ANSWERS

CHAPTER PM1

1.1 10^{-3}

1.2 1.75×10^4 N

1.3 Consider a fixed amount of water of mass M

density at surface $\rho_1 = N/V_1$

density at depth $\rho_2 = N/V_2$

$$\text{Fractional change in density} = \frac{\rho_2 - \rho_1}{\rho_1}$$

$$= \frac{M/V_2 - M/V_1}{M/V_1}$$

$$= \frac{1/V_2 - 1/V_1}{1/V_1}$$

$$= \frac{V_1 - V_2}{V_2}$$

$$= \frac{\Delta V}{V}$$

$$\text{But } k \text{ (bulk modulus)} \equiv \frac{pV}{\Delta V}$$

$$\text{So fractional change in density} = \frac{p}{k}$$

$$= \frac{4 \times 10^5 \text{ Pa}}{2 \times 10^9 \text{ Pa}}$$

$$= 2 \times 10^{-4}$$

CHAPTER PM2

2.1 $6.25 \times 10^{-4}\text{m}^2$, $4 \times 10^{-4}\text{m}^2$, $4.8 \times 10^{-4}\text{m}^2$, $7.8 \times 10^{-4}\text{m}^2$.

The circle has the greatest area.

2.2 Work done = Force $\times$ distance (parallel to the force)

= $2Tl\,b$

The increased area (both sides) is $2l$ b. Writing this increase as ΔA,

Work done = $T\,\Delta A$

that is, it is proportional to the increase in area.

<< Note that a liquid film is not identical to a rubber sheet. For a rubber sheet the force F would not be constant but would be proportional to the amount of stretching. So for the rubber sheet,

$$\text{work done} = \int_a^b kx\,\text{d}x$$

$$= \frac{1}{2}k\,(b^2 - a^2).$$ >>

2.3 The formula gives h = 45 mm. Clearly this is too small by a factor of about 3. What it means, and this is borne out in more careful measurements, is that the effective radius of the "pores" between soil grains is about l/3 of the radius of the grains themselves.

2.4 Yes

2.5 (ii) Weight of needle $= \rho Vg$

$= 7.8 \times 10 \text{ m.s}^{3} \times 40 \times 10^{-3} \text{ m} \times (0.50 \times 10\text{m}^{-3})^{2} \times 9.8 \text{ m.s}^{-1}$

$= 0.76 \text{ mN}$

(iii) Vertical component

of force associated with surface tension

$= T\ l \cos\theta$

$= 7 \times 10^{-2} \text{ N.m}^{-2} \times 2(40.5 \times 10^{-3} \text{ m}) \times \cos\theta$

$= 5.7 \times 10^{-3} \cos\theta \text{ N}$

(iv) For equilibrium with the needle lying on the surface of the water, these two vertical forces balance, so

$7.6 \times 10^{-4} \text{ N} = 5.7 \times 10^{-3} \cos\theta$

This is so for $\theta = 82°$

(v) As the weight increases, θ decreases until two vertical forces balance for $\theta = 0$. Any further increase in the weight cannot be balanced by the force associated with surface tension and the needle sinks.

(vi) Weight of big needle = 760 N

For this needle the maximum ($\theta = 0$) value of force due to surface tension = 0.57 N. The needle sinks.

(vii) The weight of an object scales as L^3.

The maximum vertical force due to surface tension scales as L.

If for a given size, the two forces balance, then increasing L leads to a disproportionate increase in the weight and the needle will sink. (Decreasing L will increase the force due to surface tension disproportionately. The needle will not be pushed out of the water; rather the angle of contact will increase leading to a decrease in the vertical component of this force.)

CHAPTER PM3

3.1 (i) Assume ferries travel at ~ 5 m.s-1 and are about 20 m in length. Then:

$$R \sim \frac{5 \text{ m.s}^{-1} \times 20 \text{ m} \times 10^{3} \text{ kg.m}^{-3}}{10^{-3} \text{ Pa.s}}$$

$$\sim 10^{8}$$

(ii) Typical household pipes are about 30 mm in diameter and the water flows at round about 10 m.s^{-1} so

$$R \sim \frac{10 \text{ m.s}^{-1} \times 30 \times 10^{-3} \text{ m} \times 10^{3} \text{ kg.m}^{-3}}{10^{-3} \text{ Pa.s}}$$

$$\sim 10^{5}$$

(iii) For the circulatory system with the figures given

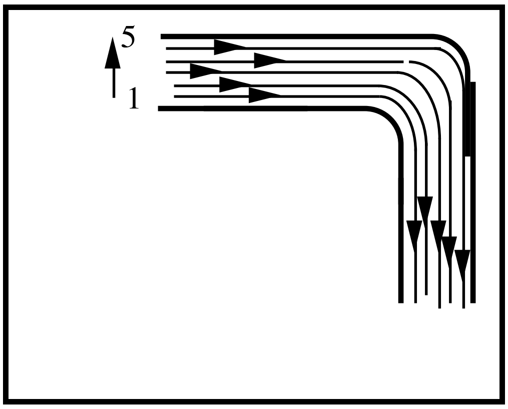
$$R \sim \frac{0.2 \text{ m.s}^{-1} \times 10 \times 10^{-3} \text{ m} \times 10^{3} \text{ kg.m}^{-3}}{10^{-3} \text{ Pa.s}} \sim 10^{3}$$

(iv) For spermatozoa

$$R \sim \frac{10^{-5} \text{ m.s}^{-1} \times 10^{-5} \text{ m} \times 10^{3} \text{ kg.m}^{-3}}{10^{-3} \text{ Pa.s}} \sim 10^{-4}$$

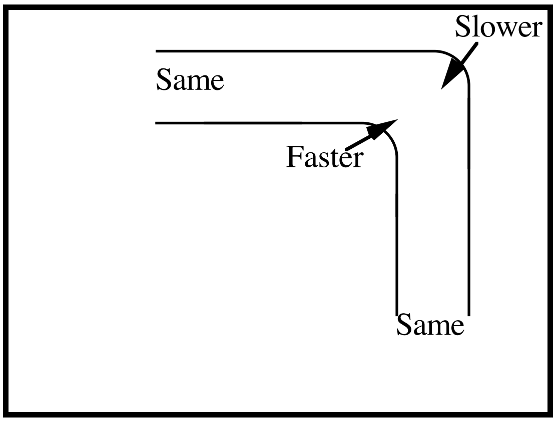
So in the first examples it would be almost impossible to keep the flow streamlined and so energy loss through turbulence must be a serious consideration. In (iv) the flow could not be anything but streamlined. In (iii) it is impossible to tell. In fact the tubes are of good enough geometrical structure that turbulence very rarely occurs, and so the heart is not called upon to supply energy which is dissipated through turbulence.

3.2



It can be observed that the streamlines hug the sharp corner. But far enough before the corner, and far enough after it, they are parallel and equally spaced.

Consider the liquid flowing between lines 1 and 2. Its cross-sectional area decreases near the corner, so the liquid speeds up there. The fluid between lines 3 and 4 has its area increased near the corner, so it slows down. So the flow velocity changes like this



3.3 The term associated with kinetic energy in Bernoulli's Equation is

$$\frac{1}{2}\rho v^2 \approx \frac{1}{2} \times (10^3\ \text{kg.m}^{-3}) \times (0.2\ \text{m.s}^{-1})^2$$

$$= 20\ \text{kg.m}^{-1}.\text{s}^{-2}$$

$$= 20\ \text{J.m}^{-3}$$

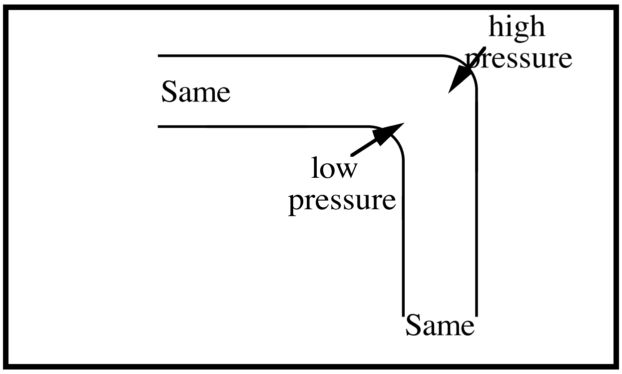
The pressure term which enters the equation is

$$p \approx 10^4\ \text{Pa}$$

$$= 10^4\ \text{J.m}^{-3}$$

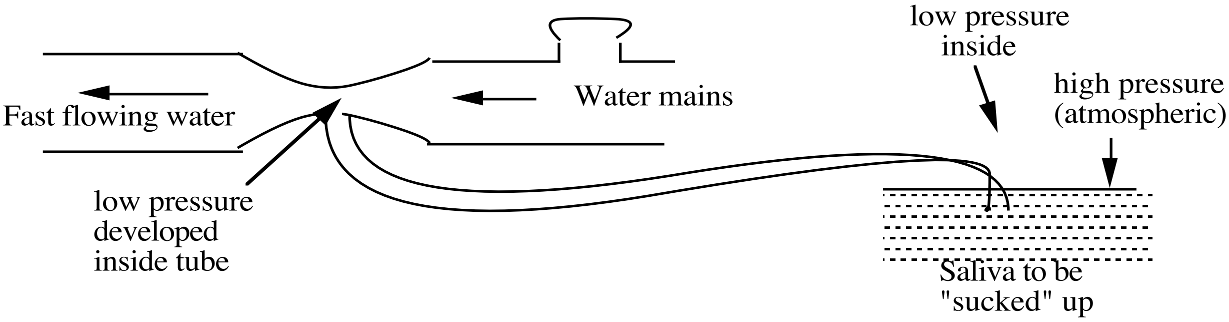
So the two terms p and ρgh are about equally important, but the kinetic energy term is a thousand times smaller, so there is no point in putting it in the equation.

3.4 Referring to answer 3.2 and applying Bernoulli's Law, the pressure in the liquid stream must be



Remembering that pressure can also be thought of as force per unit area, we see that the fluid must be exerting an extra large force on the tube right at the outside corner. This is obviously the same result of the diagram in chapter 3, 3-1, though it has a different physical "explanation".

3.5 The device is connected to a tap, as shown in the diagram. Water flows fast past the constriction, causing the pressure to drop inside the long tube. When the other end of this tube is immersed in a pool of saliva, the higher pressure outside forces the saliva up the tube, and away.



CHAPTER PM4

4.1 Turbulence means a loss of energy. The kinetic energy of the fluid (and hence its volume rate of flow) will be reduced as energy is dissipated in this way.

4.2 If you assume that the run off rain water through the down pipe is Poiseuillean, then you need to collect this information:

(i) the height of the house: this gives you the pressure head, and also length of the pipe;

(ii) the maximum recorded rainfall rate in that part of the city;

(iii) the area of the roof: this combined with (ii) will tell you the volume rate of flow that must be drained off in the worst probable circumstance.

Then plug these into Poiseuille's equation and the only unknown is the radius of the pipes.

However, the Reynolds number for this flow will obviously be something like 10^3 or 10^6 [see problem**3.1**] so the flow will be turbulent. Poiseuille's law will not be valid - and you would better get your builder to look up some more books, else you are likely to find your roof collapsing.

4.3 In rising through the "pores" in the soil the water must obey Poiseuille's Law - the flow is certainly slow enough to be streamlined. So the volume rate of flow should be 16 times as fast in the large grained soil (ratio of radii of soil grains was 2:1). However the velocity of rise would not be 16 times as fast because

(i) the area of the "pores" is down by a factor l/4 in the fine soil - and volume rate of flow = velocity × area

(ii) the effective pressure was down in the coarse grained soil - since it depends on surface tension.

All things considered, you would probably expect the rise to be somewhere around about twice as fast in the coarse soil.

4.4 Since the water is flowing at a steady rate, the pressure will reduce uniformly along the pipe.

Pressure at A (d = 0 mm)	=	5 Pa
Pressure at C (d = 1000 mm)	=	1 Pa
Pressure at B (d = 1000 mm)	=	4 Pa

4.5 (a) Resistance AB = R

Error!

Resistance of BC = $\frac{1}{2}R$

[addition of resistors in parallel]

therefore the pressure drop AB = $\frac{2}{3}$ of total drop

= 2 Pa

Error!

(b) Resistance AB = R

Resistance of one arm of BC = 16 R

[from Poiseuille's law]

Resistance of BC = 8 R

[resistors in parallel]

Pressure drop AB = $\frac{1}{9}$ of total drop

= $\frac{1}{3}$ Pa

Pressure at B = 3.7 Pa

We have already pointed out that any branching in the blood stream must be like case (b) rather than case (a) [see chapter PM3] and hence the same sort of result will apply. The resistance of the capillaries will be so much greater than the resistance of the rest of the system that by far the greater part of the pressure drop will occur there.

4.6 If the Reynolds number is too low, energy will be dissipated as a result of viscosity; if the Reynolds number is above about 2000, energy will be dissipated as a result of turbulence. A compromise between these two extremes is required.

Reynolds number for blood is somewhere between 1000 and 2000: a good compromise.

CHAPTER PM5

5.1 A couple of common examples of **plastic** materials are **butter** and **moulding clay.** The butter has to stay in place on the knife but must be spreadable. Moulding clay has to be mouldable but once moulded must retain its shape.

A less common example perhaps is a **starch paste** which is **dilatant**. If a finger is moved slowly through it little resistance is experienced but if the finger is pushed rapidly the paste hardens.

A variety of common foodstuffs show **visco-elastic** behaviour such as **honey**. If a thin vertically flowing stream of it breaks, the recoil of the thread of honey can be seen.

CHAPTER PM6

6.1 □ = tan□

6.2 This process is non-dissipative since the work done in moving the top surface upwards against the force of gravity is recovered when it moves down again.

6.3 Work is required to break the bonds holding the atoms together.

6.4 (a) Velocity gradient = $\frac{□r}{d}$

(b) Shear stress = $\frac{□r□□}{d}$

(c) Force = $\frac{2□r^2□□L}{d}$

CHAPTER PM7

7.1 Leave aside for the moment the shape and size of the ear drum, and the effect this has on impedance; and just consider the transmission of sound energy from a large body of air (or water) to a large body of skin tissue.

The density and bulk modulus of water are very much higher than air, and much closer to skin tissue. Hence

$$z_{skin} >> z_{air}$$

but: $$z_{skin} \sim z_{water}$$

Hence there is not such a bad mismatch between eardrum and water as between eardrum and air. The fraction of energy transmitted from a weak sound wave will be much greater at a water/eardrum boundary, than at an air/eardrum boundary.

7.2 Assuming the general structure of the ear is the same we would expect that the upper frequency would vary as 1/(**scale size**) simply from geometric considerations. Therefore, something the size of a mouse or a bat (400 times smaller than a man), having the right sort of ears, should have no trouble hearing 8000 kHz sounds.

CHAPTER PM8

8.1 So as to get good spatial resolution. The resolution is comparable with the wavelength which at 50 kHz is approximately 7 mm. Higher frequencies still cannot be used as the attenuation of the wave becomes too great (the attenuation increases as the square of the frequency).

8.2 Use the acoustic impedances given in the table taking that for "flesh" as "fat".

% power reflected for water-flesh boundary	=	0.2%
% power reflected for air-fresh boundary	=	99.8%