# L7 OPTICAL SYSTEMS

## OBJECTIVES

### Aims

Your aim here should be to acquire a working knowledge of the basic components of optical systems and understand their purpose, function and limitations in terms of concepts learned from earlier chapters. You should also be able to apply your knowledge of optics to describe the structure, function and limitations of a simple camera. A long term goal is that when you encounter new or unfamiliar optical instruments in future, you will be able to understand, or figure out, their function and limitations.

### Minimum learning goals

1. Explain, interpret and use the terms:

    (a) *lens system, objective, eyepiece, optical relay,*

    (b) *spherical aberration*, *chromatic aberration*, *coma*, *curvature of field*, *astigmatism*, *distortion*,

    (c) *principal planes*, *principal points*, *focal points, nodal points, cardinal points, entrance pupil, stop, aperture, focal ratio, f-number, depth of field, image brightness,*

    (d) *cornea, aqueous humour, vitreous humour, iris, retina, rods*, *cones*, *photopic vision, scotopic vision, accommodation, hyperopia, myopia, astigmatism.*
2. Describe and discuss the nature of aberrations.
3. Describe and apply ray-tracing techniques for locating images formed by lens systems whose cardinal points are given.
4. Explain how entrance pupil and aperture affect the illumination of images, and do simple calculations (photographic exposures, for example) related to these.
5. Draw a labelled diagram showing the structure of a simple camera and name its parts. Describe and discuss the function of the camera.
6. Describe the optical structure and function of the eye.

# TEXT & LECTURE

## 7-1 OPTICAL INSTRUMENTS

Optical instruments whose function is to produce images can be divided into two groups.

- **Photographic instruments** produce real images. Examples include cameras, projectors and eyes.
- **Visual instruments** produce virtual images which can be looked at with the eye. Examples are magnifying glasses, telescopes and microscopes.

Optical instruments are made up of optical components classified as objectives, eyepieces and optical relays. The component nearest the object is called the **objective** and its purpose is to form a real (intermediate) image of the object. An **eyepiece** is used, essentially as a magnifying glass, to look at the image produced by the objective. The purpose of an **optical relay** is to transfer an intermediate image from one place to another, more convenient, location. Optical relays can also change the orientation of an image, e.g. make an inverted image upright. Prismatic binoculars (figure 7.1) are an example which uses all three types of component.

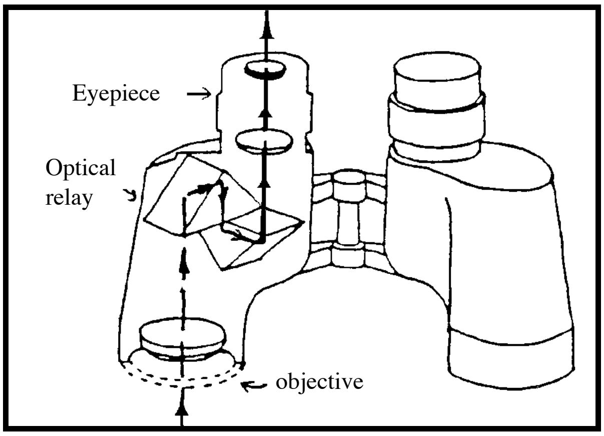


**Figure 7.1. Prism binoculars**

Objectives and eyepieces are designed to act like single ideal lenses, but they are usually made up of a number of lens elements. That is done in order to reduce lens aberrations and thus give clearer images.

## 7-2 ABERRATIONS

The equations relating image and object distances, derived earlier assuming paraxial rays, describe the performance of an "ideal" lens. If paraxial approximations are not made, the way a real lens forms images can still be calculated, although with difficulty. The differences between the performance of a real lens and an ideal lens are called **aberrations**.

There are six types of aberration. Spherical aberration and chromatic aberration were discussed in chapter L3. The remaining four are: coma, curvature of field, astigmatism, distortion.

### Coma

Coma is an aberration which shows up in the images of points well away from the principal axis. The image ($I_C$ in figure 7.2) formed by rays which pass through the central region of the lens is further from the principal axis and also further from the lens than the the image ($I_E$) formed by rays which go through the region near the edge of the lens. The net effect is that the image of an off-axis point has a comet-like or pear-shaped appearance.

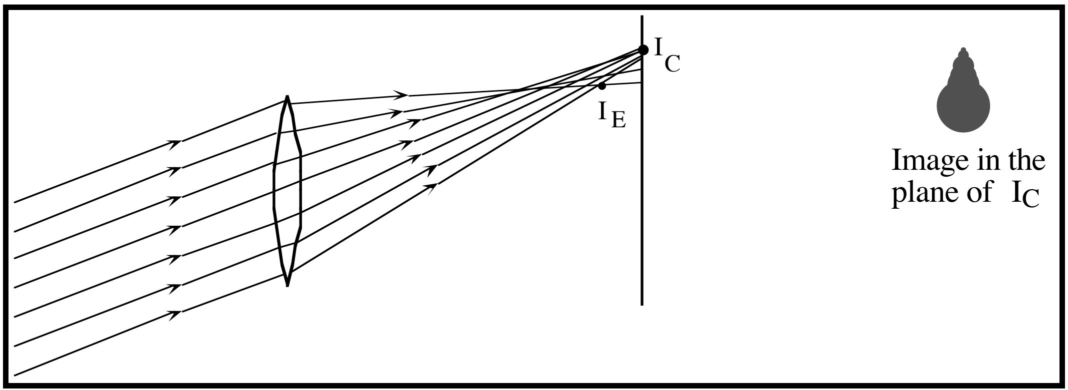


Figure 7.2. Coma

### Curvature of field

The image of a plane object, perpendicular to the principal axis, is really located on a curved surface. The effect is that if we look at the images formed in a plane perpendicular to the principal axis, part of each image will be out of focus. If we adjust the part of the image near the axis for good focus then the edges are out of focus; when the edges are well-focussed the central part is fuzzy.

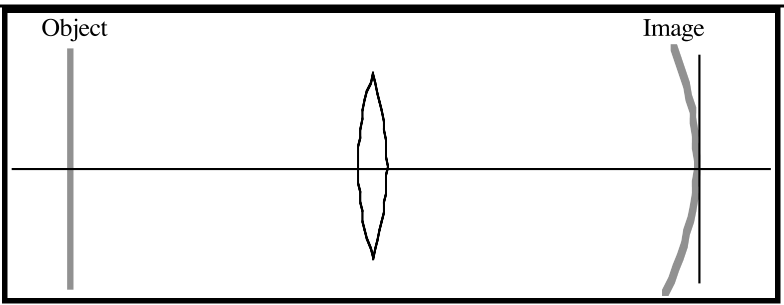


Figure 7.3. Curvature of field

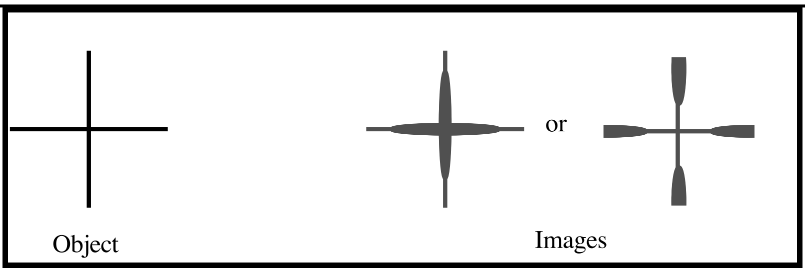


Figure 7.4. Images affected by curvature of field

### Astigmatism

Astigmatism is a complex geometrical aberration associated with the fact that the images of points off the principal axis can become elongated. Although astigmatism occurs in symmetrical lenses the effect can also be produced by asymmetries in the lens. For example rays which pass through a vertical section of the lens may come to a focus closer to the lens than do the rays passing through a horizontal section.

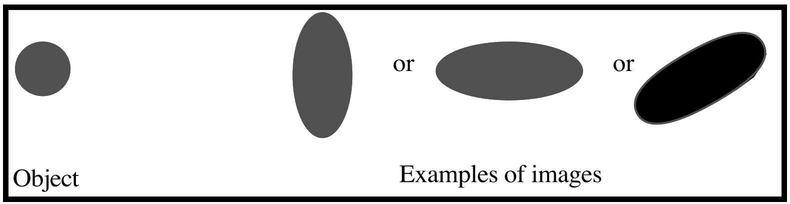


Figure 7.5. Astigmatism

### Distortion

Distortion is the alteration of the shape of an image. Two common kinds of distortion are pincushion and barrel distortion. In pincushion distortion the image of a square grid has the corners pulled out whereas in barrel distortion they are pushed in.

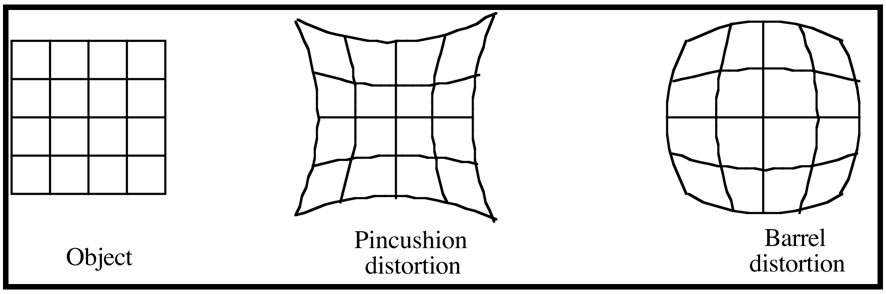


**Figure 7.6. Distortion**

## 7-3 IMAGE FORMATION BY OPTICAL SYSTEMS

### Ray tracing for a single thin lens

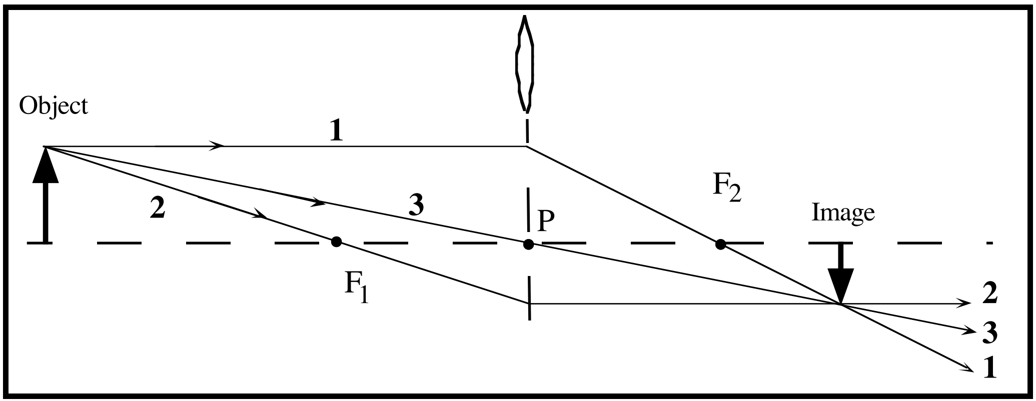


**Figure 7.7. Ray tracing for a thin lens**

In chapter L3 we discussed the following rules for ray tracing using the paraxial approximation for a thin lens (figure 7.7).

**1.** Rays incident parallel to the principal axis after passing through the lens, are deflected so that they pass through (or appear to come from) the second focal point, $F_2$.

**2.** Incident rays passing through the first focal point, $F_1$, are refracted so that they emerge parallel to the principal axis.

**3.** Rays which pass through the centre of the lens emerge in the same direction.

Since the lens is thin, all the constructions can be made by making all deflections at a plane (called the principal plane) through the centre of the lens.

### Ray tracing for a thick lens or any optical component

The function of a thick lens or a system of any number of lenses can be described in a similar manner. The properties of such a system can be described in terms of two **focal points** (as for a thin lens) as well as two **principal points** and two **principal planes** (instead of one). In addition there are two new points, called **nodal points**.

The ray-tracing rules are now as follows (see figure 7.8).

**1.** Rays which come in parallel to the principal axis are deflected at the **second principal plane** towards the second focal point.

**2.** Rays which come in through the first focal point are deflected at the **first principal plane** so that they come out parallel to the principal axis.

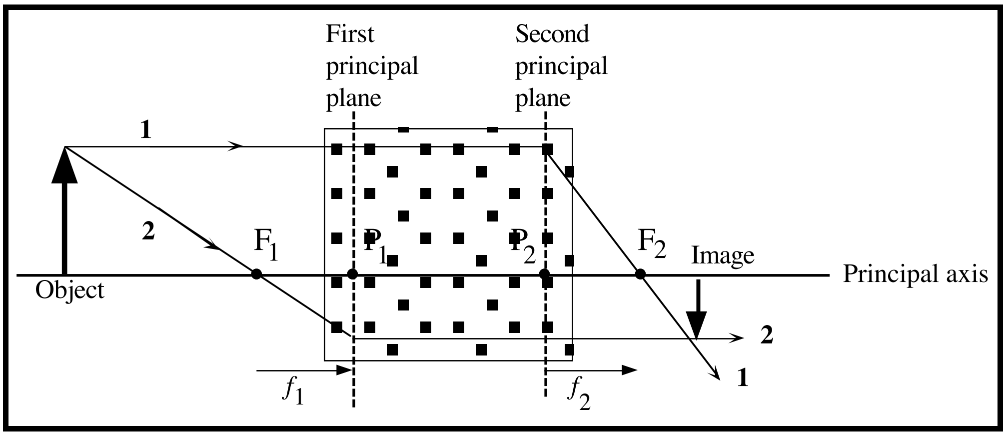


**Figure 7.8. Ray tracing using principal planes**

Note that these rules do not give the actual paths of the light rays while they are inside the lens system, but provided that the paraxial approximation is still good, they do give the correct paths of the rays which come out of the system. In the construction the complex set of deflections for the real rays is replaced by a single deflection for each construction ray, which takes place at one of two principal planes.

When the media on either side of the optical system have the same refractive index the distances $F_1P_1$, $F_2P_2$, which are called the first and second focal lengths ($f_1$,$f_2$) are equal. When the media on each side of the optical system are different (as in the eye or in oil-immersion microscopy) the two focal lengths have different values. For example the first focal length of a human eye is 17 mm while the second focal length is 23 mm.

Notice that for a single thin lens, the principal planes coincide, so a single principal plane suffices.

For a single thin lens we had a third ray-tracing rule: a ray passes through the centre undeflected. To get the equivalent of the rule for a lens system we need to define two more special points, the **nodal points**, on the principal axis of the system. The third rule is as follows.

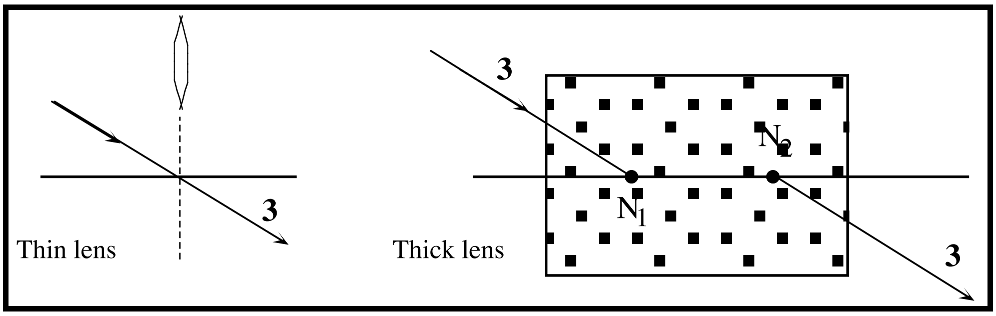


**Figure 7.9. Ray tracing using nodal points**

**3.** For a ray coming in to the first *nodal point* $N_1$, construct a ray from the second nodal point $N_2$ in the same direction.

When the medium on each side of the system is the same, the nodal points coincide with the principal points. If the media on the two sides are different the nodal points no longer coincide with the principal points.

The focal points, the principal points and the nodal points are called the **cardinal points** of a lens system.

### The lens equation

With object distance defined as the distance from the object to the first principal plane and the image distance as the distance from the second principal plane to the image (figure 7.10), the lens equation (introduced in chapter L3) still works for paraxial rays.

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \quad . \qquad \text{... (7.1)}$$

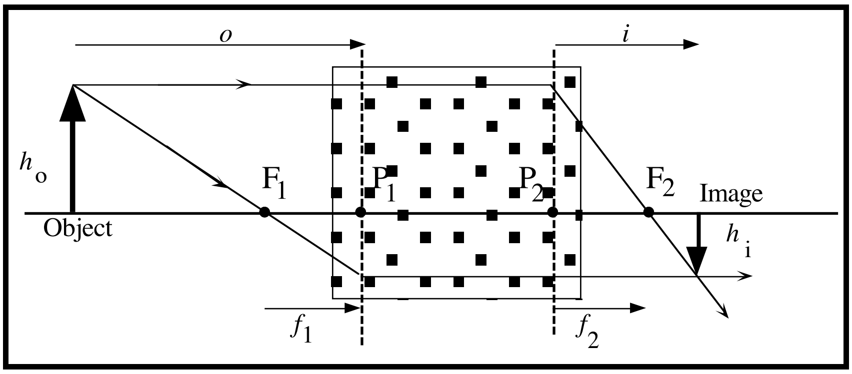


Figure 7.10. Definition of distances and lateral magnification

## 7-4 MAGNIFICATION

The **linear magnification** of an optical system is defined as the ratio of image size to object size. It is useful to distinguish two ways of specifying linear magnification. The first which we have already defined in chapter L3, is strictly the **lateral magnification**, defined as

$$m = \frac{\text{image height}}{\text{object height}} \quad .$$

The magnification is still given by the formula

$$m = -\frac{i}{o} \quad . \qquad \text{... (7.2)}$$

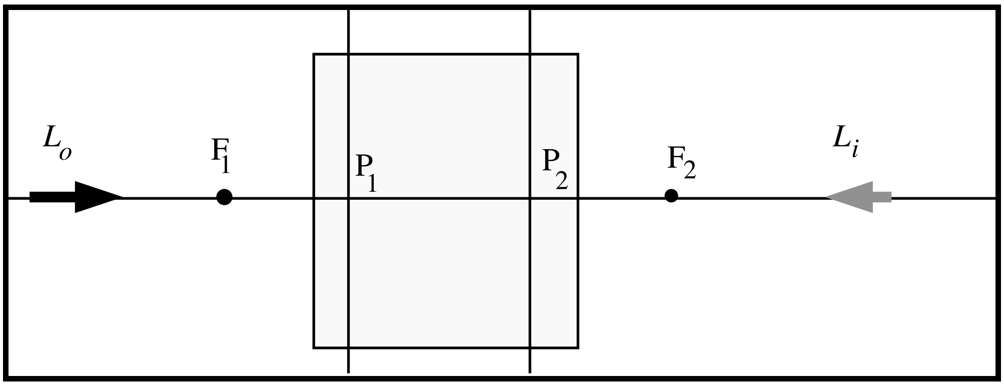


Figure 7.11. Longitudinal magnification

$$m = \frac{L_i}{L_o}$$

## 7-5 BRIGHTNESS OF THE IMAGE

The brightness of an image is determined by the amount of light passing through the optical system which in turn is determined by

(i) the diameter of the lenses, or

(ii) the diameter of the **apertures** (holes) in any opaque screens which are known as **stops** or **diaphragms**.

***Example***

For example figure 7.12 shows the formation of a point image of a distant object by a single lens. All the light collected by the lens, shown in the shaded region of the diagram, goes to form the image. The wider the lens, the more light we get, so the image is brighter.

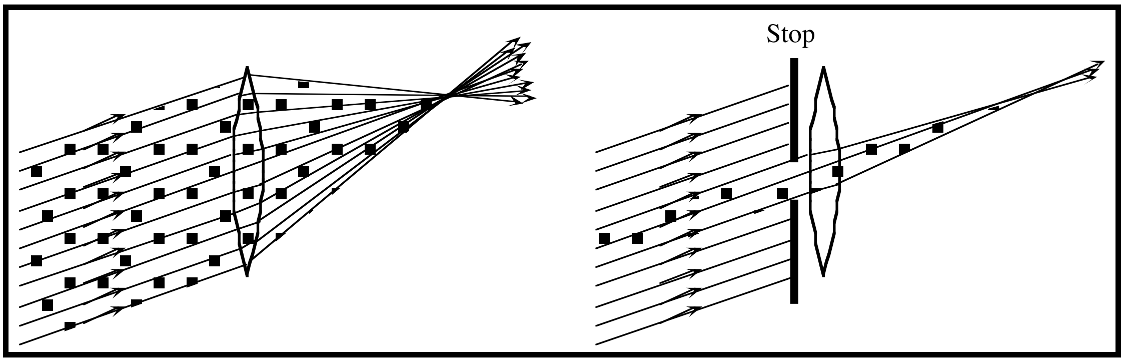


**Figure 7.12. How lens diameter affects image brightness**
All the light which goes through the lens goes into the image. The trade-off against less light is reduced aberrations.

On the other hand using all of a lens to form an image can result in noticeable aberrations, so we often deliberately restrict the amount of light using a screen with a hole, often called an **aperture stop**, to stop some of the light. The image in that case will not be so bright.

***Example***

In a two-lens system the stop is often placed between the lenses so that some of the light which enters the system does not get through to form the final image. Again the image brightness depends on the diameter of the aperture.

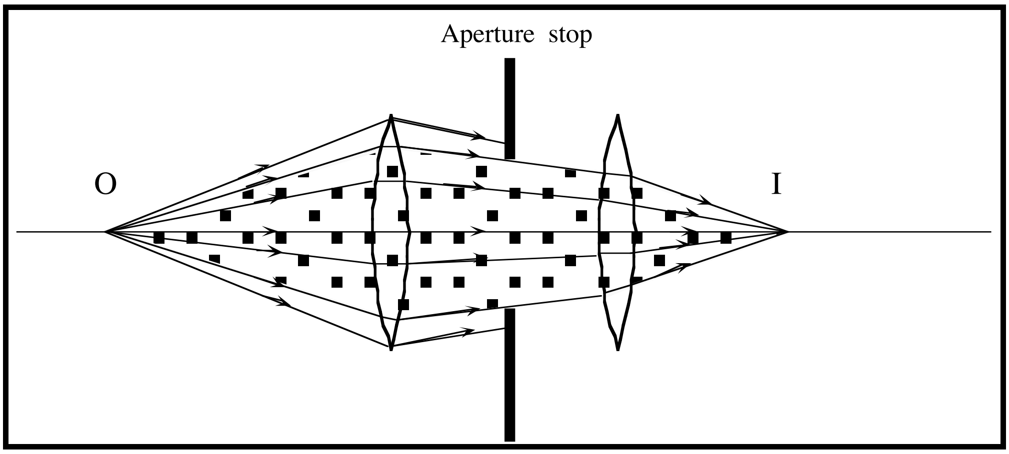


**Figure 7.13. A stop used in a multiple lens system**

If you look at the stop located between two lenses you can do so only by looking into the system from one side or the other. If you look in from the "object side" you will see an image of the aperture stop formed by the first lens. If the first lens is a converging lens, that image will be enlarged. The image of the hole (stop) seen from the object side is called the **entrance pupil** of the system (figures 7.14, 7.15). Similarly if you were to look into the system form the "image side" you would see a different image of the hole. That image is called the **exit pupil** (figure 7.15).

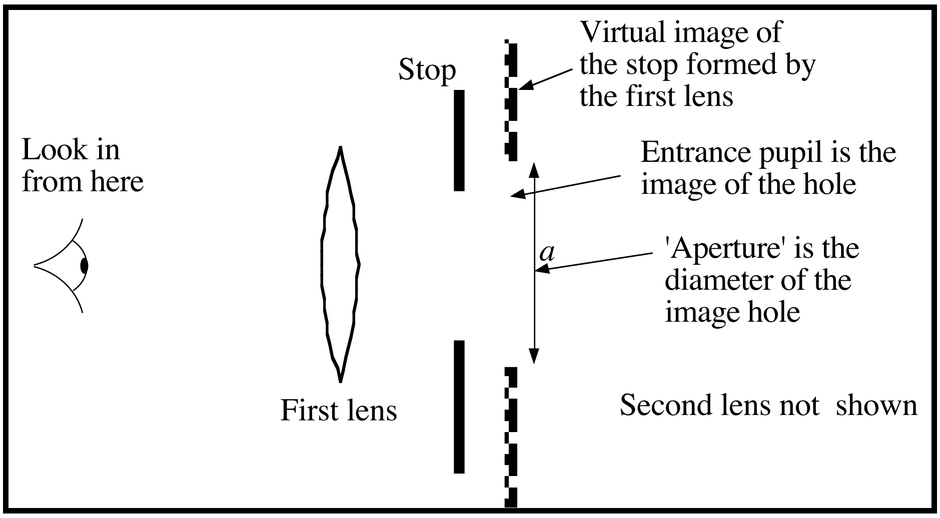


**Figure 7.14. Entrance pupil of a lens system**

We can describe how much light gets through the system in terms of the size of the actual aperture, or the size of the entrance pupil, or the size of the exit pupil. Figure 7.15 shows how the light from a point source is restricted in terms of these ideas. The bundle of rays which gets through is limited by the cone with the object point as apex and the entrance pupil as base. An alternative, equivalent, specification is the bundle of rays converging on the image point in the cone based on the exit pupil.

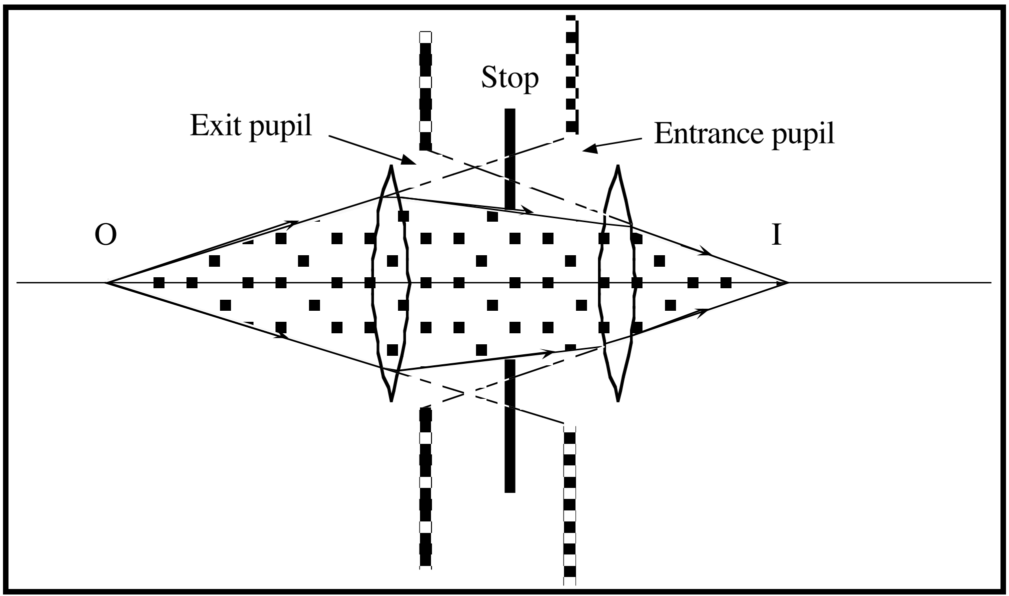


**Figure 7.15. How the light is restricted by entrance and exit pupils**

In order to use this approach we need to know, not only the location of the entrance pupil, but also its diameter. If we look at the stop through the first lens, it appears to have diameter $a$, so $a$ is the diameter of the entrance pupil. For a single lens with no aperture stop, the diameter of the entrance pupil is just the diameter of the lens.

### Aperture

The two important parameters of a lens system which affect the brightness of images are the diameter of its entrance pupil, commonly called the **aperture**, and its focal length. For a given object brightness, the image brightness is actually determined by the ratio of the aperture to the focal length. We have already seen that for a given focal length, the brightness increases with increasing aperture. But for a fixed size of aperture, a short focal length produces brighter (and smaller) images. For a given object at a reasonably large distance, systems with the same value of the ratio, aperture divided by focal length, produce images with the same image brightness. (This result breaks down at small object distances.)

Aperture is often specified as a fraction of the focal length. For example, a system which has an aperture of *f*/8 has an entrance pupil whose diameter is one eighth of its focal length. (The slash is a division sign, so the aperture is equal to *f* divided by 8.) Other terms used in connection with this idea include the following.

- The **focal ratio** ($n$) is the ratio of the focal length to the aperture, $f/a$. The terms focal ratio and **f-number** both refer to the divisor ($n$) in the expression $f/n$. Thus if the aperture ($a$) is equal to *f*/8 then the focal ratio and the f-number are both equal to 8.
- The **aperture ratio** is the ratio of the aperture to the focal length, i.e. the reciprocal of the focal ratio. For an aperture of *f*/8 the aperture ratio is 1/8.

Note that the aperture is a distance, whereas aperture ratio and focal ratio (f-number) are pure numbers without units.

**Example**

A single lens with a diameter of 10 mm and a focal length of 50 mm has an aperture of *f*/5, an f-number of 5, an aperture ratio of 0.2 and a focal ratio of 5.

## 7-6 RESOLUTION

The *theoretical* limit to the smallest objects that can be distinguished with an optical component is determined by diffraction. The smallest hole in the optical component, normally the stop, has the greatest diffraction effect.

From chapter L5 (equation 5.2), the angle between the central maximum and the first minimum of the diffraction pattern of a point object by a circular aperture of diameter $a$ is given by

$$\theta \approx \sin\theta = 1.2\frac{\lambda}{a} \quad . \qquad \text{... (7.3)}$$

The ideal image of a distant point object should be a point at a distance equal to $f$, the focal length, from the lens system. In reality the image is a small circular diffraction pattern in which the radius of the central bright region is given approximately by

$$\begin{aligned} r &= \theta f \\ &= 1.2\,\lambda\frac{f}{a} \quad . \qquad \text{... (7.4)} \end{aligned}$$

Notice that this result contains the ratio of aperture to focal length again. The larger the aperture as a fraction of the focal length, the smaller is the diameter of the diffraction pattern.

It is worth noting, however, that the practical limit to the resolution of an optical system is normally set by aberrations and other imperfections, not by diffraction.

## 7-7 THE CAMERA

In its simple form a camera consists of a light-tight box, a compound objective lens, a shutter and a film. A variable aperture controls the image brightness.

The objective, which forms a real image at the film plane, is usually specified in terms of its focal length, and its maximum usable aperture. For example a camera lens might be marked 70 mm, *f*/4.5. The image brightness is controlled by varying the size of the aperture which is normally described as a fraction of the focal length. Depending upon the lens, aperture settings can range from about *f*/32 to *f*/1.

**Focussing** is achieved by moving the objective relative to the film plane, which is fixed in the camera body. Only one object plane is in focus at any one time. The images, on the film, of point objects which are not in the object plane are small discs whose size is determined by the distance of the film plane from their true image plane and by the angle of convergence of the rays forming the image (i.e. by the focal ratio). The range of object distances for which the image discs are acceptably small is called the **depth of field**.

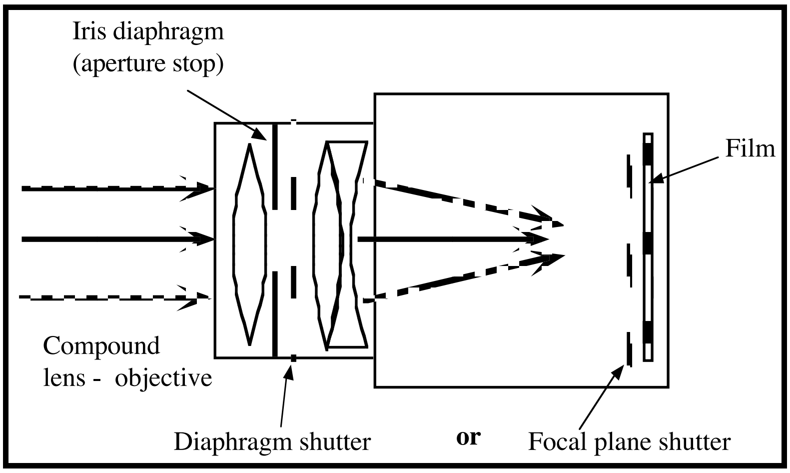


Figure 7.16. Parts of a camera (schematic)

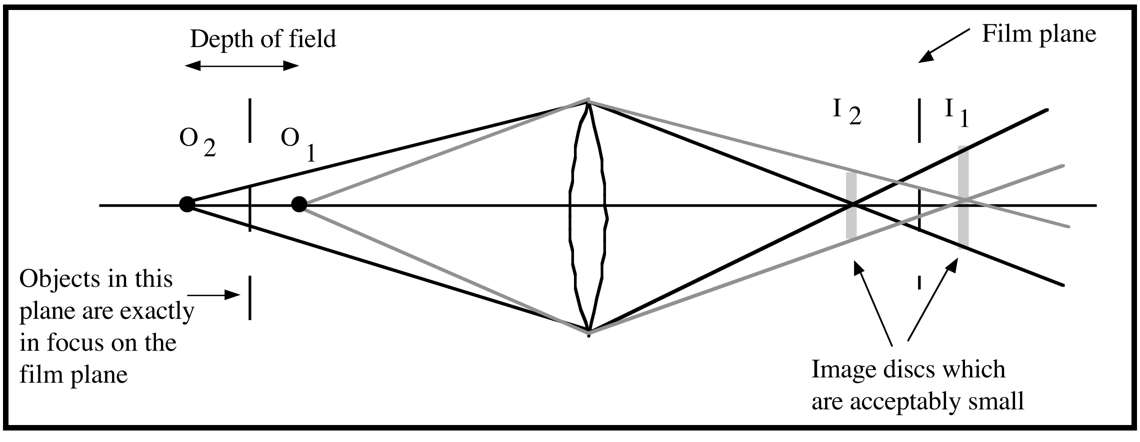


Figure 7.17. Depth of field

**Note.** The larger the aperture, the greater is the apex angle of the cone of rays coming to a focus at the image. Image points must then be formed closer to the film plane if the image is to remain acceptable. The depth of field is consequently reduced.

## 7-8 THE HUMAN EYE

The human eye is almost spherical, being about 24 mm long and 22 mm across, most of the eye being contained within a strong flexible shell called the sclera. The eye contains an optical system that produces real images on the light-sensitive **retina**. Most of the focussing is done by the outer surface of the **cornea**. The space behind the cornea, the anterior chamber, is filled with a watery liquid called the **aqueous humour** whose refractive index (1.336) is only a little less than that of the cornea (1.376), so there is very little further bending of light rays at the inside surface of the cornea. After the cornea light must pass the variable opening or **pupil** formed by the **iris** before it strikes the lens of the eye.

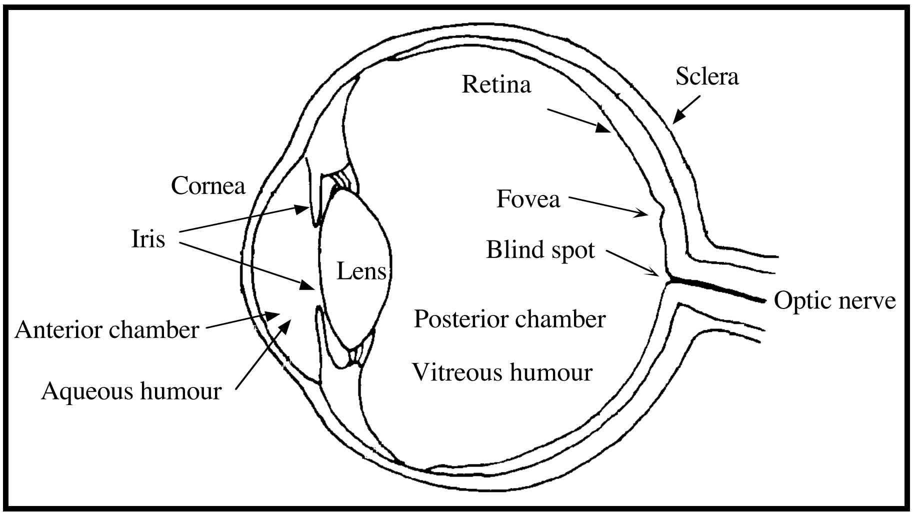


**Figure 7.18. The human right eye - horizontal section**

Variation in the focal length of the eye, and hence its ability to form images of objects at different distances - a process called **accommodation** - is achieved by altering the shape of the **lens** of the eye. The lens is a complex layered structure, whose refractive index varies within the lens. Light finally passes through the posterior chamber of the eye which is filled with a transparent jelly-like substance called the **vitreous humour** (refractive index 1.337). Inverted real images are formed on the retina at the back of the eye.

Normally the eye can focus on objects further than 25 cm from the eye, in fact most young people can focus much closer than that. However, instrument designers need to refer to a standard, close, distance at which most people can focus so the value of 25 cm has been chosen. That distance is often called the **least distance of distinct vision**.

## Rods and cones

In the retina are two kinds of sensors: **rods** and **cones** (figure 7.19). The cones function at high levels of illumination and mediate colour vision which is called **photopic vision**. The rods function at low levels of illumination when the eye has become dark-adapted and do not give the sensation of colour, a process called is **scotopic vision**. The different spectral responses of the rods and cones are sketched in figure 7.20.

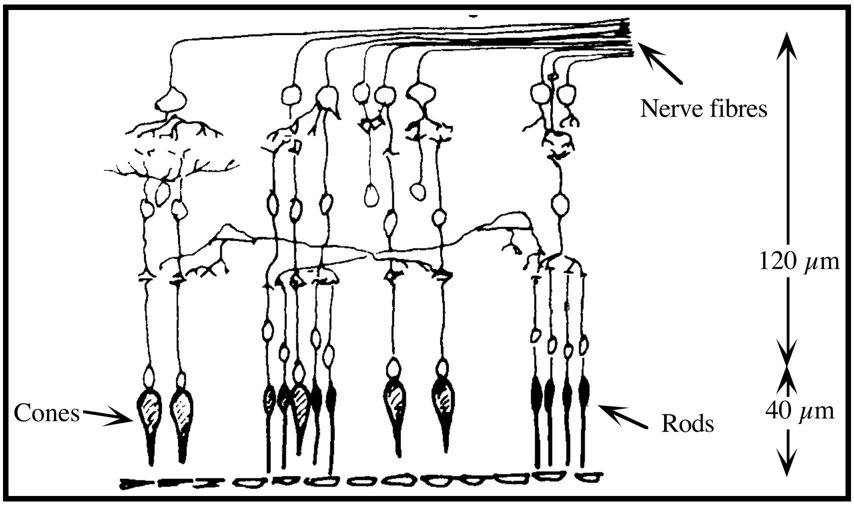


**Figure 7.19. Structure of the retina**

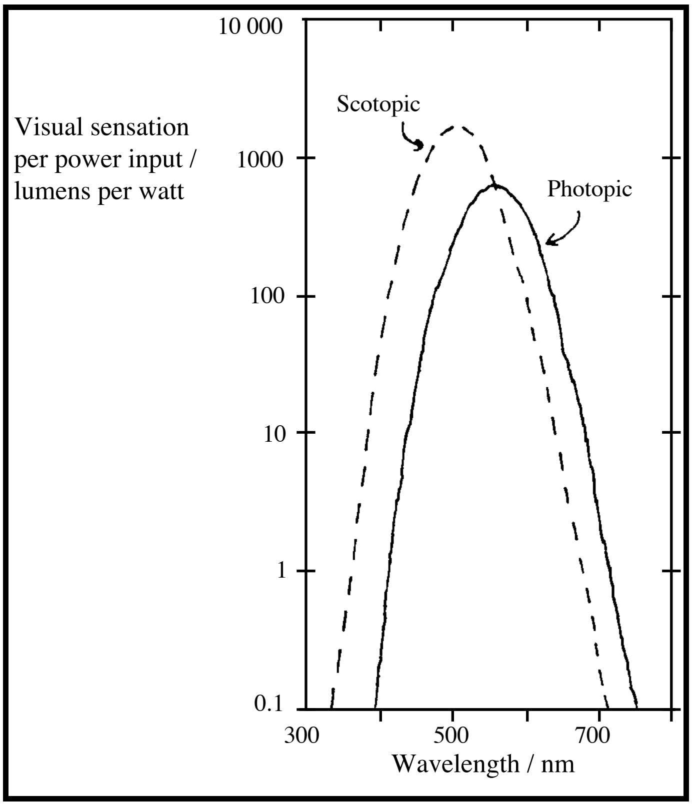


**Figure 7.20. Sensitivity of the human eye**

For intermediate light levels the spectral response is between the scotopic and photopic vision responses, which is is called **mesopic vision**.

## Resolution

The aperture stop of the eye, called the **iris**, changes its diameter as the intensity of the incident light varies. In daylight its diameter is about 3 mm. The best resolution at the centre of the eye is about 1 minute of arc ($3 \times 10^{-4}$ radian). The limit is set by diffraction and by the resolving power of the retina. The centre of the eye is populated exclusively by cones which have a diameter of about 2 $\mu$m. For two point sources to be resolved their images on the retina must be separated by about 5 $\mu$m, which is comparable to the size of the diffraction pattern (about 7 $\mu$m).

## Defects of the eye

The range of accommodation in eyes decreases with age. To make up for this loss and also to correct defects, spectacles are used. Common defects are as follows.

**Hyperopia** (or hypermetropia) is the condition in which the image of a distant object (at relaxed vision) lies behind the eye. A converging spectacle lens, which adds power to the system, is used to correct this defect.

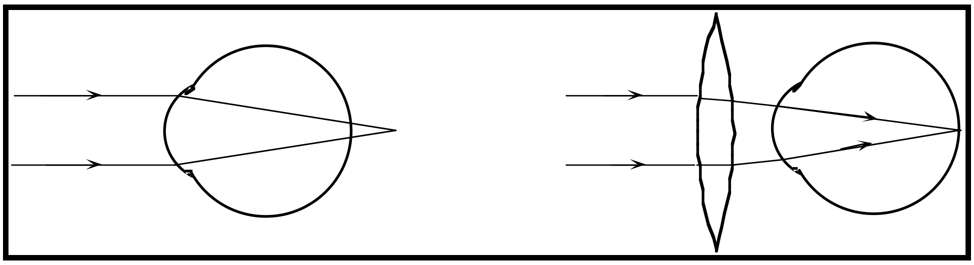


**Figure 7.21. Correction of hyperopia**

**Myopia** is the condition in which the image of a distinct object is formed in front of the retina. It is corrected using a diverging spectacle lens, i.e. by decreasing the power of the system.

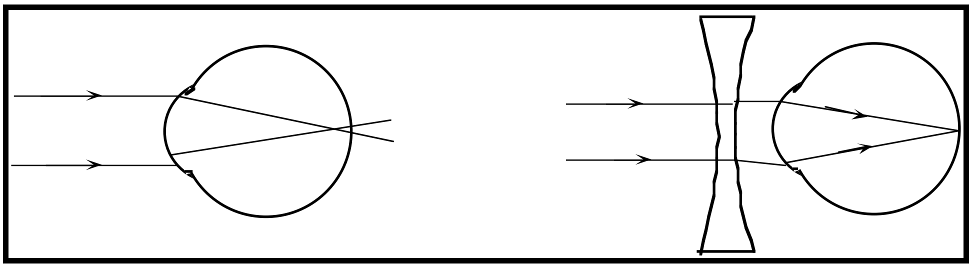
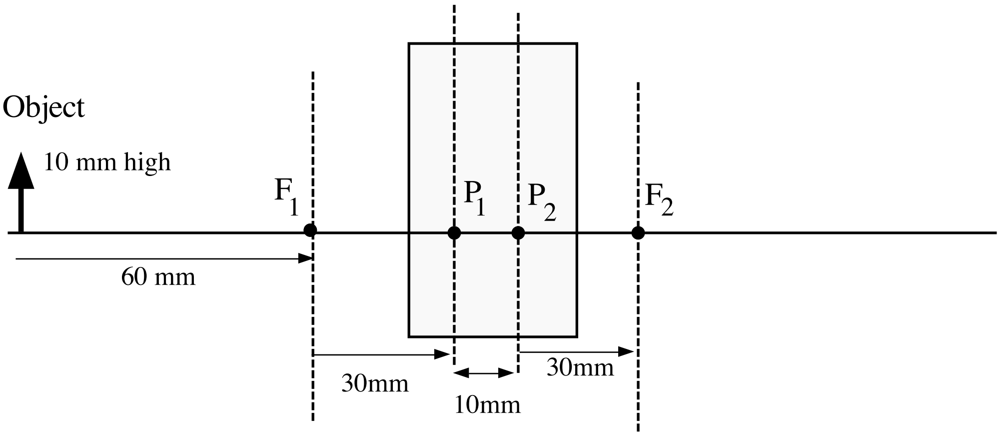


**Figure 7.22. Correction of myopia**

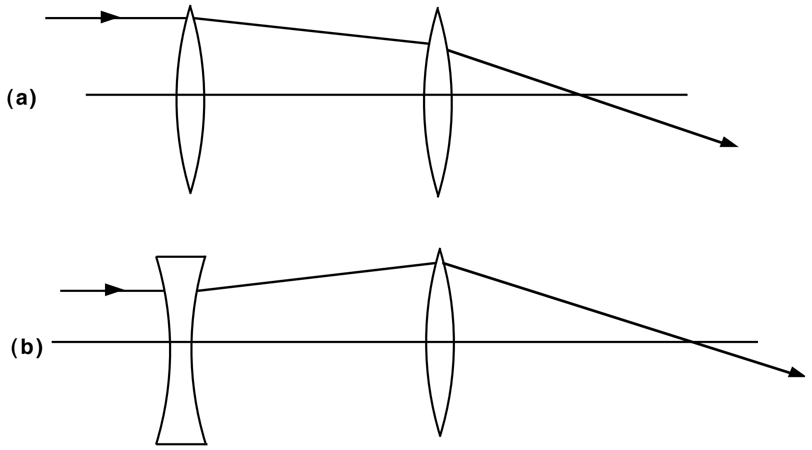
**Astigmatism** of the eye is a condition in which the radii of curvature of the cornea and the lens are not the same for all cross-sections containing the principal axis of the eye. A lens which has different curvatures can be used to compensate for the defect. The practical solution to correcting astigmatism depends on the other optic defects in the eye. If astigmatism is the only defect then a cylindrical lens can be used. If other defects are present as well, then the lens may be given one spherical and one cylindrical surface. It may be appropriate either to *add* power to the weaker axis or to *subtract power* from the stronger axis.

# QUESTIONS

**Q7.1** Use the ray tracing method described in the lecture to locate the image of the object formed by the optical component below.



**Q7.2** A ray parallel to the axis enters an optical system and passes through two lenses as shown in (a) and (b). Using only a straight-edge, locate the second principal plane in each case.



**Q7.3** What is the diameter of a lens, focal length +100 mm, aperture *f*/8?

**Q7.4** The lens of a camera has a focal length of +50 mm. What is the tallest object standing 10 m away that can give an image fitting onto the film? (The image must be smaller than 35 mm.)

**Q7.5** The lens of a camera has a focal length of 50.0 mm. Calculate how far the lens must be from the film in order to focus an object 0.20 m away. Repeat the calculation for an object at infinity. What range of travel of the lens is required to focus objects from 0.20 m to infinity.

**Q7.6** A myopic eye cannot focus on objects further away from the eye than a point called the *near point*. What are the power and the focal length of a spectacle lens which enables a person whose near point is at 3 m to see distant objects (i.e. objects at infinity)?

## Discussion questions

**Q7.7** If you move a camera while the shutter is open you get a blurred picture, but if you move your eye you can still see clearly. Discuss.

**Q7.8** In what ways are the eye and a camera similar? How to they differ?