

## L8

## VISUAL INSTRUMENTS

## OBJECTIVES

## Aims

As a climax to this unit, you should end up with a good understanding of the physical principles of visual instruments. The main topic is the principles of microscopy. The section on telescopes is included for interest - it is not examinable.

## Minimum learning goals

1. Explain, interpret and use the terms:  
*near point, least distance of distinct vision, relaxed vision, angular size, angular magnification, visual instrument, simple magnifier, compound microscope, eyepiece, objective, optical tube length, numerical aperture, field of view, condenser, resolution of a microscope, resolving power, maximum useful magnification, dark field illumination, interference microscopy.*
2. Describe, explain and discuss the operation of a simple magnifier. Solve simple quantitative problems on magnification.
3. Draw diagrams showing the essential structure and function of a compound microscope. Describe and explain how it works. Solve simple quantitative problems related to its magnifying function.
4. Describe and discuss resolution and useful magnification of microscopes.
5. Describe and discuss the brightness of images in a microscope and techniques for illuminating specimens.
6. Describe and explain the techniques of *interference microscopy*.

## Extra goals

7. Describe and explain the basic principles of telescopes.

## PRE-LECTURE

## 8-1 ANGULAR SIZE

Visual instruments are used to make an object appear larger by increasing the angle subtended at the eye by its edges. In figure 8.1, the object subtends an angle  $\alpha$  at the eye so  $\alpha$  is called the object's **angular size**. When  $\alpha$  is small its value in radians is approximately equal to the ratio of the object's linear size to its distance from the eye:

$$\alpha \approx \frac{h}{d} . \quad \dots (8.1)$$

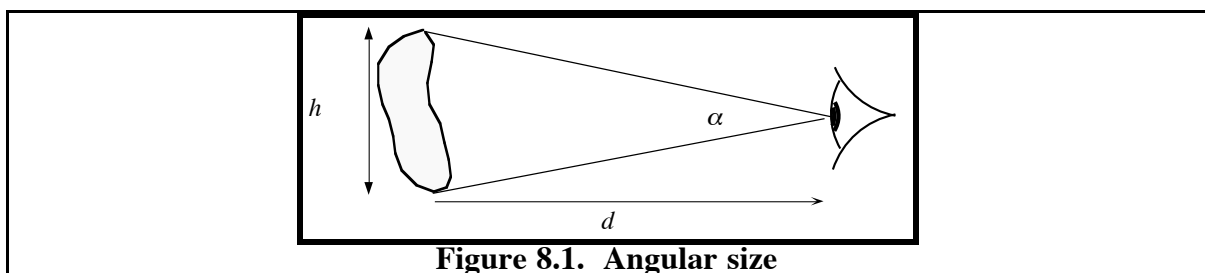


Figure 8.1. Angular size

**Q8.1** Work out the following examples.

- a) What angle is subtended at your eye by the width of your thumb, when you hold your arm outstretched?
- b) The radius of the sun is  $7.0 \times 10^5$  km and its distance from the earth is  $1.5 \times 10^8$  km. What is the angle subtended by the sun at a telescope on earth?
- c) What is the angle subtended at your eye by a tiny creature 0.1 mm long at a distance of 0.25 m?

## LECTURE

**8-2 ANGULAR MAGNIFICATION**

To see more detail in an object, we must make it look bigger. We need to make the object appear to subtend a larger angle at the eye so that the image on the retina is larger. One way of doing that is to get closer to the object. If you can't get closer you can use a **visual instrument** to achieve the magnification. For example, if the object is far away and we cannot get closer we can use a **telescope** to increase the angle. On the other hand if the object is very small we cannot bring it too close because our eyes would be unable to focus on it properly. In that case we can use a **magnifier** or **microscope** to increase the angle.

**Characteristics of the unaided eye**

The human eye is capable of focussing on objects close to the eye but how closely depends on a number of factors, including the age of the subject and the presence of optical defects in the eye. Try focussing on a small object as you gradually bring it closer to your eye. You will find that there is a location, called the **near point**, which is the closest you can bring the object while keeping it in sharp focus. Typically a young person (aged 20 or younger without optical defects) has a near point about 10 mm from the eye, whereas the best a normal sixty-year old subject can manage is to focus on objects 500 mm away. By convention, a comfortable close viewing distance for **near vision** is taken to be 250 mm (0.25 m). This distance is often called, inappropriately, the **least distance of distinct vision** denoted here by the symbol  $d_v$ . At the other extreme, an optical system can be arranged so that images are formed at infinity. In that case we have **relaxed** or **far vision**.

The finest detail that can be seen by the unaided eye, its **resolution**, can be calculated as follows. The minimum angle subtended by two points which can still be resolved is determined by diffraction at the pupil. This angle is about  $3 \times 10^{-4}$  radian. The closest distance for placing the object is about 0.25 m. Therefore, the minimum distance between two points which can still be resolved is about  $3 \times 10^{-4} \times 0.25 \text{ m} = 0.075 \text{ mm}$ .

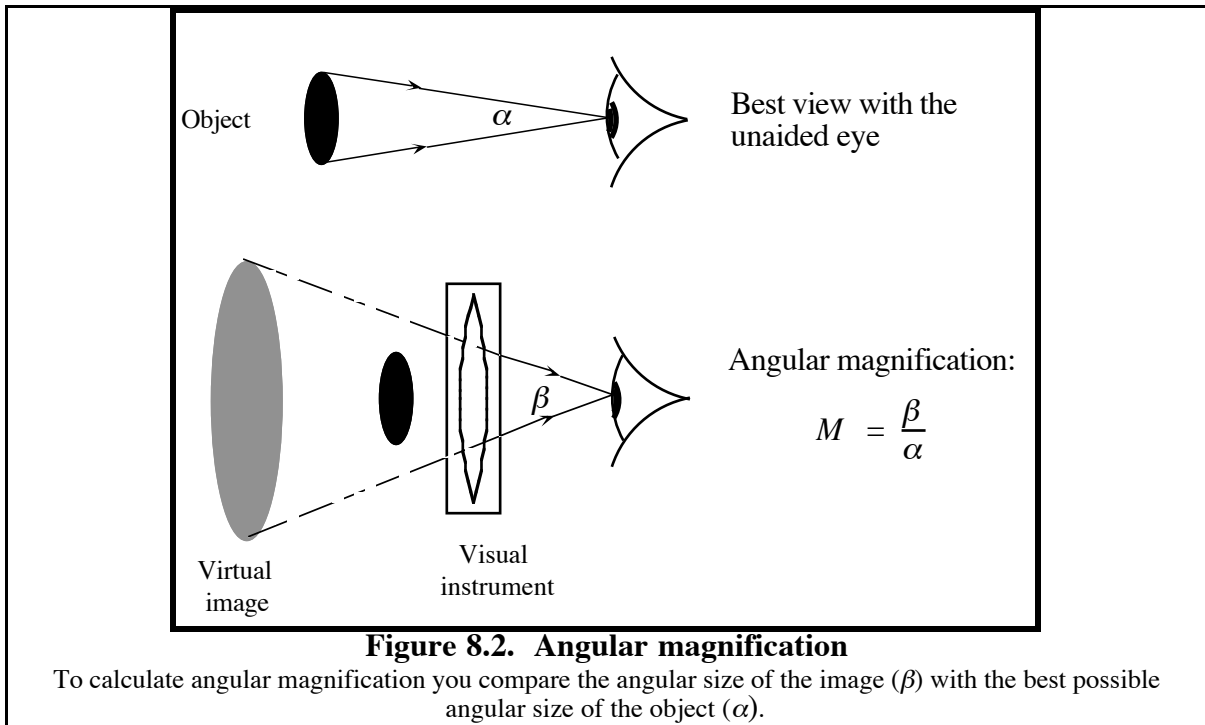
**Visual instruments**

The best detail in a small object that you can see with the naked eye is obtained by putting the object at your near point. For even more detail you need to use a visual instrument which makes the object look bigger (figure 8.2). The effectiveness of a visual instrument is described by its **angular magnification**,  $M$ , defined as the ratio:

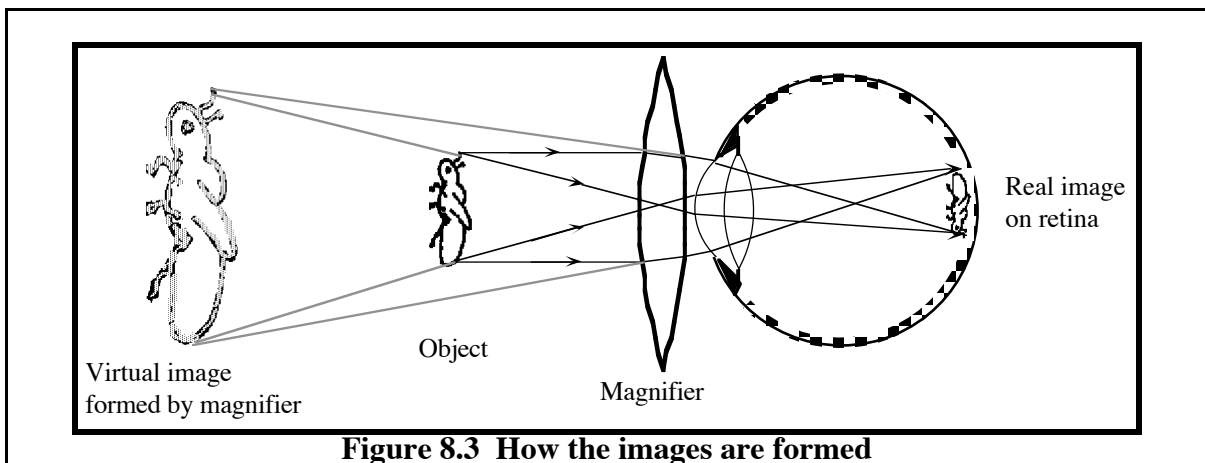
$$M = \frac{\text{angle that image subtends at eye looking through instrument}}{\text{angle that object subtends at unaided eye under the best possible conditions}}$$

$$= \frac{\beta}{\alpha}. \quad \dots (8.2)$$

For far-away objects, where a telescope is used, the denominator ( $\alpha$ ) is just the angle that the object subtends at the unaided eye. For very small objects where a magnifier or a microscope is used, the denominator is found by calculating the angle  $\alpha$  when the object is at the near point. Clearly, the magnification you get depends on how good your eyesight is. To get a nominal value for the magnification which does not depend on individual differences, the lens designer's value of magnification is obtained, by convention, by supposing that the value of  $\alpha$  is the angle that the object subtends when it is at the standard distance of 0.25 m away from the eye.

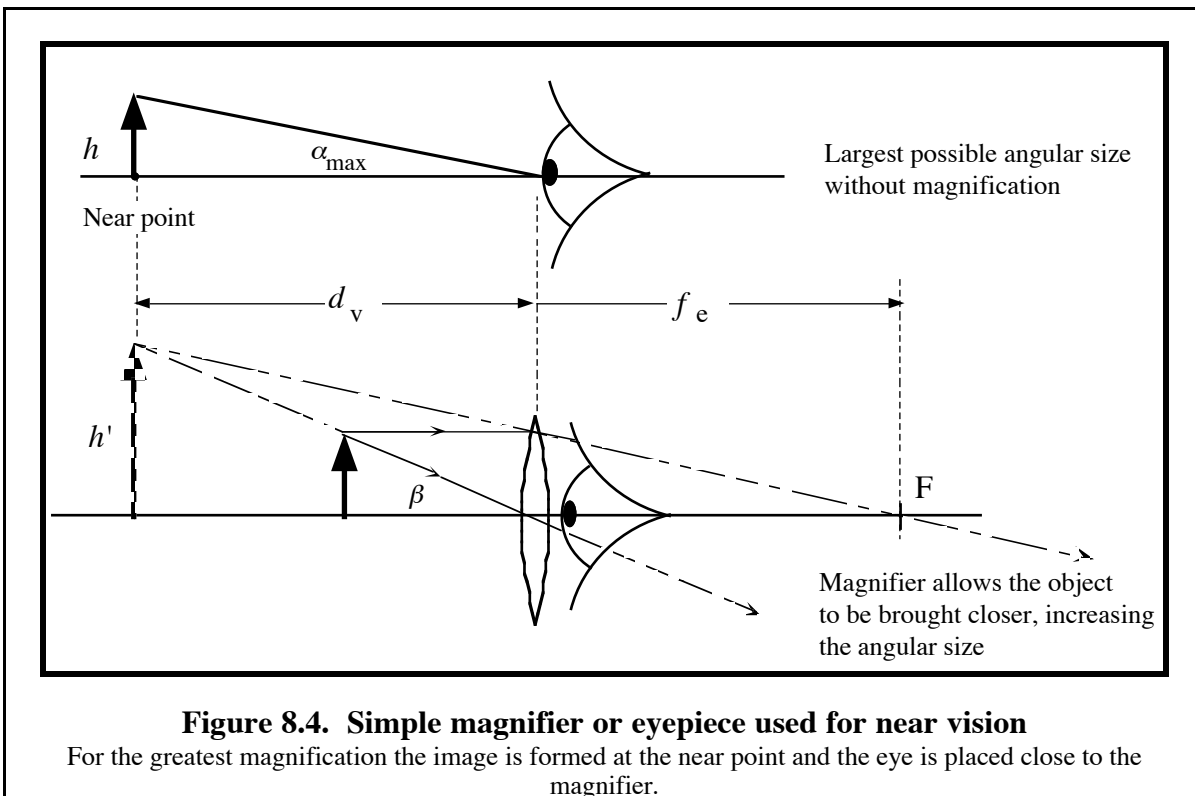


Although angular magnification is described in terms of the virtual image formed by the lens, you should remember that what you see is determined by the real image on the retina (figure 8.3). Increasing the angular size ( $\beta$ ) of the virtual image produces a bigger final real image.



### 8-3 SIMPLE MAGNIFIERS AND EYEPIECES

A single converging lens can be used as a visual **magnifier** which functions either on its own or as the **eyepiece** for a more complex instrument such as a microscope. The magnification is achieved by making the object seem to be closer to the eye than the near point. The lens produces a virtual image, which is larger than the object and therefore subtends a bigger angle at the eye. This virtual image can be located anywhere between the near point and infinity. The magnification actually achieved will depend on where the image and the eye are placed relative to the lens. In order to be able to compare the magnifications which can be achieved with different lenses we could just quote the focal length, but that does not give any immediate impression of the magnification. A more meaningful measure is the angular magnification achieved for some standard arrangement of image (or object), the lens and the eye. For the purpose of this calculation the eye is always placed as close as possible to the lens. Angles subtended by objects and images at the eye are then near enough to being the same as the angles subtended at the lens (figures 8.4 and 8.5).

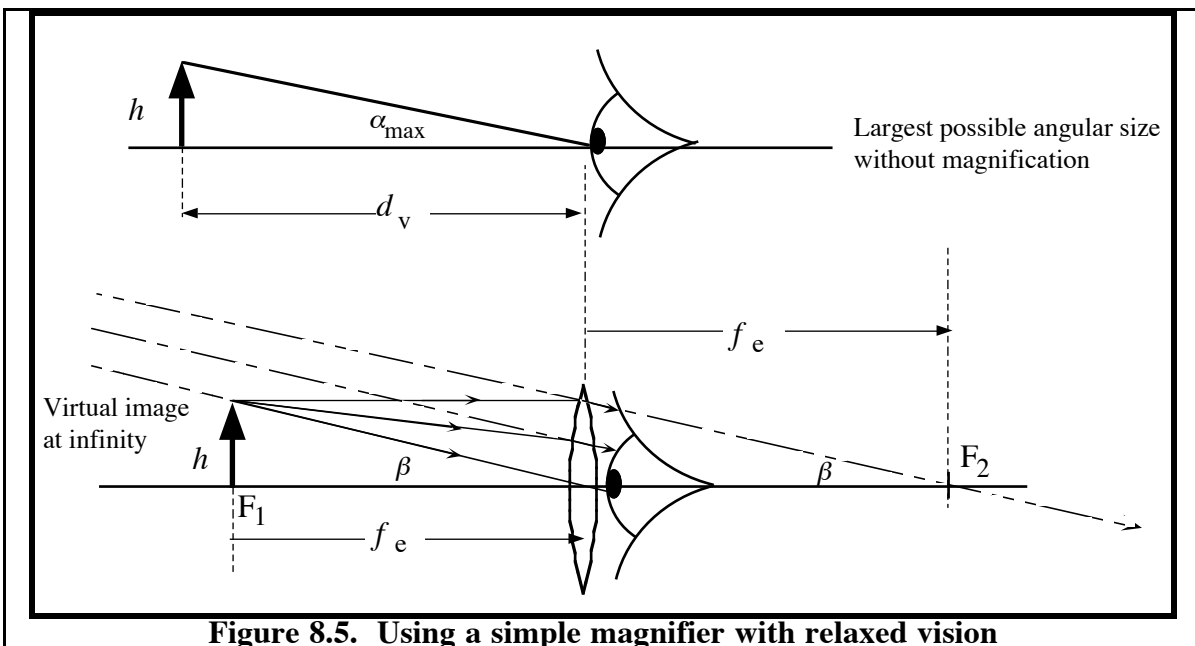


To work out the angular magnification, we compare the angular size of the image ( $\beta$ ) with the angle ( $\alpha_{\max}$ ) that the object would subtend at the eye (or lens) if it were put at the near point (distance  $d_v$ ). It is fairly easy to work out (using the paraxial approximation,  $\tan\alpha \approx \alpha$  etc.) that in this case the angular magnification is given by the formula:

$$M_e = 1 + \frac{d_v}{f_e} \quad \dots (8.3)$$

where  $f_e$  is the focal length of the lens. (The subscript e stands for 'eyepiece'.)

Another standard way of arranging things is to have the image at infinity (figure 8.5). In that case the eye is said to be **relaxed**.



For relaxed vision the object is at the focal point of the lens, so rays from a particular point on the object are parallel after refraction. In this example, rays from the bottom of the object enter the

eye parallel to the principal axis and all the rays from the top of the object enter the eye at an angle  $\beta$ . Hence the image subtends an angle  $\beta$  at the eye. With the usual paraxial approximation,

$$\beta \approx \frac{h}{f_e} . \quad \dots (8.4)$$

The object, if placed at distance  $d_v$  from the unaided eye, would subtend an angle  $\alpha$ :

$$\alpha_{\max} \approx \frac{h}{d_v} ; \quad \dots (8.5)$$

so

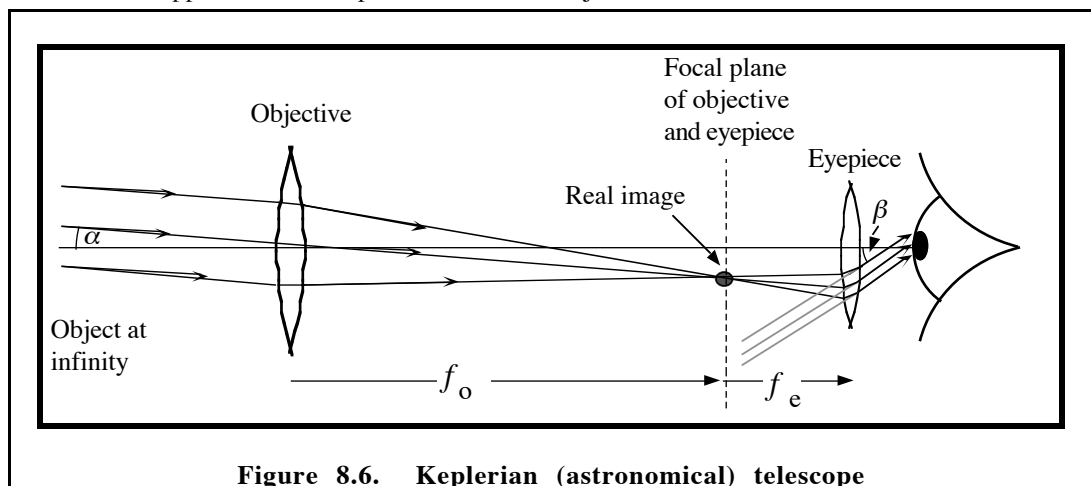
$$M_e = \frac{\beta}{\alpha_{\max}} \approx \frac{d_v}{f_e} . \quad \dots (8.6)$$

For reasonably large magnifications, this formula (8.6) is not much different from the one quoted earlier (8.3) for the case with the image at distance  $d_v$ , but it does make a difference for low-power magnifiers. Not surprisingly, you get the best possible magnification by forming the virtual image at the near point.

A magnifier is usually described by its angular magnification rather than by its focal length. For example, a magnifier using a lens with a focal length of 25 mm would be described as a "10 $\times$  magnifier".

#### 8-4 TELESCOPES

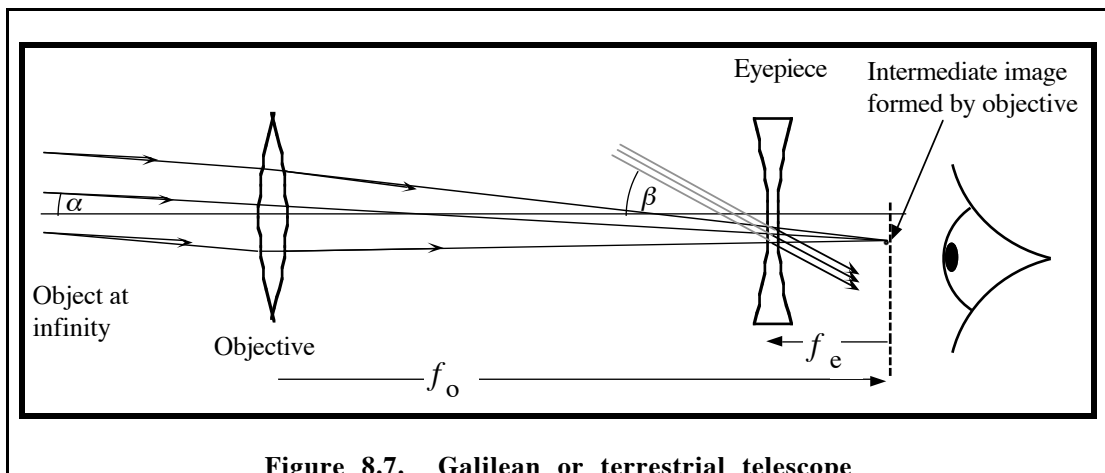
A telescope, in its basic form, consists of two components, an objective and an eyepiece. Its purpose is to increase the apparent size or separation of distant objects.



**Figure 8.6. Keplerian (astronomical) telescope**

In the astronomical telescope the objective produces a real image, which is then viewed with the eyepiece. The telescope produces an inverted image, but that is no problem when one is looking at astronomical objects. However, the same lens arrangement is used in prism binoculars in which the prisms restore the image to an upright position. (See figure 7.1 in chapter L7.)

Alternatively a diverging lens may be used as an eyepiece, as in the Galilean telescope.



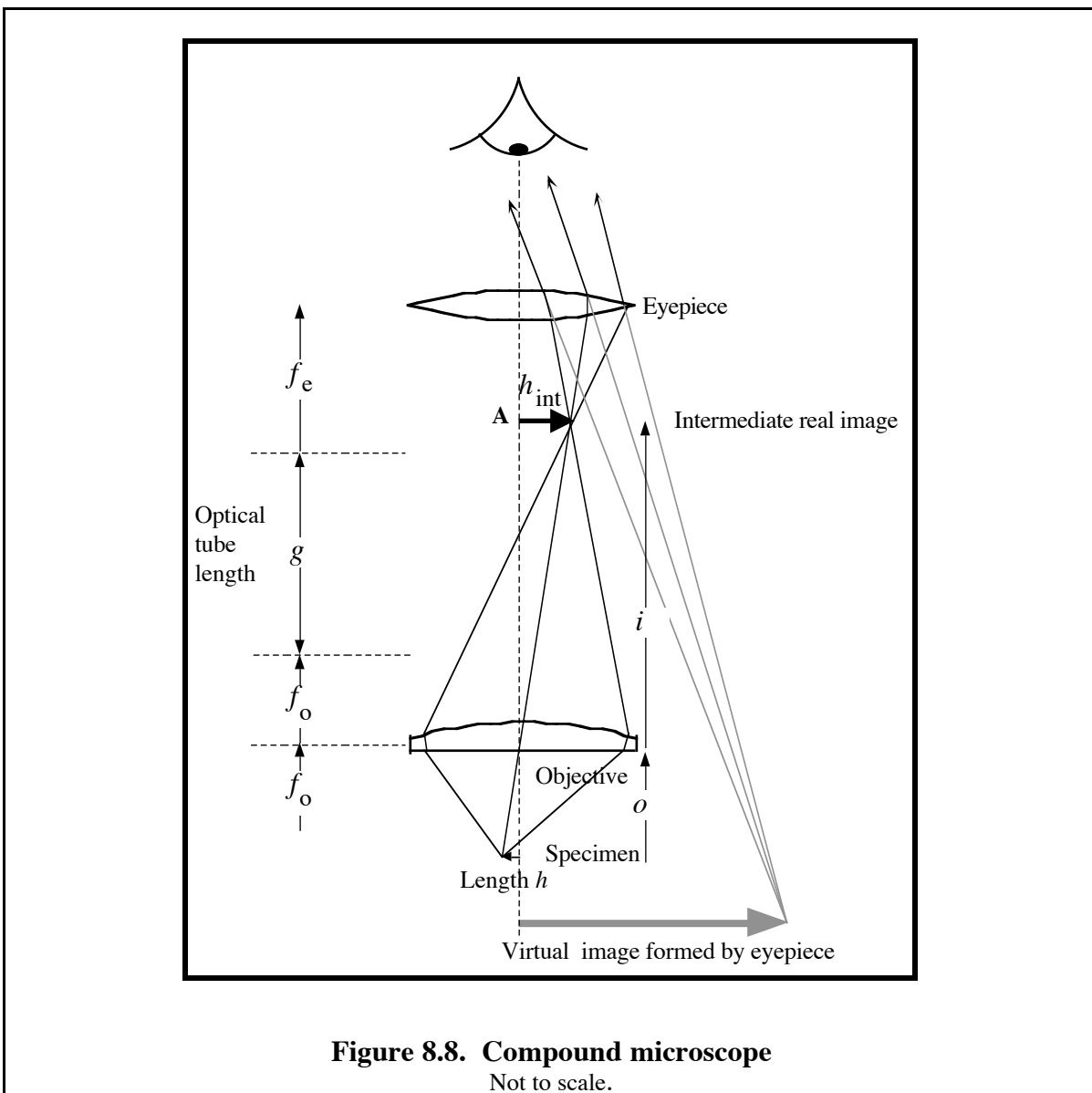
**Figure 8.7. Galilean or terrestrial telescope**

In both types of telescope the angular magnification is given by

$$M = \frac{f_o}{f_e} .$$

#### 8-4 MICROSCOPES

The maximum useful magnification obtainable with a simple magnifier is about 20 times. For greater magnification we use a **compound microscope** (figure 8.8). In its basic form it consists of an **objective** and an **eyepiece** mounted in a tube. The magnification of a microscope can be worked out in terms of the focal lengths of the objective and eyepiece and the **optical tube length**, which is defined as the distance between the second focal point of the objective and the first focal point of the eyepiece. In a high power microscope the optical tube length is much larger than either focal length so it is roughly equal to the actual separation between the objective and the eyepiece. (The single lenses in figure 8.8 may in fact be optical components each made up of several lenses.)



The specimen is placed just below the first focal point of the objective. Since the object distance is not much more than the focal length, the objective forms a much-enlarged real image, the intermediate image, at A. The eyepiece is used as a magnifier to look at the intermediate image. If that image is in the focal plane of the eyepiece, the virtual image seen by the eye will be at infinity.

The angular magnification is calculated as follows.

(i) The lateral (linear) magnification of the intermediate image can be written as the ratio of image distance to object distance, which in this case gives a magnification of

$$\frac{h_{\text{int}}}{h} = \frac{i}{o} \approx \frac{g + f_o}{f_o} .$$

Since the optical tube length  $g$  is usually very much greater than the focal length of the objective, the lateral magnification produced by the objective is

$$|m_o| = \frac{h_{\text{int}}}{h} \approx \frac{g}{f_o} . \quad \dots (8.7)$$

(ii) The eyepiece acts as a magnifier so, with relaxed vision, the angle subtended by the final virtual image is

$$\beta \approx \frac{h_{\text{int}}}{f_e} \approx \frac{gh}{f_o f_e} .$$

(iii) Now the original object, when placed at distance  $d_v$  from the unaided eye, would subtend an angle

$$\alpha \approx \frac{h}{d_v} .$$

So the total angular magnification is

$$M = \frac{\beta}{\alpha} \approx \frac{g d_v}{f_o f_e} . \quad \dots (8.8)$$

This result is just the same as saying that the total magnification is the product of the linear magnification of the objective and the angular magnification of the eyepiece:

$$M = |m_o| M_e \quad \dots (8.9)$$

with  $|m_o| \approx \frac{g}{f_o}$

and  $M_e \approx \frac{d_v}{f_e}$  .

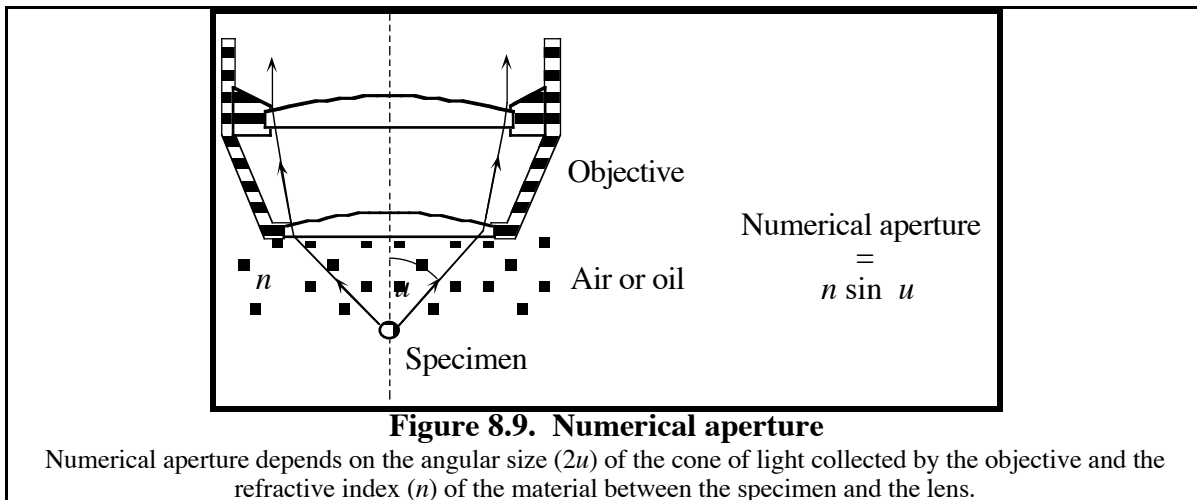
### Some typical values

Magnification of eyepieces:	up to 20×.
Optical tube length $g$ (standard value):	160 mm.
Focal length of objectives, low power:	50 to 100 mm;
medium power:	8 to 50 mm;
high power:	4 mm;
very high power:	2 mm.

From these values, the maximum magnification of a microscope is about 1600. The maximum *useful* magnification is limited by diffraction to about 200 to 400.

### Image brightness and numerical aperture

The **brightness** of the image depends on the amount of light from the specimen which enters the microscope. As we discussed in chapter L7, the brightness is determined by the entrance pupil of the objective. In microscopy another convenient way of specifying the amount of light collected is to quote the value of the angle  $u$  (figure 8.9) which describes the cone of rays collected by the objective.



However, it is not the angle  $u$  alone which matters, because the refractive index ( $n$ ) of the medium between the specimen and the objective affects the refraction of the rays as they enter the objective (and hence also, the entrance pupil). The parameter which matters is called **numerical aperture** (N.A.) which is defined as  $n \sin u$ . It turns out that numerical aperture also determines the resolving power of an objective. Hence, microscope objectives are commonly specified in terms of their lateral magnification and numerical aperture.

### Resolving power

As discussed in chapter L5, the amount of detail that can be seen in an image is limited by diffraction. Simple estimates of the limit can be made using the Rayleigh criterion. The limitations imposed by diffraction effects in a microscope which is completely free of aberrations can be described by a quantity called the **resolving power** of the microscope. Resolving power is defined as the minimum value of the distance between two points in the specimen which can just be resolved.\* Its value is given by the formula, based on the Rayleigh criterion,

$$R = \frac{0.6 \lambda}{n \sin u} = \frac{0.6 \lambda}{\text{N.A.}} \quad \dots (8.10)$$

where N.A. is the numerical aperture of the objective.

In both microscopes and telescopes, the **maximum useful magnification** is obtained when the angular separation of these two points, viewed through the microscope, is equal to the resolving power of the eye. For visible light, using a typical wavelength of 500 nm, the maximum useful magnification turns out to be about 250 times the numerical aperture.

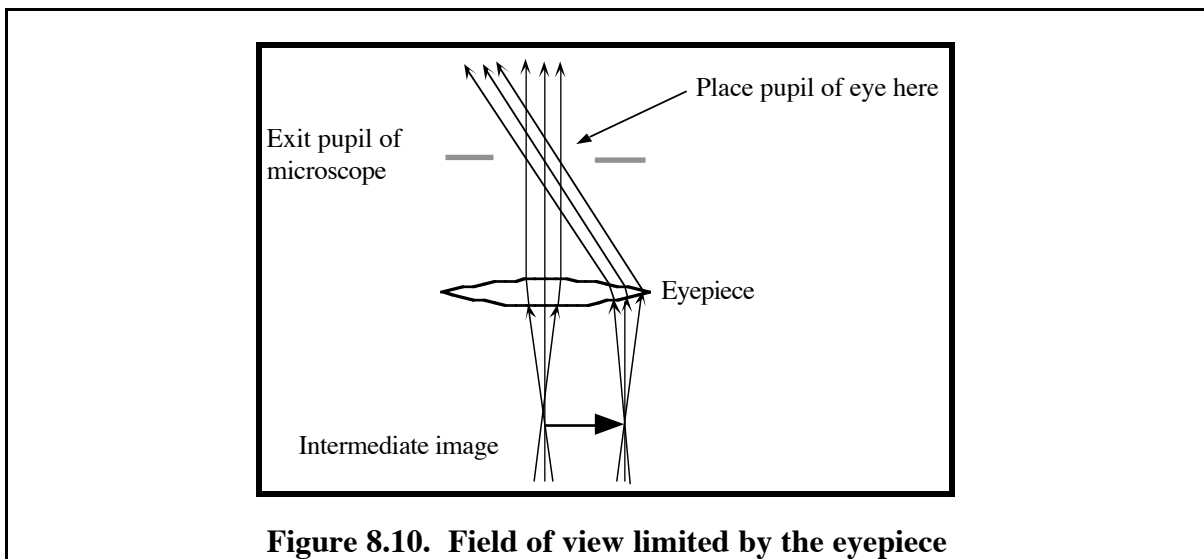
In air the maximum practical value of the numerical aperture is about 0.85, but higher values can be obtained by filling the space between the objective and the specimen with oil. With this technique, called **oil immersion microscopy**, values of numerical aperture up to about 1.4 can be obtained. Using an oil-immersion objective can increase the maximum useful magnification to about 400 $\times$ .

### Field of view

The **field of view** is the region of a specimen which can be seen at any one time. It is determined by those rays from the specimen which can go through the microscope to enter the eye. Often the limiting feature is the diameter of the eyepiece. For instance the objective may produce a large intermediate image, but the rays forming the extreme points of that image may miss the eyepiece so that they will not enter the eye. In that case the observable intermediate image is about the same size

\* The resolving power of a telescope is usually defined in terms of the *angle*, rather than the distance, subtended by object points at the objective. Similarly the resolving power of the eye is given as an angle. Whether distance or angle is meant can usually be determined from the context or from the unit used.

as the eyepiece, so the size of the field of view is given approximately by the diameter of the eyepiece divided by the magnification of the objective.



**Figure 8.10. Field of view limited by the eyepiece**

The field of view also depends on the position of the eye. For best viewing the pupil of the eye should coincide with the exit pupil of the microscope, since that is where the beam of light from the microscope is at its narrowest. For a microscope the exit pupil is the virtual image of the objective's aperture as seen through the eyepiece. It is a real image on the side of the eyepiece near the eye.

## 8-6 ILLUMINATION OF MICROSCOPE SPECIMENS

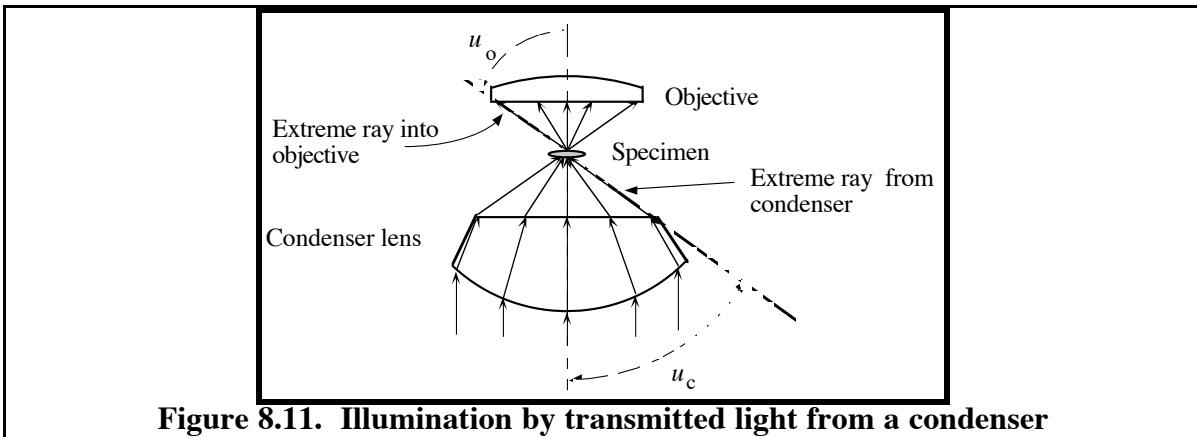
Most specimens to be viewed with a microscope are not self-luminous and so must be illuminated. For low magnification ambient lighting may be quite sufficient. At high magnifications the observed objects are small so they reflect only a small amount of light. To make such objects easily visible the intensity of light falling on them must be increased using special illuminating systems called **condensers**. Which kind of condenser is used depends on how the specimen is to be examined, which can be one of three ways:

- (a) by transmitted light,
- (b) by scattered light,
- (c) by reflected light.

For optimum performance the condenser should be such that light from each point of the specimen fills the aperture of the objective.

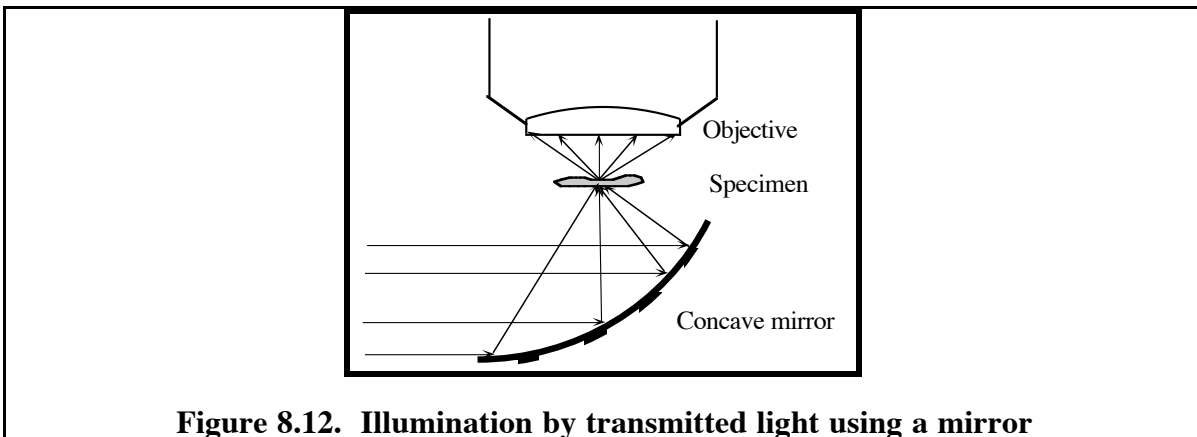
### ***Illumination by transmitted light***

For specimens viewed in transmitted light, the condenser should supply a cone of light such that all the rays in the cone can enter the objective. This means that the angle of convergence of the illuminating light onto the specimen ( $u_c$ ) should be equal to the acceptance angle ( $u_o$ ) of the objective (figure 8.11).



**Figure 8.11. Illumination by transmitted light from a condenser**

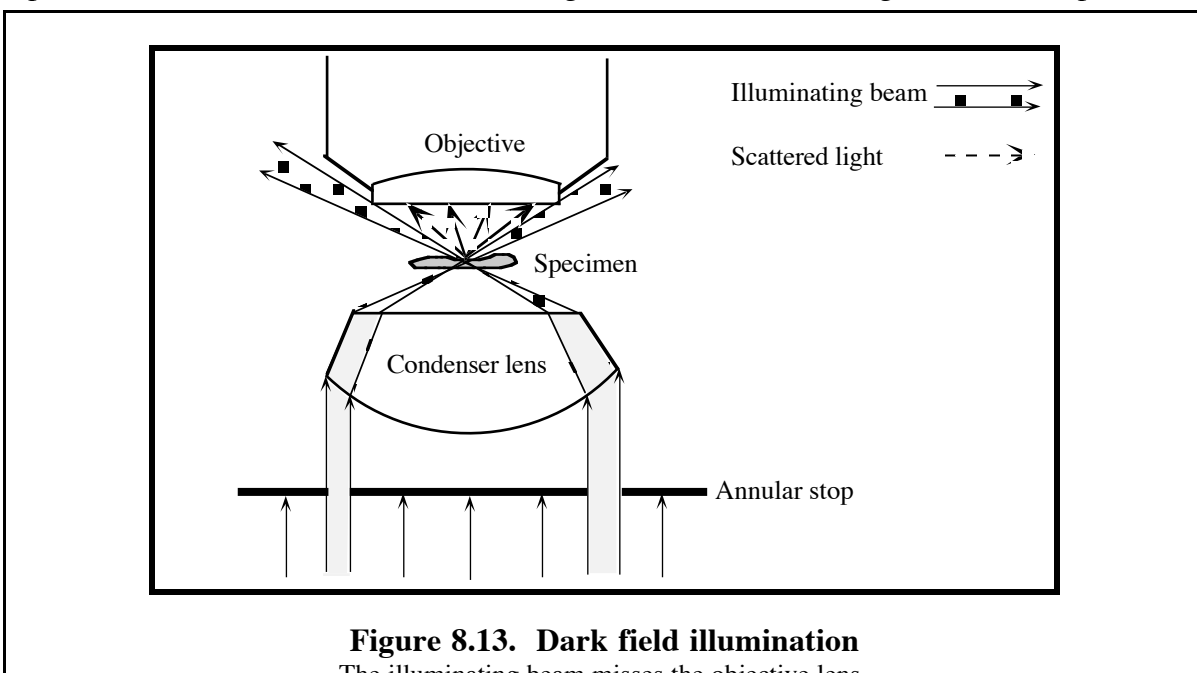
For low power objectives, (i.e. less than 10 $\times$ ) good illumination can also be made with a concave mirror reflecting the light from a convenient source into the microscope.



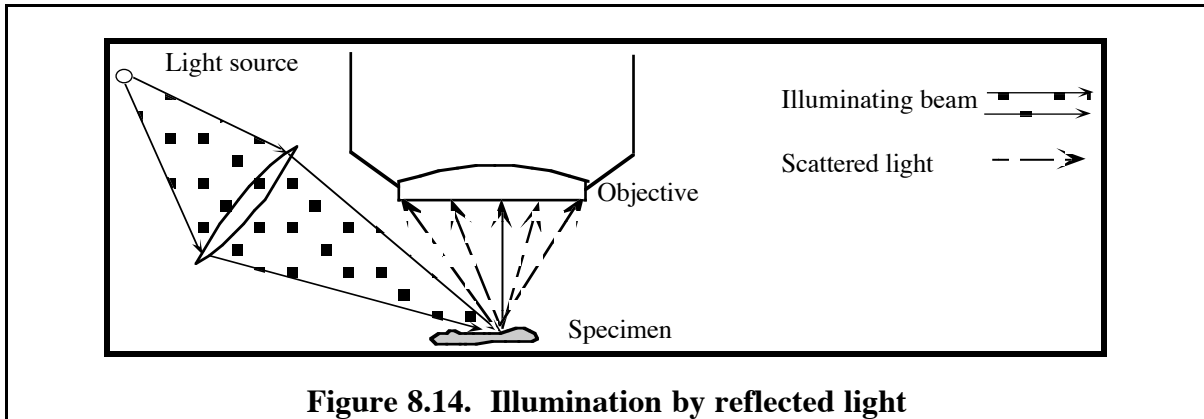
**Figure 8.12. Illumination by transmitted light using a mirror**

### *Dark field illumination*

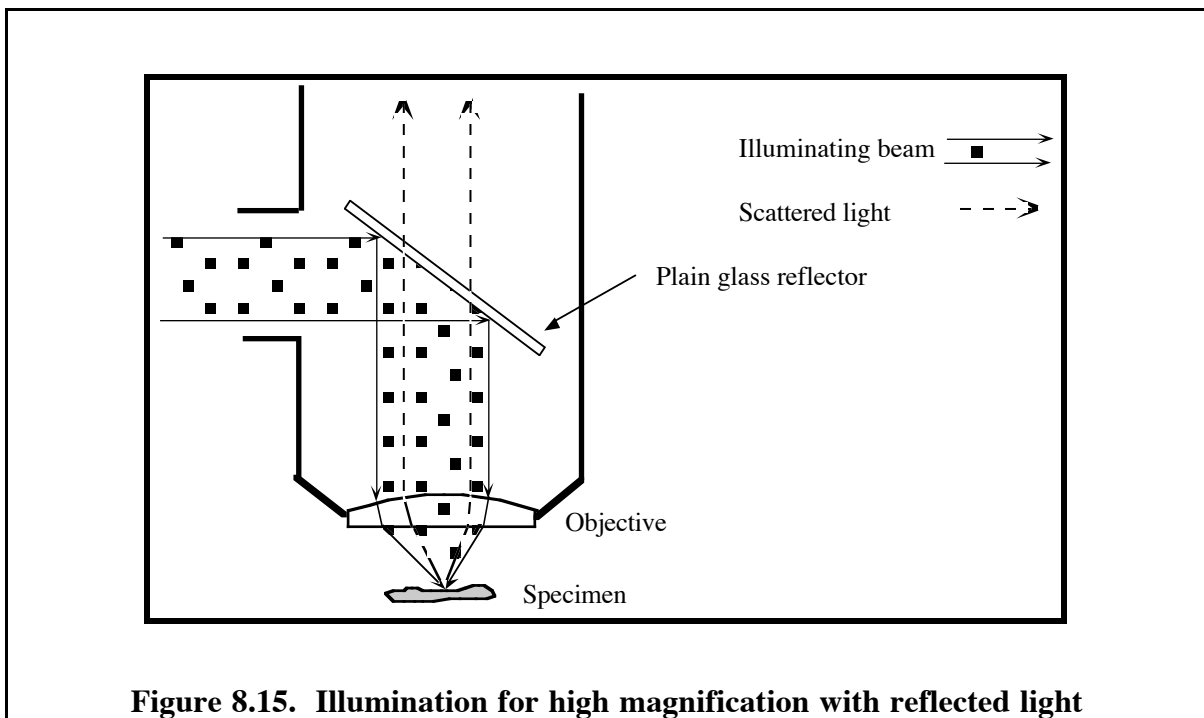
Sometimes objects can be more easily seen against a dark background. For example airborne dust becomes visible when viewed against a black cloth, by light scattered sideways out of a strong beam of sunlight. In microscopy this technique is called **dark field illumination** (figure 8.13). The condenser is equipped with a circular opaque stop so that none of the illuminating beam (shaded in the diagram) can enter the objective directly. The image is formed entirely by light scattered from the specimen, and none of the direct illuminating beam can be seen through the microscope.



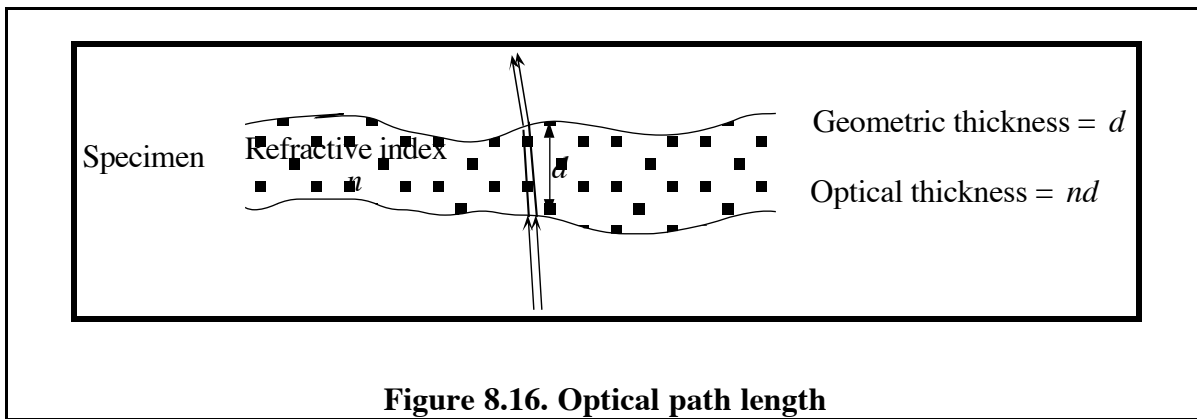
**Figure 8.13. Dark field illumination**  
The illuminating beam misses the objective lens.

***Illumination by reflected light***

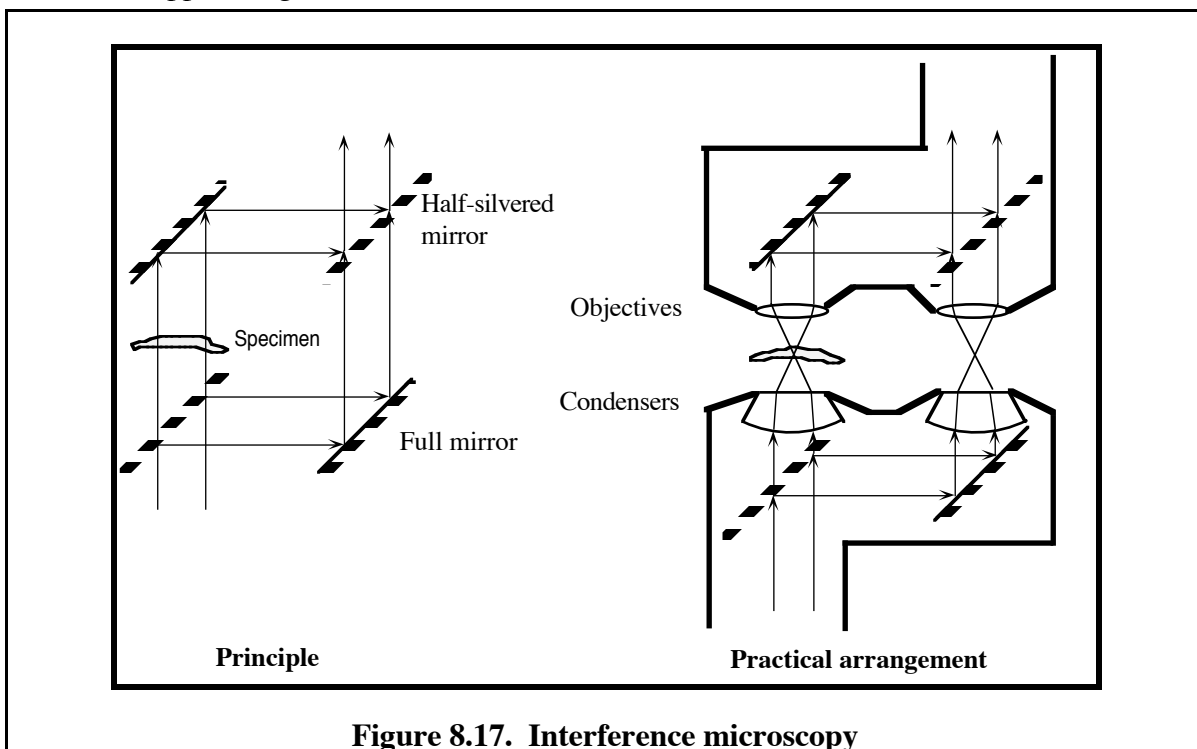
To view specimens in reflected light, illumination from above must be provided. For low power microscopes oblique top lighting may be sufficient (figure 8.14). For higher powers, more complicated systems, such as that illustrated in figure 8.15, are required.

**8-6 INTERFERENCE MICROSCOPY**

Normally when a uniform beam of light passes through a transparent material it emerges with equal intensity across the whole beam. That is true even if the refractive index has different values throughout the material or if the thickness of the specimen varies. However, different parts of the same beam of coherent light which travel through different optical path lengths will have different phases when they emerge from the specimen. Interference is a phenomenon which depends on phase relationships, so we can use interference, in what is called an **interference microscope** to "see" refractive index variations in transparent specimens of uniform thickness. See chapter L4.



In order to get coherence, a single beam of light is split into two parts, one of which goes through the specimen while the other does not. The phase of the light that has passed through a particular part of the specimen will be different from the phase of light that has taken the other path. When the two parts of the beam are recombined interference will occur. Some regions of the specimen will appear bright, others dark.



## QUESTIONS

- Q8.2** What is the angular magnification of a small magnifier, focal length 5 cm, used with relaxed vision?
- Q8.3** A magnifier with a focal length of 25 mm is held close to the eye to examine a small object. As shown in §8-2, if the object is at the focal point of the lens, the image will be at infinity.
- Calculate the angular magnification in this case.  
Now if the object is just inside the focal length, you will still get an image on the far side of the lens but it will be closer.
  - Use the lens formula to calculate where the object must be for the image to be at 0.25 m.
  - Calculate the ratio,  $\frac{\text{image height}}{\text{object height}}$ , and hence the angular magnification in this situation.
- Q8.4** (a) A microscope is made up of an objective with a focal length of 16 mm, numerical aperture 0.25 and a 10× eyepiece. The optical tube length has the standard value, 160 mm. Calculate the angular magnification.

- (b) If the eyepiece restricts the diameter of the intermediate image to 15 mm, how big is the field of view of this microscope?

**Q8.5** Suppose that the objective in the previous question has a resolving power given by

$$R = \frac{0.6 \lambda}{\text{N.A.}} .$$

What is the finest detail that we could observe on the specimen? What angle does this detail subtend at the eye when it is viewed through the microscope? Does the resolution of the eye or the resolution of the objective determine the finest detail observable with this instrument?

### Discussion questions

- Q8.6** What is the angle subtended by a TV screen for comfortable viewing? Given that the TV image is made up of 625 lines, how does the resolution of the eye compare with the angular separation of the lines?
- Q8.8** Why is the magnification of a simple magnifier defined in terms of angles rather than the actual sizes of image and object?
- Q8.9** Spectacles are not used to magnify objects. What are they used for? Discuss.
- Q8.10** Photographers alter the apertures or f-numbers of their camera lenses. Why? What is the relation between aperture setting and exposure time?
- Q8.11** The real image formed on the retina of the eye is inverted. Why don't we see things upside down?