

Workshop Tutorials for Introductory Physics

MI12: Gravity and Kepler's Laws

A. Review of Basic Ideas:

Use the following words to fill in the blanks:

solar, Galileo, all, Earth, astronomers, gravitation, areas, elliptical, Sun, Cr^3 , force, Gm_1m_2 / r^2 , Moon

Gravity and Kepler's laws

Our view of the universe and our _____ system has evolved greatly over the past five hundred years. In 1543 when Copernicus wrote a description of the universe that put the Sun rather than the _____ at the centre, he was largely ignored and his theories considered implausible. Worse than this, they were in contradiction with scripture, and the church was liable to punish any who supported Copernicus' views. About a century later his supporter _____ was interviewed by the Inquisition and put under house arrest for the rest of his life and forbidden to publish. He got off lightly. But despite the best efforts of the church, _____ could observe the motions of the planets, and soon it became known that the planets did indeed orbit the Sun.

Kepler described the orbits of the planets and was the first to recognise that the orbits were not necessarily circular, but could be, and in fact they are, _____. Kepler came up with three laws, now known as Kepler's laws. The first is that all the planets move in an elliptical orbit with the _____ at one focus. The second is that a line joining any planet to the Sun sweeps out equal _____ in equal times. The third is that the square of a planet's period, T , is proportional to the cube of its orbit's semi-major axis, r , or $T^2 = \text{_____}$, where C is a constant which has the same value for all planets.

Kepler's laws were based on observation, and offered no explanation as to why the planets behaved as they did. What _____ kept the planets orbiting? Newton's law of _____ provided the explanation. Newton postulated that there is a force of attraction between any two bodies, and this force is proportional to the masses of the bodies and inversely proportional to the square of the distance between the bodies; $F = \text{_____}$. The constant G is the universal gravitational constant. It is possible to derive Kepler's laws from Newton's law of gravitation, and it turns out the constant C is equal to $4\pi^2/GM_s$, where M_s is the mass of the Sun.

Newton's law of gravitation applies to _____ objects in the universe, and Kepler's laws apply to any orbiting body, or satellite. Kepler's laws describe the motion of our natural satellite, the _____, and all the man made satellites orbiting the Earth. They describe the motions of comets and asteroids and the moons of Jupiter.

Discussion questions

Why don't you feel the gravitational attraction of a large building when you walk near it?

Why is G so difficult to measure accurately?

B. Activity Questions:

1. Drawing orbits

Place one pin at the sun position, and another at the other focus for one of the planets.

How do you know where the other focus should be?

Now cut a piece of string to a (scaled down) length equal to twice the sum of the aphelion and perihelion distances ($l = 2(a+p)$). Tie the string in a loop and put it around the pins. Hold the pen in the loop so that it pulls the loop taut and use the string to guide the pen to draw the orbit.

Repeat for one or two other planets.

Which planets have the most eccentric orbits? Which have the least eccentric?

2. Models of the solar system

Compare the different models of the solar system.

In what ways are they different? How are they similar?

3. Kepler's Second Law

Move the “planet” around the Sun.

Note the line joining the planet to the Sun and observe the area that it sweeps out.

What happens to the velocity of the planet as it moves further away from the Sun (towards aphelion)?

What happens as it moves closer to the Sun (towards perihelion)?

C. Qualitative Questions:

1. The radius of the Earth is approximately 6,400 km. The International Space Station orbits at an altitude approximately 400 km above the Earth’s surface, or at a radius of around 6,800 km from the Earth’s centre. Hence the force of gravity experienced by the space station and its occupants is almost as great as that experienced by people on the surface of the Earth.

So why do astronauts and cosmonauts feel weightless when orbiting the Earth in a space station?

2. If you throw a ball up in the air, it falls back to the ground. However if launched at the right angle and speed, a satellite can stay in orbit indefinitely. Briefly explain in terms of the relevant physics principles how a communications satellite can stay apparently suspended high above the Earth's surface, and not fall to the ground as a ball does.

D. Quantitative Question:

Many science fiction stories feature men going to mars and meeting Martians. In some stories the space travelers are able to walk around quite comfortably and even breathe the Martian atmosphere.

a. Given the table of information below, what is the acceleration due to gravity on Mars?

(Hint: use $F = ma = Gm_1m_2 / r^2$.)

For an object to escape the gravitational attraction of a planet it needs to have a velocity away from the planet equal to or greater than the escape velocity. This velocity can be calculated using conservation of energy. Consider throwing a ball into the air with velocity v , it initially has some kinetic energy and some gravitational potential energy. When it reaches its maximum height it has only gravitational potential energy, so we can write:

$$E_{initial} = \frac{1}{2}mv^2 - GM_{Earth}m/R_{Earth} = E_{final} = -GM_{Earth}m/R_{max}$$

If we take the case of the ball actually leaving the Earth’s gravitational field totally then $R_{max} = \infty$, and substituting this into the equation above we can find the necessary velocity to achieve this. This is called the *escape velocity*.

b. Find the escape velocity at the surface of the Earth.

c. Find the escape velocity at the surface of Mars.

d. Which planet would it be easier to use as a launching base for interstellar travel?

Useful data.

	Earth	Mars
Mass ($\times 10^{24}$ kg)	5.97	0.642
Diameter (km)	12,756	6794
g (m.s^{-2})	9.8	23.1
Escape velocity (km.s^{-1})		