PM4

VISCOSITY

Come crown my brow tin leaves of myrtle
I know the tortoise is a turtle.
Come carve my name in stone immortal,
I know the turtoise is a tortle.
I know to my profound despair
I bet on one to beat a hare.
I also know I'm now a pauper
Because of its tortley turtley torpor.

OBJECTIVES

Aims

In this chapter you will look at the effect of the application of shear stresses to fluids and the associated phenomenon of fluid viscosity. The coefficient of viscosity will be defined. Poiseuille's equation, which describes the flow rate of viscous liquids through pipes is presented, discussed and applied to a number of situations

Minimum Learning Goals

When you have finished studying this chapter you should be able to do all of the following.

1. Explain and use the following terms: *shear stress, velocity gradient, viscosity, newtonian liquid.*
2. (i) Describe an experiment that shows qualitatively a relationship between shear stress and velocity gradient.

   (ii) Define the coefficient of viscosity in terms of this relationship (Newton's Law of viscosity).
3. (i) Identify the unit of viscosity as 1 Pa.s.

   (ii) Recall that the coefficient of viscosity for water is about 1×10^{-3} Pa.s.
4. (i) Describe the different response of liquids and solids to an applied shear stress.

   (ii) State Newton's law of viscosity in terms of shear stress and the rate of shear deformation.
5. (i) Explain qualitatively the dependence of the rate of streamline flow of liquid in a pipe on pressure difference, pipe length, pipe radius and the coefficient of viscosity of the liquid. (Poiseuille's equation.)

   (ii) Use Poiseuille's equation, when quoted, to do simple calculations.

   (iii) Describe three phenomena, including both water pipes and the human body, which relate to Poiseuille's equation.
6. Present the analogy between current in an electric circuit and fluid flow in a pipe system and explain what is meant by resistance in fluid flow.
7. (i) Explain how energy is dissipated by viscosity.

   (ii) Use the Reynolds number to determine whether or not viscous dissipation of energy is important in simple systems.

PRE-LECTURE

Keep in mind two particular points that have been made so far in these Properties of Matter lectures.

(i) The definition of the Reynolds number, and its importance essentially as a scaling number. In last lecture we pointed out that this number told you whether or not a particular flow system was likely to be turbulent or streamline. The same number will turn up again to decide whether or not the flow is viscous. The basic reason for the existence of this number and why it takes the form it does is perhaps one of the most important questions in the whole study of fluid flow.

(ii) The definition of the shear deformation and Hooke's Law as it applies to bodies which behave elastically under shear stresses. As we have pointed out it is the behaviour of a substance under shear which essentially distinguishes between a solid and a liquid . A solid (usually) has a large shear modulus, i.e. if you try to deform it (in shear) it will deform, but then return to its original shape afterwards. A liquid has a very, very small shear modulus. You can slide one bit of a liquid past another bit, and there will be no noticeable tendency for the two to regain their original shape when you shop pushing. Nonetheless the sliding of one bit does have an influence on the other, and this is what viscosity is all about.

(iii) Also you should recall discussion of electrical resistance and the various mathematical techniques of working with it (like Ohm's Law and Kirchhoff's Theorem). The flow of water through pipes is an important part of this lecture - and obviously much the same kind of mathematical reasoning can be used to talk about it, as was used to discuss D.C. circuits.

(iv) Recall also the meaning of the word **gradient**. If some quantity (say pressure) varies with distance (x), being big at some point and small at another, we say a pressure gradient exists.

A measure of this is the derivative $\frac{\mathrm{d}p}{\mathrm{d}x}$; or, more crudely, the ratio:

$$\frac{\text{difference in pressure at 2 points}}{\text{distance between those 2 points}} .$$

In a fluid we might expect the flow speed to change from point to point, and we could describe this variation by measuring the **velocity** gradient.

LECTURE

4-1 VISCOSITY

A feature which distinguishes one liquid from another is their "thickness" or the ease with which they pour.

Demonstration

Observe the flow of water, glycerine, oil, treacle, lava, pitch.

<< This last experiment is on show in the Physics Department, University of Queensland. The experimental record is:

1920	Pitch poured in funnel
1938 (Dec.)	First drop fell
1947 (Feb.)	Second drop fell
1954 (Aug.)	Third drop fell
1962 (May)`	Fourth drop fell
1970 (Aug.)	Fifth drop fell.

>>

The physical property which distinguishes these liquids from one another is something to do with how well the liquid molecules adhere to one another; and this molecular adhesion leads to a host of rather complicated effects.

Demonstration

(i) The rate at which solids fall through liquids (this has already been discussed in chapter FE4).

(ii) The spin-down effect, tea leaves in the bottom of a stirred cup migrate to the centre (not the outside as you might expect).

(iii) Smoke rings.

(iv) Vortex rings in liquids.

These are all traceable (in the end) to molecular adhesion, but their explanation and connection with one another is very complicated. However, we have to start somewhere. We must select **one** physical effect to measure, and try to understand the others in terms of it. We choose to concentrate on the existence of a **velocity gradient**.

When a fluid (e.g. air) flows past a stationary wall (e.g. table top), the fluid right close to the wall does not move. However, away from the wall the flow speed is not zero. So a velocity gradient exists.

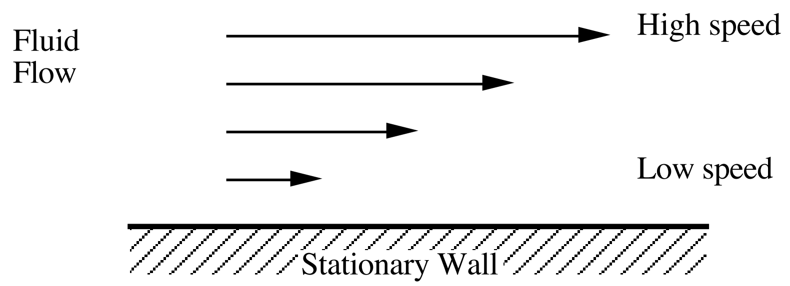


Fig 4.1 Velocity gradient in a stream of fluid moving past a stationary wall

We will find that the magnitude of this gradient (how fast the speed changes with distance) is characteristic of the fluid. We will use this fact to define **viscosity**.

Demonstration

Observe the velocity gradient in a tank of treacle.

4-2 THE COEFFICIENT OF VISCOSITY

A simple experiment set-up capable of demonstrating the law of viscosity involves a small metal plate suspended in a tank of liquid. Before the experiment starts the weight of an attached pan is adjusted so that the plate is neutrally buoyant - i.e. it does not tend to sink in the liquid or to rise.

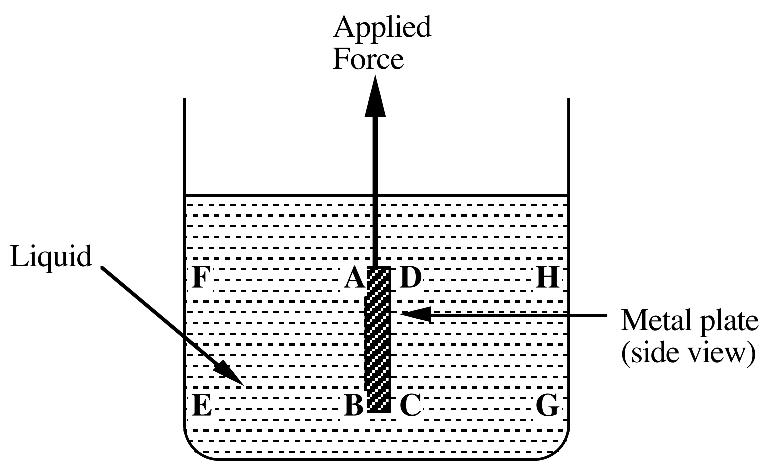


Fig 4.2 Experiment to measure coefficient of viscosity: static situation

An extra force is now applied to cause the plate to move through the liquid. When the plate is moving with speed *v* through the liquid, there will be a velocity gradient between AB and FE; also between DC and HG. The complete apparatus is

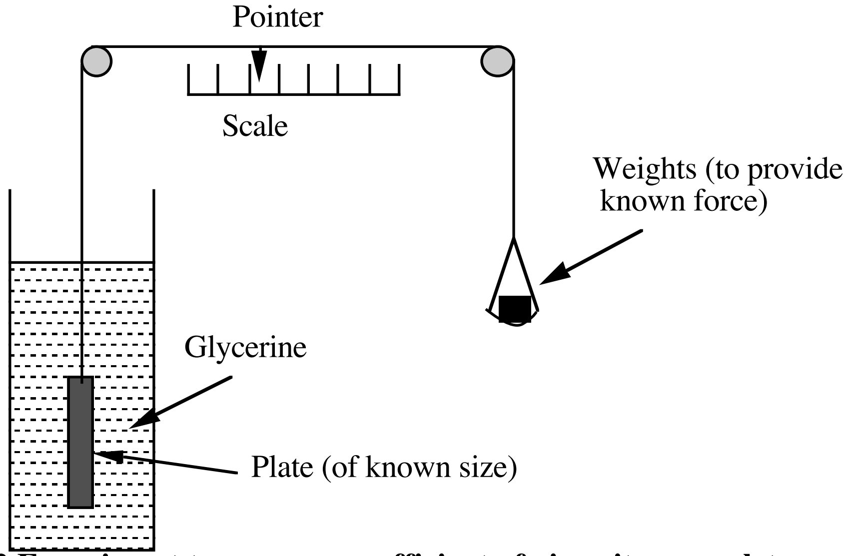


Fig 4.3 Experiment to measure coefficient of viscosity: complete apparatus

Demonstration

(i) The speed with which the plate rises, increases as the force pulling it increases:

Mass on pan/g	Time to go 100 mm/s
2	8
5	2.6

(ii) For the same force, the speed of the plate decreases as the **area** of the plate increases.

For a plate of twice the surface area

Mass on pan/g	Time to go 100 mm/s
2	13.5
5	5
7	3

We interpret these two sets of results as indicating that the speed of the plate increases with the **shearing stress** (recall the definition of stress given in chapter PM1)

(iii) For a given speed, we change the velocity gradient by moving the plate closer to the wall.

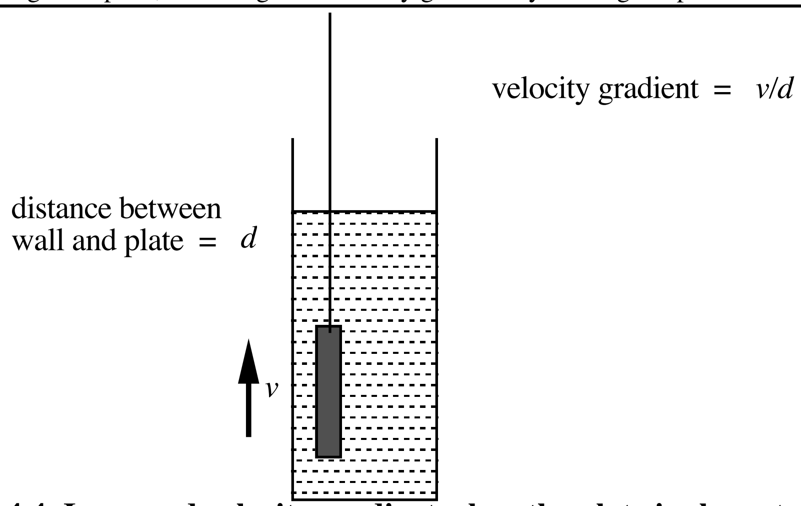


Fig 4.4 Increased velocity gradient when the plate is closer to the wall

Mass on pan	Time (close to wall)
7 g	4.2 s

We interpret this result as saying that a given shearing stress sets up a velocity gradient in the fluid.

More careful experimentation, or more detailed theoretical analysis, will clarify these conclusions into **Newton's law of viscosity** which says: When a shearing stress acts within a fluid moving in a streamlined motion, it sets up in the liquid a velocity gradient which is proportional to the stress.

$$s = \eta \frac{dv}{dx}$$

(stress) = (constant) × (velocity gradient)

The constant η is called the **coefficient of viscosity**, and is different for different liquids.

Water is obviously a lot less viscous than glycerine.

Though this equipment can roughly show Newton's Law to be plausible, it cannot be used for accurate measurement of viscosity. Some common measuring devices are:

Demonstrations

(i) Commercial oil companies use simpler viscometers. These essentially measure how fast the oil pours. The "grade" of an oil is the number of seconds it takes to pour a measured amount through a certain tap.

(ii) If not much liquid is available, or cannot be removed (e.g. protoplasm in a cell, or sap in a plant) you can observe how fast bubbles rise or particles sink in the liquid.

(iii) Laboratories usually use a torsional viscometer, which is really a very refined version of the apparatus we used above.

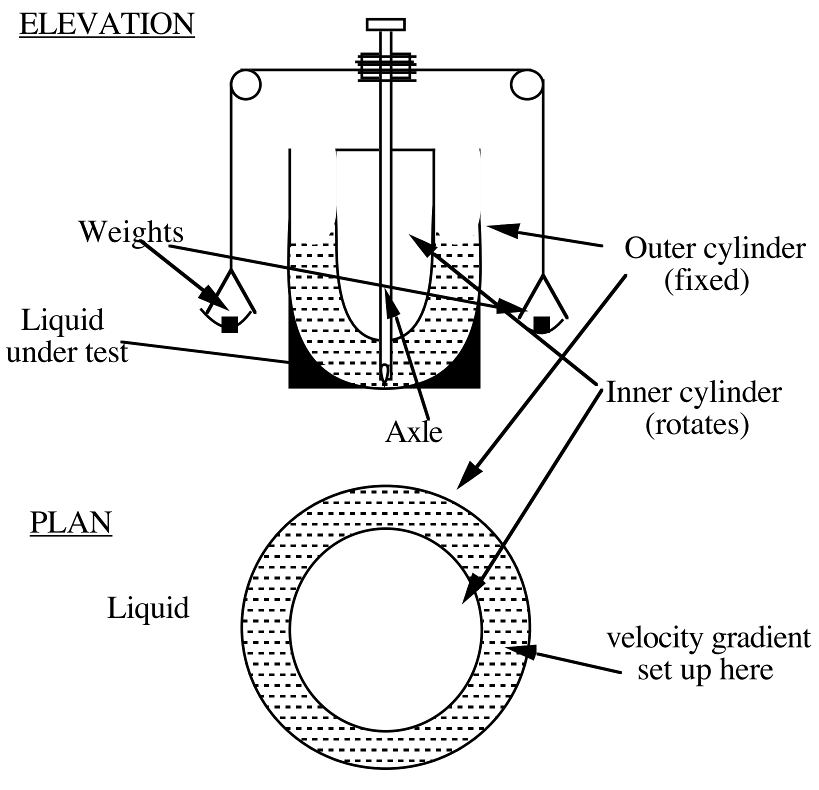


Fig 4.5 A torsional viscometer

Of course the viscosity of a liquid can change.

Demonstration

The viscosity depends on temperature, usually increasing as the temperature decreases (which is why automobiles need different oils in hot countries than in cold countries and indeed why the engine runs more freely as it heats up).

However, this kind of detail you can catch up on later, when you come to talk about viscous effects in your own discipline.

4-3 ALTERNATIVE STATEMENT OF NEWTON'S LAW

Since there are many manifestations of viscosity, there are many different statements of the basic law. We have given Newton's statement, relating velocity gradient to shear stress (pressure).

Another statement, which research workers use, specifically points up the difference between solids and liquids.

Solids

When a shearing stress is applied to a solid it suffers a shear (i.e. a shear deformation)

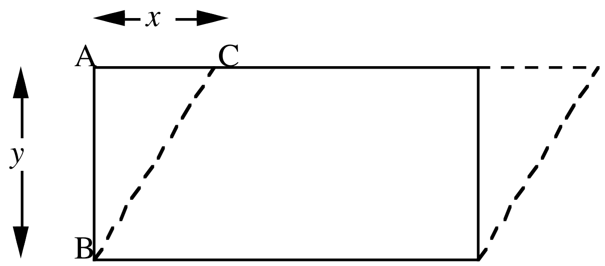


Fig 4.6 Shearing of a solid (side view)

A solid deforms instantaneously and then stops deforming. When the shearing stress is removed, if the solid is elastic the deformation recovers.

Liquids

When a shearing stress is applied to a liquid it suffers a shear deformation also, sometimes slowly sometimes fast. However, so long as the shear is applied it continues to shear. When the stress is removed, the shearing stops, but does not recover.

The basic law of behaviour of elastic solids and viscous liquids are:

Elastic solids obey Hooke's law which says

shear stress   shear deformation

{Remember: shear $= \frac{\text{length AC}}{\text{length AB}}$ }

Viscous liquids obey Newton's law which says

shear stress   velocity gradient.

However the velocity gradient is the same thing as time rate of change of shear deformation..

This can be seen as follows, with reference to figure 4.6:

$$\text{velocity gradient} = \frac{\text{speed}}{\text{transverse length}}$$

$$= \frac{\frac{dx}{dt}}{y}$$

$$\text{rate of shear} = d/dt\,(x/y) = 1/y\,(dx/dt)$$

provided y is constant, as it is.

So, Newton's law can be restated

shear stress   rate of shear deformation.

4-4 POISEUILLE'S LAW

We want to find out what effect viscosity has when fluids flow in more relevant situations. The most important one, which is the only one we will consider is flow through a long pipe.

We could go through a mathematical analysis and apply Newton's law (of viscosity) to this problem, but we will not. One thing however is obvious. Because viscosity puts restrictions on velocity gradients, it must be true that liquid will flow faster through a wide pipe than a narrow one.

Demonstration

Polystyrene chips on surface of a treacle tank.

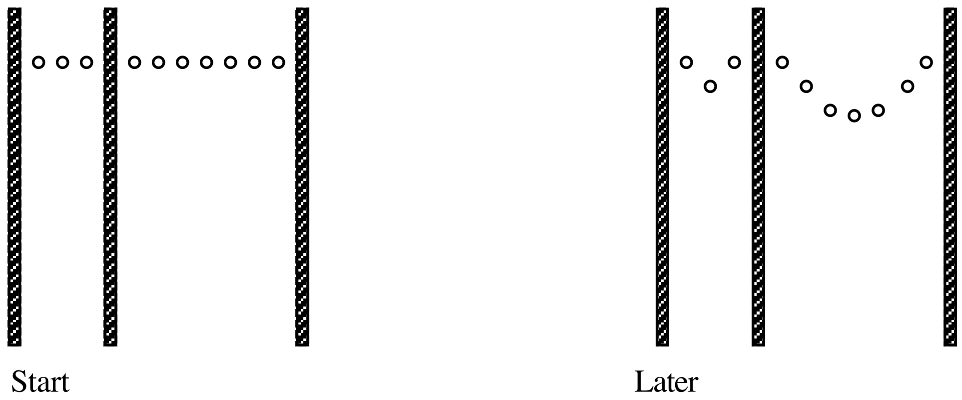


Fig 4.7 Velocity profile for moving treacle

[This technique of showing the **velocity profile** of the flow will become important later.]

In trying to find out what other factors control how fast fluids can flow through pipes, the following factors are easy to isolate:

(i) the pressure difference between the ends of the pipe. The bigger the pressure difference, the faster will be the flow;

(ii) the length of the pipe. More liquid will flow through a shorter than a longer pipe in the same time.

(iii) the radius of the pipe. More liquid will flow through a wide than a narrow pipe in the same time. This dependence is very marked. Even in our rough demonstration we get 8 times as much glycerine flowing through a pipe twice the radius of the other. (Theory says we should have got 16 times as much.)

(iv) the coefficient of viscosity of the liquid. Water flows much more easily than glycerine.

Had we gone through a mathematical analysis of the situation, we could show that Newton's law of viscosity would give the volume rate of flow, q_v, of fluid of viscosity η, through a pipe of radius r and length l, when driven by a pressure difference Δp as

$$q_{\text{v}} = \frac{\Delta p . r^4 \pi}{8 \eta l}$$

This is known as **Poiseuille's law**.

The similarity between this equation and **Ohm's law** is very marked; most workers talk about "resistance" of a pipe. See post lecture material.

Situations in which Poiseuille's law has important effects:

Examples

(i) Irrigation pipes. It is uneconomical to use spray irrigation too far from a river since the resistance of a pipe increases with its length, and you need too big a pump.

(ii) Pipes from Warragamba Dam. Here Δp and l are fixed (by geography), and the volume rate of flow is fixed by the requirements of the population of Sydney. When Sydney doubles in size, the Water Board will have to use twice as many pipes or replace the present pipes by ones of $(2)^{1/4}$ times the radius. [This is an oversimplification, see post lecture.]

(ii) Respiratory system: The flow of gas here is also Poiseuillean. The resistance to flow is determined primarily by the narrow tubes leading to the alveoli. Any general constriction of the pipes, as occurs in bronchospasm for instance, increases the resistance to flow and makes breathing much more difficult.

(iv) Circulatory system: Two points are worth making.

(a) There is a decrease in pressure across each section of the tubes. Blood pressure is highest when it leaves the heart (through the aorta) and lowest when it returns (through the inferior vena cava).

Most pressure loss occurs over the capillaries. Why?

(b)Any constriction of the tubes - for example a build up of cholesterol on the walls of the arteries - increases the resistance and hence the pressure drop (it goes as r^4 remember). So the heart has to work harder to compensate. And at times of stress, when an increased flow rate is required, there can be a breakdown.

(v) Urinary tract. You work out the relevant physics.

Anyhow, just remember that Poiseuille's law is essentially the same thing as Newton's law; and if a fluid obeys one it obeys the other. There are a large number of important liquids which do not obey these laws, and they are called **non-newtonian liquids**. One of the simplest ways to recognise a newtonian liquid is to examine its velocity profile, which should be **parabolic** for a liquid which obeys Newton's (or Poiseuille's) law.

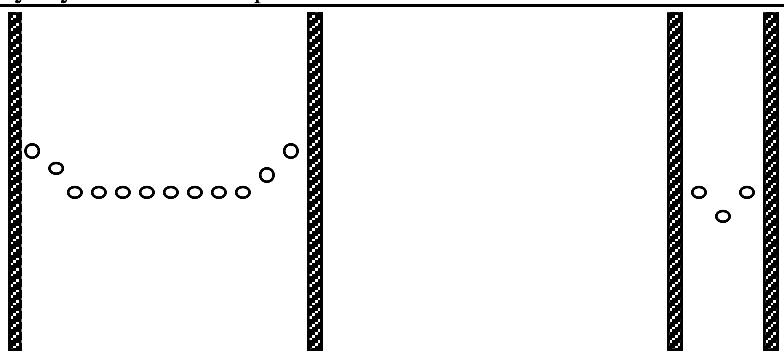
Demonstration

flow of syrup.

4-5 REYNOLDS NUMBER

Poiseuille's Law shows that viscosity is responsible for loss of pressure - and hence is an energy dissipating phenomenon. We are not talking about energy loss due to turbulence, but about energy loss which occurs even when the flow is streamlined. It comes about through friction between the streamlines moving past one another.

The criterion whether or not much energy is lost in this way, is therefore whether or not there is much of a velocity gradient throughout the whole of the liquid. Since most of this gradient occurs near a boundary (in the so called **Boundary layer**, it is the ratio of the size of the system to the size of the boundary layer which is important.



System relatively large
energy mostly conserved

System relatively small
much energy dissipated

Fig 4.8 Effect of boundary layer in velocity profile

As will be shown in the post-lecture, the ratio of total energy of flow to energy dissipated, is very closely related to the **Reynolds number** which we introduced in last lecture.

So now we can appreciate another reason why the Reynolds number is important. Viscous effects can never be neglected (i.e. the energy dissipated is appreciable) in low Reynolds number situations: in thick liquids (η large), or in small slow flow systems. On the other hand viscous effects will not be important in thin liquids or in large, fast flow systems. (However in these latter systems remember that turbulence is always possible, and energy can be lost through that means.)

In general then, flow patterns will be different in systems with low and with high Reynolds numbers. In particular, in very low Reynolds number systems, the flow is perfectly **reversible** since no turbulent effects can occur anywhere.

Demonstration

The method of swimming is quite different for fishes ($R \sim 10{,}000$) and spermatozoa ($R \sim 0.0001$).

Modes of boat propulsion which work in thin liquids (water) will not work in thick liquids (glycerine).

It is possible to stir glycerine up, and then **unstir** it completely. You cannot do this with water.

POST-LECTURE

4-6 MORE ON POISEUILLE'S LAW

Poiseuille's law can be derived from Newton's law; but to go through the complete derivation, even in post-lecture material, would be completely opposed to the philosophy of this course. If you feel you cannot do without it, there are plenty of books you can look up. But in broad outline the derivation follows these lines:

(i) The force trying to push the fluid through the tube is of course due to the pressure difference Δp. The retarding force comes from viscous drag, which acts to prevent shearing. If the flow is streamlined, then this shearing resistance acts all over the surface of imaginary tubes of fluids concentric with the tube trough which the fluid is flowing; and thus the retarding **force** will depend on the surface area of the tubes of fluid - and hence on the length of the tube.
The flow speed must therefore increase until the resisting force balances the driving force; and when that happens you will get Δp on one side of the equation and l on the other.

(ii) Trying to calculate how fast the fluid must flow to produce the necessary resisting force is where Newton's law comes in. There is most fluid-on-fluid slipping toward the outside of the tube (purely by geometry) so the velocity gradient is larges there, and is zero in the centre. In fact the **velocity profile** is a parabola.

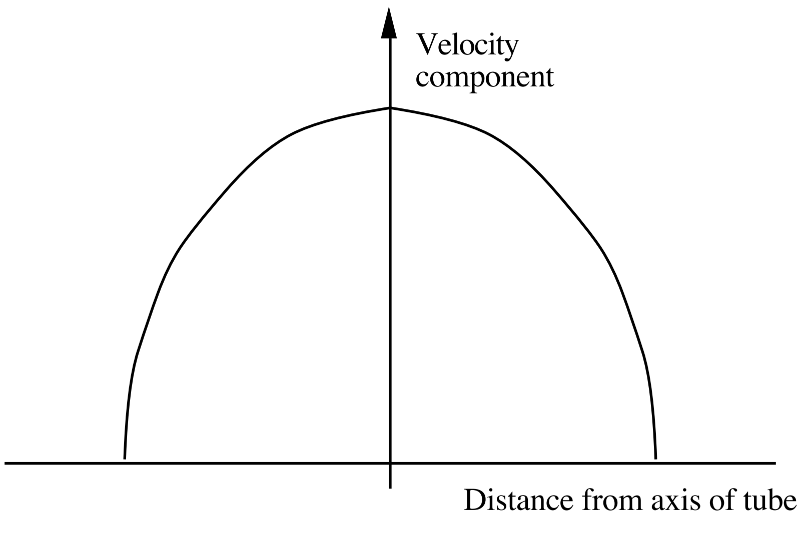


Fig 4.9 Parabolic velocity profile

You can say therefore that the average flow speed is likely to depend on r^2. And so the **volume rate of flow** (equal to the average speed × area of the tube) is likely to depend on r^4.

(iii) The factor $\pi/8$ you must take on faith, or work it out for yourself.

The only point in going through even so sketchy a "derivation" is to point out two facts:

(i) Poiseuille's law only applies to fluids that obey Newton's law, and

(ii) The assumption of streamlined flow is also built in to Poiseuille's law. If turbulence occurs than you must be very careful about using Poiseuille's law to calculate flow rates. (You will recall that in the experiment done on screen glycerine flowed through the wide pipe more slowly than would be predicted by Poiseuille's law.)

Q4.1 Why should turbulence mean that the volume rate of flow is **less** than in streamlined flow?

Q4.2 When a builder designs the drainage system for the roof of a house, what factors should influence the choice of the size of the downpipe?

Would he be correct in basing his calculations on Poiseuille's Law

Q4.3 The experiment was done in chapter PM2 in which water rose, by capillary attraction, through two columns of soils. It was observed that the water rose faster in the column with the coarser grains. Can you say now why this is so?

4-7 ELECTRICAL ANALOGUE

Ohm's Law says : $V = R\,I$

Poiseuille's Law says : $\Delta p = \frac{8\,l\,\eta}{\pi\,r^4}\,q_V$

The comparison is obvious, and hence it is most convenient to talk about flow through any kind of tube in terms of **resistance** defined thus:

$$R \equiv \frac{8 l \eta}{\pi r^4}$$

Note that the unit of this resistance is: kg. m^{-4}. s^{-1}.

It is even possible to do this when the flow is turbulent. It only means that Poiseuille's equation is not valid, and you cannot use this explicit formula for the resistance. But it is still quite possible to define a resistance.

Once you appreciate this, then you can use all the mathematical techniques of circuit analysis; in particular the rules for adding resistances in series and parallel.

Q4.4 Consider water flowing along a 1.0 m long pipe at a steady rate,

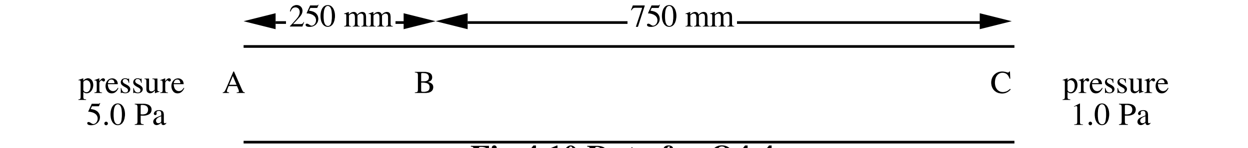


Fig 4.10 Data for Q4.4

If you measured the fluid pressure at point B, what value would you get?

Q4.5 Consider these two different streamlined flow systems,

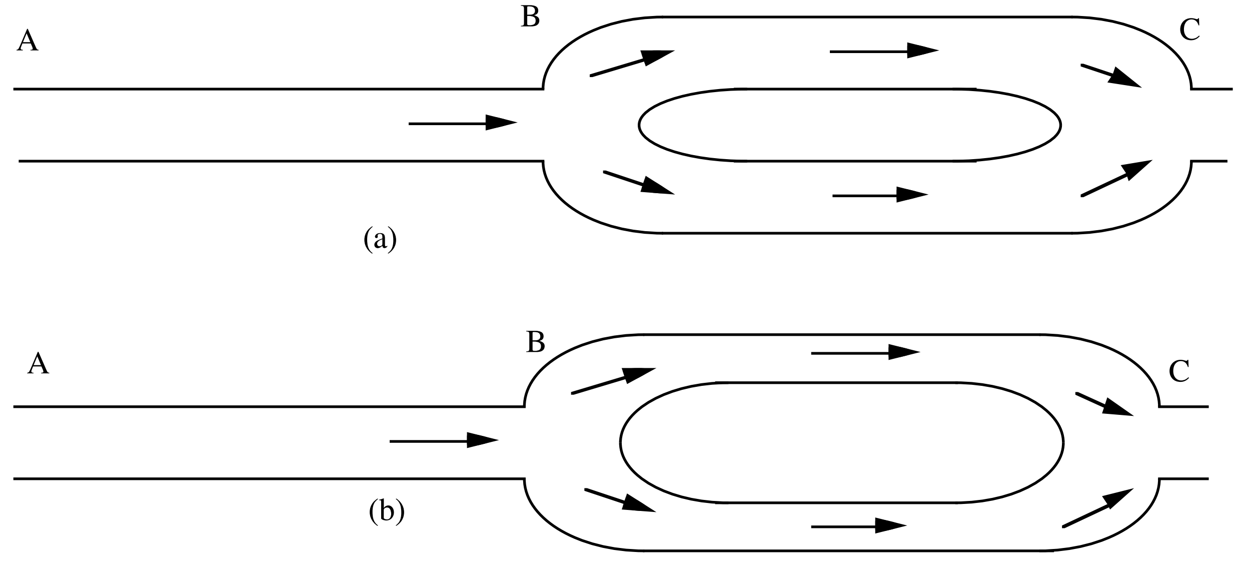


Fig 4.11 Data for Q4.5

The lengths of the two pipes in the section BC are equal to the lengths AB in both cases. The radii of all three pipes in case (a) are the same; but in case (b) the radii of the two pipes in BC are half that of pipe AB. In both cases also, the pressure at A is 4 Pa, and at C is 1 Pa.

What is the pressure at B in case (a) and case (b)?

Can you guess from these two answers, the answer to the question posed in the lecture notes: "in the circulatory system, why does most pressure drop occur over the capillaries?"

4-8 MORE ON REYNOLDS NUMBER

In the lecture, we talked about the ratio:

$$\frac{\text{Total energy of flow (per unit volume)}}{\text{Energy dissipated (per unit volume)}}$$

We can easily evaluate this ratio, since the energy dissipated is just the work done by the viscous forces divided by unit volume. And, by the same arguments used in 4-5 of chapter PM3, this must be equal to the **pressure drop**. Hence this ratio

$$= \frac{\frac{1}{2}\rho v^2}{\Delta p} \sim \frac{\frac{1}{2}\rho v^2}{\eta\, \mathrm{d}v/\mathrm{d}x} \quad \text{(by Newton's law)}$$

Now, in a system where the boundary layer is comparable in size with the scale length of the system (L), we can approximate the velocity gradient in this expression by

$$\frac{\mathrm{d}v}{\mathrm{d}x} \sim \frac{v}{L}$$

and therefore the above ratio is $= \dfrac{\rho v L}{2\eta}$

which, apart from the factor 2, is just the definition of the **Reynolds number**

Q4.6 If you were designing a circulatory system for the human body, where a prime requirement is that as little energy as possible should be dissipated, in order not to require the heart to pump any harder than absolutely necessary, what Reynolds number would you aim for?

Compare this with the Reynolds number for blood, which is somewhere between 1000 and 2000.

4-9 VALUES OF VISCOSITY

As pointed out in last lecture, and as can be checked from the equation in 4-2, the units of viscosity are those of (pressure) $\times$ (length)/(speed) or Pa.s. Some books of tables quote numbers in the old cgs unit - the **poise**. You can easily convert by remembering that

1 Pa.s = 10 poise

Purely to show you what range the values of viscosity can have, here are some common fluids:

FLUID	VISCOSITY / Pa.s
water (20°C)	1.0×10^{-3}
water (100°C)	0.3×10^{-3}
alcohol	1.2×10^{-3}
glycerine	1.5
mercury	1.8×10^{-3}
air	1.8×10^{-5}