Workshop Tutorials for Biological and Environmental Physics

Solutions to ER5B: **Capacitance**

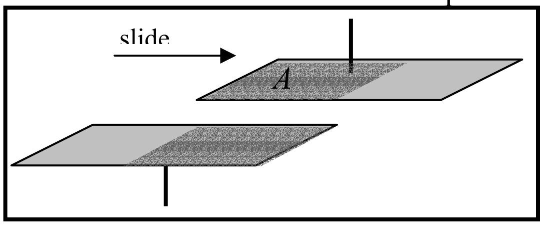
A. Qualitative Questions:

1. Capacitance of a parallel plate capacitor.

a. Since $C = \dfrac{\varepsilon_0 A}{d}$, reducing d to half its value will double the capacitance.

b. Doubling the area of both plates will again double the capacitance (assuming an ideal capacitor).

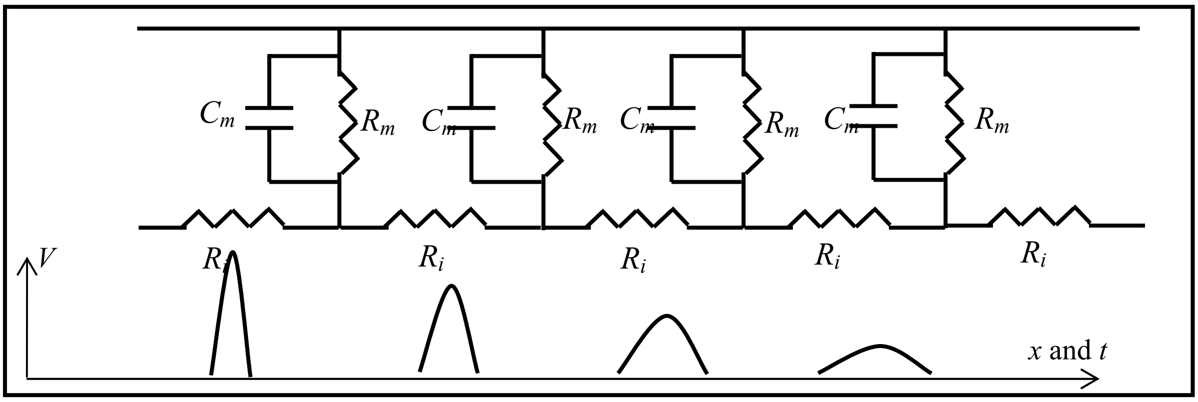
c. Doubling the area of one plate only will not change the capacitance since A is the area of overlap.



d. If the area of overlap is 50% of its original value, then the capacitance also halves.

e. Doubling the potential difference between the plates results in no change in the capacitance. The capacitance is determined by geometrical quantities.

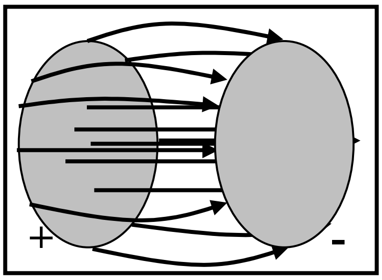
2. As an excitation, a voltage spike, moves along the membrane the resistances dissipate energy as heat, so the pulse gets smaller. The capacitors in combination with the resistors also act to smooth the peak, so that it gets smaller and spreads out, as shown in the diagram below. This is because it takes some time (determined by the time constant), to both charge and discharge the capacitor as the pulse passes along. The circuit shown below is also a good model of a lossy transmission line, such as any cable used to carry electrical signals, for example a TV antenna cable.



B. Activity Questions:

1. Variable capacitor I – giant capacitor

The capacitance is inversely proportional to the separation of the plates, moving the plates closer together increases the capacitance. The paper strips lift and align with the field lines when the field is strong enough. The strips become charged by the plate to which they are attached, and are both repelled by this plate and attracted towards the opposite plate.



2. Variable capacitor II – tuning capacitor

Notice that the capacitor is a series of leaves. Rotating the "stem" rotates one set of leaves so that the area of overlap changes. This changes the value of the capacitor.

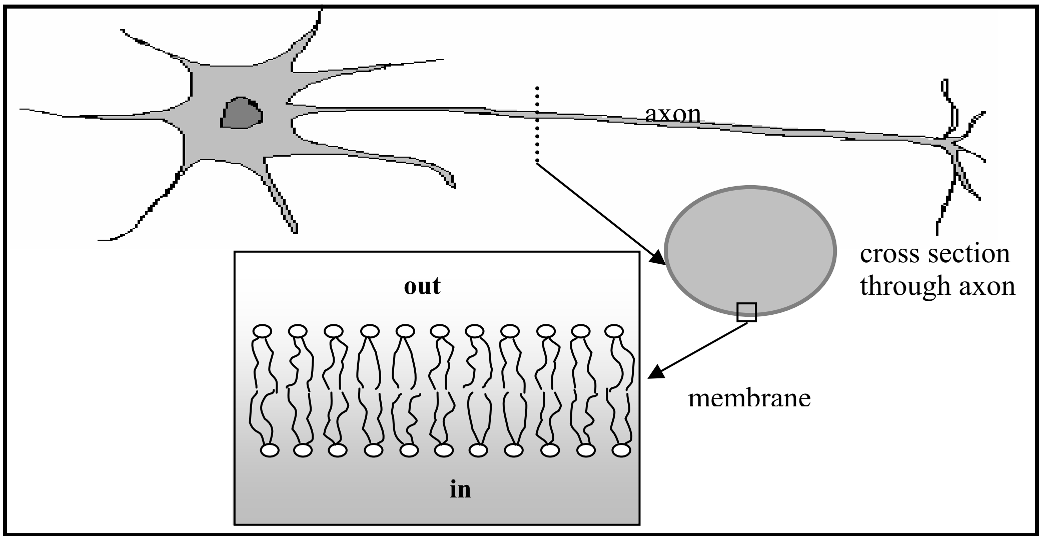
Variable capacitors can be used in tuning devices such as radios where dialing up the radio station is just twisting the "stem". The variable capacitor is part of the resonant circuit where maximum response to the transmitted signal depends on matching the resonant frequency of the circuit with the frequency of the signals carrier waves.

3. Energy stored by a capacitor

We can use the battery to charge up the capacitor and store energy $U = ½\ CV^2$ (in the form of stored charge or an electric field). If we then disconnect the capacitor from the battery and connect the leads across the small electric motor fitted with a 'propeller' – the stored electrical energy is converted into mechanical energy – in the form of rotational motion.
Changing the supply voltage does not change the capacitance, but it does change the amount of energy stored, in the same way that pouring water into a bucket does not change the capacity of the bucket, but it does change the amount of water actually in it.

C. Quantitative Questions:

1. Capacitance of an axon.
a. The radius, 5 μm, is huge compared to the thickness, 8 nm, so if you look at any small piece of membrane it will be approximately flat, and we can treat it as a parallel plate capacitor.
b. The capacitance of a parallel plate capacitor is given by $C = \varepsilon A/d$ where $\varepsilon = \varepsilon_o \times \kappa$ where κ is the dielectric constant of the material between the plates. So the capacitance per unit area is:
$C/A = \varepsilon/d = 8.85 \times 10^{-12}\ \mathrm{F.m^{-1}} \times 7 / 8.0 \times 10^{-9}\ \mathrm{m} = 8 \times 10^{-3}\ \mathrm{F.m^{-2}}$.
c. The surface area of the axon is $2\pi Rl$ where R is the radius and l is the axon length. So:
$C = \varepsilon A/d = 2\pi Rl \times 7.7 \times 10^{-3}\ \mathrm{F.m^{-2}} = 2\pi \times 5 \times 10^{-6}\ \mathrm{m} \times 1.0\ \mathrm{m} \times 8 \times 10^{-3}\ \mathrm{F.m^{-2}} = 2 \times 10^{-7}\ \mathrm{F}$.
d. We know the potential difference across the membrane and its capacitance, so we can use
$q = CV = 2 \times 10^{-7}\ \mathrm{F} \times 90 \times 10^{-3}\ \mathrm{V} = 2 \times 10^{-8}\ \mathrm{C}$.



2. A certain cardiac defibrillator consists of a capacitor charged up to 10^4 V (10,000 volts) with a total stored energy of 450 J.
a. The Electrical Potential Energy stored in a capacitor is $PE = ½\ QV$, so
450 J = ½ × 10,000 V × Q, gives Q = 0.090 C.
b. We need to know the time constant, τ, of the circuit to find how long it takes to discharge. We know the resistance, R = 1.0kΩ. The capacitance is $C = Q/V$ = 0.090C/10,000V = 9.0 μF, so
$\tau = RC = 1.0 \times 10^3\ \Omega \times 9.0\ \mu\mathrm{F} = 9.0$ ms.
As the capacitor discharges the Voltage, V, at any time, t, is given by $V = V_o(1-e^{-t/RC})$, and $Q = Q_o(1-e^{-t/RC})$. Thus $Q/Q_o = 1-e^{-t/RC}$, i.e. $0.9 = 1-e^{-t/9.0\mathrm{ms}}$. So t = -ln(0.1) × 9.0 ms = 21 ms.