

Workshop Tutorials for Physics

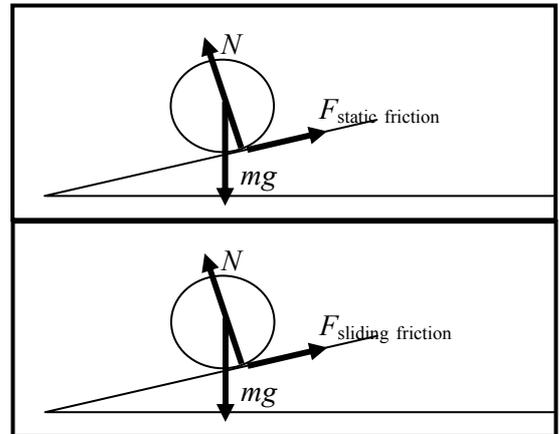
Solutions to MR11: **Rolling**

A. Qualitative Questions:

1. Friction is necessary for walking, driving and even rolling a ball or using a yo-yo.

a. See diagram opposite. Since the ball is rolling without slipping there is no sliding (kinetic) frictional force acting. However there is static frictional force acting at the point of contact with the ramp. It is this force which provides the torque about the centre of mass to cause rotation.

b. When there is no rolling there will be some energy lost due to the kinetic frictional force acting. The nature of the forces acting are no different to those acting on a box sliding down a ramp.



Sometimes when you try to accelerate a car too hard on a wet or slippery road the wheels spin and the car doesn't move. The wheels spin because the surfaces (the wheel and the wet road) have a very low coefficient of friction, μ . Hence the maximum static friction $= \mu N$ is low and so cannot oppose the driving force on the wheels from the engine and the wheels rotate over the surface, i.e. they spin, and the car goes nowhere.

2. A confectionery company is looking for a way to separate spherical lollies of the correct size from those that are too big or too small. One of their engineers suggests using a ramp at the end of the lolly making machine and holding a container under the end of the ramp at the right time for the good lollies, and putting the rubbish bin under it for the wrong sized lollies.

The lollies all have the same gravitational potential energy, mgh , at the top of the ramp. This energy is converted into translational and rotational kinetic energy $KE = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$. If they roll without slipping then their translational velocity is $v_{cm} = r\omega$ and $KE = \frac{1}{2} mv^2 + \frac{1}{2} I(v_{cm}/r)^2$. Using conservation of energy, $mgh = \frac{1}{2} mv^2 + \frac{1}{2} I(v_{cm}/r)^2$. For a uniform spherical lolly, $I = (2/5) mr^2$. Putting this into our equation for energy conservation gives:

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I(v_{cm}/r)^2 = \frac{1}{2} mv^2 + \frac{1}{2} (2/5) mr^2 (v_{cm}/r)^2 \text{ or } gh = \frac{1}{2} v^2 + (1/5)v^2$$

We can see from this that as long as the lollies are spherical, neither the mass nor size of the lollies will affect their velocity and hence the time at which they reach the bottom of the ramp.

B. Activity Questions:

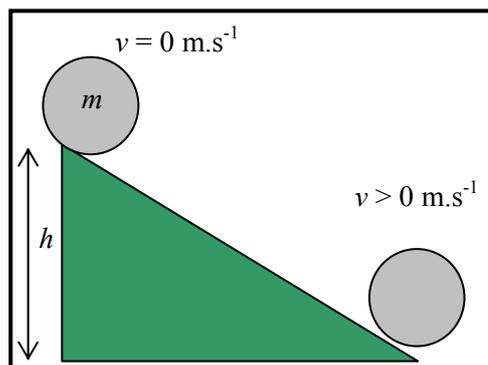
1. Rolling down a ramp

A ball will get to the bottom of a frictionless ramp faster than one with friction. On the frictionless ramp the ball slides down without rolling, so all the ball's potential energy is converted to translational kinetic energy. When the ball rolls, some of the potential energy is converted to translational kinetic energy and some to rotational kinetic energy, hence it will have a smaller translational velocity and take longer to reach the bottom. The frictional force acts to counteract the sliding that would otherwise occur. The torque means that rotational motion (rolling) occurs.

2. A loaded race

Neglecting air resistance all the solid spheres will hit the bottom at the same time. From energy conservation equations we have

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ rearranging for v gives $v = \sqrt{\frac{10}{7}gh}$ for solid spheres. Thus the velocity at the bottom of the ramp is independent of M and R , i.e. all the balls should reach the bottom at the same time. For a solid cylinder $v = \sqrt{\frac{4}{3}gh}$, so generally spheres have a higher speed than a cylinder.



3. yo-yo

As it falls, the yo-yo loses gravitational potential energy and gains both translational kinetic and rotational kinetic energy. At the end of the fall the yo-yo continues to spin as it has angular momentum. If the string is looped loosely around the central axis then the yo-yo will stay spinning at the bottom of its fall for a reasonably long time.

The yo-yo comes up again if the yo-yoist tugs on the string. The friction between the string and the yo-yo provides an upward force on the yo-yo. Since the yo-yo is spinning, it will continue to rotate as it moves upwards again.

C. Quantitative Question.

1. A car is travelling at $60 \text{ km.h}^{-1} = 16.7 \text{ m.s}^{-1}$ when the driver takes his foot off the accelerator and the car gradually rolls to a stop in 100 m. The wheels of the car are 75 cm in diameter.

a. We can find the linear acceleration of the car's wheels using $v_f^2 = v_o^2 - 2as$, and rearranging for a :

$$a = (v_f^2 - v_o^2) / 2s = [(16.7 \text{ m.s}^{-1})^2 - (0 \text{ m.s}^{-1})^2] / 2 \times 100 \text{ m} = 1.4 \text{ m.s}^{-2}.$$

This is the linear acceleration of the centre of mass of the car's wheels.

b. The angular acceleration of the wheels is $\alpha = a / r = 1.4 \text{ m.s}^{-2} / 0.375 \text{ m} = 3.7 \text{ rad.s}^{-2}$.

c. The rotational inertia of each wheel is 8 kg.m^2 , the torque exerted on each wheel about its axis is:

$$\tau = I\alpha = 8 \text{ kg.m}^2 \times 3.7 \text{ rad.s}^{-2} = 30 \text{ N.m}.$$

This torque is provided by the frictional forces between the road and the tyres and the frictional forces in the wheel bearings.

2. Lothar's record bicycle sprint.

a. From Lothar's point of view, he is moving with the bicycle and so the average linear speed of the centre of the wheel is zero.

b. For Lothar the wheel is rotating uniformly about him. Thus for him all points on the rim of the bicycle have the same speed. We assume there is rolling without slipping then the instantaneous speed of the point of contact with the ground must be zero for a spectator on the ground. The total speed at any point is the magnitude of the vector sum of the velocity of the center of mass and the velocity of the wheel relative to the center of mass. From a spectators frame of reference, the average speed = distance covered / time taken = $1000 \text{ m} / 62.955 \text{ s} = 15.88 \text{ m.s}^{-1}$. For a point on the ground the velocity of the wheel relative to the center of mass must thus be $-\vec{v}_{cm}$. Since the top of the wheel is traveling in the opposite direction its velocity is $+\vec{v}_{cm}$ and the speed is v_{cm} . Hence the speed is 15.88 m.s^{-1} .

c. The average linear acceleration will be zero since he sees no change in the average speed.

d. From Lothar's frame of reference the top of the wheel is undergoing circular motion and there will be a radial acceleration $a_r = v^2/r = (15.88 \text{ m.s}^{-1})^2 / 0.60 \text{ m} = 420 \text{ m.s}^{-2}$.

e. As the spectator is in a different frame of reference and the velocity of the center of mass is not zero, they will not see the same speeds and accelerations as Lothar. As explained above the velocity of a point on the ground for the spectator will be zero and the velocity of a point on the top of the wheel will be $2v_{cm}$. The acceleration of the center of mass will be zero, but the acceleration of the point on the top of the wheel will be $4v_{cm}^2/2r$, which is double that seen by Lothar.