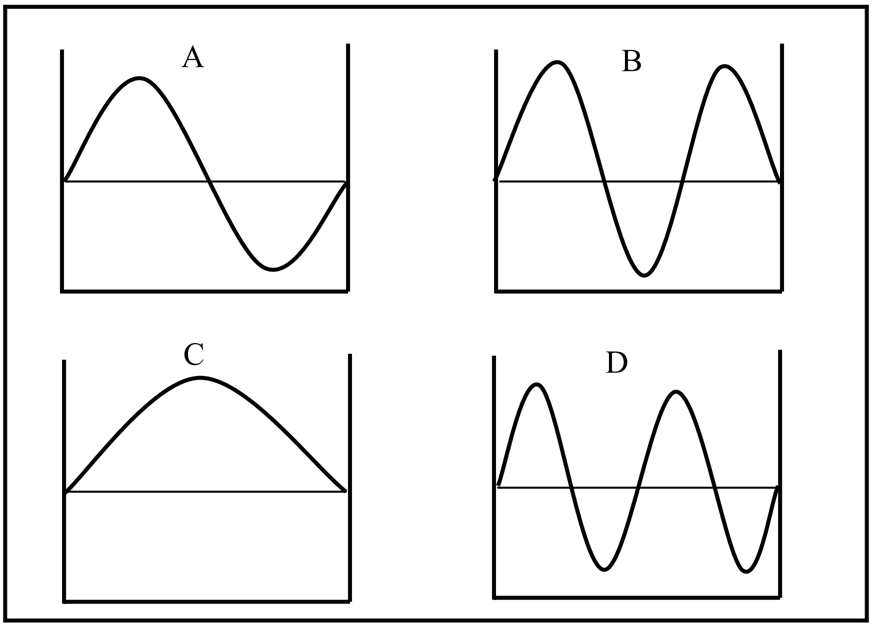
Workshop Tutorials for Physics

# QR3: **Wave Functions II - Particles in Boxes**

## A. Qualitative Questions:

**1.** Electrons show both wave like and particle like behaviour, and we need to take into account both aspects to understand their behaviour.
Shown below are four wave functions of an electron in an infinite potential well.
**a.** Rank the wave functions in order of increasing energy of the electron. Justify your answer.




**b.** Sketch the probability density for the electron in the first excited state.
The electron is now replaced with a proton.
**c.** Is the proton's zero point energy higher or lower than the electron's?
**d.** Sketch, to the same scale, the ground state wave functions of the proton and the electron.

**2.** When an electron is part of an atom it is confined to an orbital. We can model a bound electron as a particle trapped in a potential well.
**a.** How does confinement of a particle, such as an electron, account for discrete energy levels for that particle?
**b.** Why is it not possible for the ground state energy of a confined electron to be zero?

## B. Activity Questions:

**1. Potential Wells and Wave functions**
Examine the drawings of the wave functions for particles in potential wells.
**a.** What do the axes represent?
**b.** What does the wave function represent?

**2. Classical particle in an elastic potential energy well**

Send the glider along the air track and allow it to bounce off the spring at the end.

**a.** Sketch the elastic potential energy of the system (glider and track, including springs) as a function of glider position.

Allow the glider to bounce back and forth.

**b.** Where does it spend most of its time?

**c.** Sketch the probability of finding the glider at a position on the track as a function of position.

**d.** How does this compare to the probability density for an electron trapped in a potential well?

**3. Waves on a string**

Why are only certain wavelengths of the standing wave possible?

**a.** Discuss the terms "trapped inside an atom" and "electron in a potential well". What is the "well"?

**b.** Use your answers to part **a** to build a simple quantum model of an electron in an atom. (Your model should not be a simple analogy to the planets orbiting the sun in the solar system.)

**c.** What is the role of standing waves in your model? Why does the existence of standing waves require quantisation of energy?

## C. Quantitative Questions:

**1.** A pollen grain of mass 2.0 mg moves back and forth under a microscope between two glass slides. The slides are separated by 0.05mm and the pollen grain moves so slowly that it takes 90s to move from one slide to the other. Think of this motion as that of a quantum particle trapped in a one dimensional infinite potential well.

**a.** What energy quantum number ($n$) describes its motion?

Quantum mechanics says that the wave function describing the motion will be positive at some points and negative at others. Furthermore, if $n$ is an even number ($n$ = 2,4,6…), the wave function will be negative as many times as it is positive.

**b.** It could be argued therefore that that the average probability of being able to see the pollen grain with the microscope at any point is zero. Is this argument correct? (yes or no only)

**c.** If you answered yes, explain why your answer is apparently in contradiction to the classical result that the pollen grain must be seen somewhere. If you answered no, explain why you think this apparently straight forward argument is wrong.

**2.** One of the puzzles of early models of atomic structure was why the electrons didn't simply go into the nucleus, to which they are attracted by electrostatic (Coulomb) force.

**a.** Calculate the smallest allowed energy of an electron were it trapped inside an atomic nucleus (diameter about $1.4 \times 10^{-14}$m).

**b.** Calculate the smallest allowed energy of a proton were it trapped inside an atomic nucleus.

**c.** Comparing these energies, should we expect to find electrons inside nuclei?