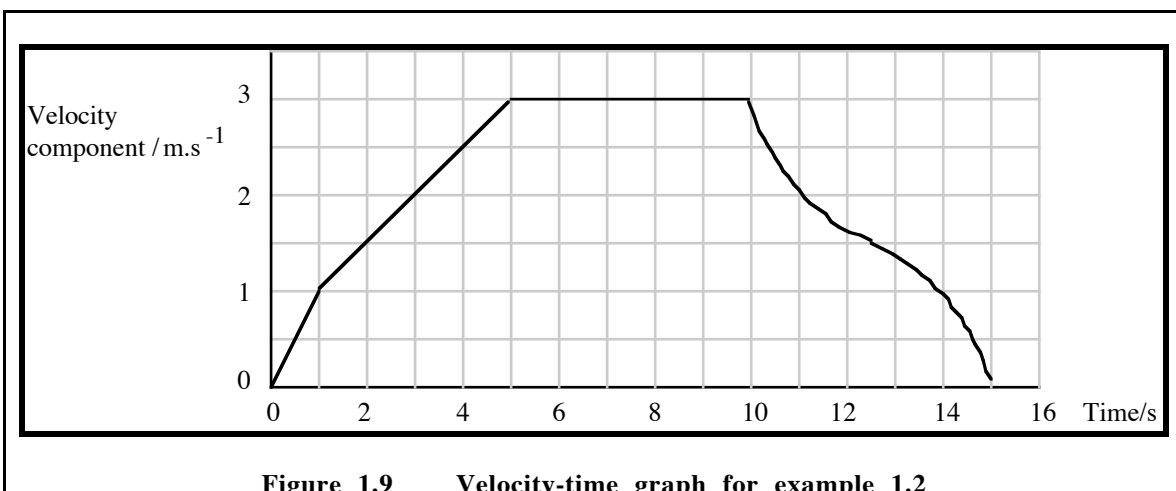


Figure 1.9 Velocity-time graph for example 1.2

Then by estimating slopes on the velocity-time graph (figure 1.9) we obtain the acceleration-time graph (figure 1.10).

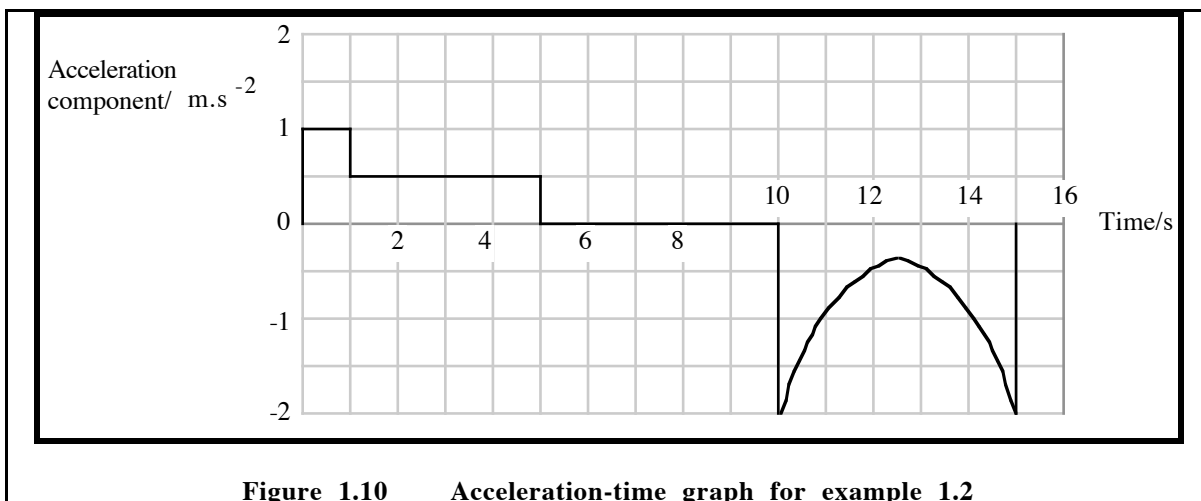


Figure 1.10 Acceleration-time graph for example 1.2

Example 1.3: A velocity-time graph

Suppose that we had been given velocity-time information instead of position-time information. We can perform a reverse process and find displacement as a function of time.

time /s	velocity component/m.s ⁻¹
0	10
1.0	7
2.0	-2
3.0	-17
4.0	-38
5.0	-65

We want to find the displacement component as a function of time, using the starting point as the reference point. First, we draw the velocity-time graph, figure 1.11.

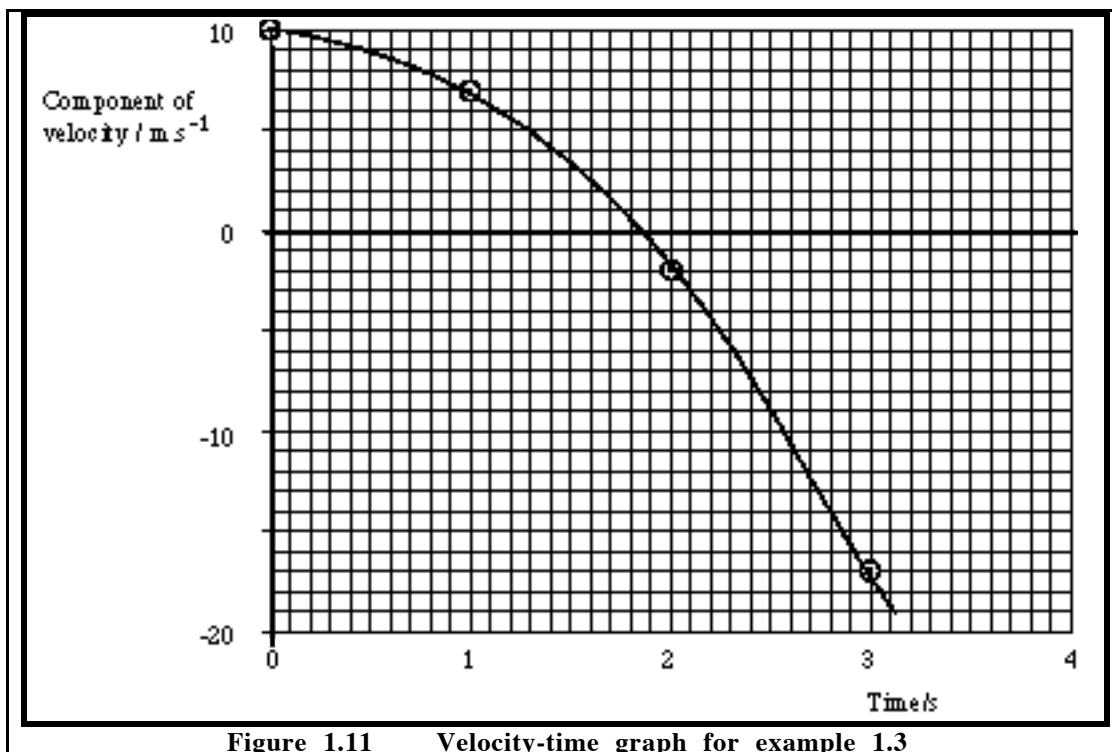


Figure 1.11 Velocity-time graph for example 1.3

The displacement component after a certain time is given by the area under the velocity-time graph up to that time. By counting squares, the area under the curve up to $t = 1.00$ s, say, is 90 small squares which represents a displacement of

$$90 \times (1.0 \text{ m.s}^{-1}) \times (0.10 \text{ s}) = 9.0 \text{ m.}$$

i.e. we find that the displacement after the first second is 9.0 m. We can continue this process and plot a displacement-time graph (figure 1.12). Note that areas under the time axis in figure 1.11 are to be subtracted since the body is moving in the opposite direction; values of the velocity component are negative, so each contribution to the displacement is negative.

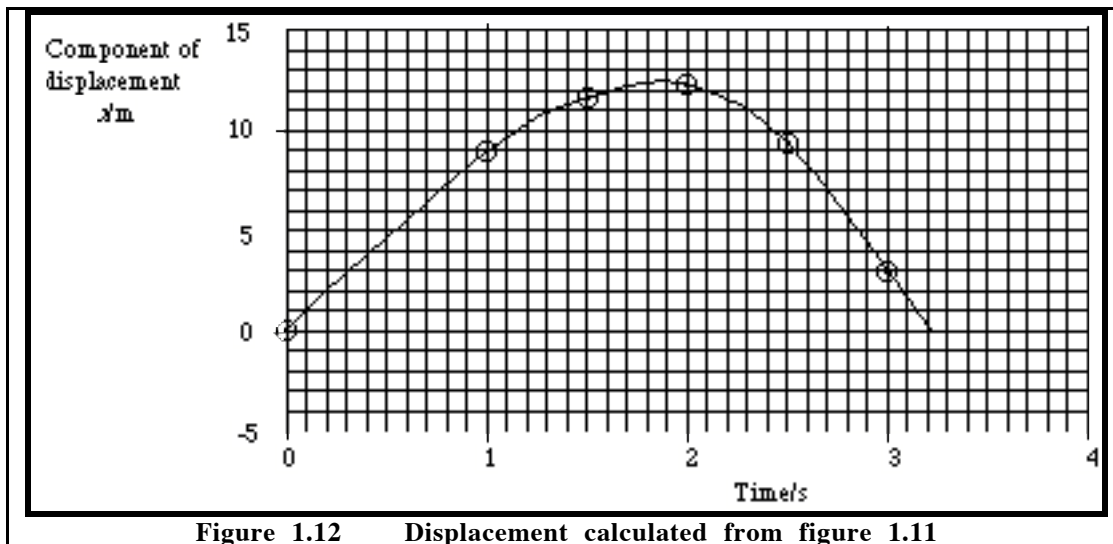


Figure 1.12 Displacement calculated from figure 1.11

In calculus terminology this operation is an integration:

$$x = \int_0^t v_x dt ,$$

i.e. displacement from the starting point is the integral of the velocity with respect to time.

It is also possible to find velocities from an acceleration-time graph in a similar manner if the starting velocity is known.

1-5 MOTION IN MORE THAN ONE DIMENSION

So far our discussion has been restricted to one-dimensional motion. In three dimensions directions are obviously important. Displacement, velocity and acceleration are all quantities which need both their magnitudes and directions to be specified in order to describe them completely. They are **vectors**. At any instant, the velocity is in the direction that the object is moving.

Demonstrations: Ball thrown into the air

(Draw your own diagrams. They should show the path of the object together with arrows to represent velocity and acceleration at some instant.)

When the ball is thrown vertically upwards, the acceleration is 9.8 m.s^{-2} downwards along the same vertical line, and the problem is one-dimensional.

When the ball is thrown upwards at some angle, the problem is two-dimensional since there is horizontal motion as well as vertical motion. The acceleration is still 9.8 m.s^{-2} vertically downwards, but now the acceleration is not in the same direction as the velocity. The velocity of the ball changes in both magnitude and direction; its path curves.

The acceleration is in the vertically-down direction which means that only the vertical component of velocity changes. The horizontal component of velocity remains constant.

Uniform circular motion

An object moves with constant speed, v , in a circle of radius R . The instantaneous velocity is along a tangent to the circle and the direction of the acceleration is along the radius towards the centre of the circle. This is called a **centripetal acceleration**. Its magnitude is

$$a = \frac{v^2}{R} . \quad \dots (1.5)$$

In this special case the magnitude of the velocity (i.e. the speed) remains constant and only its direction changes. The magnitude of the acceleration is also constant but its direction keeps changing, so that it always points towards the centre of the circle. (If the speed of a circular motion changes there is a component of acceleration tangential to the path but the radial component of the acceleration is still described by equation 1.5.)

1-6 FORCES

Having described motion (kinematics), we can move on to examine the causes of changes in motion or causes of acceleration (dynamics). These are **forces**.

Firstly, we have the **law of inertia**: *if there are no forces acting, the motion remains unchanged*. If more than one force is acting and these forces cancel or balance each other, this has the same effect as no force. (This law is also known as Newton's first law of motion.)

Demonstrations

- Air track with glider - glider at rest, glider moving with constant velocity.
- Dry ice puck.
- Motion in a spacecraft cabin.

Secondly, if there is an unbalanced force acting, there is an acceleration in the direction of the net force.

Since both magnitude and direction are needed for the complete specification of a force, force is a vector quantity.

Demonstrations

What is the direction of the force in each of the following cases?

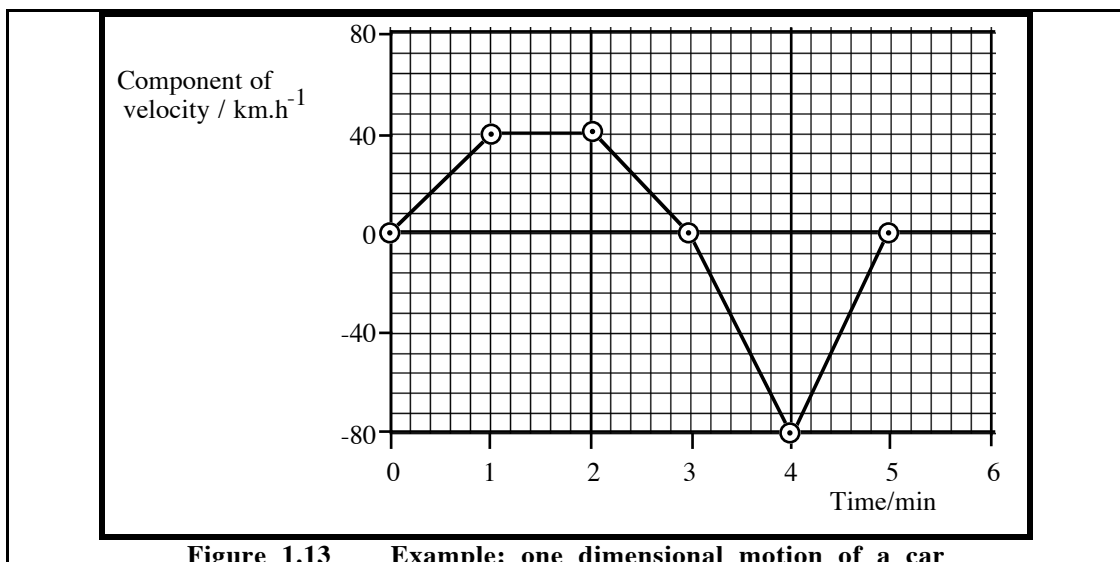
- A ball thrown into the air.
- Circular motion examples.

POST-LECTURE

1-7 QUESTIONS

Motion in one dimension

- Q1.5** a) Consider the velocity and acceleration-time graph in example 1.2 above. Give an account of how the velocity and acceleration change during each segment.
- b) Consider the velocity-time graph in example 1.3 above. Note that areas between the graph and the time axis represent positive contributions to the displacement if the curve is above the axis and negative contributions if the curve is below.
- What happens to the displacement when the velocity goes through zero to become negative ?
- What can you say about the areas above and below the time axis when the displacement is zero (i.e. when the object has returned to its starting point)?
- What does a negative displacement mean?
- Q1.6** Here is a velocity-time graph (figure 1.13) for a car journey along a straight road.



Give a qualitative description of the motion of the car during the journey.

How far is the car from its starting point at the end of its journey? [Hint: Accelerations for each segment are given by slopes, distances are given by areas.]

Q1.7 The driver of a car wants to average 60 km.h^{-1} for a 20 km trip. The car travels the first 10 km at a constant speed of 40 km.h^{-1} . How fast must it go over the last 10 km?

Q1.8 This exercise is inspired by the N.S.W. Motor Traffic Handbook. It concerns stopping distances.

Assume that the reaction time to apply the brakes of a car is 1.0 s, and assume that the brakes give a constant acceleration of magnitude 5 m.s^{-2} .

Compare the distances required to stop the car travelling at

(a) 9 m.s^{-1} and (b) 18 m.s^{-1} . (9 m.s^{-1} is approximately 32 km.h^{-1} .)

(Hint: use velocity-time graphs.)

Motion in more than one dimension

Q1.9 An object is accelerating if

- (a) the magnitude and direction of its velocity both change, or
- (b) only the magnitude of its velocity changes, or
- (c) only the direction of its velocity changes.

Give examples of each.

Forces

Q1.10 Suppose you are standing in front of a frictionless table and an object on it is moving from left to right at constant velocity. What happens to the motion if:

- (a) you apply a force directed to the right;
- (b) you apply a force directed to the left;
- (c) you apply a force directed towards yourself?

Q1.11 Consider the motion of an object on which a single force acts. What can you say about the magnitude and direction of the forces that produce the following effects:

- (a) object moving from left to right with constant velocity;
- (b) object moving from left to right with velocity increasing;
- (c) object moving from left to right irregularly (i.e. stopping and starting);
- (d) object moving in circle at constant speed;
- (e) object moving in circle at variable speed?

APPENDIX

1-8 EXAMPLES OF MATHEMATICAL DESCRIPTIONS OF MOTION

In the lecture you have seen how motion can be described by a table of numbers or by a graph. In some special situations a mathematical equation can be used.

For example, if an expression for the displacement is given, it may be differentiated to give velocities and accelerations. Or, if the velocity is given, it may be differentiated to give accelerations.

Conversely, if the acceleration is given, together with some statement about what the moving object is doing at time $t = 0$, it may be integrated to give velocities and displacements. Or if the velocity is given together with some similar statement, it may be integrated to give the displacement.

Q1.12 The displacement of an object is given by

a)
$$x = A + Bt + Ct^2 + Dt^3 \text{ or}$$

b)
$$x = A \cos(\omega t),$$

where A, B, C, D, ω are constants.

In each case,

- i) find the displacement of the object at time $t = 0$, and
- ii) differentiate to find the velocity and the acceleration as functions of time.

Q1.13 At $t = 0$, an object is at position $x = x_0$ and has velocity component $v = v_0$. Its acceleration is

a)
$$a = c$$

b)
$$a = -kt$$

where c and k are constants.

In each case, integrate to find velocities and displacements at any subsequent time t .