Workshop Tutorials for Physics

Solutions to MR12: **Gravity and Kepler's laws**

A. Qualitative Questions:

1. The radius of the Earth is approximately 6,400 km. The International Space Station orbits at an altitude approximately 400 km above the Earth's surface, or at a radius of around 6,800 km from the Earth's centre. Hence the force of gravity experienced by the space station and its occupants is almost as great as that experienced by people on the surface of the Earth. Astronauts and cosmonauts feel weightless when orbiting the Earth because there is no contact force between them and the space station. An astronaut standing on the floor in the space station is accelerating towards the Earth at the same rate and with the same velocity as the space station, hence he or she is not "pushed" against the floor the way someone on earth is, and they feel "weightless" because they are in free fall, although the astronaut still has almost the same weight as on Earth.

2. If you throw a ball up in the air it falls back to the ground.
a. The communications satellite is falling and a gravitational force is acting on it. The difference between it and the ball is that the satellite has sufficient tangential speed so that it moves sideways at such a rate as to fall "past" the earth and so continue in an orbit around the earth.
b. Increasing the mass of a space station, such as Mir, does not have an effect on its orbit. The gravitational force, $F = Gm_{\mathrm{Earth}}m / r^2$ provides the force to keep Mir in orbit, and not disappear into space. The net force acting to keep an object in circular motion is the centripetal force, $F = mv^2 / r$. Equation this to the gravitational force gives $Gm_{\mathrm{Earth}}m / r^2 = mv^2 / r$ or $Gm_{\mathrm{Earth}} / r = mv^2$, from which we can see that the velocity of the satellite depends on the mass of the Earth and the radius of its orbit, but not on the satellites mass.

B. Activity Questions:

1. Drawing orbits
The planets move in elliptical orbits around the sun. The closest point to the sun is called the perihelion and the furthest point is called the aphelion. The sun is at one focus of the ellipse. The string should be a length equal to the sum of the aphelion and the perihelion, so that the pencil is at most the aphelion distance from the sun. The distance between the foci is equal to the difference between the aphelion and perihelion distances. The eccentricity of the orbit is the ratio of the distance between the foci to the length of the major axis, which is (aphelion – perihelion) / (aphelion + perihelion), which is equal to 0 for circle and is between 0 and 1 for an ellipse.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Aphelion	70	109	152	249	817	1515	3004	4546	7304
Perihelion	46	108	147	206	741	1353	2741	4445	4435

Distances × 10^6 km

2. Models of the solar system
Our understanding of the motion of the planets has changed greatly over the last few hundred years.
You may see some of these changes by noting the differences between the models – the most obvious is that early models had the Earth at the centre of the solar system with everything else, including the sun, orbiting around it. Modern models of the solar system place the Earth at the centre.

3. Kepler's Second Law
The area swept out per unit time by the line joining the planet and the sun is constant. The distance between the sun and planet varies because the orbit is elliptical, hence the length of this line varies in time. For the area swept out per unit time to be constant the velocity must vary, decreasing as the planet moves further from the sun (towards aphelion) and increasing as it moves closer (towards perihelion).

C. Quantitative Questions:

1. Many science fiction stories feature men going to mars and meeting Martians. In some stories the space travelers are able to walk around quite comfortably and even breathe the Martian atmosphere.

a. The gravitational force between two bodies is given by $F = Gm_1m_2 / r^2$. Using Newton's second law, $F = ma$, we can write for the acceleration due to gravity at the surface of Mars:

$g_{\text{Mars}} = GM_{\text{Mars}} / r_{\text{Mars}}^2 = (6.67 \times 10^{-11}\ \text{N.m}^2.\text{kg}^{-2} \times 0.642 \times 10^{24}\ \text{kg}) / (3397 \times 10^3\ \text{m})^2 = 3.7\ \text{m.s}^{-2}$.

b. The escape velocity can be calculated using conservation of energy. Consider throwing a ball into the air with velocity v, it initially has some kinetic energy and some gravitational potential energy. When it reaches its maximum height it has only gravitational potential energy, so we can write:

$E_{initial} = ½\, mv^2 – GM_{\text{Mars}}m/R_{\text{Mars}} = E_{final} = -\, GM_{\text{Mars}}m/R_{max}$.

If we take the case of the ball actually leaving the Earth's gravitational field totally then $R_{max} = \infty$, and substituting this into the equation above gives: $½\, mv^2 – GM_{\text{Mars}}m/R_{\text{Mars}} = -\, GM_{\text{Mars}}m/\infty = 0$.

Rearranging for v^2 gives:

$v^2 = 2GM_{\text{Mars}}/R_{\text{Mars}} = 2 \times 6.67 \times 10^{-11}\ \text{N.m}^2.\text{kg}^{-2} \times 0.642 \times 10^{24}\ \text{kg} / 3397 \times 10^3\ \text{m} = 2.52 \times 10^6\ \text{m}^2.\text{s}^{-2}$.

and $v = 5.02 \times 10^3\ \text{m.s}^{-1} = 5.02\ \text{km.s}^{-1}$.

c. The lower escape velocity on Mars means that high velocity gas molecules will be able to escape the atmosphere. In a gas the molecules have a range of velocities, and the average velocity depends on the temperature of the gas. On Earth, at normal atmospheric temperatures, helium molecules have enough energy to escape the atmosphere, and when a helium balloon deflates the helium is lost into outer space. On Mars, some molecules of oxygen and nitrogen would escape at normal atmospheric temperatures.

d. To give Mars an (unenclosed) Earthlike atmosphere the gas which made up the atmosphere would have to be quite cold to keep it from escaping. Living on Mars with an unenclosed atmosphere would be more like living in Antarctica than in Cairns.

2. Your job is to get a satellite in orbit around Jupiter above the source of the signal to monitor it. The space shuttle Kookaburra is to put a communications satellite at the right orbit, with the right velocity, to keep it directly above the signal.

a. To keep a satellite above a point on the surface of a planet we require that the orbital period of the satellite matches the rotational period of the planet. We can then use Kepler's third law for a small satellite orbiting a much larger body: $T^2 = (r^3 4\pi^2) / GM$ where T is the period of orbit, and r is the radius of the orbit. In this case M is the mass of Jupiter. Rearranging for r^3 gives, and using $T = 9.9\ \text{h} = 35600\ \text{s}$:

$r^3 = (T^2GM) / (4\pi^2) = [(35600\ \text{s})^2 \times 6.67 \times 10^{-11}\ \text{N.m}^2.\text{kg}^{-2} \times 1899 \times 10^{24}\ \text{kg}] / (4\pi^2) = 4.07 \times 10^{24}\ \text{m}^3$

$r = 1.60 \times 10^8\ \text{m} = 160{,}000\ \text{km}$. This is the distance from the center of Jupiter, so the height above the surface of jupiter is $= 160{,}000 – R_{\text{Jupiter}} = 160{,}000 – 71492 = 88{,}500\ \text{km}$.

b. The total distance the satellite travels per orbit is $x = 2\pi r = 2 \times \pi \times 1.60 \times 10^8\ \text{m} = 1.01 \times 10^9\ \text{m}$.

It does this in a time $T = 32400\ \text{s}$, hence it must have a (linear) velocity

$v = x / T = 1.01 \times 10^9 / 35600\ \text{s} = 2.82 \times 10^4\ \text{m.s}^{-1} = 28\ \text{km.s}^{-1}$.

	Earth	Mars	Jupiter
Mass ($\times 10^{24}$ kg)	5.97	0.642	1899
Diameter (km)	12,756	6794	142,984
g (m.s^{-2})	9.8	**3.7**	23.1
Escape velocity (km.s^{-1})	11.2	**5.02**	59.5
Period of rotation (h)	23.9	24.6	9.9