Workshop Tutorials for Introductory Physics

Solutions to MI6: **Work, Power and Energy**

A. Review of Basic Ideas:

Work energy and power

We use the words work, energy and power a lot in everyday life. You might tell someone to be quiet because you're trying to **work** on an assignment. Sports drinks give you **energy** to keep going for longer, and people brag about how **powerful** their cars are. We also use these words in physics, but with more precise meanings.

Work is the transfer of energy, to or from an object via a **force**. When energy is transferred to an object then the work done on the object is **positive**, if energy is transferred from an object then the work done on the object is negative. (This means that the object has done positive work on something else.)

If a force changes the speed at which an object is moving then the object's **kinetic** energy changes. The kinetic energy of an object with mass m moving at speed v is defined as $KE = \frac{1}{2}\boldsymbol{mv^2}$. If the kinetic energy is the only type of energy changed by the action of a force then the work done is equal to the change in kinetic energy.

From Newton's laws we know that force is mass times **acceleration**, $F = ma$, and from kinematics we know that $v^2 = v_0^2 + 2as$. Using this we discover that

$$W = \Delta KE = \tfrac{1}{2} m (v^2 - v_0^2) = \tfrac{1}{2} m (2as) = mas = Fs$$

To be strictly correct, because force is a vector quantity (it has **direction**) we should say that

$$\vec{\mathbf{W}} = \Delta KE = Fs\cos\theta = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}}$$

where θ is the angle between the direction of the force F and the displacement x.

Lifting things can be very hard work. In this case if the object is lifted from rest to a higher position and put down so it is again at rest, such as carrying a suitcase upstairs, then the work done is the change in the object's **gravitational** potential energy. If you lift a heavy suitcase, carry it around an airport for a while, then put it down again, you haven't actually done any work, although you may feel like you have.

Power is the rate at which work is done, or energy is transferred. The power rating on the back of an appliance tells you how quickly it turns electrical energy into some other form of energy, such as light or heat.

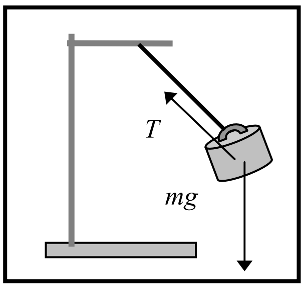
Discussion questions

The total work done on the TV really is zero. It starts with no kinetic energy, ends with no kinetic energy, and assuming the floor is flat it will not change gravitational potential energy. However this does not mean that Brent does no work on the TV. He must accelerate it to move it, and increase its potential energy if he lifts it. This energy comes from chemical potential energy stored in Brent. When he decelerates the TV and puts it down he must absorb the energy he has put in. However this energy is not converted back to chemical potential energy, but is lost as heat. In addition, if Brent slides the TV then the energy he has put in to accelerate it and give it kinetic energy will be dissipated as heat due to friction by the floor and TV. So while the energy of the TV has not changed, and no work has been done on it, Brent must do work to move it.

B. Activity Questions:

1. Pendulum

The forces acting on the pendulum are its weight (gravity), and the tension in the string. The tension is always at right angles to the path, hence it does no work. Ignoring friction, only the weight of the pendulum does work as it swings, converting gravitational potential energy into kinetic energy and back again.



2. Falling

When you drop an object it falls due to gravity, losing gravitational potential energy. As it falls this gravitational potential energy is converted to kinetic energy, and the object gains speed. The change in kinetic energy is equal to the work being done on the falling object, and is done by gravity. Air resistance may also do some negative work on the object, acting to reduce its kinetic energy and slow it down.

3. Power

The power used by the appliances is written on the back, and is measured in watts, W, or sometimes volts × current or VI. If an appliance is rated at *X* watts it converts *X* joules per second of energy.

A hairdryer converts electrical energy into thermal energy (heat) and kinetic energy, a lamp produces heat and light. All appliances convert at least some electrical energy into thermal energy.

C. Qualitative Questions:

1. Work is defined as the change in kinetic energy. When you decrease an object's kinetic energy you do negative work, for example catching a ball.

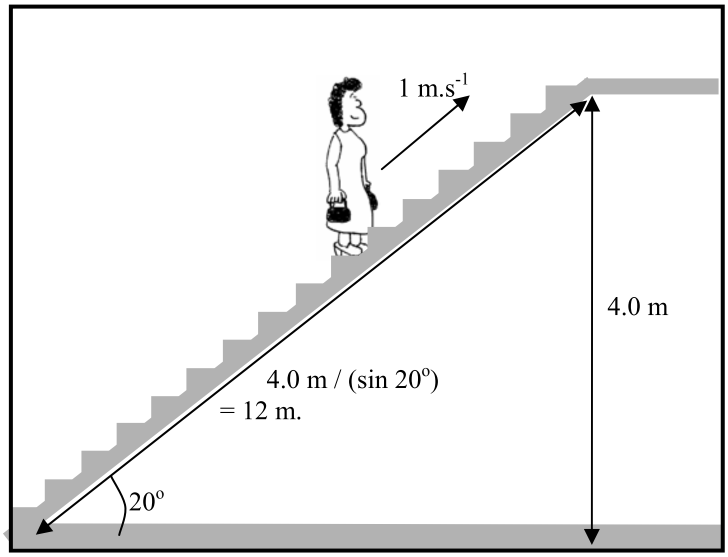
2. Two approximately frictionless water slides come down from a single platform and finish in the same pool. One is steep and straight, the other spirals down. Rebecca and Brent start at the same time, Brent goes down the steep slide and Rebecca goes down the other one.

a. Brent will land in the pool first. His slide is steeper hence the component of gravity acting along the direction of the slide to accelerate him is greater, his acceleration is greater (over a shorter distance) and he reaches the bottom first.

b. They both start with the gravitational potential energy mgh, and as there is negligible friction they will have kinetic energy $\frac{1}{2}mv^2$ when they reach the bottom. They will have the same velocity, $v = (2gh)^{1/2}$, at the bottom of the slide. (Brent has a greater acceleration, but it acts over a shorter time.)

c. The amount of work done by the gravitational force is the change in kinetic energy, $\frac{1}{2}mv^2$. They have the same velocity, v, so gravity does the greater work on which ever of them is heavier, ie the one with greater m.

D. Quantitative Question.



Rebecca rides an escalator at 1 m.s^{-1}. The escalator makes an angle of 20^o with the horizontal and takes her up a height of 4 m. Rebecca plus her shopping weigh 62 kg.

a. The work done is that required to lift Rebecca plus shopping through a height of 4 m, which is equal to the change in potential energy.

$W = \Delta mgh = mg\Delta h$
$\quad = 62 \text{ kg} \times 9.8 \text{ m.s}^{-2} \times 4.0 \text{ m}$
$\quad = 2400 \text{ J} = 2.4 \text{ kJ}$

b. The power required to take her to the next floor at a rate of 1 m.s^{-1} is the energy used per second. We know the total energy used, so we want to find the time taken.

She has a velocity of 1 m.s^{-1} at an angle of 20^o to the horizontal. The distance she must travel is $d = 4.0 / (\sin 20^o) = 12$ m. At a velocity of 1 m.s^{-1} this will take $t = d/v = 12 \text{ m} / 1 \text{ m.s}^{-1} = 12$ s.

The power required is therefore $P = E/t = 2400 \text{ J} / 12 \text{ s} = 200 \text{ J.s}^{-1} = 200$ W.

Note that this is only the power required to move Rebecca, the escalator must also move itself which requires a lot of power.