ANSWERS TO REVIEW QUESTIONS

IMPORTANT NOTE

In many cases the notes below are merely indications of the main points of a suitable answer. In an ideal exam answer, more detailed descriptions, explanations and arguments are required.

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1 (i) Stress $= \dfrac{\text{Load}}{\text{Area}}$

$= \dfrac{40\ \text{kg} \times 10\ \text{m.s}^{-2}}{5.0 \times 10^{-5}\ \text{m}^{3}}$

$= 8.0 \times 10^{6}$ Pa

Strain $= \dfrac{\text{Stress}}{\text{Young's Modulus}}$

$= \dfrac{8.0 \times 10^{6}\ \text{Pa}}{2.0 \times 10^{11}\ \text{Pa}} = 4.0 \times 10^{-5}$

(ii) Increase in length = Strain × length

= 20 μm

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2 (a) Tensile stress is equal to the tensile force divided by the cross-sectional area at right angles to the force. Tensile strain is equal to the ratio of the extension to the original length.

(b) See (more complicated) diagram page 5 and description page 5.

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3 (a) An elastic moduli is a ratio of stress to strain; elastic limit is the maximum stress that exists before a material flows plastically; strength is the maximum stress that exists before a material fractures.

(b) The strength of brick is the same, no matter which way it is placed. Maximum force will be associated with maximum cross-sectional area (80 mm by 200 mm).

So F_{max} = A_{max} × strength

$= (80 \times 10^{-3}\ \text{m} \times 200 \times 10^{-3}) \times 4.0 \times 10^{7}$ Pa

$= 6.4 \times 10^{5}$ N

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4 (i) F_{max} = A_{max} × strength

$= \pi(15 \times 10^{-3})^{2} \times 1.7 \times 10^{8}$ Pa

$= 1.2 \times 10^{5}$ N

(ii) Max. length change = Strain × length

Strain $= \dfrac{\text{Stress}}{\text{modulus}}$

$= \dfrac{\text{Tensile strength}}{\text{Young's modulus}}$

Max. length change $= \dfrac{1.7 \times 10^{8}\ \text{Pa} \times 0.50\ \text{m}}{9.4 \times 10^{9}\ \text{Pa}}$

= 9.0 mm

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5 (i) Young's modulus

(ii) Modulus $= \dfrac{\text{Stress}}{\text{Strain}}$

$= \dfrac{10\ \text{kg} \times 10\ \text{m.s}^{-2}}{10.0 \times 10^{-3}\ \text{m} \times 1.00 \times 10^{-3}\ \text{m}} \times \dfrac{0.20\ \text{m}}{0.01 \times 10^{-3}\ \text{m}}$

$= 2.0 \times 10^{11}$ Pa.

6 Stress $= \dfrac{5.0 \times 10^{5}\ \text{N}}{1.0 \times 10^{-10}\ \text{m}^2}$

Strain $= \dfrac{\Delta l}{l}$

$= 1.00000$

Young's modulus $= \dfrac{\text{Stress}}{\text{Strain}}$

$= \dfrac{5.0 \times 10^{5}\ \text{N}}{1.0 \times 10^{-10}\ \text{m}^2}$

$= 5.0 \times 10^{5}$ Pa

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7 Consider the 1.025×10^{3} kg of sea water.

At the surface where the density is 1.025×10^{3} kg.m^{-3}, the volume occupied by this amount is 1.000 m^{3}. The pressure at this level is 1.01×10^{5} Pa.

Now calculate the volume of this amount of water where the pressure is 5.0×10^{7} Pa.

Strain $= \dfrac{\text{Stress}}{\text{Bulk modulus}} = \dfrac{5.0 \times 10^{7}\ \text{Pa}}{2.2 \times 10^{3}}$

$= 2.3 \times 10^{-2}$

$\Delta V = 1.00\ \text{m}^3 \times 2.3 \times 10^{-2}$

$= 2.3 \times 10^{-2}\ \text{m}^3$

Error!

$\rho = 1.05 \times 10^{3}$ kg.m^{-3}

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8 Left to itself a liquid will assume a shape that minimises the surface area and hence the surface energy.

If the film within the loop of thread is punctured, then this loop takes up the largest possible area (a circle) to minimise the total film area within the wire loop.

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9 (a) See diagram page 18.

(b) $L = \dfrac{2T}{\rho g r}$

$= \dfrac{2 \times [illegible] \times 10^{-1}\ \text{N.m}^{-1}}{1.0 \times 10^{3}\ \text{kg.m}^{-3} \times 10\ \text{m.s}^{-2} \times [illegible] \times 10^{-5}\ \text{m}}$

$= 2$ m.

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10 See page 17.

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11 (i) On expansion the surface area of the bubble will increase; work must be done to increase this area.

(ii) $W = \Delta U$

= Surface tension (new area - old area)

$= 0.07\ \text{J.m}^{-2}\ 4\pi[(1.0 \times 10^{-3}\ \text{m})^2 - (0.5 \times 10^{-3}\ \text{m})^2]$

$= 0.7\ \mu$J

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12 (i) F = Surface tension × length of contact

$= 0.07\ \text{N.m}^{-1} \times 42 \times 10^{-3}\ \text{m}$

= 3 mN

(ii) Downwards

(iii) It will be reduced.

13 (a) See page 29

(b) The rate of flow of water through the original vessel equals the total rate of flow of water through the five vessels.

Rate of flow is given by multiplying the flow velocity by the cross-sectional area.

So, $\pi r^2 v \quad = \quad 5\pi\left(\frac{r}{4}\right)^2 v'$ (v' velocity in the small vessels)

$v' \quad = \quad \frac{16}{5} v$

$\quad = \quad 3.2\ v$

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14 (i) Turbulence will occur when

$R \quad \geq \quad 2000$

$R \quad = \quad \frac{vL\rho}{\eta}$

$v_{crit} \quad = \quad \frac{2000 \times 1 \times 10^{-3}\ \text{Pa.s}}{10 \times 10^{-3}\ \text{m} \times 1 \times 10^{3}\ \text{kg.m}^{-3}}$

$\quad = \quad 0.2\ \text{m.s}^{-1}$

(ii) Pipes B and C have the same pressure drop along them, have the same length but pipe C has a radius $\frac{1}{2}$ that of pipe B.

So, from Poiseuille's equation, the flow rate in pipe C will be $\left(\frac{1}{2}\right)^4$ that of pipe C.

(iii) If Q is the volume rate of flow

$Q_A \quad = \quad Q_B + Q_C$

$Q_B \quad = \quad 16 Q_C$

So $Q_A \quad = \quad 17 Q_C$

and hence $v_A A_A \quad = \quad 17 v_C A_C$

$v_C \quad = \quad \frac{0.01\ \text{m.s}^{-1} \times \pi \times (5 \times 10^{-3}\ \text{m})^2}{17 \times \pi \times (2.5 \times 10^{-3}\ \text{m})^2}$

$\quad = \quad 2\ \text{mm.s}^{-1}$

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15 $R \quad = \quad \frac{vL\rho}{\eta}$

We want $R_{model} \quad = \quad R_{real}$

$\frac{Lv}{R} \quad = \quad \frac{5.0 \times 10^{-3}\ \text{m} \times 20 \times 10^{-3}\ \text{m.s}^{-1}}{10^{-3}}$

$\quad = \quad 10^{-1}\ \text{m}^2.\text{s}^{-1}$

Of the liquids suggested, sucrose would be the most suitable.

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16 (a) See page 32.

(b) If turbulence is present, not all the work done by the force associated with the pressure difference appears as kinetic energy of the flow.

17 (ii) $v_B = \dfrac{A_A}{A_B} v_A$

$= \dfrac{(10\text{mm})^2}{(5.0\text{mm})^2} \times 1.0 \times 10^{-2}\ \text{m.s}^{-2}$

$= 4.0 \times 10^{-2}\ \text{m.s}^{-1}$.

(iii) Turbulence is more likely to occur near B then near A. [Reynolds number is inversely proportional to scale length.]

(iv) $R = \dfrac{vL\rho}{\eta}$

So $v = \dfrac{2000 \times (1.0 \times 10^{-3}\ \text{Pa.s})}{10^3\ \text{kg.m}^{-3} \times (10 \times 10^{-3}\ \text{m})}$

$= 0.2\ \text{m.s}^{-1}$

This estimate uses a critical Reynolds number of 2000 and a characteristic length equal to the tube diameter at B.

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18 (i) The pressure is higher where the velocity is lower; the velocity is lower where the cross-sectional area is higher.

So the pressure is higher at A.

(ii) (a) Conservation of energy : the work done by the force associated with the pressure difference appears as an increase in the kinetic energy of the fluid.

(b) Conservation of fluid : the mass rate of flow (and hence, for an incompressible fluid, the volume rate of flow) must be constant.

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19 (i) Q (flow rate) $= vA$

$v = \dfrac{8.3 \times 10^{-5}\ \text{m}^3.\text{s}^{-1}}{\pi(9 \times 10^{-3}\ \text{m})^2}$

$= 0.33\ \text{m.s}^{-1}$

(ii) $R = \dfrac{\rho L v}{\eta}$

$= \dfrac{10^3\ \text{kg.m}^{-3} \times (18 \times 10^{-3}\ \text{m}) \times (0.33\ \text{m.s}^{-1})}{4 \times 10^{-3}\ \text{Pa.s}}$

$= 1 \times 10^3$

This is < 2000, so the flow is probably non-turbulent.

(Onset of turbulence is at a Reynolds number of about 2000.)

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20 See page 39.

21 Across each of the n holes in the worm's membrane there is the same pressure difference Δp.

Each hold carries $\frac{1}{n}$ of the total volume flow rate of water.

The number of holes, n, is given by

$$\frac{1}{2} \times \frac{2 \times 10^{-3}\ \text{m}^2}{10^{-12}\ \text{m}^2}$$

i.e. $n = 10^9$.

<u>For any one hole</u>

$$Q = \frac{1.2 \times 10^{-3}\ \text{m}^3.\text{h}^{-1}}{3600\ \text{s.h}^{-1} \times 10^9}$$

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{10^{-12}\ \text{m}^2}{\pi}}$$

$l = 10 \times 10^{-6}$ m (the thickness of the membrane)

$\eta = 10^{-3}$ Pa.s

So
$$\Delta p = \frac{1.2 \times 10^{-3}\ \text{m.s}^{-1} \times 10^{-5}\ \text{m} \times 10^{-3}\ \text{Pa.s} \times 8}{3600 \times 10^9 \times \pi \times \left(\frac{10^{-12}\ \text{m}^2}{\pi}\right)^2}$$

$= 84$ Pa

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22 $$Q = \frac{50 \times 10^{-6}\ \text{m}}{t}$$

Also, from Poiseuille's equation

$$Q = \frac{\pi \times (0.3 \times 10^{-3}\ \text{m})^4 \times 1.2 \times 10^4\ \text{Pa}}{8 \times 80 \times 10^{-3}\ \text{m} \times 4 \times 10^{-3}\ \text{Pa.s}}$$

So $t = 4.2 \times 10^2$ s

$= 7.0$ min

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23 (a) Q is proportional to r^4.

If r decreases from r_1 to $0.8\ r_1$, Q decreases from Q_0 to $(0.8)^4 Q_0$.

To counteract this (raise Q to Q_0) the pressure difference must increase from p_0 to $\frac{p_0}{(0.8)^4}$

i.e. to $2.44\ p_0$

Percentage increase of 144% needed.

(b) We are assuming there is no turbulence, that is, that the Reynolds number is less than 2000

$$R = \frac{vL\rho}{\eta} = \frac{1.0\ \text{m.s}^{-1} \times 100 \times 10^{-3}\ \text{m} \times 10^3\ \text{kg.m}^{-3}}{10^{-3}\ \text{Pa.s}} = 10^5$$

So flow is definitely turbulent, and a calculation such as in (a) is inappropriate.

24 The pressure drop is the same across every capillary (because they act in parallel to convey blood).

The rate of flow through each of the 10^{10} capillaries is 10^{-10} of the total rate of flow.

So $\Delta p = \dfrac{8Ql\eta}{\pi r^4}$

$$= \frac{8 \times (8 \times 10^{-5}\,\mathrm{m^3.s^{-1}} \times 10^{-10}) \times 2 \times 10^{-3}\,\mathrm{m} \times 4 \times 10^{-3}\,\mathrm{Pa.s}}{\pi \times (4 \times 10^{-6}\,\mathrm{m})^4}$$

$= 0.6$ kPa

25

Pressure difference	:	increase
radius	:	increase
length	:	decrease
viscosity	:	decrease

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26 See pages 48, 49, 50, 51.

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27 See pages 48, 49, 50, 51.

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28 (a)

Solid	:	deformation (strain) occurs
Liquid	:	flow (a continuous shear) occurs
Visco-elastic material	:	a mixture of solid and liquid behaviour

(b) See page 54.

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29 (a) See page 3.

(b) Modulus (constant) = shear stress/shear strain

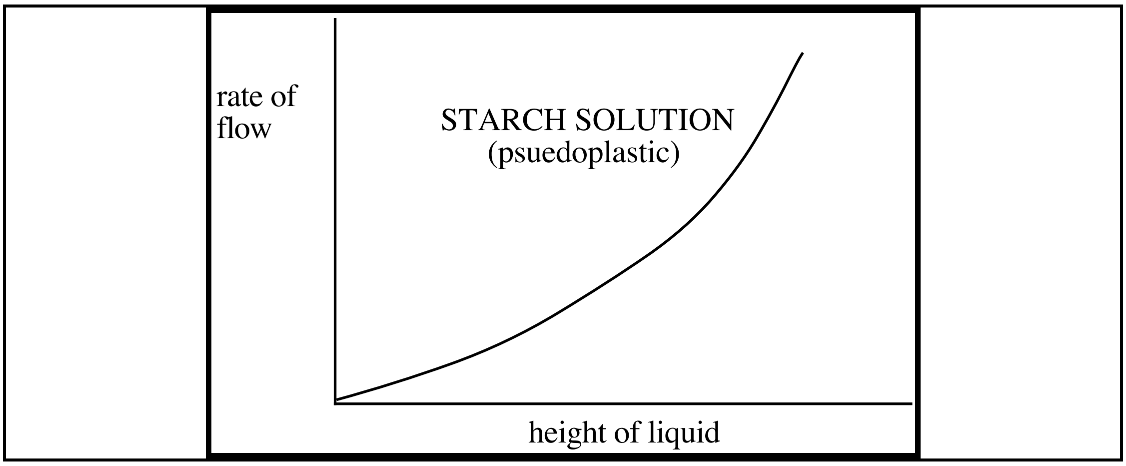
(c) shear stress $\propto$ time rate of change of shear deformation

(d) stress = coefficient of viscosity $\times$ rate of shear deformation
or
stress = coefficient of viscosity $\times$ velocity gradient

(e) For a Newtonian liquid, η is constant as the shear rate changes.
For a dilatant liquid, η increases as the shear rate increases
For a pseudoplastic liquid, η decreases as the shear rate increases

(f) See page 50.

30 See pages 49 and 50.



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31 The paint becomes easier to stir as the rate of stirring is increased. Or, more technically, the viscosity decreases as shear rate increases.

This means paint is a non-newtonian fluid (pseudoplastic).

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32 (a) See page 58.

(b) Since the surfaces are rough, they will touch only at a few points. Thus the real area of contact is much less than the apparent area of contact.

(c) As the load is increased, plastic flow will occur at the contact points and thus the real area of contact will increase.

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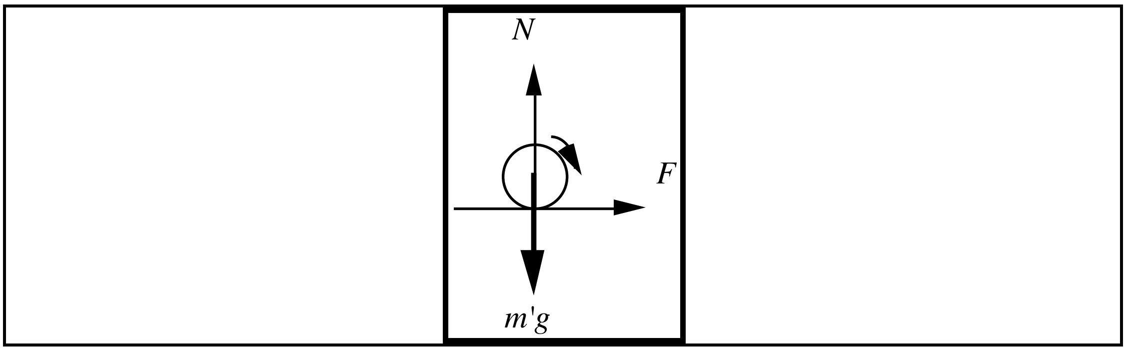
33 See pages 62 and 63.

34



$F = \mu N,$
$= \mu m'g,$

where m' = 0.2 of total car mass, M.

This frictional force acts on both back wheels so $2\mu m'g = Ma$

$a = 2\mu 0.2g,$
$= 2.0 \times 0.8 \times 0.2 \times 10 \text{ m.s}^{-2},$
$= 3.2 \text{ m.s}^{-2}.$

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35 (a) See pages 59 and 60.

(b) The oil separates the two surfaces; relative motion is now governed by viscous forces which are usually less than surface-to-surface friction forces.

36 After $F > 4$ kN, the frictional force is its maximum value of 4 kN and the sled accelerates with an acceleration given by net force (F - 4 kN) divided by mass (1000 kg).

(ii) Frictional force $= \mu N$

$= \mu mg$

and also $=$ 4 kN

so $\mu = \dfrac{4 \times 10^3 \text{ N}}{1000 \text{ kg} \times 10 \text{ m.s}^{-2}}$, $= 0.4$.

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37 (i) The frictional force, f, is proportional to the weight of the barrel.
So the applied force, F, must be at least f, that is μmg, to move the barrel.

(ii) The frictional force (and hence the necessary applied force to move the barrel) does not depend on the apparent area of contact.

If the barrel is rolled, the necessary propulsive force is less than if the barrel is dragged. In the case of rolling the point of contact is not moving relative to the floor.

38

$$z = \sqrt{k\rho};$$

$$z = \rho c;$$

$$c = \sqrt{\frac{k}{\rho}}.$$

$$z_{iron} = \sqrt{17 \times 10^{10} \text{ Pa} \times 7.9 \times 10^{3} \text{ kg.m}^{-3}}$$

The other material with a similar product of bulk modulus and density is tin.

$$z_{tin} = \sqrt{7.3 \times 10^{10} \text{ Pa} \times 19.6 \times 10^{3} \text{ kg.m}^{-3}}$$

$$c_{tin} = \sqrt{\frac{7.3 \times 10^{10} \text{ Pa}}{19.6 \times 10^{3} \text{ kg.m}^{-3}}} = 1.9 \times 10^3 \text{ m.s}^{-1}$$

$$c_{iron} = \sqrt{\frac{17 \times 10^{10} \text{ Pa}}{7.9 \times 10^{3} \text{ kg.m}^{-3}}} = 4.6 \times 10^3 \text{ m.s}^{-1}$$

The speed of sound is 2.4 times faster in iron than in tin.

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39 (a) See pages 78 and 79.

(b) One would expect the midline to be midway between the right and left side of the skull.
i.e. that reflections would be obtained after a time 131 μs.
So the echo comes 3 ms early, indicating that the midline is shifted towards the right by a distance Δx given by $\frac{1}{2} v\Delta t$, where v is the speed of sound, Δt is 3 μs and the factor of $\frac{1}{2}$ arises from the fact that the signal travels to the reflection region and back.

So $\Delta x = \frac{1}{2} \times 1540 \text{ m.s}^{-1} \times 3 \times 10^{-6} \text{ s}$

$= 2.3$ mm

(ii) As indicated above, the left-hand hemisphere of the brain is enlarged.

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40 (a) (i) amplitude
(ii) frequency
(iii) number and relative amplitude of harmonies

(b) See page 71 and 74.

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41 (i) See pages 78 and 79 (echoscopy).
(ii) See pages 77 and 78 (Doppler technique)

42 (a) Specific acoustic impedance $= \sqrt{\text{bulk modulus} \times \text{density}}$

$$z = \sqrt{k\rho}$$

(b) The sound power reflection coefficient is given by

$$a_r = \left[\frac{z_2 - z_1}{z_2 + z_1}\right]^2$$

so, if the specific acoustic impedances are equal, all the sound is transmitted; while the more dissimilar the specific acoustic impedances, the more sound is reflected.

(c) See pages 78 and 79

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43 See page 74.

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44 (i) Suppose the depth of the object is d

then
$$c = \frac{2d}{t}$$

$$d = \frac{1}{2} \times 1.5 \times 10^3 \text{ m.s}^{-1} \times 60 \times 10^{-6} \text{ s}$$

$$= 45 \text{ mm.}$$

(ii) The object must be about the size of the wavelength to be able to reflect the sound wave.

$$l = \lambda$$

$$= \frac{c}{f}$$

$$= \frac{1.5 \times 10^3 \text{ m.s}^{-1}}{10 \times 10^6 \text{ s}^{-1}}$$

$$= 0.15 \text{ mm}$$