ANSWERS TO REVIEW QUESTIONS

IMPORTANT NOTE: READ THIS FIRST.
There are three different kinds of answer here.

- The usual form for quantitative questions is a short entry giving the final numerical answer. Sometimes there will be also a brief indication of how to get to that result. The amount of detail given is usually much less than that required in a model answer. For descriptive answers ("bookwork") references to the text may be all that is given. These entries have no special label.
- **Model answers** are written out in full as a guide to the kind of response which would get full marks in an exam. However, a model answer given here, memorised and presented in an exam, may not score very well because it lacks originality.
- **Notes** are intended to indicate the features of a good answer or give a commentary on which an answer might be based. They often contain background information which would not need to be reproduced in a model answer.

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1 **Notes**

a) See chapter L1. The light-globe spectrum should show a continuous distribution with a single peak (in the infra-red if wavelength values are marked.) The spectrum for a discharge tube should show a set of narrow spikes and the laser spectrum should show a single very narrow spike.

b) See chapters L1 (pp 4-5) and L4. A good answer should include the idea that light is a complex mixture of individual simple harmonic waves (harmonics) with different frequencies. If two sources are to be coherent they must emit similar mixtures of harmonics. For each frequency the phase difference between the harmonic oscillations of the two sources must remain constant over a reasonable period of time - long enough for observations to be made. In the case of incoherent sources all the individual harmonics are completely unrelated - their phases change rapidly and randomly.

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2 **i)** 492 nm.

ii) 2.3×10^8 m.s^{-1}.

iii) 4.7×10^{14} Hz.

Note that the frequency does not change.

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3 **a)** $I = \dfrac{P}{4\pi r^2} = 0.16$ W.m^{-2}.

b) **Notes.** The answer must bring out the essential features that line spectra have light restricted to a finite number of narrow bands of frequencies (or wavelengths) whereas continuous spectra have some contribution over a wide band (or bands) of frequencies. See chapter L1.

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4 **a)** 7.5×10^{14} Hz to 4.3×10^{14} Hz. **b)** Red.

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5 $\sin\theta_c = \dfrac{n_{air}}{n_{glass}} = \dfrac{1.00}{1.45}$; $\theta_c = 43.6°$.

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6 See chapter L8.

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7 a) 0.75 m from the lens on the other side of the lens.

c) Real and inverted.

d) 1.50.

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8 There are many possible good answers.

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9 a) Speed = 1.97×10^8 m.s^{-1}. Wavelength = 3.9×10^{-7} m. Frequency = 5.00×10^{14} Hz .

b) Refracted rays should be bent towards the normal.

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10 a) 3.3 μm.

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11 b) 0.6 μm.

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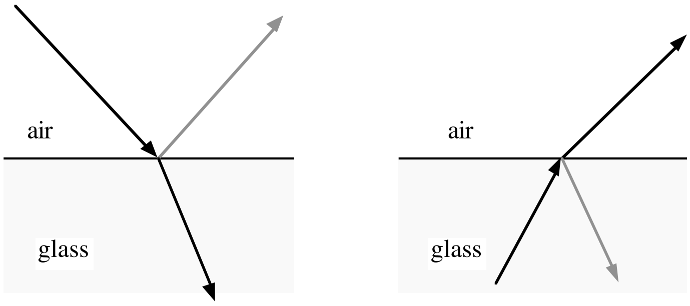
12 b) 0.13 m. c) -72 mm.

13 a,b)



c) 42°.

14 Model answer

a) Power = $\frac{1}{f} = \frac{1}{-200\text{ mm}} = -5.00\text{ m}^{-1}$.

The power of two thin lenses in contact is the sum of their powers, so

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}\ .$$

So the focal length of the combination is

$$f = \frac{f_1 f_2}{f_1 + f_2}$$

$$= \frac{(-200\text{ mm}).(100\text{ mm})}{-200\text{ mm} + 100\text{ mm}}$$

$$= +200\text{ mm.}$$

b) The magnification is $m = \frac{\text{image size}}{\text{object size}} = \frac{50\text{ mm}}{5\text{ mm}} = \frac{i}{o}$.

So $o = \frac{i}{10}$.

The lens equation is

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$$

so

$$\frac{1}{f} = \frac{1}{i} + \frac{10}{i} = \frac{11}{i}$$

and

$$f = \frac{i}{11} = \frac{200\text{ mm}}{11} = 18\text{ mm.}$$

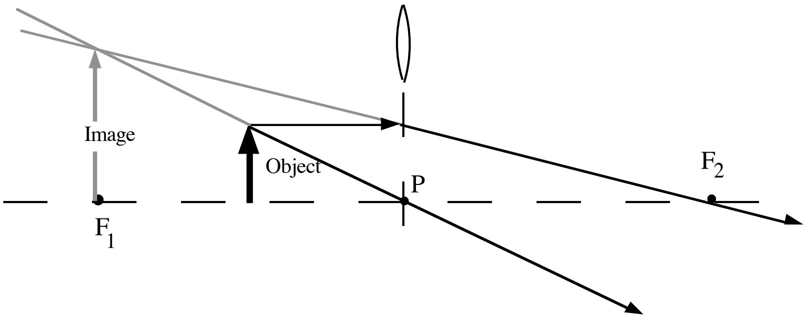
15 a) The image is 0.67 m from the lens on the side away from the object.

b) Real.

c) 1.7 or 1.67.

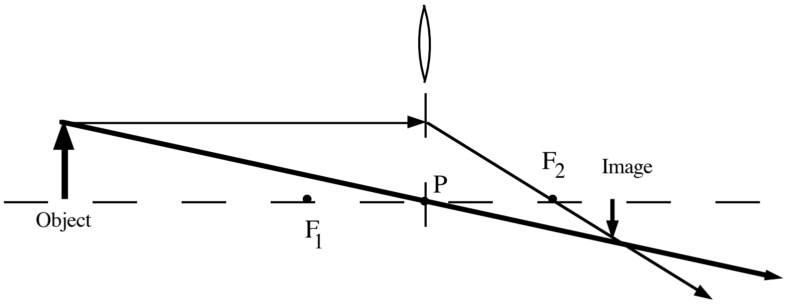
16 Model answer

i)



The image is virtual, erect and magnified.

ii)



The image is real, inverted and diminished.

17 Notes

i) 0.15 m . [Since $i \gg o$, $o \approx f$.] A sledge-hammer approach using the lens equation yields an unrounded value of 0.1456 m.

ii) 2.7 m^{-1}, or 2.9 m^{-1} if you use the unrounded answer from (i).

[Total power = sum of powers; so $P_2 = P - P_1 = \frac{1}{0.15\ \text{m}} - 4.0\ \text{m}^{-1} = 2.7\ \text{m}^{-1}$.]

iii) 1.2 m (or 1.17 m).

18 Notes

i) The image is located 1.20 m on the side of the lens away from the object. [It is not good enough to just write i = 1.20 m.]

ii) It is real.

iii) 2.0.
Note that the value of the refractive index was not needed.

19 Notes

a) 25 dioptres (in SI it is 25 m^{-1}).

b) Converging.

c) See chapter L3. Any lens with spherical surfaces suffers from spherical aberration. Any single-element lens suffers from chromatic aberration.

20 Model answer

i) The lensmaker's formula gives the focal length, f:

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) .$$

The ratio of the two focal lengths is therefore

$$\frac{f_b}{f_r} = \frac{n_r - 1}{n_b - 1} ;$$

so $$f_b = \frac{0.50}{0.51}(0.100\ \text{m}) = 0.098\ \text{m}.$$

ii) Due to spherical aberration the best possible image of a point source will be a smeared out circle of light. Since chromatic aberration produces different focal lengths, the nature of the best image will also depend on the frequency of the light. The best image for high frequency (blue) light will be found near a point 98 mm from the lens, but this image, which still suffers from spherical aberration, will be spoiled by the presence of unfocussed low frequency (red) light surrounding the bluish image. Similarly, at 100 mm from the lens there will be a blurred reddish image surrounded by a blue halo.

21 Notes

A phase change of π occurs at both reflections so the condition for a minimum is

$2nd = \left(m + \frac{1}{2}\right)\lambda$. The smallest thickness, d, which will do the trick is found by putting $m = 0$.

This gives a thickness of 0.12 μm.

22 Model answer

Camera lenses are coated or 'bloomed' with thin films of transparent material in order to reduce reflections from the surfaces of the lens; this lets more light into the camera. Reflection is reduced by thin-film interference. For each layer of blooming, the interference condition for no reflection at normal incidence can be met exactly for only one frequency of light. For neighbouring frequencies the reflection will be reduced but not completely eliminated. Hence for visible white light which contains components with a wide range of frequencies some components will still be reflected to some degree. The appearance of this reflected light is white light minus the components whose reflection has been noticeably reduced. For example, a film which practically eliminates reflection in the green part of the spectrum will appear as white minus green which looks purple. Lenses with several layers of blooming can produce other colours.

23 Notes

You need to realise that in this diagram the angle between the glass slabs is exaggerated. The fringes are produced by thin film interference in the wedge of air between the two glass slabs. This film is really much thinner than either slab. Reflections take place at the bottom of slab B and at the top of slab A. Since the angle of incidence is practically zero for both reflecting surfaces the condition for a bright fringe is that $2nd = (m + \frac{1}{2})\lambda$. [There is a phase change of π at the lower surface, but not at the upper one.]

The first bright fringe ($m = 0$) will occur near the contact point D and the 150th fringe ($m = 149$) will occur somewhere near E. Taking the height of the block to be h and the refractive index of air to be 1.00 we can put $2h = 150\lambda$ (near enough) which yields $h = 30$ μm.

24 Model answer

Phase reversal will occur for light reflected from the top surface of the oil, but not for that reflected from the bottom surface. So strong reflection (constructive interference) will occur for harmonic components with wavelength λ given by

$$2nd = \left(m + \frac{1}{2}\right)\lambda .$$

where n is the refractive index of the oil, d is the thickness of the oil film and m is a non-negative integer. Trying different values of m we get

m	λ
0	572 nm
1	191 nm

and shorter wavelengths for higher m. The only visible wavelength in this sequence is 572 nm, so that is the most strongly reflected component. Other components with wavelengths near this one (in the range, say, 0.5 μm to 0.65 μm) will also be reflected strongly.

25 25 mm.

26 Model answer

a) For complete constructive interference (maximum intensity) the difference in optical path lengths must be an integer multiple of the wavelength in vacuum (or zero). For complete destructive interference (minimum intensity) the difference in path lengths must be an odd integer multiple of half a wavelength: $\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc.

b) Since the distance (x) from the slits to the screen is large, the positions (y) of the dark bands are given by:

$$y = \frac{(m + \frac{1}{2})\lambda}{d} x .$$

Since the first dark band will occur when $m = 0$, the third band will be given by $m = 2$, so

$$\lambda = \frac{d\,y}{2.5x}$$

$$= \frac{0.15 \times 10^{-3}\text{m} \times 9.5 \times 10^{-3}\text{ m}}{2.5 \times 0.90\text{ m}}$$

$$= 0.63\ \mu\text{m}.$$

27 Model answer

i) The colours are produced by thin-film interference. Reflected light reaching the eye comes partly from the front surface and partly from the back surface of the soap film. These two beams of white light are coherent. When the two beams meet and interfere in the eye some harmonic (monochromatic) components will interfere constructively while others will interfere destructively, since the conditions for interference depend on the relation between the wavelength in vacuum and the difference in the optical paths travelled by the two beams. The resulting interference pattern can be considered as white light minus those components which interfere destructively. For example if the thickness of the film is such that a component with wavelength near 550 nm interferes destructively, a range of components in the green part of the spectrum will be reduced, leaving relatively enhanced components near the red and blue ends.

ii) As the water drains away, the film near the top becomes thinner. Just before it breaks, when it is thinner than any visible wavelength, the differences in optical path lengths are nearly zero for all wavelengths. Since there is a phase change of π on reflection at the front surface of the film, but not at the back surface, the condition for destructive interference is satisfied for all components. So the film appears to reflect no light at all - it looks dark.

28 Notes

i, ii) See chapter L5.

iii) Measure the angular position, θ, of the nth-order maximum and calculate λ from the grating equation:

$$d \sin\theta = n\lambda ,$$

where d is the spacing between adjacent lines on the grating.

iv) The angular spread of the spectrum can be made much greater, giving better resolution.

29 Notes

To find the positions of the first-order bright fringes use the grating equation:

$$\sin\theta = \frac{\lambda}{d} .$$

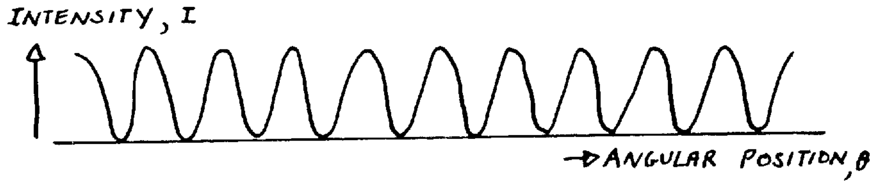
Take typical values of wavelength, say 700 nm for red and 450 nm for blue.

i) 0.09 rad (5°).

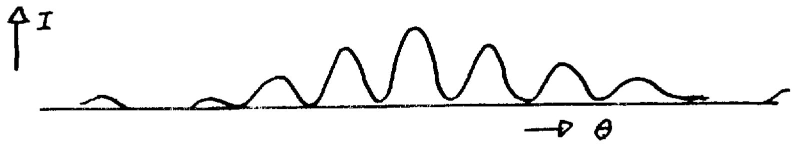
ii) 0.06 rad (3°).

30 Model answer

a)



b)



c) The separation Δy between adjacent fringes is given by

$$\Delta y \approx \frac{\lambda x}{d}$$

where d is the distance between the midlines of the slits and x is the distance from the slits to the screen. So

$$\Delta y \approx \frac{500 \times 10^{-9}\ \text{m} \times 1.5\ \text{m}}{0.40 \times 10^{-3}\ \text{m}} = 1.9\ \text{mm}.$$

31 Model answer

a) $$\Delta y \approx \frac{\lambda x}{d}$$

where Δy is the distance between adjacent fringes;
x is the distance from the slits to the screen;
λ is the wavelength of the light;
d is the distance between the midlines of the slits.

b) (i) When monochromatic blue light is substituted for monochromatic red light the fringe spacing decreases.

(ii) If the slit separation is decreased, then the fringe separation increases.

c) If the width of both slits is much less than one wavelength then the intensities of the bright fringes are fairly uniform. Using wider slits increases the overall brightness because they let more light through, but the *relative* intensities of the bright fringes on either side of the central maximum are reduced. The intensities are modulated by the diffraction pattern of a single slit of finite width. The spacing of the fringes is not affected unless one of them coincides with a minimum in the diffraction pattern.

32 Notes

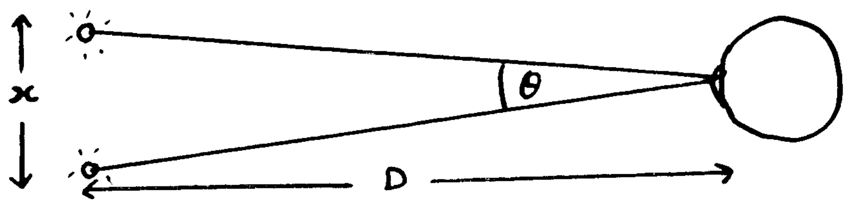
Using the Rayleigh criterion with wavelength 700 nm and pupil diameter 5 mm gives a maximum distance of 60 m.

33 Model answer

Assuming that the limitation on distinguishing the lights is imposed by diffraction at the pupil of the eye, the lights can be resolved if their angular separation is greater than a value θ_{min} given by the Rayleigh criterion:

$$\sin\theta_{min} \approx \frac{1.2\lambda}{a}$$

where a is the diameter of the pupil.



This angle is related to the separation x of the lights and their distance D from the eye by

$$\theta_{min} \approx \frac{x}{D} .$$

Since the angle is small, $\sin\theta_{min} \approx \theta_{min}$, so

$$D \approx \frac{x\,a}{1.2\lambda} .$$

Using a typical value, 500 nm, for the wavelength of visible light we get

$$D = \frac{1.5 \text{ m} \times 4.0 \times 10^{-3} \text{ m}}{1.2 \times 500 \times 10^{-9} \text{ m}}$$

$$\approx 10 \text{ km}.$$

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34 See chapter L6.

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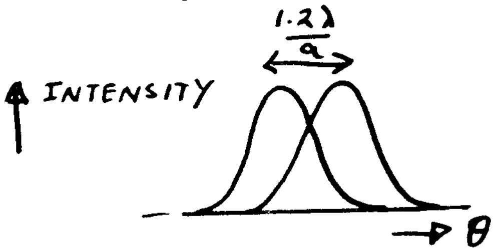
35 Model answer

a) As the slit width is reduced, the width of the central maximum increases.

b) The image of a point object formed by any optical system with a finite circular aperture is a diffraction pattern with a finite size. The Rayleigh criterion specifies that two such images can be regarded as being just resolvable when their angular separation θ is given by:

$$\sin\theta \approx \frac{1.2\lambda}{a}$$

where a is the diameter of the aperture and λ is the dominant wavelength of the light.



The reason for this criterion is that the value of θ is also equal to the half the angular width of the first minimum in the Fraunhofer diffraction pattern of the image formed by a distant point source and the circular aperture. Thus the criterion states that when the separation of the two smeared images is such that the centre of one lies on the edge of the other, they can just be resolved as separate entities. If the images are close you can't decide whether they are two separate images or a single blurred image.

c) The wavelength can be found from the frequency f and the speed of light c : $\lambda = \frac{c}{f}$.

If y is the lateral separation of the objects and x is their distance from the antenna, and $x \gg y$ then $\frac{y}{x} \approx \theta$ so

$$x \approx \frac{ay}{1.2\,\lambda} = \frac{a\,y\,f}{1.2\,\lambda c}$$

$$= \frac{(15 \text{ m}) \times (150 \text{ m}) \times (9 \times 10^{9} \text{ Hz})}{1.2 \times (3.0 \times 10^{8} \text{ m.s}^{-1})}$$

$$\approx 60 \text{ km} \quad [0.06 \times 10^{3} \text{ km}].$$

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36 Notes

a-d) See chapters L1, L6.

e) The optical path difference for the E and O waves must be a quarter of a wavelength.

$$\frac{\lambda}{4} = (n_1 d - n_2 d)$$

So

$$d = \frac{\lambda}{4(n_1 - n_2)} = 10\ \mu\text{m}.$$

37 Model answer

As light penetrates a dichroic material absorption occurs at different rates for different linear polarisations. Hence unpolarised incident light will become partially plane polarised, the degree of polarisation depending on the thickness of the material. If the slab of material is thick enough, the emerging light could be effectively linearly polarised.

In a uniaxial birefringent crystal, different linear components of the wave polarisation, either parallel to or perpendicular to the optic axis of the crystal, travel at different speeds. Hence the crystal has different refractive indices for the two components and they can be bent through different angles as they enter and leave the crystal. Hence an unpolarised beam of light can become two linearly polarised beams travelling in different directions.

If the incident light is linearly polarised and the two linear polarisation components are made to travel in the same direction (e.g. by having zero angle of incidence) the two components will get out of phase, giving elliptically polarised light. If the thickness of the crystal is just right the emerging light could then be circularly polarised (phase difference $\pi/2$, $3\pi/2$, etc.) or plane polarised (phase difference, π, 2π, etc.).

38 (i) Parallel to the polarising axis of the last polariser (horizontal in this case). (ii) $0.13I_0$.

39 Model answer

a) The stretched sticky tape becomes birefringent. As a consequence, incident plane polarised light (from the first polariser) will emerge from the other side of the tape elliptically polarised. Since a beam of elliptically polarised light can be considered to be a superposition of two plane polarised beams (with different phases) the second linear polariser is unlikely to block both components. (It could do so if the "elliptical" polarisation were, in fact, a plane polarisation with its axis perpendicular to the second polariser's axis, but that is unlikely.)

b) For complete plane polarisation by reflection the angle of incidence must be equal to the Brewster angle, which is given by

$$n = \tan\theta.$$

So the angle of incidence is $\theta = \tan^{-1} n = \tan^{-1} 1.3 = 52°$.

40 Model answer

i) View each lamp through each plastic sheet and rotate the sheet about the viewing axis. For one of the sources and one of the sheets there should be a waxing and waning of the transmitted light as the sheet is rotated. That source is the polarised source and that sheet is the Polaroid.

ii) Look through each sheet at the blue sky (at about 90° to the direction of the sun) and, as before, rotate the sheet. The sky-light, as viewed through the sheet, will change in intensity as the Polaroid is rotated.

41 See chapter L6.

42 a) Intensity is halved.

b) The transmitted beam is plane polarised with its polarisation parallel to the Polaroid's polarising axis.

43 Notes

a) See chapter L5. Note that the outer bright rings are much fainter than the central bright disc.

b) The angle θ between the the central maximum and the first minimum is is given by

$$\sin\theta \approx \frac{1.2\lambda}{a} .$$

The radius of this disc on a screen at distance x will be given by

$$r \approx x\theta .$$

If we take the diameter of the central maximum to be equal to the diameter of the dark ring which is the first minimum, then the diameter will be $\frac{2.4\lambda x}{a} \approx 0.05$ m. If we take the diameter to be, say, half-way out the answer would be $\frac{1.2\lambda x}{a} \approx 0.02$ m.

44 Model answer

a) The setting *f*/16 means that the diameter of the entrance pupil of the lens is equal to one sixteenth of the focal length.

b) Since the intensity *I* is proportional to the area of the aperture and the area is proportional to the diameter we can write

$$\frac{I_1}{I_0} = \frac{\text{area}_1}{\text{area}_0}$$

$$= \left(\frac{\text{diameter}_1}{\text{diameter}_0}\right)^2 = \left(\frac{f/4}{f/16}\right)^2 = \left(\frac{16}{4}\right)^2 = 16.$$

The new image intensity is equal to 16 times the old image intensity.

45 Notes

a) See chapters L5, L7.

b) Taking the "diameter of the central part of the diffraction pattern" to be diameter of the first minimum, its value is given by

$$2f\theta \approx 2f\,\frac{1.2\lambda}{a} = 5\ \mu\text{m}.$$

46 Notes

i) The maximum aperture is equal to *f*/2.8. (Diameter = 50 mm/2.8 = 18 mm.)

ii) The image brightness decreases because less light gets through the lens.

[This isn't really worth 6 marks.]

47 See chapter L8.

48 Notes

For an object at infinity, lens-film (image) distance = focal length = 50 mm.

For an object at 150 mm, calculate image distance = 75 mm.

So the range of movement must be 25 mm.

49 Notes

See chapter L7. In the camera the lens moves to alter the image distance. In the eye, the curvature of the lens is altered and that changes its focal length.

50 Model answer

i) The power of two thin lenses in contact is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

so

$$f_2 = \frac{f_1 f}{f_1 - f}$$

$$= \frac{43.0\text{ mm} \times 100.0\text{ mm}}{43.0\text{ mm} - 100.0\text{ mm}}$$

$$= -75.4\text{ mm.}$$

ii) Diameter = *f*/8 = 100 mm/8 = 12.5 mm.

iii) The lens-film distance (i) can be found from the lens equation:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

so

$$i = \frac{of}{o - f}$$

$$= \frac{1.10\text{ m} \times 0.1000\text{ m}}{1.10\text{ m} - 0.1000\text{ m}}$$

$$= 0.110\text{ m.}$$

51 Notes

a) See chapter L8.

b) The usual convention is that the magnification is quoted for the case of relaxed vision (image at infinity). The angular magnification is then given by the formula:

$$M = \frac{d}{f}$$

where d is the standard value for the viewing distance (0.25 m) and f is the focal length of the magnifier. So

$$f = \frac{d}{M} = \frac{0.25 \text{ m}}{5} = 0.050 \text{ m}.$$

c) This extra magnification can be obtained by bringing the image closer. Recalling the formula for angular magnification in the case where the image is formed at distance d,

$$M = 1 + \frac{d}{f} \quad ,$$

we can see that this fits the requirement: 6 = 5 + 1. So $i = -d = -0.25$ m

To find the object distance, use the lens equation, which becomes

$$o = \frac{i\,f}{i - f}$$

$$= \frac{-0.25 \text{ m} \times 0.050 \text{ m}}{-0.25\text{m} - 0.05 \text{ m}}$$

$$= 0.04 \text{ m}.$$

The object should be placed at 0.04 m from the lens, just inside the focal length.

An alternative approach is to argue that, since the image distance is now finite, the magnitude of the lateral magnification is equal to the angular magnification so that

$$\frac{i}{o} = -6 \quad \text{or } i = -6o$$

and put that into the lens equation.

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52 See chapter L8.

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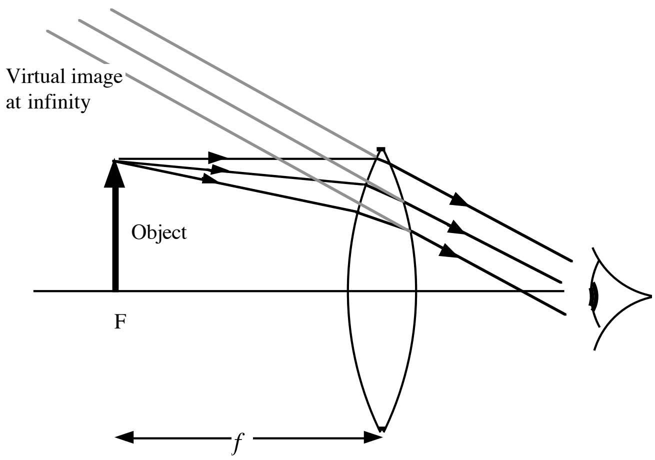
53 Notes

a)



b) Since the linear size of the object is much smaller than its distance from the eye, the angular size α (the angle subtended at the eye) is small so

$$\alpha \approx \frac{\text{linear size}}{\text{distance}} = \frac{4 \text{ mm}}{250 \text{ mm}} = 0.02 \text{ rad}.$$

c) In this case the angular magnification is given in terms of the standard viewing distance (d = 0.25 m) and the focal length f by the formula:

$$M = \frac{d}{f} \quad .$$

So the angular size of the image, the "apparent angular size", is

$$M\alpha = \frac{0.25\ \text{m}}{0.050\ \text{m}} \times \frac{4\ \text{mm}}{250\ \text{mm}} = 0.08\ \text{rad.}$$

[Note that using the rounded value 0.02 rad from answer a) will give an angular size of 0.10 rad here.]

54 Notes

a) See chapter L8 and the answer to Q53 above.

b) For viewing with the unaided eye, the standard maximum angular size α of the object with height h will be achieved when it is viewed from the standard distance, d = 0.25 m:

$\alpha = \dfrac{h}{d}$. The angular size of the image is then

$$\beta = M\alpha = M \times \frac{h}{d} = 8 \times \frac{2\ \text{mm}}{250\ \text{mm}} = 0.06\ \text{rad.}$$

55 Notes

See chapter L8 and the answer to Q53 above.

$$\beta = M\alpha\ ; \quad M \approx \frac{d}{f}\ ; \quad \alpha \approx \frac{h}{d}\ ; \quad \text{so } \beta \approx \frac{h}{f} = 0.25\ \text{rad.}$$

56 Notes

a) See chapter L8 and the answer to Q53 above.

b) $M = m_o M_e\ ; \quad M_e \approx \dfrac{d}{f_e}\ ; \quad \text{so } f_e \approx \dfrac{dm_o}{M} = 21\ \text{mm.}$

57 a) See chapter L8. **b)** (i) 5×. (ii) 75×.