**Chapter L1**

**1.1** Distance = speed × time interval = \((3.0 \times 10^8 \text{ m.s}^{-1}) \times (1.0 \times 10^{-6} \text{ s})\) = 0.30 km.

**1.2** Use \(v = f\lambda\).

\[ f = \frac{v}{\lambda} = \frac{3.0 \times 10^8 \text{ m.s}^{-1}}{500 \times 10^{-9} \text{ m}} = 6.0 \times 10^{14} \text{ Hz.} \]

For glass \(n = 1.50\). This means \(\frac{c}{v_g} = 1.50\);

\[ v_g = \frac{3.00 \times 10^8 \text{ m.s}^{-1}}{1.50} = 2.0 \times 10^8 \text{ m.s}^{-1}. \]

The frequency remains unchanged. Therefore the wavelength will change.

\[ \lambda_g = \frac{v_g}{f} = \frac{2.0 \times 10^8 \text{ m.s}^{-1}}{6.0 \times 10^{14} \text{ Hz}} = 0.33 \text{ µm}. \]

**1.3** See the example in the text.

One source has irradiance \(I\) and \(I = kE^2\).

For 3 incoherent sources: \(I_{\text{total}} = kE^2 + kE^2 + kE^2 = 3kE^2 = 3I\).

For 3 coherent sources: \(I_{\text{total}} = k(E + E + E)^2 = 9kE^2 = 9I\).

**1.5** Use \(I = \frac{P}{4\pi^2} = \frac{10 \text{ W}}{4\pi(20 \text{ m})^2} = 2.0 \times 10^{-3} \text{ W.m}^{-2}\).

**1.6** The maximum sensitivity occurs at about 555 nm. The half maximum points are at about 510 nm and 615 nm. The 1% points are at about 425 nm and 690 nm.
Chapter L2

2.1 Since the angle of incidence for the total internal reflection is equal to 45°, the refractive index must be at least 1.41 \((1/\sqrt{2})\).

2.2 a) Little.
b) At glancing angle, almost all is reflected.
c) Little.
d) All, if the angle of incidence is greater than the critical angle.

2.3 Use \(\sin \phi_c = \frac{n_{\text{liquid}}}{n_{\text{glass}}}\)

\[n_{\text{liquid}} = \sin 59° \times 1.56 = 1.34.\]

The refractive index of the liquid must be less than the refractive index of the glass in order that total internal reflection should be possible.

2.4

Rays from a mark X on the bottom of the pool are bent at the surface so that they appear to come from a shallower point Y. Hence the pool seems to be shallower than it really is.

2.5

\[n_1 \sin i = n_2 \sin r.\]

\[1.00 \sin 45° = 1.50 \sin r; \quad 1.33 \sin 30° = 1.00 \sin r; \quad r = 28°; \quad r = 42°.\]
2.6 (a)

For a field of view of 30° fish can see objects above the water surface that lie within a cone of half angle 42°. The shaded area represents the fish's field of view.

(b) Total internal reflection occurs at 49°. (Show this.) So a fish can see objects that lie anywhere above the water surface. The view will be compressed into a cone of half angle 50°. This will be surrounded by a ring of reflections from the water-air boundary.

2.7 If we consider the refraction at one air-glass boundary it is easy to see that it is the difference between the values of the refractive index at either end of the spectrum that determines how spread-out the spectrum will be. The refractive index of dense flint glass goes from about 1.65 to 1.70 while for crown glass it goes from about 1.52 to 1.53. The difference for dense flint glass is greater (about 0.05 as opposed to 0.01) so it would give a more dispersed spectrum.

2.8 \[5.11 \times 10^{14}\text{ Hz. 395.4 nm.}\]

2.9 As well as the main reflection from the metal backing, there is another reflection from the front surface of the glass. Images of the same object formed by these two reflections are in different locations.

2.10 **Hint.** Think in terms of variation of refractive index with temperature.

2.11 Why not? The laws of reflection and refraction can be understood in terms of wave behaviour.

2.12 No.

Maybe - if I already know how refractive index varies with wavelength, I could measure refractive index. But that is not the most accurate way of measuring wavelength.

2.13 No. The refractive index on the other side has to be smaller.

2.14 Yes. No. Yes.

2.15 **Notes.** No, but you can get only a lateral displacement of the light. You would not normally notice any spectrum using a thin piece of glass and a relatively wide beam of light. The advantage of a prism is that the final directions of the light rays for different frequencies are different, so they continue to diverge after they leave the prism.

2.16 Use lenses to focus the light. See chapter L3.
Chapter L3

3.1

Since the distances from you to the mirror and from the image to the mirror are equal, by similar triangles ABC, ADE, the minimum height of the mirror must be half your height.

3.2 A lens can never give a magnification of 1.00, because that would require zero power. For a magnification of -1.00 use a converging lens and place the real object at a distance of |2f| from the lens.

3.3

The ray travelling parallel to the principal axis is refracted through the focal point, and the ray travelling along the line joining the object and the focal point is refracted parallel to the axis. The image is virtual and upright. The rays do not actually pass through the image point.

3.4 The focal length of a convex lens is positive.

i) \[
\frac{1}{i} = \frac{1}{f} - \frac{1}{o}
\]

If \(o\) is very large, \(\frac{1}{o} \approx 0\) and \(\frac{1}{i} = \frac{1}{f}\), or \(i = f\).

The image is real and lies on the other side of the lens.

For parts (ii), (iii) and (iv) make \(o\) the subject of the equation

\[i = \frac{fo}{o-f}\]

ii) \(o = 2f\). This gives \(i = 2f\). The image is real and appears at a distance \(2f\) on the other side of the lens.

iii) \(o = f\). This gives \(i = \infty\). The image lies at an infinite distance.

iv) \(o < f\). The denominator of our equation for \(i\) is always negative so \(i < 0\). The image is virtual and lies on the same side of the lens.

3.5 Make \(i\) the subject of the equation

\[i = \frac{fo}{o-f}\]

The lens is concave so \(f\) is negative. Consequently, the numerator is always negative, the denominator is always positive and \(i\) is always negative. The image is always virtual and lying on the same side of the lens.

3.6 (a) Suppose the light is travelling towards the lens in the \(\rightarrow\) direction.
The first surface is like this. It is convex. So \( R_1 = +1 \text{ m} \).

The second surface is like this: It is concave. So \( R_2 = -1 \text{ m} \).

\[
\frac{1}{f} = (1.5 - 1) \left( \frac{1}{(+1 \text{ m})} - \frac{1}{(-1 \text{ m})} \right) = 1 \text{ m}^{-1}.
\]

\[ f = +1 \text{ m}. \]

The lens is converging.

(b) \( R_1 \) is negative and \( R_2 \) is positive so \( 1/f \) is negative.

(c) The flat surface means that \( R_1 \) is infinite, giving zero for \( 1/R_1 \). \( R_2 \) is negative, so \( 1/f \) is positive.

3.7

(a) \[ \frac{1}{f} = (\frac{1.5}{1.33} - 1) \left( \frac{1}{(+1 \text{ m})} - \frac{1}{(-1 \text{ m})} \right) = 0.13 \times 2 \text{ m}^{-1} = 0.26 \text{ m}^{-1}. \]

\[ f = 3.9 \text{ m}. \]

(b) \[ \frac{1}{f} = (\frac{n'}{n} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Since the lens is an air-filled bag, \( n \) is the refractive index of air (= 1.00) and \( n' \) that of water (> 1), so the term \( \left( \frac{n'}{n} - 1 \right) \) is negative. The radii of curvature of the bag will be of roughly the same magnitude on both sides. The first side \( R_1 \) will be positive, the second side \( R_2 \) will be negative so the term \( \frac{1}{R_1} - \frac{1}{R_2} \) will be positive. So \( f \) will be negative and the lens will be diverging.

3.8

![Wall](image)

First lens: \( f = +0.1 \text{ m}; \quad P = +10 \text{ m}^{-1}. \)

Second lens: \( f = +0.2 \text{ m}; \quad P = +5 \text{ m}^{-1}. \)

3.9

From figure 2.19:

refractive index of crown glass for 700 nm light \( n_r = 1.50 \); 
refractive index of crown glass for 400 nm light \( n_v = 1.51 \)

Use \[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

The factor \( \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \) is the same in both cases, so

\[ \frac{f_v}{f_r} = \frac{n_r - 1}{n_v - 1} \]

\[ \therefore \quad f_v = \frac{n_r - 1}{n_v - 1} f_r = \frac{0.50}{0.51} f_r . \]

The focal length for violet light is \( f_v = 0.098 \text{ m}. \)
3.10

First consider any object. By sketching the rays from the edges of the object which go straight through the principal point of the lens you can see that the angles subtended at the lens by the object and its image are equal. Looking at the smaller triangle you can see that the angle is approximately equal to $d/i$. For the sun which is very far away $i = f$ so $\alpha \approx d/f$. Hence

$$d = \alpha f = 0.01 \times 150 \text{ mm} = 1.5 \text{ mm}.$$

The diameter of the lens does not affect the size of the image, but it will affect its brightness because a wider lens gathers more light. What do you need for a burning glass that could be used to set the card alight?

3.11  (a) The intermediate image is located 0.2667 m from the first lens on the far side. If the second lens were not there the image would be real and its magnification would be -0.3333. (Keep extra figures in intermediate results.) The second object distance is real, with $o = +0.3333$ m. The final image is real and is located at 1.00 m from the second lens. The magnification produced by the second lens is -3.00, so the total magnification is +1.00. The final image is 35 mm high and is upright with respect to the object.

(b) Details for the intermediate image are the same. The second object distance is virtual (negative), with $o = -0.1667$ m. The final image is real and is located at a distance of 0.10 m on the far side of the second lens (0.20 m from the first lens). The second magnification is +0.600, so the total magnification is -0.200 and the image height is 7.0 mm. The final image is inverted.

3.12  For the first lens, since the object is at infinity the image is at the focal point:

$$i_1 = f_1 = 400 \text{ mm}.$$  

This image lies on the side of the first lens away from the object. It is the object for the second lens.

![Diagram](image)

When the second lens lies only 200 mm from the first, this image serves as a virtual object 200 mm from the second lens. So we put $o_2 = -200$ mm.

$$\frac{1}{o_2} + \frac{1}{i_2} = \frac{1}{f_2},$$

$$\frac{1}{-200 \text{ mm}} + \frac{1}{i_2} = \frac{1}{+600 \text{ mm}}$$

$$i_2 = +150 \text{ mm}$$

The final image lies 150 mm on the real image side of the second lens.

![Diagram](image)

The incident beam was parallel so this image point is the second focal point of the simple optical system made by combining two lenses.

3.13  Hint. Dispersion depends on refractive index. There is no dispersion on reflection.

3.14  The linear magnification produced by a plane mirror is always exactly 1 (or -1 if you use the convention associated with lenses.)
**3.15**

![Diagram](image)

**3.16** The beam will continue to converge but the point of convergence will be in front of the mirror. You could say that the object point is virtual and the image point is real.

**3.17** Whenever the object is more than one focal length away from the lens, it is always inverted.

**3.18** Upside down. No.

**3.19** When the image distance is infinite. Maybe.

**3.20** Not with a real object. But consider a virtual object.

**3.21** No. You can photograph a virtual image in the same sense that you can see one, by forming a real image of the virtual image on the film or retina.

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**Chapter L4**

**4.1** Use the expression, fringe separation \( \approx \frac{\lambda x}{d} \).

(a) If the wavelength is doubled, the separation is doubled.

(b) If the spacing \( d \) between the two sources is doubled, the separation is halved.

**4.2** Use the expression for the fringe separation: \( \frac{\lambda x}{d} \).

\[
\text{Separation of red light fringes} = \frac{(600 \times 10^{-9} \text{ m}) \times (1.0 \text{ m})}{0.10 \times 10^{-3} \text{ m}} = 6 \text{ mm}.
\]

\[
\text{Separation of blue light fringes} = \frac{(400 \times 10^{-9} \text{ m}) \times (1.0 \text{ m})}{0.10 \times 10^{-3} \text{ m}} = 4 \text{ mm}.
\]

Notice that at 12 mm and 24 mm the blue and red bright fringes coincide but that at 6 mm and 18 mm a blue minimum coincides with a red maximum.

In white light many wavelengths are present. Coincidence of bright fringes occurs only at the centre of the pattern. At either side of the central white fringe a few bright fringes with coloured edges may be seen. Beyond this the fringes become indistinct.

**4.3** For a Young's double slit set-up it is easy to see that light travelling to a point in the centre of the screen equidistant from the two sources will still be out of phase by \( \phi \). So there will not be a bright fringe in the centre of the screen.

The condition for a bright fringe will be satisfied for a point on the screen slightly off-centre. The two sources produce an interference pattern identical to the pattern described earlier except that it is shifted slightly to one side.
4.4 The sketch shows a soap film viewed in reflected light.

The dark region at the top. Here the film thickness $t$ is very small so the effective path difference is almost zero. However, the wave reflected from the front surface suffers a phase shift of $\pi$ while the one reflected from the back surface does not. The two reflected waves interfere with each other destructively and the region appears dark.

The first bright region. Here the film must provide a half wavelength effective path difference to cancel the effect of the phase change on reflection, i.e.,

$$D = 2nb = \frac{\lambda}{2}.$$  

The thickness here is $b = \frac{\lambda}{4n}$.

The two reflected waves interfere constructively and the region appears bright.

4.5 This results from interference between light reflected from the front surface and from the back surface of the oil. The thickness at the edge would be of the order of one or, at most a few, wavelengths of light.

As the film becomes thicker, constructive and destructive effects for different frequencies of light overlap and colours are blurred.

4.6 589 nm. The separation would be doubled.

4.7 The peak sensitivity of the eye is at a wavelength of 550 nm. (See figure 1.7). The thickness of the coating should be 0.10 $\mu$m or some small odd integer multiple of that.

4.8 Why not? Provided that the dimensions are right it should work with any kind of waves.

4.9 Assuming that the lamp is properly waterproofed, the fringe pattern will shrink to about 70% of its former size. This happens because the conditions for interference, properly stated in terms of phase differences, translate into optical path differences. When the space is filled with water (refractive index 1.3) the optical paths all become longer by a factor of 1.3. If you put this into the condition for an interference maximum, the rest follows.

4.10 No. The lamps are not coherent sources. [You should be able to explain why they are not coherent.]

4.11 The original fringe pattern would disappear. Since the twin slits are now emitting light with different wavelengths there is no correlation between them and the waves are not coherent. It doesn’t even make sense to talk about coherence between waves with different frequencies. However you should see two diffraction patterns - which are really interference patterns- one for each slit. The nature of these patterns is described in the next chapter.

4.12 The original fringe pattern will probably disappear because the waves coming out of the two slits are no longer coherent. But see the answer to Q4.11 and chapter L5!

4.13 The refractive index of the oil must be between 1.0 and 1.3. (There is no net phase change at reflection when the film thickness approaches zero. Since there is a phase change at the air-oil boundary there must also be one at the oil-water boundary.)
5.1 One way of specifying the width of the central maximum is to quote the angular separation of the minima on either side. The angular position (from the centre) of each of these minima is given by

\[ \theta \approx \sin \theta = \frac{\lambda}{a} = \frac{500 \times 10^{-9} \text{ m}}{0.10 \times 10^{-3} \text{ m}} = 5 \times 10^{-3} \text{ rad.} \]

Hence the width of the central maximum is twice this: \( 10 \times 10^{-3} \text{ rad.} \). This is sufficient answer but if you want the linear distance you can use \( \theta \approx \tan \theta = \frac{y}{x} \) where \( y \) is the linear position of a minimum on the screen and \( x \) is the distance from the slit to the screen. This gives \( y = \frac{\lambda x}{a} \).

The width of the central maximum is twice this so

\[ \text{width} = 2 \frac{\lambda x}{a} = 2 \left( \frac{500 \times 10^{-9} \text{ m}}{0.10 \times 10^{-3} \text{ m}} \right) = 10 \text{ mm}. \]

5.2 A single slit of width \( a \) gives a pattern whose half width is \( \frac{\lambda x}{a} \).

A single slit width \( 2a \) gives a pattern whose half width is \( \frac{\lambda x}{2a} \)

The pattern is narrower.

A rectangular aperture gives a pattern which might be regarded as a product of these.

The long side of the aperture gives the short side of the diffraction pattern.
5.3 The angle between the central maximum and the first minimum is

\[ \theta \approx \sin \theta = 1.2 \frac{\lambda}{a} \cdot \]

The angular width of the central maximum is twice this, so

\[ \text{angular width} = 2 \times 1.2 \frac{\lambda}{a} \]

\[ = 2 \times 1.2 \times \frac{500 \times 10^{-9} \text{ m}}{2 \times 10^{-3} \text{ m}} \]

\[ = 6.0 \times 10^{-4} \text{ rad.} \]

5.4 The geometry of the situation is shown in the figure.

![Diagram](image)

According to the Rayleigh criterion, the two sources will be just resolvable if the central maximum of source 2 falls on the first minimum of source 1.

When this is so, the angle between the central maximum and the first minimum

\[ \theta = \text{angle between central maxima of the two sources} \]

\[ = \text{angle between the sources subtended at the eye.} \]

\[ \theta \approx 1.2 \frac{\lambda}{a} = \frac{\lambda}{a} \]

Call the distance to the car \( x \) and the separation of its headlights \( y \) and use \( \frac{y}{x} \approx \theta \).

\[ \frac{\lambda}{a} \approx \frac{y}{x} \]

Distance, eye-to-car: \( y \approx \frac{a y}{\lambda} = \frac{5 \times 10^{-3} \text{ m}}{500 \times 10^{-9} \text{ m}} \times 1 \text{ m} = 10 \text{ km.} \)

5.5 The locations of the first-order maxima are

(a) \( \text{Red: } \theta_1 = \frac{\lambda_1}{d} \); violet: \( \theta_2 \approx \frac{\lambda_2}{d} \).

\( \therefore \theta_1 - \theta_2 = \frac{(\lambda_1 - \lambda_2)}{d} = \frac{300 \times 10^{-9} \text{ m}}{(1/400) \times 10^{-3} \text{ m}} = 0.12 \text{ rad.} \)

(b) Similarly, \( \theta_1 - \theta_2 = 0.24 \times 10^{-3} \text{ rad.} \)
Chapter L6

6.1(a)

At A the light is linearly polarised and \( I_A = \frac{1}{2} I_{in} \).

No light reaches B.

When we look at any source through two plane polarisers at right angles to each other, no light passes through.

\( \text{First sheet} \quad \text{Second sheet} \)

(b) At A the light is linearly polarised and \( I_A = \frac{1}{2} I_{in} \).

At B the polarisation is parallel to the polarising axis of the second polariser. Only a component of amplitude \( E_B = E_A \cos \theta \) passes through. By Malus's law:

\[
I_B = I_A \cos^2 \theta = \frac{1}{2} I_{in} \cos^2 \theta .
\]

(c) Look at the answer to question b. After passing through the second sheet,

\[
I = \frac{1}{2} I_{in} \cos^2 (30^\circ) .
\]

After passing through the third sheet the polarisation is horizontal. Using Malus's law again:

\[
I = \frac{1}{2} I_{in} \cos^2 (30^\circ) \cos^2 (60^\circ) = 0.094 I_{in} .
\]

Are you surprised that the intensity is not zero?

6.2 Let the ordinary-wave refractive index be \( n_o \), the extraordinary-wave refractive index be \( n_e \), the wavelength in vacuum be \( \lambda \) and that in the medium be \( \lambda_m \).

A slab with thickness \( d \) has \( \frac{d}{\lambda_m} \) wavelengths between its boundaries.

The number of ordinary-wave wavelengths between the boundaries will then be \( \frac{d n_o}{\lambda} \) and the number of extraordinary-wave wavelength will be \( \frac{d n_e}{\lambda} \).

For the thinnest possible quarter-wave plate, the difference between these two numbers must be \( \frac{1}{4} \)

i.e. \( \frac{d}{\lambda}(n_e-n_o) = \frac{1}{4} \).

For a wavelength of 600 nm

\[
d = \frac{1}{4} \times 600 \times 10^{-9} \text{ m} = 1.67 \times 10^{-5} \text{ m}.
\]

Calculation for a wavelength of 500 nm shows that a thinner plate is needed at that wavelength.

... continued over
The general expression for thicknesses for quarter-wave plates is
\[ \frac{d}{\lambda}(n_e - n_o) = m + \frac{1}{4} \]
where \( m \) is any integer.

Our plate of thickness of \( 1.67 \times 10^{-5} \) m would be a quarter-wave plate for wavelengths of
- 600 nm, \( m = 0 \);
- 120 nm, \( m = 1 \);
- 67 nm, \( m = 2 \), etc.

Only the 600 nm wavelength lies in the visible range.

A much thicker slab, say \( 1.67 \times 10^{-4} \) m, would be a quarter wave plate at 6000 nm, 1200 nm, 667 nm, 462 nm, 355 nm, etc. Of these, two wavelengths, 667 nm and 462 nm, lie within the visible range. Even thicker slabs would act as quarter wave plates at even greater numbers of visible wavelengths.

6.3 Gray arrows show the electric field vectors for the two component polarisations. The sketches are spaced at intervals of 1/8 of a wave period.

6.4 55°.

6.5

Generally, more of the reflected light will be polarised with \( \mathbf{E} \) parallel to the water surface. If the sunglasses have their polarising axis perpendicular to this direction, the reflected light will not enter the eye. Hence the reflected glare is reduced.
6.6 25% of the original intensity.

6.7 Quartz.

6.8

6.9 Yes.

6.10 Hint. An iceblock is not a single crystal.

6.11 Send in a beam of plane polarised light. Look at the light coming out with a plane polariser (analyser). Find the orientation of the quarter wave plate which gives no variation in intensity when the analyser is rotated.

6.12 It becomes linearly polarised. It becomes circularly polarised the other way.

6.13 The axes of the polarisers on windscreen and headlamps would be crossed.

6.14 Hint. Try detecting the polarisation of the light reflected from the top of the counter.

6.15 Two images on the screen would have polarisations at right angles to each other. The viewer would have to wear polarising (analysing) glasses with two different polarising axes.

Chapter L7

7.1

We have considered two rays from the top of the object.
(i) a ray parallel to the axis which passes through the second focal point and
(ii) a ray passing through the first focal point which emerges parallel to the axis.

The top of the image is at the point where these two rays cross. The image lies 15 mm beyond \( F_2 \).
7.2

(a) 

(b) 

In each case we extend the lines for the incident and emerging rays where necessary. They intersect at the second principal plane.

7.3 \( D = f/8 = 100 \text{ mm}/8 = 12.5 \text{ mm}. \)

7.4

\[
\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \quad ; \quad i = \frac{o \cdot f}{o - f}.
\]

Linear magnification: \( m = \frac{i}{o} = \frac{f}{o-f} = \frac{f}{o} \) since \( o \gg f. \)

also: \( m = \frac{a}{h} \) where \( a = \) image height.

\[ h = \frac{a}{m} = \frac{a \cdot o}{f} = \frac{35 \text{ mm} \times 10 \text{ m}}{50 \text{ mm}} = 7 \text{ m}. \]

7.5 The image distance for an object 200 mm away is given by

\[
\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \quad ; \\
\frac{1}{0.200 \text{ m}} + \frac{1}{i} = \frac{1}{0.050 \text{ m}} \quad ; \\
\frac{1}{i} = 15 \text{ m}^{-1} \quad ; \\
i = 67 \text{ mm}.
\]

Then find the image distance for an object at infinity:

\[ i = f = 50 \text{ mm}. \]

So the lens must be able to move from a position 50 mm in front of the film to a position 67 mm in front of the film. The range of travel is therefore 17 mm.

7.6 The lens required is one that can produce an upright image that is only 3 m away from the eye, for an object at infinity. A diverging lens will do that.

\[ i = f = -3 \text{ m}. \]

The power is \( \frac{1}{(-3 \text{ m})} = -0.3 \text{ m}^{-1} \) (or -0.3 dioptres as the optometrist would say).
8.1 a) Width of thumb \( \approx 1 \times 10^{-2} \text{ m}. \)
   Distance eye-thumb \( \approx 1 \text{ m}. \)
   \[ \alpha \approx \frac{1 \times 10^{-2} \text{ m}}{1 \text{ m}} = 1 \times 10^{-2} \text{ rad}. \]

b) Diameter of sun \( = 1.4 \times 10^6 \text{ km}. \)
   Distance earth-sun \( = 1.5 \times 10^8 \text{ km}. \)
   \[ \alpha = \frac{1.4 \times 10^6 \text{ km}}{1.5 \times 10^8 \text{ km}} \approx 1 \times 10^{-2} \text{ rad}. \]
   (The angle subtended by the moon is similar.
   Radius of moon \( = 1.7 \times 10^3 \text{ km}. \)
   Distance earth-moon \( = 3.8 \times 10^5 \text{ km}. \))

c) Length of creature \( = 1 \times 10^{-4} \text{ m}. \)
   Distance eye-creature \( = 0.25 \text{ m}. \)
   \[ \alpha = \frac{10^{-4} \text{ m}}{0.25 \text{ m}} = 4 \times 10^{-4} \text{ rad}. \]

8.2 The magnification is \( M_e = \frac{d_v}{f_e} = \frac{0.25 \text{ m}}{0.05 \text{ m}} = 5. \)

8.3 a) With the object at the focal point: \( M_e = \frac{0.25 \text{ m}}{0.025 \text{ m}} = 10. \)

b) \[ \frac{1}{o} + \frac{1}{i} = \frac{1}{f} \]
   \[ \frac{1}{o} + \frac{1}{(-0.25 \text{ m})} = \frac{1}{0.025 \text{ m}} \]
   \[ \frac{1}{o} = 44 \text{ m}^{-1} \]
   \[ o = 23 \text{ mm} \]

c) \[ \frac{\text{image height}}{\text{object height}} = \frac{1}{o} = \frac{250 \text{ mm}}{23 \text{ mm}} = 11. \]

The angles are
   \[ \beta = \frac{11 \text{ h}}{0.25 \text{ m}} \]
   \[ \alpha = \frac{h}{0.25 \text{ m}}. \]
   So
   \[ M = \frac{\beta}{\alpha} = 11. \]

We obtain a slightly greater magnification by placing the object just inside the focal length of the lens. The calculation with the image at infinity is easier and 10 is not significantly different from 11.
8.4 (a) Use \( M = \frac{m_o}{M_e} \) with \( \frac{m_o}{M_e} = \frac{g}{f_o} \) to get

\[
M = \frac{m_o}{M_e} = 10 \times \frac{160 \text{ mm}}{16 \text{ mm}} = 100.
\]

(b) \[
h_{\text{int}} = 15 \text{ mm.}
\]
\[
h = \frac{f_o}{g} h_{\text{int}} = \frac{0.016 \text{ m}}{0.16 \text{ m}} \times 15 \text{ mm} = 1.5 \text{ mm.}
\]

8.5 The finest detail has dimensions:

\[
R = \frac{0.6 \times 600 \times 10^{-9} \text{ m}}{0.25 \text{ m}} = 1.44 \times 10^{-6} \text{ m.}
\]

When viewed from a distance of 0.25 m this detail subtends an angle \( \alpha = R/0.25 \text{ m} \). When viewed through a microscope the angular size of the detail is observed as

\[
\beta = M \times \alpha = 100 \times (1.44 \times 10^{-6} \text{ m})/0.25 \text{ m} = 6 \times 10^{-4} \text{ rad.}
\]

Since the eye can resolve more detail than this, (resolution of the eye is \( 3 \times 10^{-4} \text{ rad} \)) the resolution of the microscope is determined by the objective.