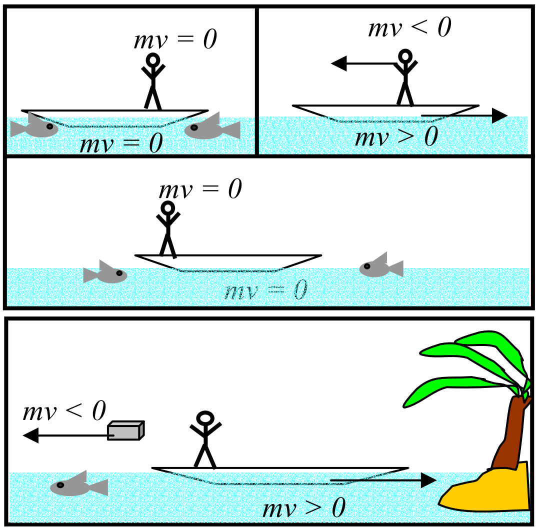
Workshop Tutorials for Biological and Environmental Physics
Solutions to MR7B: **Conservation of Momentum**

A. Qualitative Questions:

1. Collisions.
a. The amount of damage caused during a collision will be determined by the magnitude of the forces acting during that collision. This will depend on the change in momentum on impact and the time for which the colliding bodies are in contact.
b. To miminise damage for a given momentum change of an object, it is better to maximize the time of collision. Change in momentum equals average force × time of collision. For example, a "soft" landing onto a mattress causes less damage than a "hard" landing onto concrete as the time of impact is longer and hence the average force exerted by the surface on the body is less.

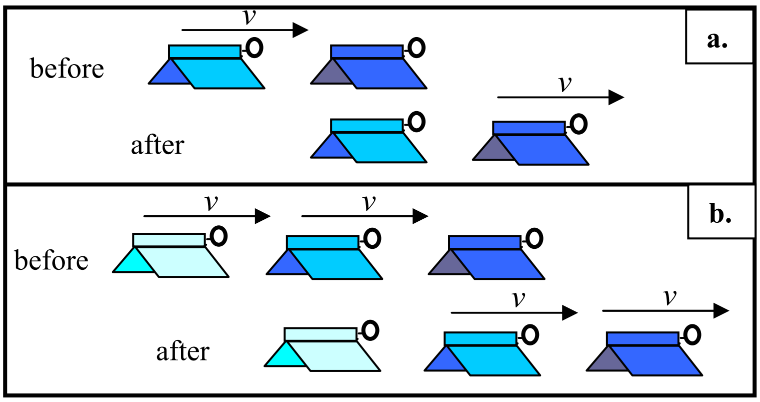


2. Rebecca and Brent are on the lake without a paddle.
a. Brent will not get the boat to shore by walking in the opposite direction. In the absence of external forces the centre of mass will not move, and when he gets to the end of the boat and stops it will be less than one boat lengths closer to shore. See diagram opposite.
b. They could throw things overboard as hard as they could in a horizontal direction away from shore. By conservation of momentum, whatever momentum they give the thrown objects must also be given to them and the boat, but in the opposite direction. This maintains conservation of momentum, so that the total momentum of the system is still zero.

B. Activity Questions:

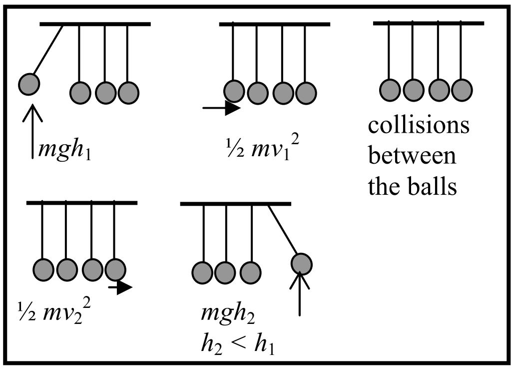
1. Pendulum on Trolley
When you raise the bob and hold the base still the total momentum of the pendulum-trolley system is zero. When you release the bob it swings down, gaining momentum. In order for momentum to be conserved the trolley must move the opposite way, which it does. As the pendulum swings back and forth the trolley will roll back and forth in the opposite direction, until friction eventually stops it.

2. Air track
a, b. See opposite.
c. The metal loops help make the collisions elastic and prevent the gliders sticking to each other.
d. The frictional forces are much greater with the air flow turned off and the gliders can no longer be considered an isolated system as the external forces due to friction become significant.



3. Newton's cradle

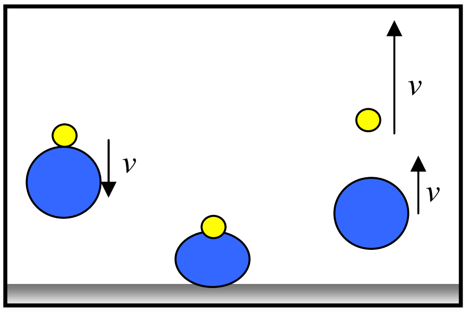
Steel balls have (almost) elastic collisions, in which both kinetic energy and momentum are conserved. The lead balls have inelastic collisions in which only momentum is conserved. Both energy and momentum conservation are needed to explain the behaviour of the balls. Energy conservation is needed to account for the kinetic energy of the balls before and after the collision, and any thermal energy produced in an inelastic or partially inelastic collision - $KE_{initial} = KE_{final} + E_{\mathrm{thermal}}$. The collisions obey conservation of momentum.



4. Bouncing balls II

a. The small ball held over the big ball bounces off higher as some momentum is transferred from the big ball to the small ball, increasing its velocity. Momentum has been conserved during the collision and the change in momentum of the small ball is large.

b. If the balls are switched around the momentum is still conserved, but the transfer of momentum from the small to the big ball makes little difference to the big ball's velocity due to its large mass.



C. Qualitative Questions:

1. The canoe has a mass of 60 kg and Brent has a mass of 70 kg. The canoe is 1 m from the shore. Brent jumps with a horizontal velocity of 2.5 m.s^{-1} towards the shore.

a. See diagram opposite.

b. We can use conservation of momentum to find the velocity of the canoe.

$p_i = p_f$

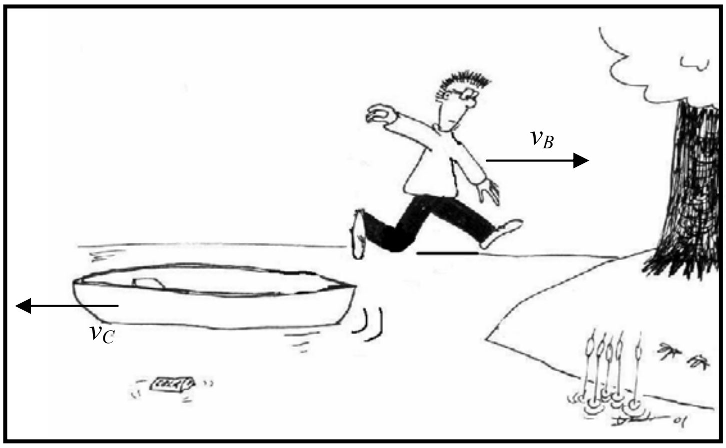
$p_i = p_B + p_C = m_B v_B + m_C v_C$

$= 70\text{ kg} \times 0\text{ m.s}^{-1} + 60\text{ kg} \times 0\text{ m.s}^{-1} = 0\text{ kg.m.s}^{1}$.

$p_f = m_B v_B + m_C v_C = p_i = 0\text{ kg.m.s}^{1}$.

so $v_C = - m_B v_B / m_C = -70\text{ kg} \times 2.5\text{ m.s}^{-1} / 60\text{ kg}$

$v_C = -2.9\text{ m.s}^{-1}$



2. Heading a soccer ball.

The change in momentum is $\Delta p = p_f - p_i$. See diagram opposite.

Looking at the components:

In the y direction:

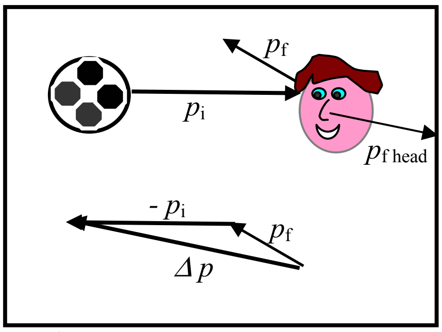
$\Delta p_y = p_{fy} - p_{iy} = 0.450\text{ kg} \times 8\text{ m.s}^{-1} \sin 45^{o} - 0 = 2.54\text{ kg.m.s}^{-1}$

In the x direction:

$\Delta p_x = p_{fx} - p_{ix} = - 0.450\text{ kg} \times 8\text{ ms}^{-1} \times \cos 45^{o} - (0.450\text{ kg} \times 20\text{ m.s}^{-1})$

$= - 2.54\text{ kg.m.s}^{-1} - 9.0\text{ kg.m.s}^{-1} = - 11.54\text{ kg.m.s}^{-1}$.

$\Delta p^2 = \Delta p_x^2 + \Delta p_y^2 = (2.54^2 + 11.54^2)\text{ kg}^2.\text{m}^2.\text{s}^{-2}$



and $\Delta p = 11.8\text{ kg.m.s}^{-1}$ at angle $\theta = \tan^{-1}(2.54/11.54) = 12^{o}$ above the horizontal.

The change in momentum of the head will be equal and opposite, i.e. 11.8 kg.m.s^{-1} at 11^{o} below the horizontal. The mass of the head = 0.07 × 80 kg = 5.6 kg.

Hence the velocity of the head will be 11.8 kg.m.s^{-1} / 5.6 kg = 2.1 m.s^{-1} in a direction 23^{o} *below* the horizontal.

The kinetic energy to be absorbed = $\frac{1}{2}mv^2 = \frac{1}{2} \times 5.6\text{ kg} \times (2.1\text{ m.s}^{-1})^2 = 12\text{ J}$.