PM3

HYDRODYNAMICS

Some fish are minnows
Some are whales.
People like dimples.
Fish like scales.
Some fish are slim,
And some are round.
They don't get cold,
They don't get drowned
But every fish wife
Fears for her fish.
What we call mermaids
and they call merfish.

OBJECTIVES

Aims

In this chapter you will look at the behaviour of fluids in motion and the explanation of that behaviour both in terms of forces, energy and the continuity of the fluid. The distinction between smooth and turbulent flow is investigated.

Minimum Learning Goals

When you have finished studying this chapter you should be able to do all of the following.

1. Explain, interpret and use the terms:

   thrust force, lift force, streamline, turbulence.

2. (i) Explain why the description of mutual forces between a moving fluid and a stationary object is identical to that for a stationary fluid and a moving object.

   (ii) Draw a diagram showing the origin of thrust and lift forces in such situations.

   (iii) Explain why it is preferable in discussing liquid flow, to consider the liquid as a continuous substance rather than individual molecules.

3. Describe how energy is dissipated in turbulent motion.

4. (i) Recall the definition of Reynolds number

   $$R = \frac{vL\rho}{\eta}$$

   and state how L is determined in different situations.

   (ii) Recall that it is experimentally found that turbulent flow occurs if $R \geq 2000$.

   (iii) Do simple calculations and interpretations involving Reynolds number.

5. (i) Explain how, for streamline motion in a tube (or channel) of variable cross-section, the flow speed depends on the cross-sectional area. (Equation of continuity.)

   (ii) Give a quantitative description of the branching effect at pipe junctions.

   (iii) Explain why flow speed must increase where streamlines are crowded together.

6. (i) Use an energy argument to explain why, for constricted streamline flow, the fluid pressure decreases as the flow speed increases. (Bernoulli's Principle.)

   (ii) Describe and explain the following phenomena : the venturi effect, the chimney effect, the working of an atomiser.

PRE-LECTURE

Recall the following background information from earlier chapters, particularly chapters FE3, FE4 and FE5.

(i) All fluids (liquids and gases) exert a pressure on the walls of any container which contains them - pressure being defined as force per unit area. This same pressure is exerted by each part of the fluid on neighbouring parts. The **kinetic theory** is a school of thought which seeks to understand how this pressure arises through the collision of individual molecules of the fluid with the walls and with each other. We will not in fact pursue kinetic theory any further - but concentrate on **experimentally** observable laws concerning fluid pressure and its effects.

(ii) The simply established laws concerning fluid pressure are these:

(a) The pressure at any point in a fluid is the same in all directions (Pascal's Principle).

(b) The pressure within a fluid can vary from point to point; in a fluid **at rest** the pressure varies with vertical height according to the law.

$$p = \text{constant} + \rho g h.$$

It will be the concern of this lecture to establish how the pressure varies inside a fluid which is **in motion**.

(iii) In general, mechanical forces can be classified as either dissipative or conservative forces, according to whether or not they result in the dissipation of energy (usually as conversion into thermal energy). Typically forces such as electromagnetic or gravitational are conservative and frictional forces are dissipative.

Workers in hydrodynamics (or aerodynamics) try to classify pressures and fluid forces similarly. However since the origin of these effects has a more complicated microscopic explanation, this classification is not always so straightforward. The basic criterion employed is whether or not the **equation of conservation of mechanical energy is obeyed**.

LECTURE

3-1 THRUST AND LIFT FORCES

The study of hydrodynamics involves the study of the interaction of fluids and solid bodies. Three apparently different kinds of interaction can be distinguished:

(a) moving fluids with stationary objects

(b) stationary fluids with moving objects

and (c) moving fluids with moving objects

From work you have done already you can understand in a general way where the forces of interaction come from.

(a) A moving fluid exerts a force on a stationary object because each molecule of the fluid, on bouncing, is accelerated by the solid. The solid exerts a force on the fluid.

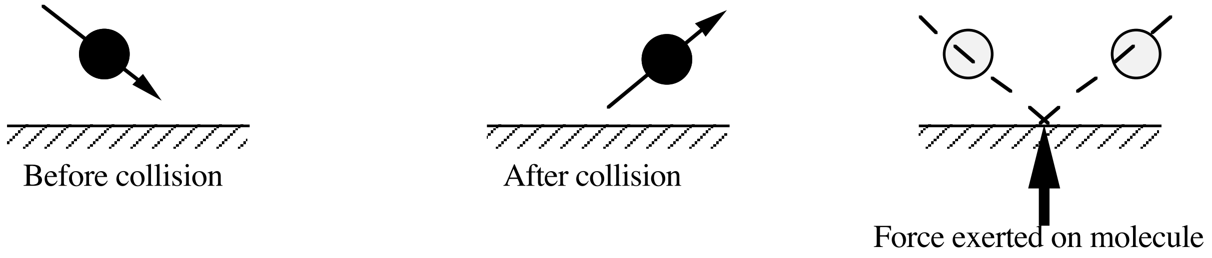

Figure 3.1 Collision of fluid molecules with a solid surface

and, by the fact that forces occur in pairs, the fluid exerts an equal and oppositely directed force on the solid.

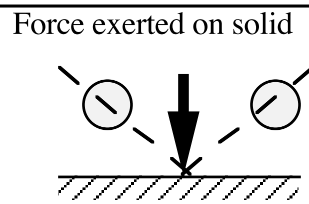

Figure 3.2 Force exerted by fluid molecules on solid surface

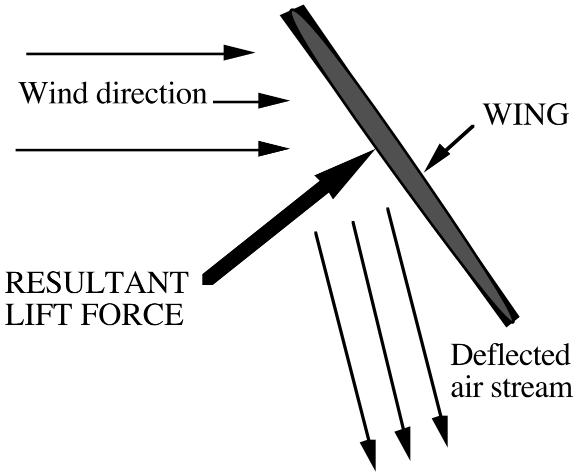

Figure 3.3 Lift force exerted by horizontal wind on an inclined wing

Examples: Hovering birds, gliders, kites.

(b) A moving solid exerts a force on a stationary fluid by exactly the same mechanism, by giving a velocity to (i.e. accelerating) each molecule of the fluid.

Example : Fish tails.

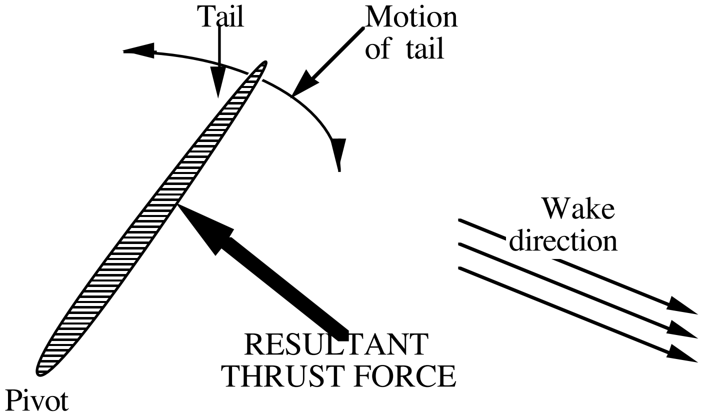

Figure 3.4 Thrust force on flipping fish tail

(This example is in fact too complicated to worry about too much for now; suffice it to say that the backward and forward motion of the tail results in an **average** forward thrust.)

From all this we want to draw two simple conclusions:

(1) This way of analysing things is too simplistic. Yet the main conclusion is correct: if you want to move up through a fluid, you must push the fluid down; if you want to move forward, you must push the fluid backwards.

(2) The physics of what happens is the same whether it is the fluid or the solid or both which is moving.

Application

Aeronautical engineers can predict how an aeroplane will behave in flight by observing it at rest in a wind tunnel (or even in a water tank).

3-2 STREAM LINES AND TURBULENCE

The preceding analysis is obviously too simplistic as can be seen from a very easy observation. When a stream of (gently) flowing fluid is diverted by the presence of a wall, the particles of fluid do **not** all bounce off the wall, most bounce off other fluid particles.

Demonstration

Glycerine solution flowing in a flow tank.

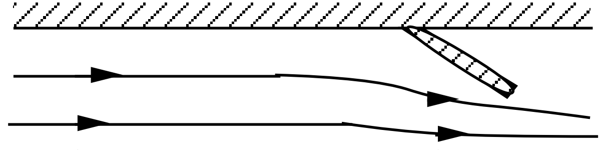

Figure 3.5 Streamlines for fluid flowing past a solid obstacle

Since the stream is diverted (accelerated) the wall must be exerting a force on the fluid, and the fluid on the wall. The origin of this force must be that the fluid molecules bounce off one another, causing those next to the wall to bounce off it more violently. That means the fluid **pressure** must increase near the corner. More of this later.

Whatever the means whereby force is exerted on the wall, it is clear that for some parts of their motion particles of the fluid do not travel in straight lines but in curved paths.

It turns out that it is more helpful in describing fluid flow to think of the fluid as a **continuous** substance rather than to concentrate on the motion of individual molecules. Particles of this continuous fluid can be considered to travel along these smooth continuous paths which are given the name **streamlines**. These stream lines can of course be **curved** or **straigh**t, depending on the flow of the fluid.

This continuous substance can be regarded as being made up of bundles or tubes of streamlines. The tubes have elastic properties:

(a) A tensile strength, which means that the parts of the fluid along a particular streamline stick together and do not separate from one another,

(b) zero shear modulus, which means that each streamline moves independently of any other.

Streamline motion is not the only possible kind of fluid motion. When the motion becomes too violent, eddies and vortices occur. The motion becomes **turbulent**.

Demonstrations

Wakes of boats

Liquid tank demonstration.

Turbulence is important because it is a means whereby energy gets dissipated.

When a body is moved through a stationary fluid in streamline motion some kinetic energy is given to the fluid, but only temporarily. When the body has passed, the fluid is still again; no net energy has been given to it.

But when turbulence is established, a net amount of kinetic energy is left in the fluid after the body has passed.

Application

This is very important in aeronautical engineering. Air turbulence means increased fuel consumption in aircraft, and many cunning and intricate devices are used to reduce turbulence.

The shape of a body will, to some extent, decide whether it will move through a fluid in streamline or turbulent motion.

Demonstration

Shapes of marine animals, specially shaped corks.

3-3 REYNOLDS NUMBER

What factors determine whether a fluid will flow in streamlined or in turbulent motion? You could guess some of these more or less easily.

(i) Speed of flow - faster flow gets turbulent more easily.

(ii) Stickiness of fluid - thick, sticky liquids like glycerine become turbulent less easily than thin liquids like water. [Just what physical quantity is involved here is not obvious. It is called the **kinematic viscosity** and we cannot say anything about it till next lecture. The symbol for it is □/□ (see post lecture).]

(iii) A more unexpected result which turns up is that the size of the system is important.

For water flowing at the same speed through narrow pipes, the flow becomes turbulent more easily in the tube of larger radius.

More thorough experimental investigation will collect all these results thus. We define for any system a number R, called the **Reynolds number**

$$R \equiv \frac{vL\square}{\square}$$

where v is a typical flow speed of the fluid, L is a typical length scale and □/□ the kinematic viscosity of the fluid.

Then it is found experimentally that if this number is not too large (smaller than about 2000) the motion will be streamline; whereas if $R \geq 2000$ then turbulence can set in.

There is no theoretical explanation of this value of 2000, it is just found to be the case.

3-4 THE EQUATION OF CONTINUITY

For fluids which are flowing in streamlined motion, what laws do they obey? Firstly there is the so called equation of continuity:

for an incompressible fluid moving in streamline motion in a tube of variable cross-section, the flow speed at any point in inversely proportional to the cross sectional area

$$\text{Speed} \quad \frac{1}{\text{area}} \cdot$$

The reason behind this is very easy to grasp. If you want a more rigorous statement, see the post-lecture material.

One sees many applications of this. Four examples follow.

Demonstrations

(i) In flowing rivers, when going from deep to shallow, the flow speed increases (often becoming turbulent).

(ii) In the circulatory system of the blood there is a **branching effect**.

When a fluid flows past a Y-junction made up of pipes of the same diameter, the total cross-sectional area after the branch is twice that before the branch, so the flow speed must fall to half.

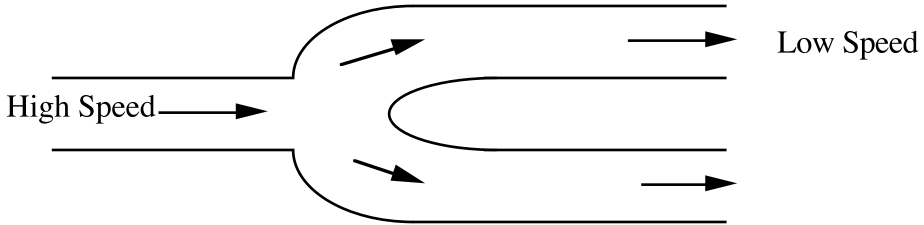

Figure 3.6 Y-junction with pipes of same diameter

Conversely, if it is important to keep the flow speed up, the pipes after the branch must have half the cross-sectional area of those before.

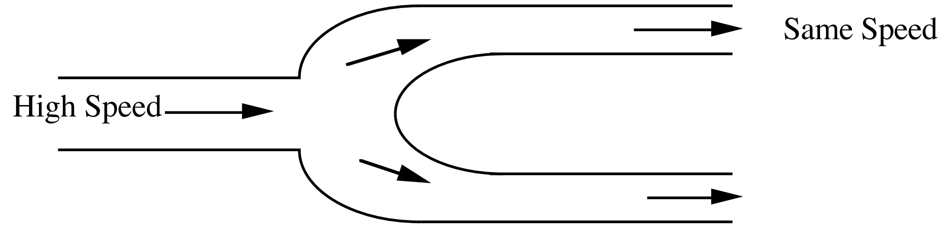

Figure 3.7 Y-junction with pipes of half the original cross-section

(Note: blood will clot if its speed falls too low.)

Most gases behave like incompressible fluids provided their flow speed is less than the speed of sound. The bulk modulus of a gas, while lower than that of a solid, is still large enough for the equation of continuity to describe its motion.

Demonstrations

Air conditioning systems must also be built with consideration for the branch effect.

Also the tube structure of the respiratory system is remarkably similar to that of the circulatory system.

In complicated patterns of streamline flow, the stream lines effectively define flow tubes. So the equation of continuity says that where streamlines crowd together the flow speed must increase.

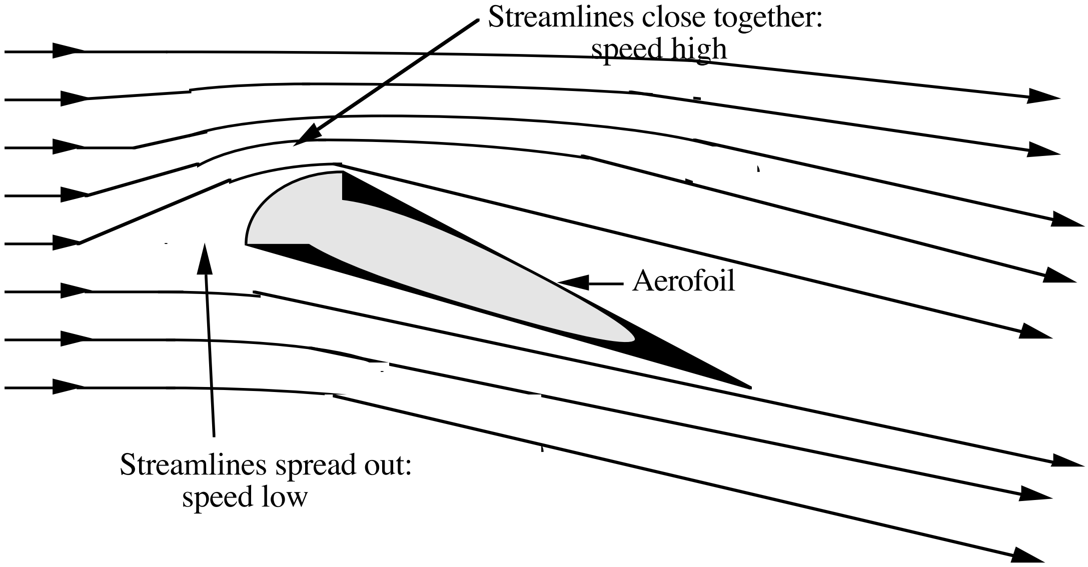

Fig 3.8 Streamline pattern around an aerofoil

3-5 BERNOULLI'S PRINCIPLE

Demonstration

An interesting effect which is easy to show is that, for a fluid (e.g. air) flowing through a pipe with a constriction in it, the fluid pressure is lowest at the constriction.

In terms of the equation of continuity, the fluid pressure falls as the flow speed increases.

The reason is easy to understand. The fluid has different speeds and hence different kinetic energies at different parts of the tube. The changes in energy must result from work being done on the fluid and the only forces in the tube that might do work on the fluid are the driving forces associated with changes in pressure from place to place.

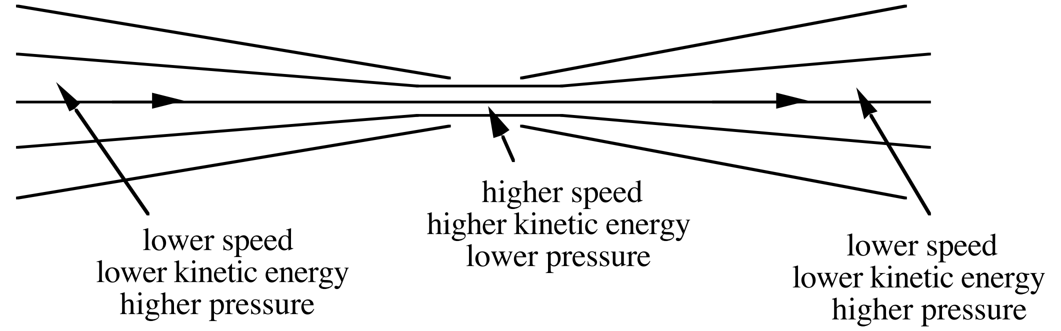

Figure 3.9 Application of Bernoulli's Principle

The units of pressure, N.m^{-2}, might be rewritten as J.m^{-3}; that is, pressure is dimensionally equivalent to work/volume.

Since the fluid is driven from regions of high pressure to those of low pressure and thus increases its kinetic energy, we can write

kinetic energy/volume + work/volume is constant,

i.e. $\frac{1}{2}\rho v^2 + p = \text{constant.}$

In cases where the flow is not horizontal, we should add in the gravitational potential energy/volume also: $\frac{1}{2}\rho v^2 + p + \rho g h = \text{constant.}$

This is known as Bernoulli's equation. For the very simple cases it says what we had before - the fluid pressure is lowest where the flow speed is highest.

Demonstrations

The venturi effect: a fast jet of air emerging from a small nozzle will have a lower pressure than the surrounding atmospheric pressure. You can support a weight this way:

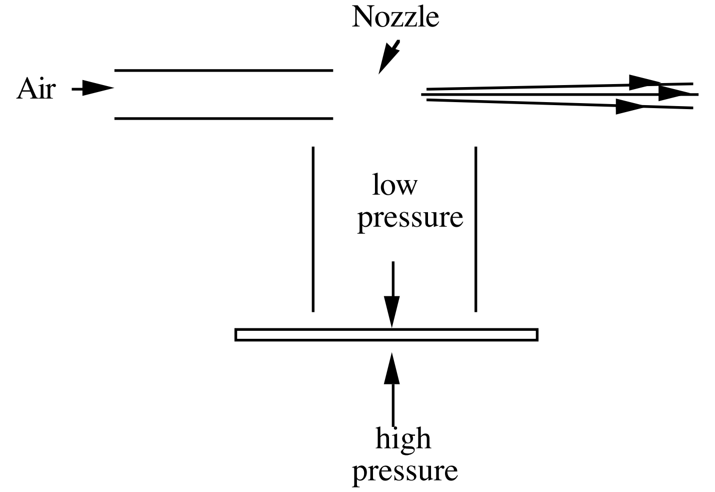

Figure 3.10 Example of the Venturi Effect

The chimney effect: just the venturi effect being used to suck material up. [Note in most automobiles, petrol is sucked into the carburettor in this way.]

Atomiser: This same effect makes atomisers and spray guns work.

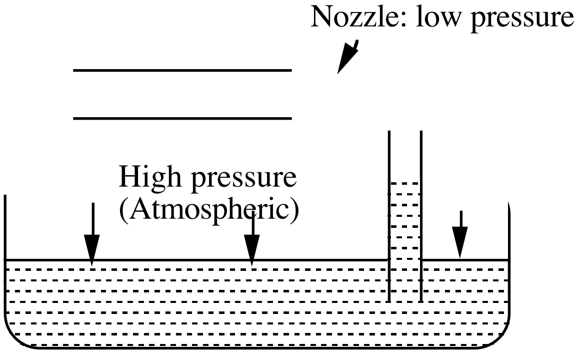

Figure 3.11 An "Atomiser"

It is most important that the free surface of the liquid should be open to the atmosphere, else the high pressure outside the container and the low pressure inside will result in the container being crushed. {Fly sprays always have a small air hole.]

A spinning ball or cylinder moving through a fluid experiences a sideways force.

There is a high pressure on one side (so a big force) and low pressure (small force) on the other. The ball experiences a net sideways thrust. This is one of the ways players can get cricket or ping pong balls to swerve.

Demonstration

Spinning cylinder.

POST-LECTURE

3-6 MORE ON REYNOLDS NUMBER

There are several points to note about the definition of the Reynolds Number.

(a) It is not a precise physical quantity. The quantities L and v are only typical values of size and speed. It is often not possible even to say *which* length you are talking about. For a body moving through a fluid it might be either length or breadth or thickness - or any other dimension you might think of. For a fluid flowing through a channel or a tube, it turns out that it is the diameter of the tube which enters. It is not until you learn more about the Reynolds Number that you can really hazard an intelligent guess at which one you should use.

This imprecision in its definition reflects the fact that the basic physical law is itself rather vague - indeed it can often only be stated as we did: "The flow of fluid in a system is more likely to be turbulent if the system is large, than if it is small". It is not surprising then that the magic number of 2000 is also only rough.

b) The "stickiness" index, the kinematic viscosity, is given the strange symbol η/ρ for the following reason. There are many ways in which this "stickiness" or **viscosity** manifests itself. Basically, how fast the fluid flows determines one measure of stickiness known as the coefficient of viscosity (η) - see next lecture. How easily the fluid becomes turbulent is related to this but to the density (ρ) as well - or if you like, it defines a different measure of stickiness. It is pointless to say any more at this stage, except to give units.

η is measured in units of Pa.s; to give you a feeling for what numbers occur, for water $\eta \sim 10^{-3}$ Pa.s.

(c) The Reynolds number is a dimensionless number as you can see from its definition:

$$[R] = \frac{[\mathrm{m.s^{-1}}][\mathrm{m}][\mathrm{kg.m^{-3}}]}{[\mathrm{Pa.s}]}$$

This will be understood when you come to see where the Reynolds Number comes from. It is a **ratio** of two quantities - essentially a scaling number.

Q3.1: Work out the Reynolds Number for the following flow systems, and say in which ones you might expect there to be a lot of energy dissipated through turbulence.

(i) A Sydney Harbour ferry

(ii) Household plumbing pipes

(iii) The circulatory system. [Take an average sort of figure for the flow speed of blood to be 0.2 $\mathrm{m.s^{-1}}$ the diameter of the largest blood vessel, the aorta, to be ~ 10 mm; and guess that the viscosity of blood probably is not very different from that of water.]

(iv) Spermatozoa swimming. [They are typically about 10 μm in length with speeds of about $10^{-5}\mathrm{m.s^{-1}}$.]

3-7 CONTINUITY

A careful derivation of the equation of continuity goes like this.

Consider a fluid flowing through an irregular tube like this

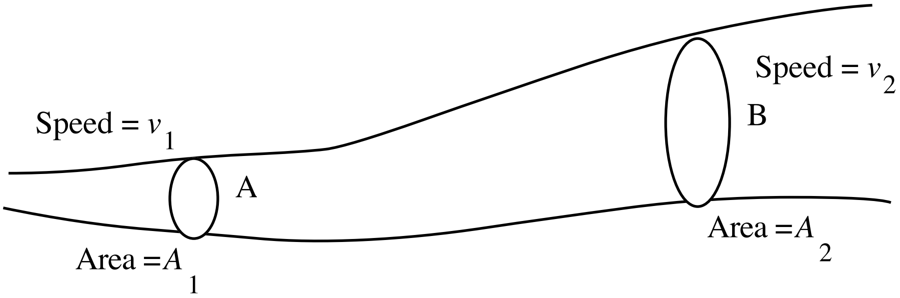

Figure 3.12 Figure for derivation of Equation of Continuity

The volume of fluid flowing past A in a very small time Δt = $A_1 v_1 \Delta t$.

So the mass which flows past A is $\rho_1 A_1 v_1 \Delta t$. Similarly the mass of fluid flowing past B in time Δt is $\rho_2 A_2 v_2 \Delta t$.

Now, when the flow is steady all the material which goes past A must go past B in the same time (or else it will continually piling up somewhere) so

$$\rho_1 A_1 v_1 \Delta t. = \rho_2 A_2 v_2 \Delta t.$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Then if the fluid is incompressible, its density does not change, so

$$A_1 v_1 = A_2 v_2$$

which is the result stated earlier.

Notice that for the final statement to be true, incompressibility is important. But notice also that if the fluid is approximately incompressible, i.e. if its density never changes by very much, then the equation of continuity, as we quoted it, is approximately true.

The quantity appearing in this equation, Av, measures the volume of the fluid which flows past any point of the tube divided by time. It is given the name **volume rate of flow**, and is usually denoted by the symbol q. See, for example, Poiseuille's law on page 43.

Q3.2 From observations you have made, either from the TV screen or in the real world, draw in the stream lines for a liquid flowing in streamline motion through a drain with a corner in it.

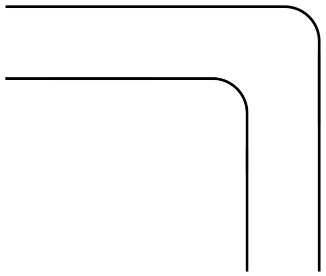

Figure 3.13 A drain with a corner in it!

Use continuity to decide where the flow speeds up, and when it slows down.

You cannot apply this to water flowing around a bend in the river. A Reynolds number calculation shows that the situation is quite different.

3-8 BERNOULLI'S EQUATION

If you really want a more careful derivation of Bernoulli's equation, you can look it up in another book. It goes along the same lines as the proof of the equation of continuity. Just remember that, because you are using the equation of conservation of energy, it is important that there should be no energy dissipation through turbulence. Bernoulli's equation only really applies when the motion is strictly streamline.

Nonetheless, provided there is not too much turbulence, the law will approximately apply. Certainly, in all of the experiments we did on screen the flow must have been pretty turbulent, yet they all showed the characteristic effect of pressure drop.

Q 3.3 Medical textbooks often quote Bernoulli's equation simply as

$$p + \rho g h = \text{constant}$$

implying that the kinetic energy term ($\frac{1}{2}\rho v^2$) is not important but the gravitational potential energy term is.

Use the average speed for blood flow quoted above and a typical human blood pressure of 10^4Pa to explain why this is so.

Q 3.4 In section 2 above, you analysed streamlined flow round a corner. Using the result of that analysis show how the **pressure** changes as the liquid goes round the corner. Can you reconcile this with the kind of simple minded diagrams drawn for the lift force on wings drawn in figure 3.3 above?

Q 3.5 When you are in the dentist's chair, the dentist uses a device based on the venturi effect to suck saliva out of your mouth. Discuss.