

Workshop Tutorials for Introductory Physics

Solutions to EI5: Circuits

A. Review of Basic Ideas:

Power, energy and electrical circuits.

When you switch on a battery-powered radio, you're using electrical **energy**. You can often find the rate of energy consumption written on the back of the radio as so many **watts**. This rate at which the radio uses energy is called **power** (P) and it depends on both the **voltage** (V) supplied by the battery and the total **current** (I) from the battery, through the relation: $P = VI$. If the power consumption is constant then the energy (E) used in a time interval (Δt) is given by $E = P\Delta t$.

Similar relations hold for appliances which run on mains electricity which supplies an alternating current, AC. Although the AC voltage and current vary very rapidly, the average voltage, average current and average power consumption are still connected via the relation $P = VI$.

A radio uses electrical energy to produce **sound** and quite a lot of **thermal** energy which leaves the radio as heat. As far as energy consumption goes, we can treat most appliances, like radios and toasters, as pure resistors. You can work out the effective resistance of an appliance by using the relation that voltage equals current \times resistance, $V=IR$.

From this relation you can see that for a given voltage, the more resistance you have, the **less** the current will be, which seems very sensible. A resistor is like a fatty deposit in an artery, slowing down the flow of **blood**, or speed humps slowing down the flow of traffic. When you have resistors in parallel, their total resistance is less, because it's like having two lanes instead of one, so more traffic can flow. When you have two parallel electrical paths the **electrons** can flow down two paths, so you get more **current**, even though there are more resistors!

Discussion questions:

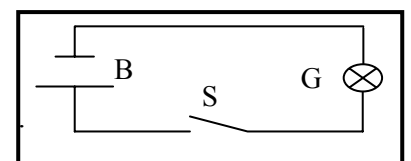
You *can* have a voltage without a current. Any time charges are separated there is a voltage, for example in a battery or a cell membrane. Current only flows if there is a conducting path for it to flow along.

You cannot *cause* a current to flow without a voltage. The voltage supplies the force which pushes the charges, and makes them flow, which is a current. In normal materials there must be an accompanying voltage to allow current flow to continue. In a superconductor the voltage can be removed once the current is established, and the current will continue to flow, but the voltage is necessary to start the flow.

B. Activity Questions:

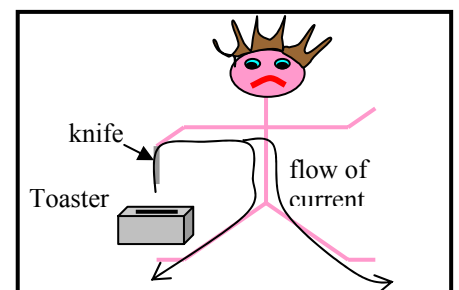
1. Torch – a simple circuit

The torch has a battery, B, which supplies the voltage, a globe, G, which converts electrical potential energy into light, and a switch, S, which completes the circuit allowing current to flow when the torch is turned on.



2. Toaster Man – resistors in series

The current is inversely proportional to the resistance, the greater the resistance the less current can flow, and the less likely toaster man is to be electrocuted. Electricians wear rubber soled shoes to increase the resistance between themselves and the Earth.



3. Measuring current and voltage

The ammeter measures the current, which is the rate at which charge passes a given place in the circuit. To do that, the ammeter must be part of the circuit. If you want the measured current to be the same as the current that would have been there without the meter then its resistance must be very small.

The voltmeter is connected in parallel, because it measures the difference in potential between two points. Connecting the voltmeter to those two points creates a parallel path. If you want the measured voltage to be the same as the voltage without the meter, you need a meter with very high resistance, so that it takes practically no current.

4. Simple membrane model – resistors in parallel

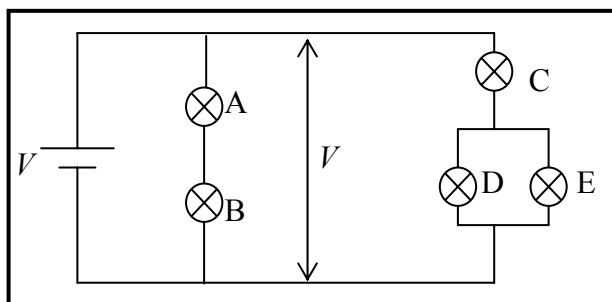
When resistors are connected in parallel the total resistance is less than any individual resistance. There are more paths for the current to flow along, and so the total current is greater. Resistance is the voltage divided by the current, $R = V/I$, so a larger current means a smaller resistance for a given voltage supply.

C. Qualitative Questions:

1. You can redraw the circuit as shown.

The voltage is the same across each arm of the circuit. The potential difference across each of A and B will be $\frac{1}{2}V$. In the second arm we have globes, which we can treat as resistors in a combination of series and parallel. The total resistance of D and E will be half that of each of them individually, which is also half that of globe C.

Hence the resistance of this arm is $\frac{3}{2}$ times that of a single globe. Globe C will have a potential difference of $\frac{2}{3}$ of the voltage V , and D and E will each have $\frac{1}{3}V$. Brightness increases with power which goes like the V^2 , so we can rank the brightness by ranking the voltages. Hence the order of brightness will be $C \rightarrow A \& B \rightarrow D \& E$ (brightest to dimmest).

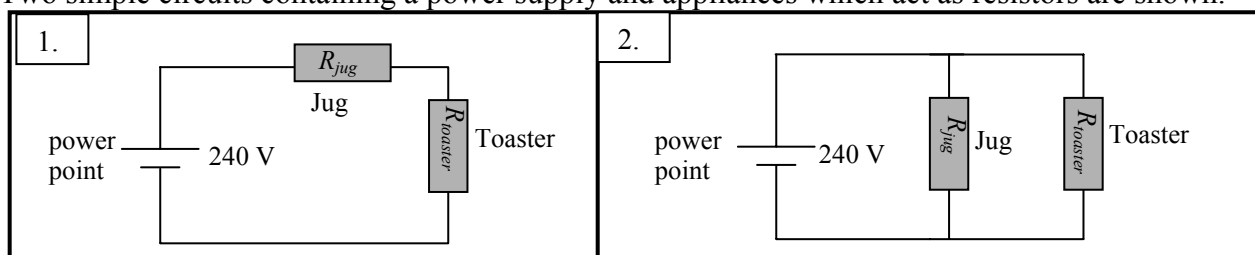


2. Kirchhoff's rule for junctions states that the total currents going into a junction must be equal to the total currents coming out of a junction. This is a statement of conservation of charge, as current is just a flow of charge. Charge must be conserved, so whatever flows into a junction must flow out again. If it didn't come out again, or if more came out than went in, then charge is either being created or destroyed.

Each point in space has only one value of electrical potential at any time. Work your way mentally around *any* closed path, noting and adding up all the changes in potential as you go, making sure to count decreases as negative changes and increases as positive. When you get back to the starting point the sum must be the potential that you started with. In a circuit with just one battery, any loop which includes the battery will include a rise in potential across the battery and drops (or no change) everywhere else. The connection with energy conservation is that if you imagine that you were to take a little test charge around the loop, then the potential energy of the system at the end would be the same as it was when you started. Remember that potential is defined as potential energy per unit charge.

D. Quantitative Question:

Two simple circuits containing a power supply and appliances which act as resistors are shown.



a. The total resistance of circuit 1 is simply $R_{\text{toaster}} + R_{\text{jug}} = 30\Omega + 40\Omega = 70\Omega$

In circuit 2 the resistors are in parallel, so we need to use

$$(1/R_{\text{total}}) = (1/R_{\text{toaster}}) + (1/R_{\text{jug}}) = 1/30 + 1/40 = 0.0583 \text{ so } R_{\text{total}} = 1/0.0583 = 14.1\Omega$$

b. In circuit 1 the same current is flowing through both appliances.

Using $V=IR$, we get $I = V/R = 240/70 = 3.4 \text{ A}$.

In circuit 2 the current is different in each "arm" of the circuit. $I_{\text{toaster}} = V/R = 240/30 = 8 \text{ A}$.

c. In circuit 1: $I_{\text{jug}} = I_{\text{toaster}} = 3.4\text{A}$, and in circuit 2: $I = V/R = 240/40 = 6 \text{ A}$.

d. If the toaster burnt out in circuit 1, the circuit would be incomplete, or open, and no current could flow, so you could not operate the jug either. This would be very inconvenient, so houses are wired in parallel. Another reason is that appliances are usually designed to operate at a specific voltage, wiring everything in parallel means that everything has the same voltage across it.