Workshop Tutorials for Physics
Solutions to ER3: **Flux and Gauss' Law**

A. Qualitative Questions:

1. A Gaussian surface.

a. The flux through the surface shown is $\Phi = \dfrac{q_{inside}}{\varepsilon_0} = \dfrac{q_2 + q_3}{\varepsilon_0}$.

b. All of the charges contribute to the electric field at point P, a field at any point is the sum of the fields due to all charges present. Suppose you have a Gaussian surface which contains no net charge.

c. Gauss's law does not require that there is no electric field at all points on the surface, it requires that the integral of E over the entire surface is zero. Consider for example a dipole consisting of two charges of equal magnitude and opposite sign, enclosed in a Gaussian surface. The points on the surface which are closer to one charge than the other will have a non-zero field, however the integral of the field over the entire surface will still be zero.

If the electric field is zero everywhere then the total charge contained must be zero. If there was a net charge contained there would be points at which there was a field.
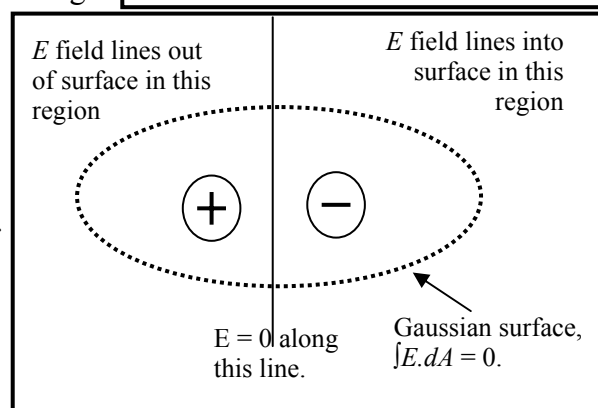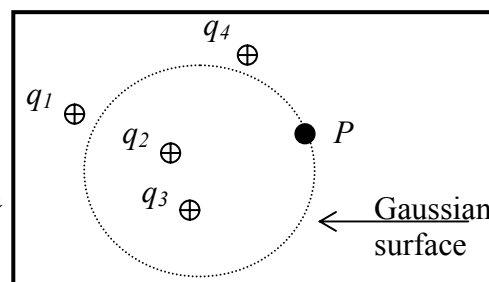
2. A "Faraday cage" is a metal cage used to shield devices from electromagnetic fields.
a. A conducting wire cage can shield its contents from external electric fields. Consider a metal shell in an electric field. According to Gauss's law if there is no contained charge then the flux through the surface must be zero and the electric field anywhere within the conductor must also be zero. Physically, if there is an electric field outside the conductor then there will be charge separation on the outside of the conductor. The charges will move until there is no longer a force acting on them, and hence the field must be zero. As long as the wires of the cage are close together compared to the size of the cage the field inside the cage will be approximately zero. This is why when you pass over a bridge which has metal scaffolding on it you lose radio reception, particularly on the AM band. Radio waves are electromagnetic waves, and the scaffolding shields you against them. FM has shorter wavelengths so needs a finer mesh to shield against it.
**b.** You cannot shield against electric fields by putting the devices that generate the fields into metal cages. The field induces charge separation on the cage, such that the field outside the cage is the same as if the cage were not there.

B. Activity Questions:

1. Faraday's Icepail
The ball is lowered into the container, and charges are induced on the container walls. When the ball is touched to the inner surface all its excess charge is transferred to the container and appears on the outer wall of the container. All the charge is transferred because when the ball is in contact with the container they act as a single conductor.

The electric field within a conductor is zero, and if you draw a Gaussian surface inside the conductor it will contain zero charge because $E = 0$ everywhere on the surface. Hence the ball can contain no charge. When the ball is removed, we can see that it has no charge, thus confirming Gauss's law.
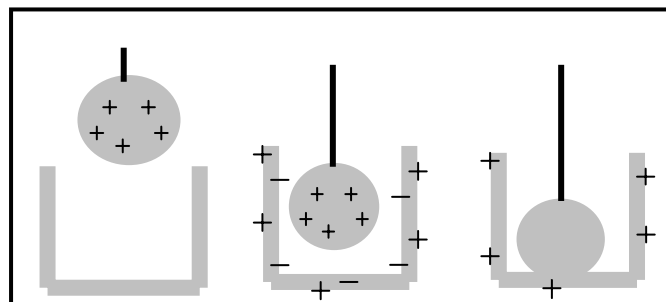
2. Gauss' law

When the generator is turned on the metal becomes charged. On a conductor all the charge goes to the outside of the conductor, so the balls inside do not become charged. When the plastic cup is placed on the generator the balls fly out. The plastic is an insulator, so the charge does not flow easily to the outer surface. The balls inside become charged and repel each other and are light enough to fly out of the cup.

3. Flux

The flux onto the panel is $\Phi = -AI\cos\theta$ where I is the intensity of the light, A is the area of the panel and θ is the angle between the normal vector to the surface and the direction of the incident radiation. Note the minus sign because the flow lines and the vector representing the area are in opposite directions.

To maximise the flux you need to maximise the area that the light "sees", so the surface should be perpendicular to the light. To minimise the flux you turn the area parallel to the light. Doubling the A would double the flux.

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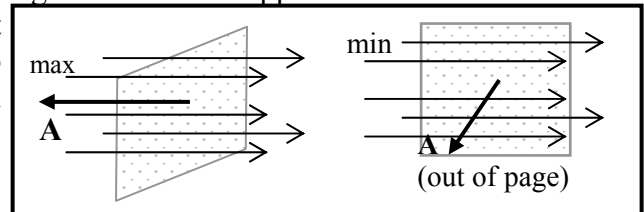
C.   Quantitative Questions:

1. Wayne Frankenstein's cat re-animator.
a. The flux through the top surface will be:

$\Phi = \oint E.dA = E.A = 60 \text{ N.C}^{-1} \times 100 \text{ m} \times 100 \text{ m} = 6.0 \times 10^5 \text{ N.C}^{-1}.\text{m}^2$.

The flux through the bottom surface is $\Phi = \oint E.dA = E.A = 100 \text{ NC}^{-1} \times (100\text{m})^2 = 1.00 \times 10^6 \text{ N.C}^{-1}.\text{m}^2$.

The sides are parallel with the field so there will be no flux through the sides, so the net flux through the cube is $1.00 \times 10^6 \text{ N.C}^{-1}.\text{m}^2 - 6.0 \times 10^5 \text{ N.C}^{-1}.\text{m}^2 = 4.0 \times 10^5 \text{ N.C}^{-1}.\text{m}^2$.

b. For any closed surface (Gauss' law), $\Phi = \oint E.dA = q/\varepsilon_0$ where q is the charge enclosed by the surface.

$\Phi = \oint E.dA = 4.0 \times 10^5 \text{ NC}^{-1}\text{m}^2 = q/\varepsilon_0$ and $q = 4.0 \times 10^5 \text{ N.C}^{-1}.\text{m}^2 \times 8.85 \times 10^{-12} \text{ C}^2.\text{N}^{-1}.\text{m}^{-2} = 3.5 \times 10^{-6} \text{ C}$

c. If 3.5×10^{-6} C flows through the cat in 1 ms, the current will be:
$I = \Delta q/\Delta t = 3.5 \times 10^{-6} \text{ C}/1 \times 10^{-3}\text{s} = 3.5$ mA. (Not very much, the cat will definitely stay dead.)

2. Charge inside a shell.
a. See diagram opposite.
b. Inside the shell we choose an imaginary spherical Gaussian surface of radius r centred on the charge q. According to Gauss' law $\Phi = \oint E.dA = q/\varepsilon_0$.

$E.dA = EdA$ in this case because E is perpendicular to the A at all points. So:

$E\oint dA = E \, 4\pi r^2 = q/\varepsilon_0$. And $E(r) = \dfrac{q}{4\pi\varepsilon_0 r^2}$ If we choose a Gaussian surface outside the sphere the charge contained will be the same, so the expression for E will be the same.

c. The thin shell has no effect on the field due to q, as the field inside and outside the shell are described by the same equation. If the shell had finite thickness the field would be zero inside the metal.
d. Since the shell is a conductor the charges on the shell will distribute themselves such that there will be a charge $+q$ on the outer surface of the shell and $-q$ on the inner surface.
e. A test charge outside the sphere will experience a force because it is in an electric field.
f. The inner charge will not experience a force because it is shielded by the shell. The charges on the outside of the shell will redistribute themselves, but those on the inside will not, so the field inside the shell does not change.
g. There is no conflict with Newton's third law. The forces on the point charge in the shell and the test charge are not an action-reaction pair. The force on the test charge is due to the charges on the outside of the shell.