

## L4

## INTERFERENCE

## OBJECTIVES

**Aims**

When you have finished this chapter you should understand how the wave model of light can be used to explain the phenomenon of interference. You should be able to describe and explain, with words and a minimal use of mathematical formulas, some idealised examples of interference, such as that produced by two coherent monochromatic point or line sources or monochromatic fringes in a thin film or wedge.

**Minimum learning goals**

1. Explain, interpret and use the terms:  
*phase, phase difference, in phase, superposition, interference, interference pattern, path difference, optical path difference, coherence, coherent sources, incoherent sources, coherent waves, incoherent waves, fringe, order (of a fringe), amplitude, angular position, fringe separation, thin film, thin film interference.*
2. State and explain the principle of superposition.
3. Describe and explain how interference between light waves can produce spatial patterns of varying intensities of light. Describe the conditions which are necessary for the formation of such interference patterns.
- 4(a) Use words, diagrams and graphs to describe interference patterns produced on a plane screen by two monochromatic, coherent, point or parallel-line sources.
  - (b) State and apply the relations among (i) wavelength, (ii) slit separation, (iii) angular and linear positions of light and dark fringes, and (iv) the distance from the slits to the screen.
- 5(a) Describe and explain interference of monochromatic light produced by reflection from thin films of uniform and non-uniform thickness.
  - (b) State and apply the conditions for maxima and minima in reflected monochromatic light for thin films and wedges.
6. Describe examples and applications of thin film interference
7. Describe and explain the appearance of interference patterns produced by double slit and thin-film arrangements with white light.

## PRE-LECTURE

**4-1 SUPERPOSITION OF WAVES**

So far we have described the behaviour of light in terms of the behaviour of light rays which were usually straight lines although they could change direction at a boundary between two media. In this chapter and the next we look more closely at the wave nature of light and in doing so we will see some of the limitations of the ray model.

**Revision**

You should make sure that you still understand the idea of an oscillation and the terms *amplitude*, *phase* and *frequency* - see chapter FE7. Also re-read §1-2 in chapter L1.

**Addition of waves**

Interference, the topic of this chapter, is just the combination of waves. Interference of light waves can be described in terms of electric field (see chapter E1). To see how to calculate the combined effect of two waves think of two simple harmonic waves with the same angular frequency  $\omega$  and equal amplitudes  $A$  as they both pass through the same point in space. Suppose that they have the

same polarisation, which means that their electric fields are parallel (or antiparallel) and their electric fields at the point of interest can be described by the components  $E_1$  and  $E_2$  referred to the same direction. The two waves may, however, have different phases. The equations describing how these field components change with time at *one* fixed point in space can be written as

$$E_1 = A \sin(\omega t)$$

and

$$E_2 = A \sin(\omega t + \phi)$$

where  $\phi$  is the phase difference between the waves. (These equations match the wave equation 1.1 with  $x = 0$ ) The total field is found using the **principle of superposition** which says that the total field is equal to the vector sum of the individual wave fields. 'Vector sum' means that we have to take proper account of directions, by using components for example. In this simple example the directions are chosen so that each field can be described using only one component; hence simple algebraic addition gives the answer:\*

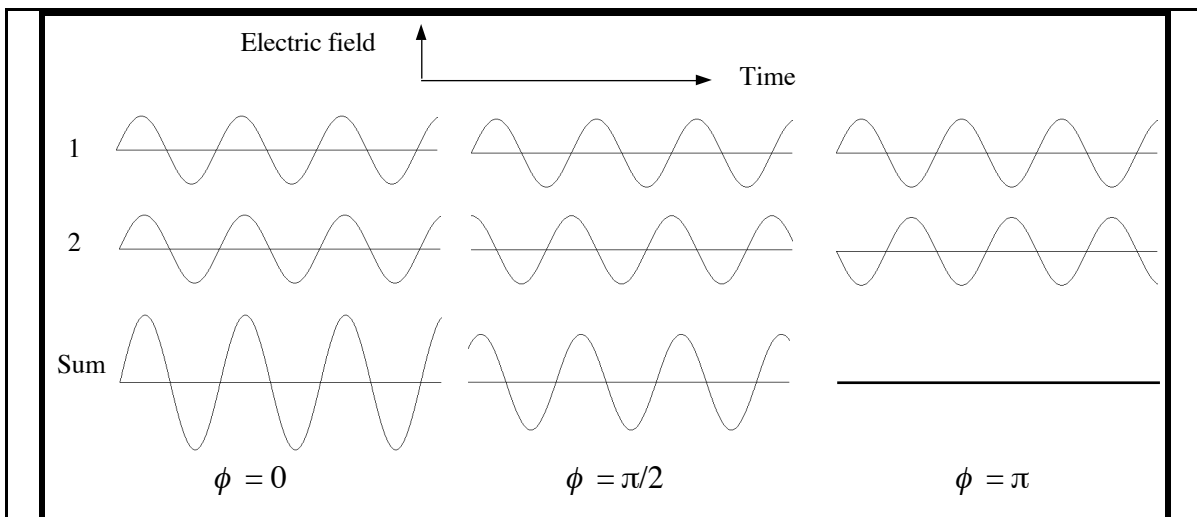
$$E = E_1 + E_2 = 2A \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right).$$

The equation can be rewritten as

$$E = A_t \sin\left(\omega t + \frac{\phi}{2}\right)$$

where the new total amplitude is

$$A_t = 2A \cos\left(\frac{\phi}{2}\right).$$



**Figure 4.1. Addition of two electromagnetic waves**

The amplitudes of the two elementary waves are equal. The sum of waves 1 and 2 is shown below.

Important things to note about this example are that the amplitude of the total electric field depends on  $\phi$ , and the angular frequency of the resultant wave is still  $\omega$ . Two special cases are of interest.

- If the waves are in phase, then  $\phi = 0$ , so the resultant amplitude is twice the amplitude of one of the waves:

$$2A \cos\left(\frac{\phi}{2}\right) = 2A.$$

Since the 'intensity' (irradiance in the case of light) of a wave is proportional to the square of its amplitude, the intensity of the resultant wave is four times the intensity of one of the original waves.

\* The addition is carried out using the identity:

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right).$$

- If the waves are out of phase by half a cycle, then  $\phi = \pi$  so the total amplitude is

$$2A \cos\left(\frac{\phi}{2}\right) = 0$$

and the intensity of the resultant wave is zero.

**Note**

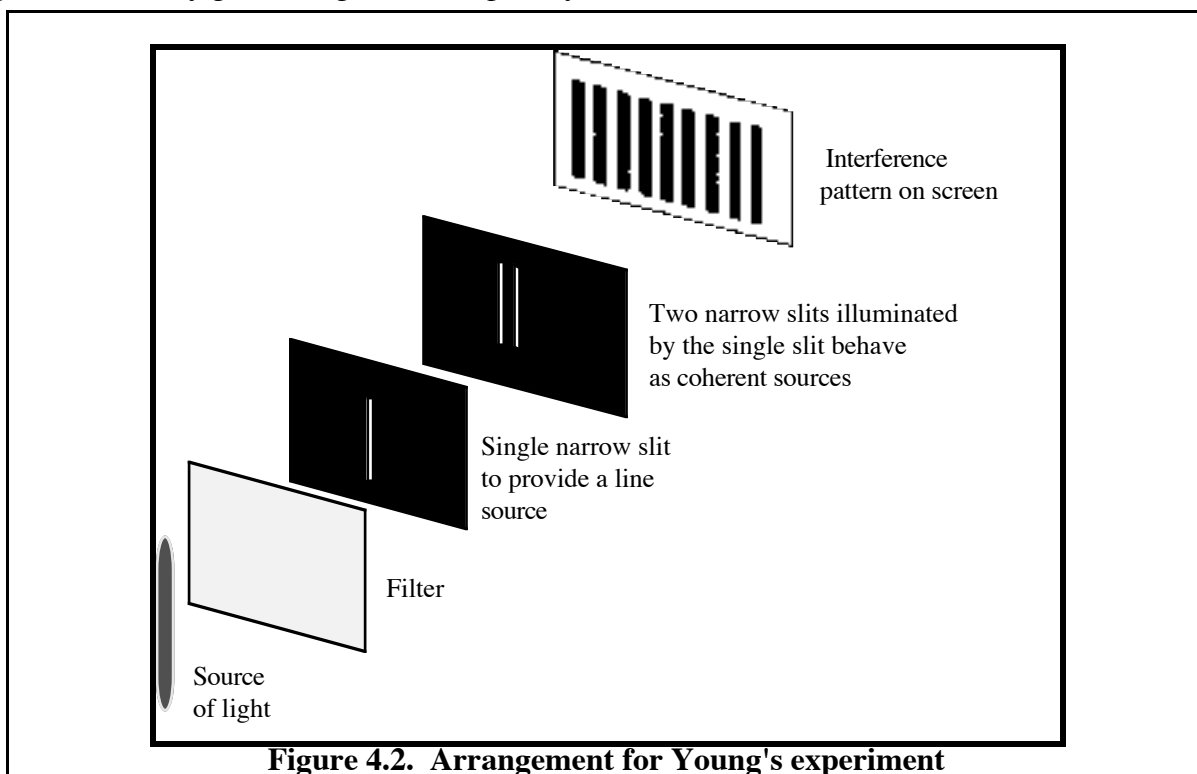
The terms 'constructive' and 'destructive' are sometimes used to describe these interference maxima and minima. Those names are avoided here because they can be misleading. Nothing is actually destroyed in wave interference; rather the effect of two waves is always additive, as expressed by the principle of superposition. Certainly energy is never destroyed - if energy seems to be missing from some place it always shows up somewhere else.

**TEXT & LECTURE**

#### 4-2 YOUNG'S DOUBLE SLIT EXPERIMENT

The most famous of all demonstrations of the wave nature of light is Thomas Young's double slit experiment. Here we describe a modern version of Young's experiment as demonstrated in the video lecture. The arrangement (figure 4.2) consists of a source of light, a coloured filter, an opaque plate with a narrow slit cut into it, another plate with two narrow parallel slits in it and a white screen for viewing the light. Each of the slits is quite narrow, typically less than a tenth of a millimetre, and the two slits in the second plate are usually separated by only a fraction of a millimetre.

Light travels from the source through the filter to the first slit. From there it travels to the plate with two slits where much of it is blocked off but some can get through both the slits. Some of the light which finally gets through is intercepted by the screen.

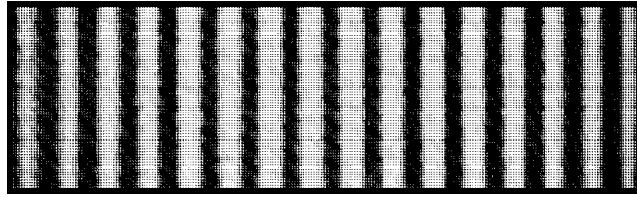


**Figure 4.2. Arrangement for Young's experiment**

When only one of the slits in the second plate is open there is a diffuse pool of light on the screen. This patch of light is wider than the slit that the light came through. This spreading out is called **diffraction**, a topic which will be taken up in the next chapter.

When light passes through each of the pair of slits in turn (keeping the other slit covered) you can see a pool of light on the screen. The areas covered by these two pools of light overlap considerably, so one would expect naively that with two slits open the resulting pool of light would

just be a merging of the two pools already seen, bright in the middle and falling off at the edges. This expectation turns out to be quite wrong - instead of a continuous patch of light there is a pattern of light and dark stripes, called interference **fringes**. The ray model of light has no hope of accounting for that!



**Figure 4.3. A twin slit interference pattern**

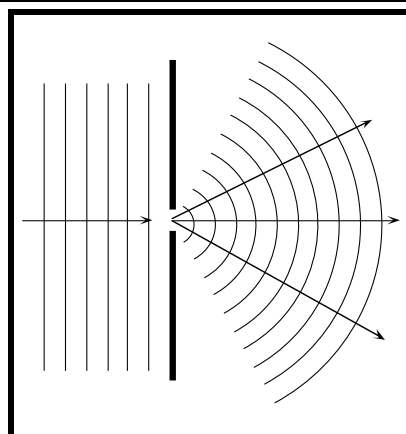
At some places where there used to be light from each slit separately there is now darkness, but the energy in the light has not been destroyed. The brightest parts of the fringe pattern are now more than twice as bright as the brightest part of the light pool from one slit. The energy of the light has just been redistributed.

Although Young's original experiment used white light, the contrast between light and dark fringes is enhanced if a suitable coloured filter is used to restrict the range of frequencies in the light. The fringes are sharpest when a very narrow range of frequencies - monochromatic light - is used. (On the other hand using a broad range of frequencies produces some pretty multi-coloured effects - more about that later.)

The explanation of Young's experiment needs the wave model of light. To see how the wave model works it is useful to study a similar experiment using water waves instead of light, in which the superposition of waves can be seen directly.

### **Interference in water waves**

Many aspects of wave behaviour can be observed in water waves. In a simple direct analogue of Young's experiment straight-line water waves (analogous to plane waves in three dimensions) are generated by a long paddle. The waves travel to a barrier with two narrow slots in it. When only one of the slots is open (figure 4.4) diffraction can be observed; the waves spread out instead of forming a sharp shadow. Each of the wave fronts coming out the other side has a roughly circular shape, but the amplitude of the waves is weaker at the sides than it is in the straight-through direction.

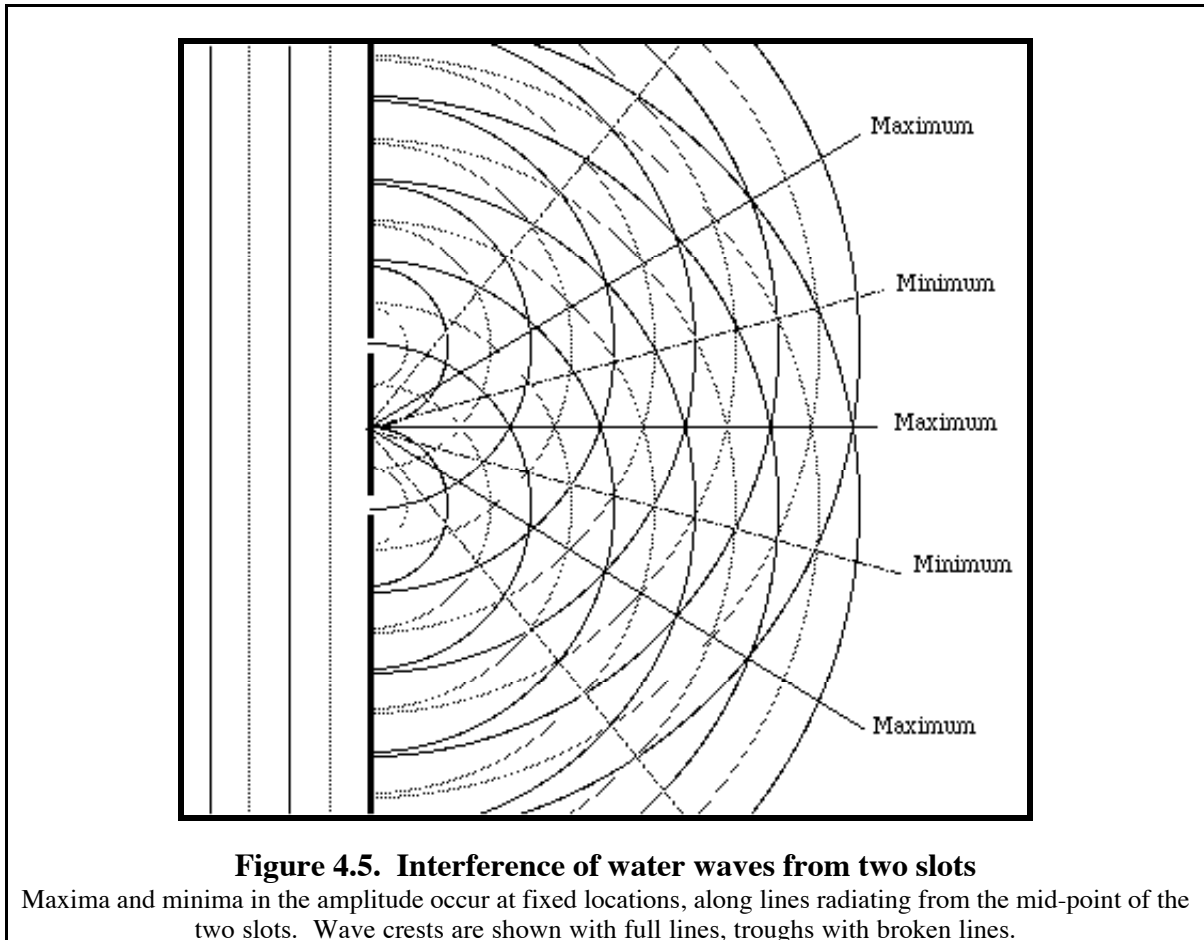


**Figure 4.4.  
Diffraction of  
water waves at  
a hole**

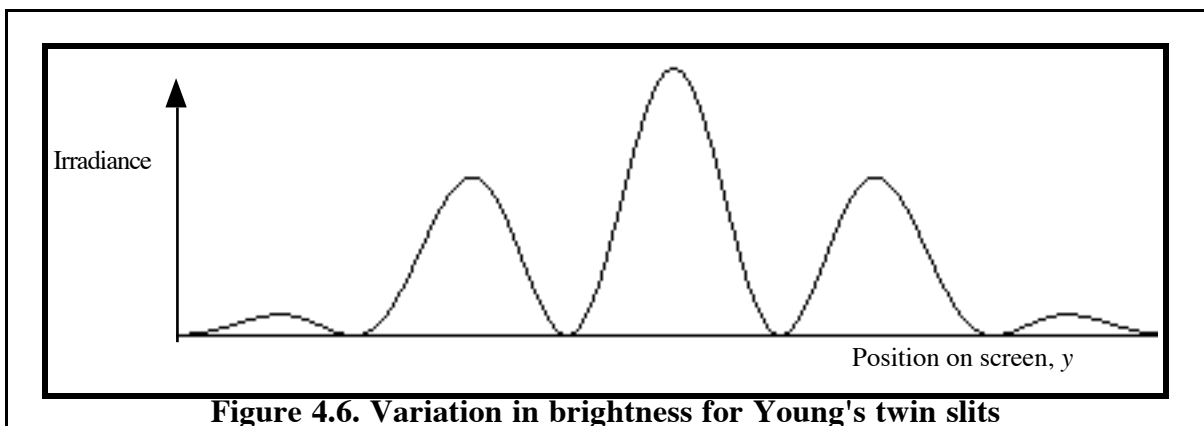
When both slots are open a new feature, called interference, is seen: there are some places where there is practically no wave disturbance and others where it is quite strong. See figure 4.5.

In the case of water waves the wave disturbance can be taken to be the displacement of the surface of the water from its equilibrium level. The amplitude of the resultant wave pattern varies from a minimum of zero at some places to a local maximum at other places. Furthermore, although the wave disturbance at any place varies in time, the amplitude at any single place is fixed. At a place

where the amplitude is a maximum, say  $A$ , the wave disturbance varies from a minimum of  $-A$  to a maximum of  $+A$ . At a point where the amplitude is equal to zero, there is no net wave disturbance at any time. The amplitude of the resultant wave changes continuously from point to point between the locations of the maxima and minima.



These maxima and minima correspond to the fringes in Young's experiment.



In this example the locations of the points of maximum and minimum amplitude all lie on approximately straight lines radiating from a point mid-way between the slits. If you look at the resultant wave at places in between the positions of the maxima and minima you will see that the amplitude varies smoothly with position. Figure 4.6 shows how the intensity varies with position on the screen for a typical Young's slits experiment using light.

### 4-3 SUPERPOSITION

The key to understanding interference is the **principle of superposition** which says simply that the combined effect of several waves at any place at a particular instant of time is given by the sum (vector sum if the wave property is a vector) of the wave property for the individual waves. The contribution from one wave is just that which would occur if the other waves were not there. In the case of the water waves the appropriate wave property is the linear displacement (change in position) of the surface of the water from its equilibrium position; for example if one wave produces an upward displacement of 2.0 mm and the other gives a downward displacement of 1.5 mm, the net effect is 0.5 mm up. In the case of light waves the appropriate wave property is the electric field (which can point in any direction perpendicular to the direction of propagation).

The interference pattern can be understood in terms of superposition. There are some places where a crest of one water wave arrives at exactly the same time as the trough of the wave from the other slot. At any one of those points the net displacement for these two waves is a minimum - zero if the amplitudes are equal. Although the crest of wave 1 and the trough of wave 2 move on, the sum of the two wave disturbances at the same fixed point in space remains zero. There are other points where a crest always arrives in step with a crest and a trough with a trough. At those points the amplitude of the resultant wave turns out to be the sum of the individual amplitudes. There are many other places where the individual waves add to give other values of the resultant amplitude. The resultant wave at any point depends on the phase difference between the individual waves (as well as their amplitudes).

Note that interference occurs only at places where both waves are present. Outside the region where the waves overlap there is no interference.

It is very important to note that although the two waves add up at any point in space that does not stop the progress of the waves. Each wave is quite unaffected by the other!

### 4-4 ANALYSIS OF TWIN SLIT INTERFERENCE

In Young's experiment with light, the function of the single slit is to ensure that the light falling on each of the pair of slits is **coherent**. Although the light consists of a continuous distribution of component waves with different frequencies and wavelengths, light reaching each of the twin slits from the narrow single slit has the same composition. If the pair of slits is placed symmetrically then any change in any component of the light, including any change in phase, occurs simultaneously at both slits. So the slits behave as **coherent sources**.

In the water wave experiment, the waves are much less complex, being essentially composed of only one frequency component. Since the original wave had straight wavefronts the waves emanating from the two slots are exactly in phase at all times. (Their amplitudes at the slots are also equal provided that the slots are equally wide.)

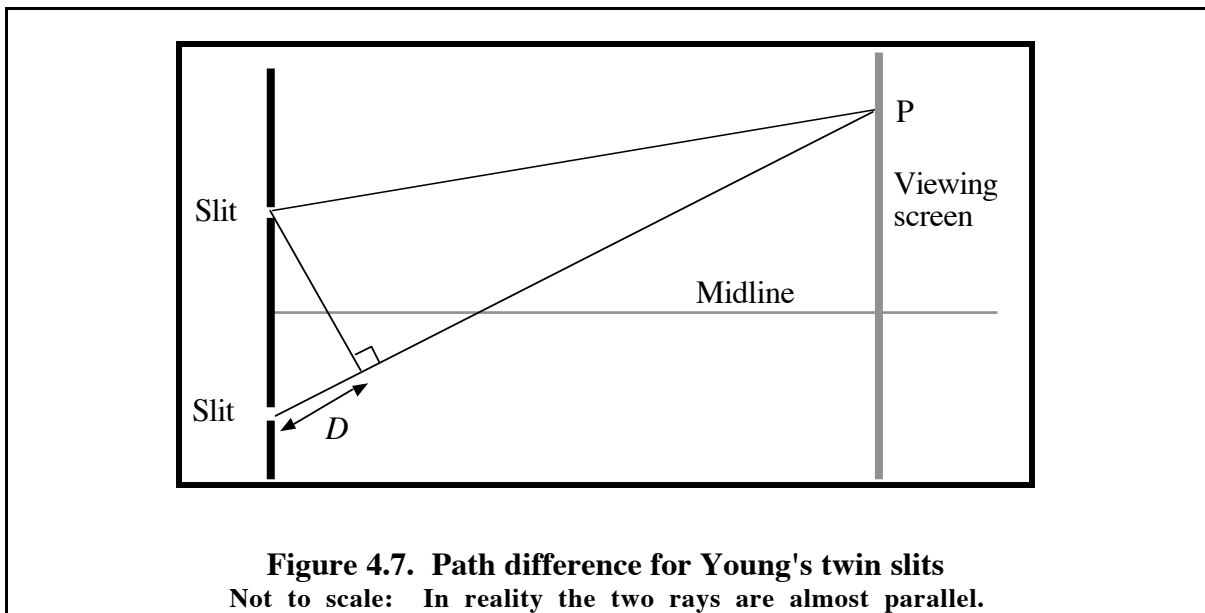
#### Conditions for interference maxima and minima

It is easy to calculate the points in space where maxima and minima in the interference pattern occur. The analysis is essentially the same for the water waves example and for the Young's twin slits because both can be treated in two dimensions. (It is assumed that the Young's slits are very long compared with their width and separation.)

The wave amplitude at some point P depends on the phase difference between the two interfering waves. If the waves are in phase at their sources (the slits in the case of Young's experiment), then the phase difference at P is determined by the difference in times taken for the light to get from the sources to P. That time difference, in turn, depends on the speed of the waves and the difference in the distances, called the **path difference**, travelled by the two waves. In the case of light, we can say that the phase difference is proportional to the **optical path difference**, which is the product of the actual path difference and the refractive index of the medium.

$$\text{Optical path length} = n \times (\text{geometrical path length}).$$

Since we usually consider Young's experiment in air, the optical and geometrical path differences are essentially equal. The path difference for Young's slits is labelled  $D$  in figure 4.7. It is very important to note that this figure is not to scale. This analysis is valid only if the screen is a long way from the slits and if the point P is close the central axis. If those conditions are satisfied rays from each of the slits to P are almost parallel and it is said that the experiment satisfies Fraunhofer conditions. (The general case in which angles and distances are not so small, Fresnel conditions, is very difficult to analyse.)



If the optical path difference,  $D$ , is equal to a whole number ( $n$ ) of wavelengths then the phase difference will be  $n$  times  $2\pi$  (corresponding to  $n$  wave cycles) and the two waves will be exactly in phase. That produces a maximum in the amplitude of the resultant wave.

For a maximum: 
$$D = m\lambda \quad (\text{for } m = 0, \pm 1, \pm 2, \pm 3, \dots) \quad \dots (4.1a)$$

The value of  $m$  is called the **order** of the bright fringe; the fringe in the middle is the zero-order fringe.

A minimum in the amplitude will occur if the optical path difference is equal to an odd number of half-wavelengths.

For a minimum: 
$$D = (m + \frac{1}{2})\lambda \quad (\text{for } m = 0, \pm 1, \pm 2, \pm 3, \dots) \quad \dots (4.1b)$$

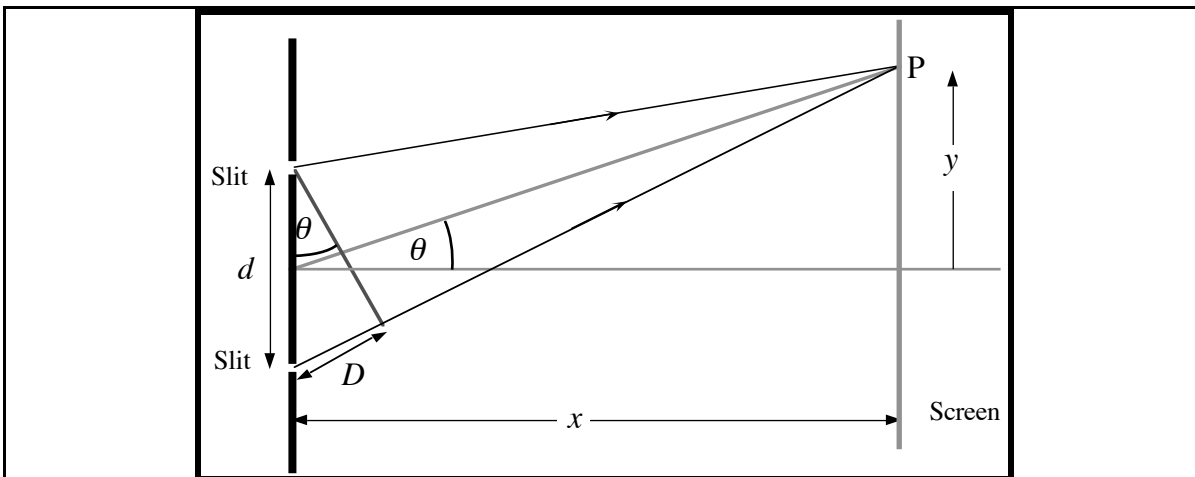
Since the irradiance ("intensity") in the interference pattern is proportional to the square of the wave amplitude, maxima and minima in the intensity occur at the same places as the maxima and minima in the wave amplitude. The relation between phase difference  $\phi$  and path difference  $D$  which applies at all points (including those between the intensity maxima and minima) is

$$\frac{\phi}{2\pi} = \frac{D}{\lambda} \quad \dots (4.2)$$

These results apply to all kinds of interference between two elementary waves, not just the Young's slits experiment.

### Location of maxima and minima

The location of points in the interference pattern is most conveniently specified in terms of the angular position  $\theta$  of the point P (see figure 4.8). The **angular position** of P is measured from the midline between the slits.



**Figure 4.8. Geometry of the twin slits arrangement**

Not to scale: distances along the y-axis are grossly exaggerated.

It can be seen from the diagram that  $D \approx d \sin \theta$  so the conditions become:

$$\text{for a maximum} \quad m\lambda \approx d \sin \theta \quad \dots (4.3a)$$

$$\text{and for a minimum} \quad (m + \frac{1}{2})\lambda \approx d \sin \theta . \quad \dots (4.3b)$$

These formulas give the angular positions of the bright and dark fringes in the space beyond the slits. They are perfectly accurate only for small values of the angle  $\theta$ .

If the fringes are viewed on a screen at a long distance from the slits these formulas (4.3) can be rewritten approximately in terms of the linear position  $y$  of the point P on the screen. First note that the position coordinate,  $y$ , of P can be written as  $y = x \tan \theta$ . For small values of the angle, as considered here,  $\tan \theta \approx \sin \theta$ , so the conditions can be rewritten:

$$\text{for a maximum} \quad y \approx \frac{m\lambda x}{d} \quad \dots (4.4a)$$

$$\text{and for a minimum} \quad y \approx \frac{(m + \frac{1}{2})\lambda x}{d} . \quad \dots (4.4b)$$

It follows from these equations that the fringe spacing, the distance between two successive light fringes or two successive dark lines on the screen, is given by

$$\Delta y \approx \frac{\lambda x}{d} . \quad \dots (4.4c)$$

These results illustrate why you can see good fringe patterns only if the range of wavelengths in the light is small. The angular positions of the maxima and minima depend on the wavelength, so if the wavelength doesn't have a well defined value then the fringes are not well-defined either.

### Coherence of light sources

In the water wave experiment there is no problem with the coherence of the two sources. Both sets of waves are produced by splitting one continuous wave. On the other hand light from an ordinary source can normally be described as a superposition of a vast number of elementary waves, which have a continuous range of frequencies and wavelengths. These elementary waves can be related to photons emitted by atoms or molecules in the light source. Although each elementary wave has a fairly well-defined frequency, it does not last for long. Since the emission of elementary waves from the light source is entirely uncorrelated, the source is said to be incoherent. If the Young's slits were

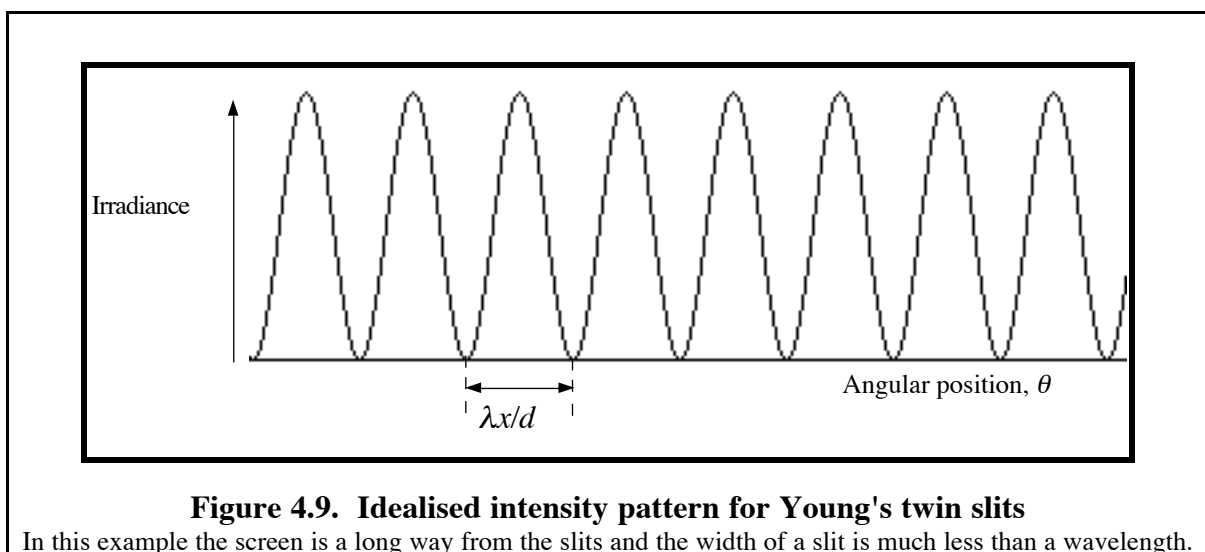
illuminated directly by an ordinary lamp, coherence between individual waves arriving at the two slits would exist only for extremely short times, and the interference fringe pattern would jump around very rapidly. Although an interference pattern would exist it would not stay in one place long enough to be seen. The important feature of Young's experiment is the production of long-term coherence by splitting the wavefronts of each and every elementary wave so that the same phase relationship between waves from the two slits persists for a relatively long time. The coherence is achieved by using the single narrow slit as a common source which illuminates both the twin slits. Each wave from the first slit produces two coherent parts at the twin slits, so that whatever phase fluctuations there are among the elementary waves, exactly the same fluctuations occur at both slits.

### **Laser light**

One of the special features of light from a laser is that it is highly coherent. Therefore Young's experiment can be done by sending light from a laser directly onto the twin slits, without using the single slit. Another advantage of laser light is that it is highly monochromatic - the spread of wave frequencies (the bandwidth) is much smaller than anything that can be obtained from a lamp and coloured filters. In these respects the water wave interference experiment is analogous to Young's experiment with a laser.

### **Behaviour of the interference pattern**

If the fringes are observed on a screen a long way from the slits ( $x \gg d$ ) the irradiance of the fringes is fairly uniform (figure 4.9). In that case the maximum irradiance is about four times the irradiance due to one source alone. The "missing" intensity from the dark fringes has gone into the extra intensity in the bright fringes, so that there is no violation of energy conservation. On a distant screen the fringes are uniformly spaced and the separation is approximately equal to  $\lambda x/d$ .

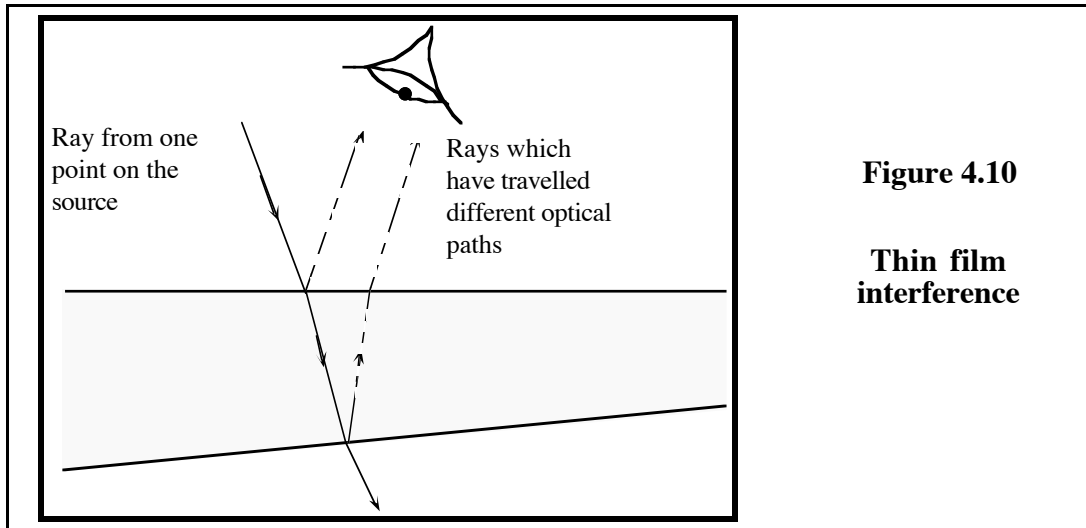


### **Points to note**

- For a fixed wavelength the fringe spacing varies inversely with the separation of the slits. If the slits are moved further apart then the fringes get closer together.
- For a fixed separation of the slits, the fringe spacing is proportional to the wavelength (provided that the approximations stated above are satisfied).
- There is always a bright fringe (order zero) on the central axis. So when white light is used the only bright fringe which shows up strongly is the central one because its location does not depend on wavelength. Since the fringe spacing depends on wavelength, the total pattern for white light is a continuous mess of overlapping fringe patterns. The overall effect is white light in the middle and "washed-out" coloured fringes on the sides.

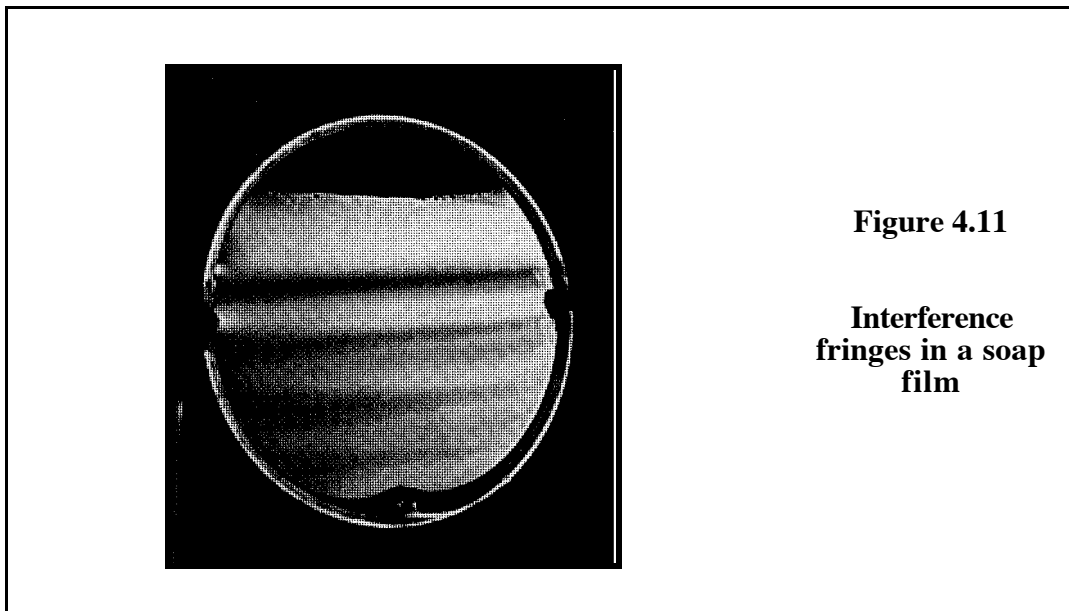
#### 4-5 THIN FILM INTERFERENCE

Interference patterns can be observed whenever waves from two or more coherent sources come together. In Young's experiment the waves came from two separate sources but in **thin film interference**, the waves come from one source. One wavefront is split into two parts which are recombined after traversing different paths. Examples of thin film interference occur in oil slicks, soap bubbles and the thin layer of air trapped between two glass slabs. Here thin film means a layer of transparent material no thicker than several wavelengths of light.



**Figure 4.10**

**Thin film  
interference**



**Figure 4.11**

**Interference  
fringes in a soap  
film**

When light strikes one boundary of the film, some of it will be reflected and some will be transmitted through the film to the second boundary where another partial reflection will occur (figure 4.10). This process, partial reflection back and forth within the film and partial transmission, continues until the reflected portion of the light gets too weak to be noticed. The interference effects come about when parts of the light which have travelled through different optical paths come together again. Usually that will happen when the light enters the eye.\* Thus for example, light

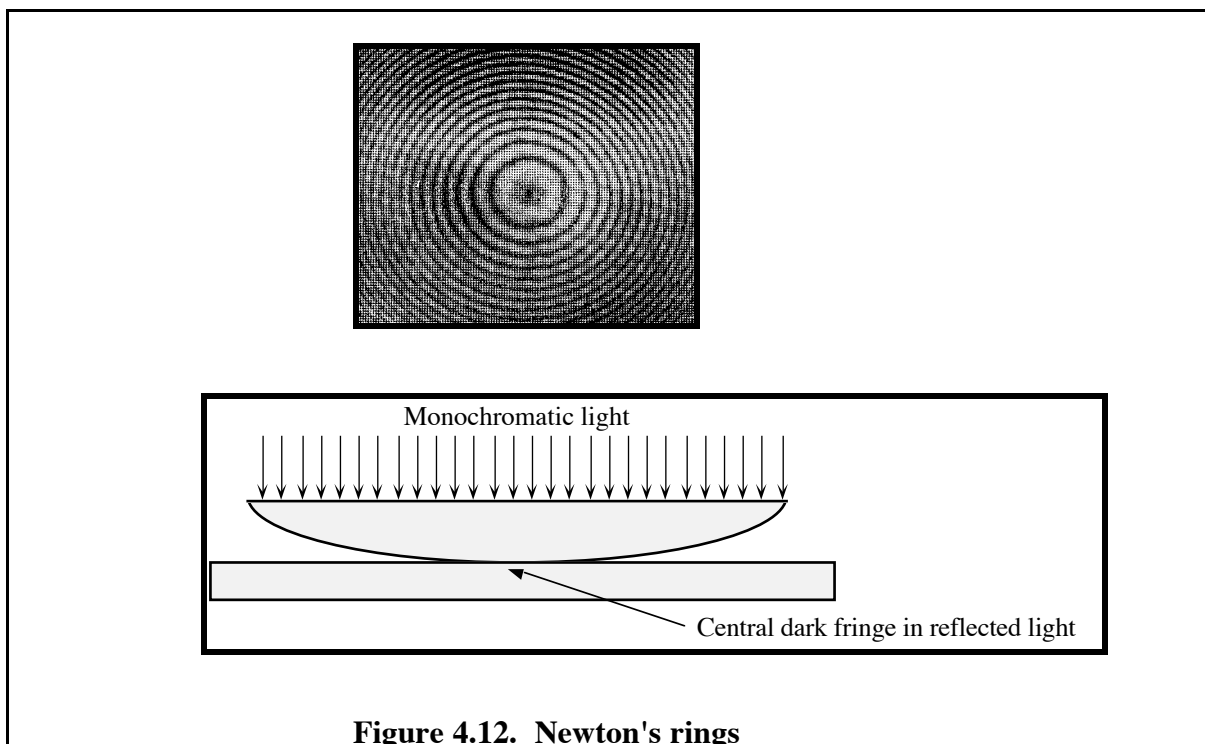
\* When light rays are brought to a focus either by the eye or a lens, there is no extra optical path difference introduced so the focussing has no effect on the conditions for the location of the interference fringes.

reflected back from the top surface of the film can interfere with light which has been reflected once from the bottom surface and is refracted at the top surface.

The interference effect for monochromatic light, light or dark or somewhere in between, is determined by the amplitudes of the interfering waves and their phase difference. The conditions for a maximum or minimum in the irradiance are the same as before: a phase difference of  $m(2\pi)$  gives a maximum and a phase difference of  $(m + \frac{1}{2})(2\pi)$  produces a minimum.

### Change of phase at reflection

A new phenomenon reveals itself here. A straightforward interpretation of the conditions for interference maxima and minima solely in terms of optical path difference gives the wrong answer! Two examples illustrate this point. In a very thin soap film it is possible to get a film thickness which is much less than one wavelength. So the path difference between light reflected from the two surfaces of the film is much less than a wavelength and the corresponding phase difference will be almost zero. A zero phase difference should produce brightness, but the opposite is observed - when the film is very thin there is no reflection at all! The explanation is that whenever a light wave is reflected at a boundary where the refractive index increases, its phase jumps by  $\pi$  or half a cycle. In the case of the soap film, the light reflected from the first surface, air to soapy water, suffers a phase change, but light reflected at the water-air boundary has no phase change. You can observe this effect yourself in soap bubbles. Carefully watch the top of a bubble as the water drains away. As the film gets thinner you will see a changing pattern of coloured fringes. Just before the bubble breaks, the thinnest part of the film looks black - indicating no net reflection.

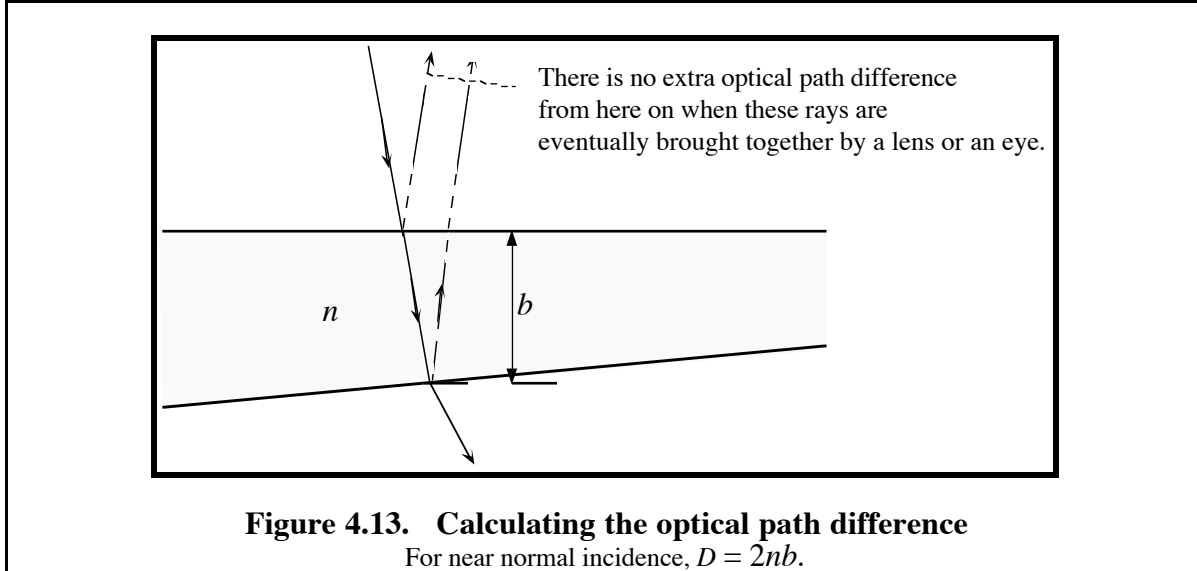


**Figure 4.12. Newton's rings**

The other example is a thin film interference pattern called Newton's rings which are formed using a curved glass lens resting on a flat glass slab (figure 4.12). The thin film is the air between the lens and the slab. The important feature is that where the optical path difference is zero, right in the middle of the pattern where the lens actually touches the slab, there is darkness instead of a bright fringe. The dark spot can be explained by saying that there is a phase change of  $\pi$  in the light reflected at the boundary between air and glass.

### Analysis of thin film interference

The conditions for finding bright or dark fringes in a thin film clearly depend on the angle of incidence of the light, but a useful approximation can be worked out assuming that the incident light rays are normal to the surface, or almost so. In that case the optical path difference between parts of an elementary wave reflected from the top and bottom surfaces of a film is just  $2nb$ , where  $b$  is the thickness and  $n$  is the refractive index (figure 4.13).



To work out the conditions for bright and dark fringes you have to include the effect of phase changes at reflection. Each phase change of  $\pi$  has the same effect as the addition of an extra half wavelength of optical path.

#### *No net phase change at reflection*

If there is no phase change at either boundary or a phase change at both boundaries (for example: a film of water on glass), the conditions for maxima and minima are

for a bright fringe: 
$$2nb = m\lambda \quad \dots (4.5a)$$

and for a dark fringe: 
$$2nb = \left(m + \frac{1}{2}\right) \lambda \quad (m = 0, 1, 2, 3, \dots) \quad \dots (4.5b)$$

#### *Phase change at one boundary*

Where there is a phase change at only one boundary (for example an air film trapped between two glass plates or a soap bubble) the interference conditions depend on both the thickness and the phase change at reflection. The conditions are simply interchanged:

for a bright fringe: 
$$2nb = \left(m + \frac{1}{2}\right) \lambda \quad \dots (4.6a)$$

and for a dark fringe: 
$$2nb = m\lambda \quad (m = 0, 1, 2, 3, \dots) \quad \dots(4.6b)$$

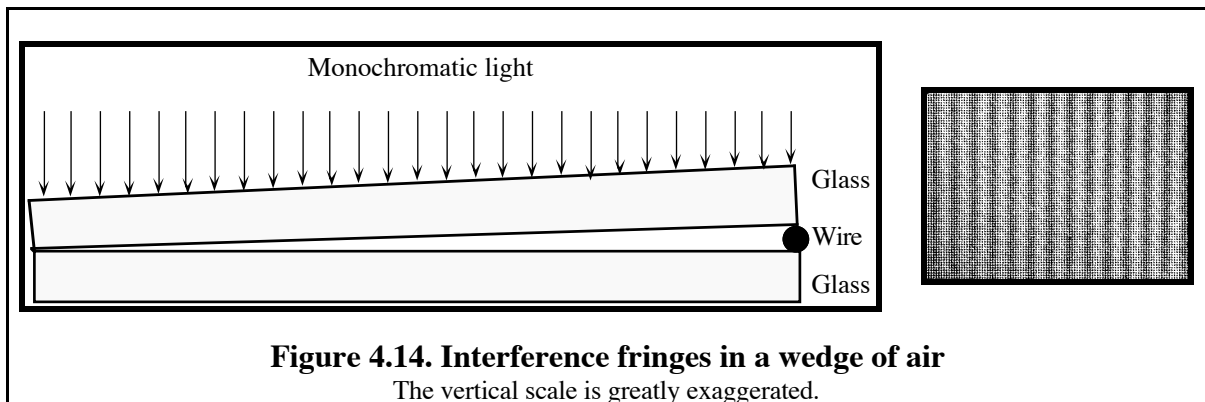
#### *Notes*

- There is no point in trying to memorise these equations. It is better to work them out when you need them by combining the conditions expressed in terms of phase difference (equations 4.1a and 4.1b) with the phase changes at reflection and the relation between optical path and phase difference.
- It is important to remember that the value of wavelength to be used in these relations is the wavelength in vacuum (or air). If you need to know the value of the wavelength,  $\lambda_m$ , in the medium with refractive index  $n$  it can be calculated using the relation

$$\frac{\lambda}{\lambda_m} = n .$$

### Example: fringe patterns in wedges

If two flat glass plates are allowed to touch at one edge and are separated by a small object such as a thin wire at the opposite edge, the space between the plates contains a wedge-shaped thin film of air (figure 4.14).



When monochromatic light is shone down on to this arrangement, interference fringes will be observed in the reflected light. Since the existence of a bright or dark fringe depends on the thickness of the film at a particular place, fringes will be seen at various places across the air wedge. The analysis above shows that the spacing of the fringes is proportional to the wavelength. For a given wavelength each fringe follows a line or contour of constant thickness in the air film. If you follow across the fringe pattern, the thickness of the film will change by  $\lambda/2n$  as you go from one fringe to the next. If the medium in the wedge is air then  $n = 1.000$ , so the fringe spacing corresponds to a change in thickness of  $\lambda/2$ . This gives a way of measuring the thickness of the thin object used to prop the plates apart if you already know the wavelength: just count the total number of fringes across the whole wedge and multiply by  $\lambda/2$ . The resolution in this measurement is about half a wavelength, or better, depending on how well you can estimate fractions of a fringe. Alternatively, you could use this method and a wire of known diameter to find the wavelength.

### Localisation of the fringes

Although a narrow light source (the single slit) is needed to produce coherence in Young's experiment, thin film fringes can be formed using extended light sources, even daylight from the sky. The difference is that in thin film interference every incident wavefront, no matter where it comes from, is split into two wavefronts when it meets the first surface of the film. When the two waves meet again they have a definite phase relationship so that interference is seen to occur. The phase difference between the waves is locally constant and the fringes are said to be localised. You can see that when you look at thin film fringes - they appear to be located in (or just behind) the film.

### Coloured fringes

If a thin film is illuminated with white light the reflected light will contain a continuous range of fringe patterns corresponding to the spectrum of wavelengths in the light. You do not, however, see the same colours as the pure spectrum like a rainbow. Instead the colours are formed by subtraction from the white light. For example, at a place where the film thickness is just right for a dark fringe in the green you will see white light minus green, which leaves the red end and the blue end of the spectrum; the resulting visual sensation is purple.

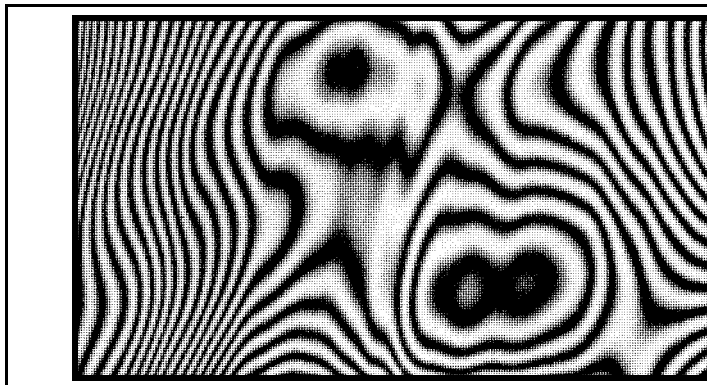
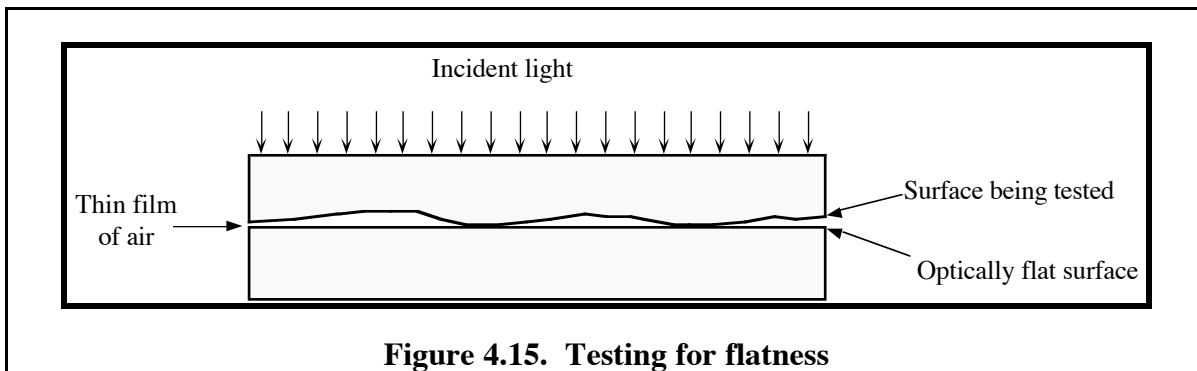
### Where does the energy go?

There is a puzzle that needs to be answered: what happens to the energy of the light when the reflected light is removed by interference? The energy cannot be destroyed so it must go somewhere else - it is transmitted through the film instead of being reflected. As in the case of Young's experiment, the energy is rearranged in space but it is never destroyed. If you are used to thinking of energy as a kind of fluid, then that idea may be hard to understand. However experimental evidence supports the wave theory, so the "fluid" model of energy needs to be abandoned. Energy is *not* like matter, it does not have to flow continuously through space. Another way of resolving the problem is to say that the principle of superposition (just adding things up) works for electric fields but it does not apply to energy or wave intensity.

## 4-6 APPLICATIONS OF THIN FILM INTERFERENCE

### Testing for flatness

Given a slab with a very accurately flat surface, thin film interference can be used to test the flatness of the another surface. (At least one of the two objects needs to be transparent.) Interference fringes formed by the thin film of air between the surfaces gives a contour map of variations in the height of the surface being tested. The contour interval is equal to half a wavelength of the light in the gap.



**Figure 4.16.**  
**Thin film contour**  
**fringes**

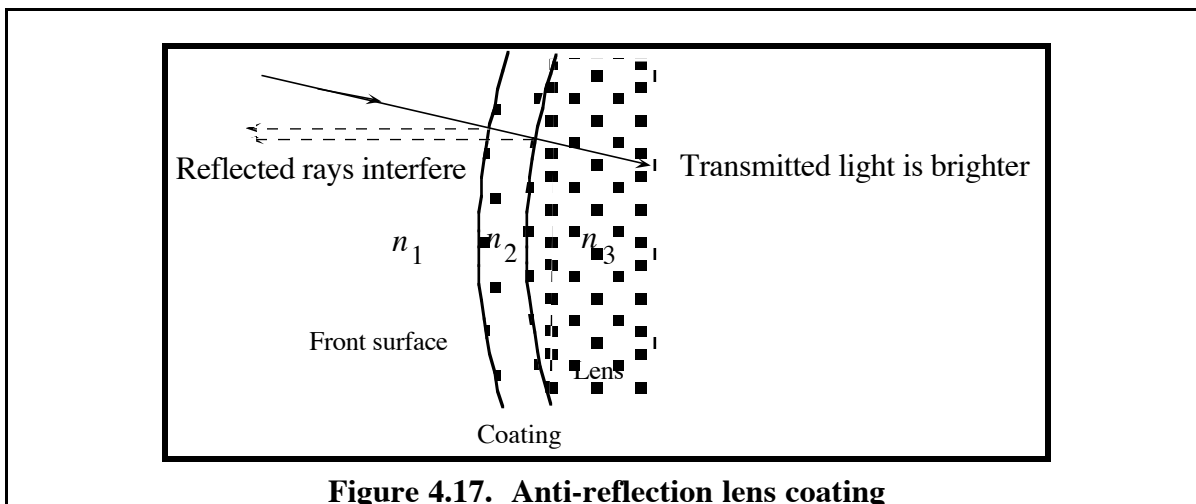
### Blooming of lenses

A common application of thin film interference is in anti-reflection coatings on lenses that are used in cameras, microscopes and other optical instruments. A modern lens system may have as many as ten glass surfaces each with a reflectivity of about 5%. Without some kind of treatment about half the light entering such a lens system would be reflected instead of going on to form the final image. Apart from the loss of brightness involved, multiple reflections in an optical system can also degrade the quality of an image.

The amount of light reflected from each surface can be greatly reduced using the technique of **blooming**, that is the deposition of an anti-reflection coating. Interference in the reflected light means that light is transmitted instead of being reflected. The choice of material for the coating is important. Clearly it must be transparent, but it should also result in approximately equal reflectivities at both surfaces, so that the reflected waves (at a chosen wavelength) can completely

cancel each other. Cancellation is achieved exactly when the refractive index of the coating is equal to the geometric mean of the refractive indices of the air and the glass:  $n_2 = \sqrt{n_1 n_3}$ . See figure 4.17. However it is not easy to find materials with exactly the right properties, so in practice a compromise is needed. Magnesium fluoride, which has a refractive index of 1.38, is often used.

The thickness of the coating is chosen to work best for light of a wavelength near the middle of the visible spectrum, for example a wavelength of 500 nm corresponding to yellow-green light. In that case the lens still reflects some light in the blue and red so it looks purple in reflected light. The refractive index of the coating is between that of air and glass so there is a phase change at both reflections. At the chosen wavelength we require  $2n_2b = (m + \frac{1}{2})\lambda$  for no reflection. With  $m = 0$ , the film thickness is a quarter of a wavelength.



### THINGS TO DO

Look for examples of interference in your environment. The colours in oil slicks are an example of thin-film interference. Next time that you see one make a note of the colours and their sequence. Are they the same as the colours of the rainbow? Can you explain the differences or similarities? Other examples of thin-film interference may be found in soap bubbles, the feathers of some birds and opals.

You can make a thin film using two sheets of transparency film like that used on overhead projectors. Just place the sheets together and look at the reflected light. A dark background behind the sheets will help. You should be able to see coloured contour fringes which map the thickness of the air between the sheets. To enhance the effect place the two sheets on a hard surface and by rubbing something like a handkerchief over them, try to squeeze the air out of the gap. What do you see now? See what happens when you press your finger on one part of the top sheet. Does the angle at which you look make any difference? Does the angle of the incident light matter? Look through the sheets and try to see the interference in the transmitted light; why is that harder to see?

Observe the colour of the light reflected from various camera lenses. Can you explain the colour? Is the colour the same for all lenses?

## QUESTIONS

- Q4.1** Two coherent monochromatic sources produce an interference pattern on a screen. What happens to the pattern if
- the wavelength is doubled,
  - the distance between the two sources is doubled?
- Q4.2** A Young's double slit experiment consists of two slits 0.10 mm apart and a screen at a distance of 1.0 m.
- Calculate the separation of blue light ( $\lambda = 400$  nm) fringes.  
Calculate the separation of red light ( $\lambda = 600$  nm) fringes.  
Sketch the pattern near the centre of the screen.  
Can you deduce anything about interference patterns in white light?
- Q4.3** Suppose that two coherent sources have a constant phase difference  $\phi$  which is not equal to zero. How do the conditions for interference maxima and minima change?
- Q4.4** If you dip a wire frame into soapy water and take it out, you will see colours in the thin film of water. If the light source is behind you, you will see thin film interference.
- As the water drains away from the top, the colours there will disappear. Can you explain why and estimate the thickness of the soap film at the top?
- Q4.5** The edges of an oil patch on the road appear coloured. Can you explain why and estimate the thickness of the film there?
- Why is it not possible to see fringes over the whole of the film?
- Q4.6** The wavelength of a spectral line was measured using a Young's twin slit set-up with a suitable filter to select the appropriate line. The separation of the twin slits was 0.523 mm and the screen was placed 1.22 m from the slits. The distance between the two second-order bright fringes on the screen was measured as 5.50 mm.
- Calculate the wavelength of the spectral line. What would happen to the separation of the fringes if the distance to the screen were doubled?
- Q4.7** A lens with refractive index 1.53 is to be coated with magnesium fluoride (refractive index 1.38) in order to eliminate reflections at the peak sensitivity of the human eye. (See chapter L1.) How thick should the coating be?

### Discussion questions

- Q4.8** Could Young's twin slit experiment be done with sound waves? Discuss.
- Q4.9** What would happen to the fringe separation if Young's experiment were done entirely inside a big tank of water? Explain.
- Q4.10** Is it possible that you could observe interference fringes in the light from the two headlamps of a distant car? Explain.
- Q4.11** Suppose that instead of putting a filter between the lamp and the single slit in Young's experiment, a red filter were put over one of the twin slits and a blue filter over the other. What effect would that have on the fringes?
- Q4.12** What happens if you remove the screen with the single slit in figure 4.2 ?
- Q4.13** When an oil slick spreads out on water, reflections are brightest where the oil is thinnest. What can you deduce from that?
- Q4.14** Bloomed lenses look coloured. Does that mean that the lens coating is made of a coloured material?